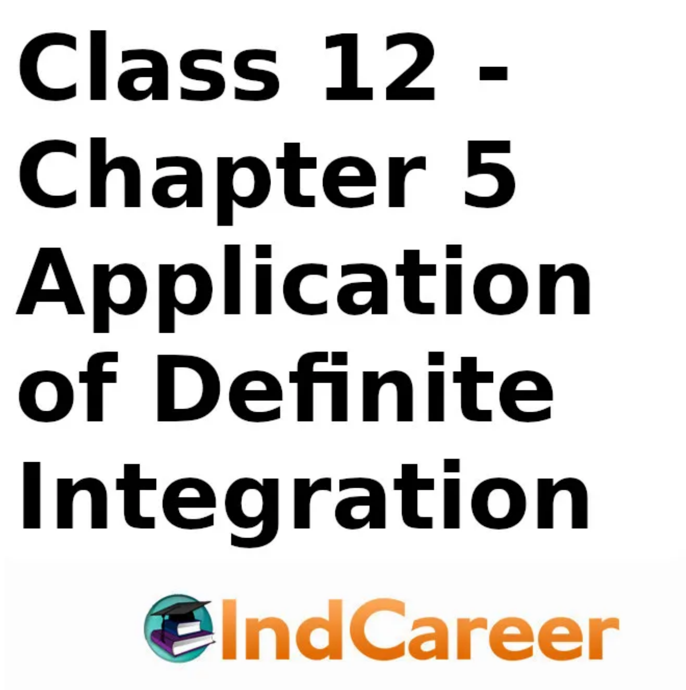
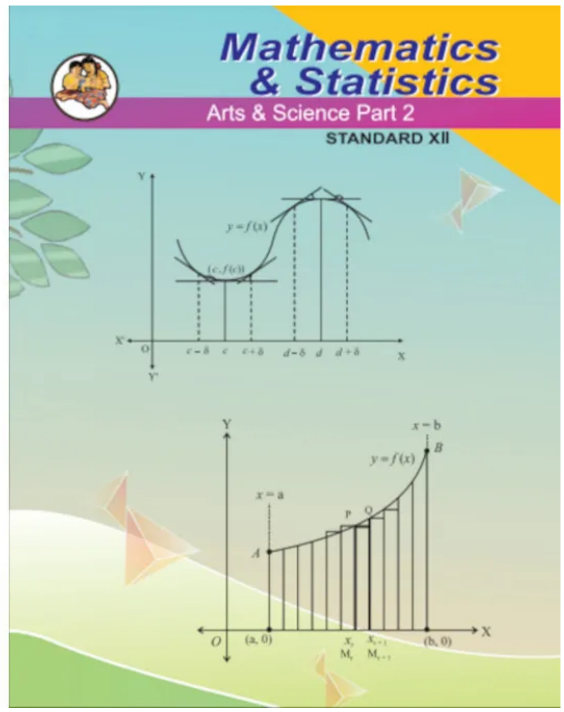
Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 5- Application of Definite Integration





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Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 5- Application of Definite Integration

Class 12: Maths Chapter 5 solutions. Complete Class 12 Maths Chapter 5 Notes.

Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 5- Application of Definite Integration

Maharashtra Board 12th Maths Chapter 5, Class 12 Maths Chapter 5 solutions

Ex 5.1

1. Find the area of the region bounded by the following curves, X-axis, and the given lines:

(i) y = 2x, x = 0, x = 5.

Solution:

Required area = $\int_0^5 y dx$, where y = 2x

= $\int_0^5 2x dx$

= $\left[\frac{2x^2}{2}\right]_0^5$

= 25 – 0

= 25 sq units.

(ii) x = 2y, y = 0, y = 4.

Solution:

Required area = $\int_0^4 x dy$, where x = 2y

= $\int_0^4 2y dy$

= $\left[\frac{2y^2}{2}\right]_0^4$

= 16 – 0

= 16 sq units.

(iii) x = 0, x = 5, y = 0, y = 4.

Solution:

Required area = $\int_0^5 y dx$, where y = 4

= $\int_0^5 4 dx$

= $[4x]_0^5$

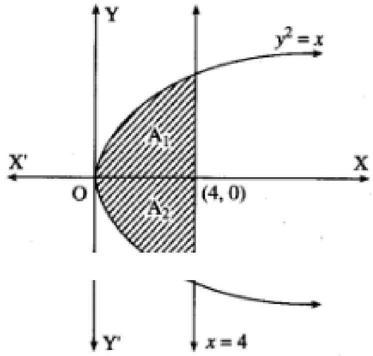
= 20 – 0

= 20 – 0
= 20 sq units.

(iv) y = sin x, x = 0, x = $\frac{\pi}{2}$
Solution:
Required area = $\int_0^{\pi/2} y dx$, where y = sin x
= $\int_0^{\pi/2} \sin x dx$
= $[-\cos x]_0^{\pi/2}$
= -cos $\frac{\pi}{2}$ + cos 0
= 0 + 1
= 1 sq unit.

(v) xy = 2, x = 1, x = 4.
Solution:
For xy = 2, y = $\frac{2}{x}$
Required area = $\int_1^4 y dx$, where y = $\frac{2}{x}$
= $\int_1^4 \frac{2}{x} dx$
= $[2 \log |x|]_1^4$
= 2 log 4 – 2 log 1
= 2 log 4 – 0
= 2 log 4 sq units.

(vi) y^2 = x, x = 0, x = 4.
Solution:



The required area consists of two bounded regions A1 and A2 which are equal in areas.

For $y^2 = x$, $y = \sqrt{x}$

Required area = $A_1 + A_2 = 2A_1$

$$= 2\int_0^4 y\,dx, \quad \text{where } y = \sqrt{x}$$

$$= 2\int_0^4 \sqrt{x}\,dx$$

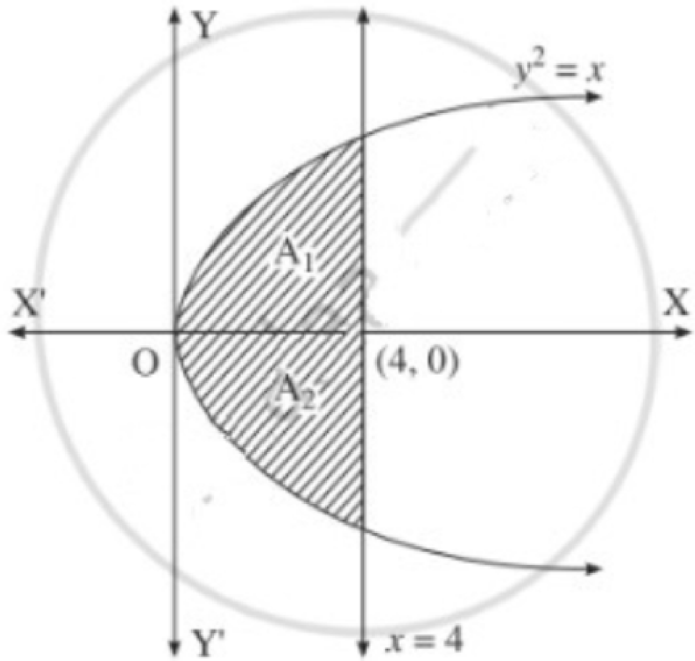
$$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^4$$

$$= 2\left[\frac{2}{3}(4)^{3/2} - 0\right]$$

$$= 2\left[\frac{2}{3}(2^2)^{3/2}\right] = \frac{32}{3} \text{ sq units}$$

(vii) $y^2 = 16x$, $x = 0$, $x = 4$.

Solution:



The required area consists of two bounded regions A1 and A2 which are equal in areas.

For $y^2 = x$, $y = \sqrt{x}$

Required area = $A_1 + A_2 = 2A_1$

$= 2\int_0^4 y \cdot dx,$ where $y = \sqrt{x}$

$= 2\int_0^4 \sqrt{x}\cdot dx$

$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= 2\left[\frac{2}{3}(4)^{\frac{3}{2}} - 0\right]$

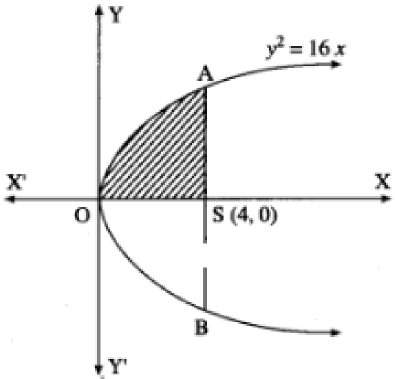
$= 2\left[\frac{2}{3}\left(2^2\right)^{\frac{3}{2}}\right]$

$= \frac{128}{3}$ sq units.

2. Find the area of the region bounded by the parabola:

(i) $y^2 = 16x$ and its latus rectum.
Solution:
Comparing $y^2 = 16x$ with $y^2 = 4ax$, we get
$4a = 16$
$\therefore a = 4$
$\therefore$ focus is $S(a, 0) = (4, 0)$

∴ focus is S(a, 0) = (4, 0)



For $y^2 = 16x$, $y = 4\sqrt{x}$

Required area = area of the region OBSAO

= 2 [area of the region OSAO]

$$= 2\int_0^4 y\,dx, \quad \text{where } y = 4\sqrt{x}$$

$$= 2\int_0^4 4\sqrt{x}\,dx$$

$$= 8\left[\frac{x^{3/2}}{3/2}\right]_0^4 = 8\left[\frac{2}{3}(4)^{\frac{3}{2}} - 0\right]$$

$$= 8\left[\frac{2}{3}(2^2)^{\frac{3}{2}}\right] = \frac{128}{3} \text{ sq units.}$$

(ii) $y = 4 - x^2$ and the X-axis.

Solution:

The equation of the parabola is $y = 4 - x^2$

$\therefore x^2 = 4 - y$

i.e. $(x - 0)^2 = -(y - 4)$

It has vertex at P(0, 4)

For points of intersection of the parabola with X-axis,

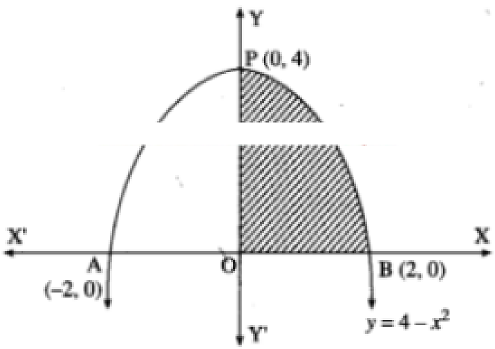
we put y = 0 in its equation.

$\therefore 0 = 4 - x^2$

$\therefore x^2 = 4$

$\therefore x = \pm 2$

$\therefore$ the parabola intersect the X-axis at A(-2, 0) and B(2, 0)



Required area = area of the region APBOA

= 2[area of the region OPBO]

$$= 2\int_0^2 y\,dx, \text{ where } y = 4 - x^2$$

$$= 2\int_0^2 (4 - x^2)dx$$

$$= 8\int_0^2 1dx - 2\int_0^2 x^2dx$$

$$= 8[x]_0^2 - 2\left[\frac{x^3}{3}\right]_0^2$$

$$= 8(2-0) - \frac{2}{3}(8-0)$$

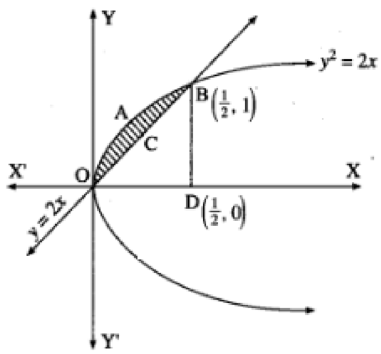
$$= 16 - \frac{16}{3} = \frac{32}{3} \text{ sq units.}$$

3. Find the area of the region included between:

(i) $y^2 = 2x$ and $y = 2x$.

Solution:

The vertex of the parabola $y^2 = 2x$ is at the origin O = (0, 0).



To find the points of intersection of the line and the parabola, equaling the values of 2x from both the equations we get,

$y^2 = y$

$\therefore y^2 - y = 0$

$\therefore y = 0$ or $y = 1$

When $y = 0$, $x = \frac{0}{2} = 0$

When $y = 1$, $x = \frac{1}{2}$

$\therefore$ the points of intersection are $0(0, 0)$ and $B(\frac{1}{2}, 1)$

Required area = area of the region OABCO = area of the region OABDO – area of the region OCBDO

Now, area of the region OABDO = area under the parabola $y^2 = 2x$ between $x = 0$ and $x = \frac{1}{2}$

$$= \int_0^{1/2} y\,dx, \quad \text{where } y = \sqrt{2x}$$

$$= \int_0^{1/2} \sqrt{2x}\,dx = \sqrt{2}\left[\frac{x^{\frac{3}{2}}}{3/2}\right]_0^{1/2}$$

$$= \sqrt{2}\left[\frac{2}{3}\left(\frac{1}{2}\right)^{3/2} - 0\right] = \sqrt{2}\left[\frac{2}{3} \cdot \frac{1}{2\sqrt{2}}\right] = \frac{1}{3}$$

Area of the region OCBDO = area under the line $y = 2x$ between $x = 0$ and $x = \frac{1}{2}$

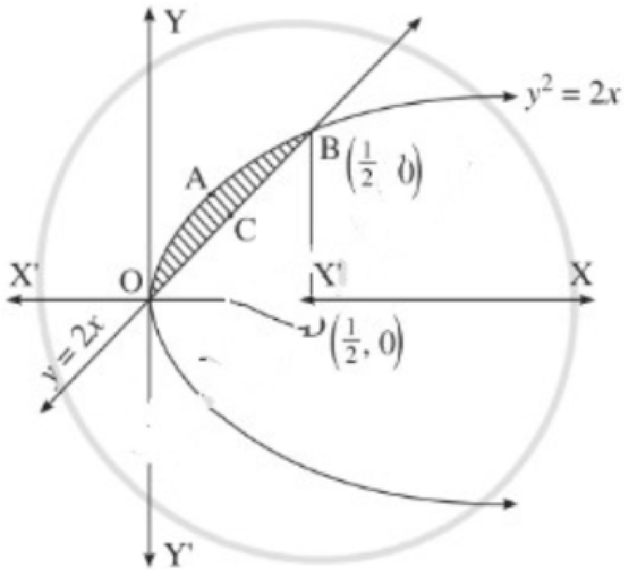
$$= \int_0^{1/2} y\,dx, \quad \text{where } y = 2x$$

$$= \int_0^{1/2} 2x\,dx = \left[\frac{2x^2}{2}\right]_0^{1/2} = \frac{1}{4} - 0 = \frac{1}{4}$$

$$\therefore \text{ required area} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \text{ sq unit.}$$

(ii) $y^2 = 4x$ and $y = x$.

(ii) $y^2 = 4x$ and $y = x$.
Solution:
The vertex of the parabola $y^2 = 4x$ is at the origin O = (0, 0).



To find the points of intersection of the line and the parabola, equaling the values of 4x from both the equations we get,
$\therefore y^2 = y$
$\therefore y^2 - y = 0$
$\therefore y(y - 1) = 0$
$\therefore y = 0$ or $y = 1$
When $y = 0$, $x = \frac{0}{2} = 0$

When y = 1, x = $\frac{1}{2}$

∴ the points of intersection are O(0, 0) and B($\frac{1}{2}$, 1)

Required area = area of the region OABCO = area of the region OABDO – area of the region OCBDO

Now, area of the region OABDO = area under the parabola y^2 = 4x between x = 0 and x = $\frac{1}{2}$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{x}x$$

$$= \int_0^{\frac{1}{2}} \sqrt{2}xdx$$

$$= \sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{1}{2}}$$

$$= \sqrt{2}\left[\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} - 0\right]$$

$$= \sqrt{2}\left[\frac{2}{3} \cdot \frac{1}{2\sqrt{2}}\right]$$

$$= \frac{1}{3}$$

Area of the region OCBDO = area under the line y = 2x between x = 0 and x = $\frac{1}{2}$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{where } y = x$$

$= \int_0^{\frac{1}{2}} 2x \cdot dx$

$= \left[\frac{2x^2}{2}\right]_0^{\frac{1}{2}}$

$= \frac{4}{1} - 0$

$= \frac{4}{3}$

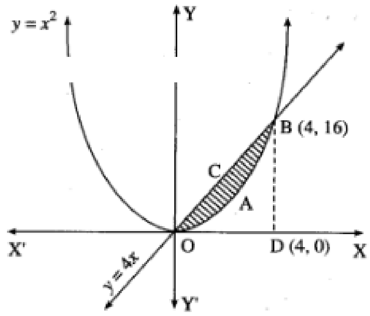
∴ required area

$= \frac{4}{1} = \frac{4}{3}$

$= \frac{8}{3}$sq units.

(iii) $y = x^2$ and the line $y = 4x$.
Solution:
The vertex of the parabola $y = x^2$ is at the origin 0(0, 0)
To find the points of the intersection of a line and the parabola.

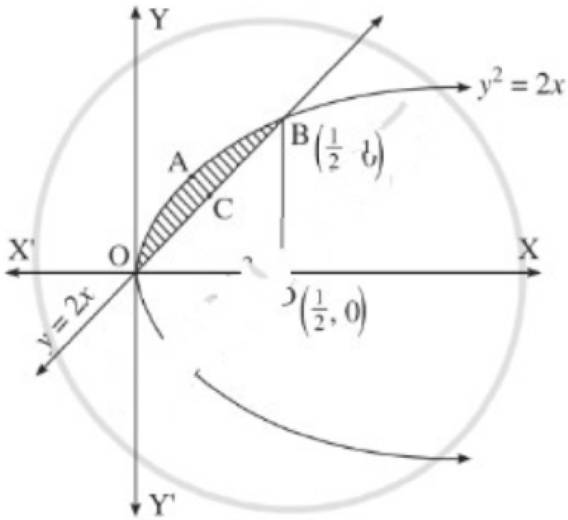
To find the points of the intersection of a line and the parabola.



Equating the values of y from the two equations, we get
$x^2 = 4x$
$\therefore x^2 - 4x = 0$
$\therefore x(x - 4) = 0$
$\therefore x = 0, x = 4$
When x = 0, y = 4(0) = 0
When x = 4, y = 4(4) = 16
∴ the points of intersection are 0(0, 0) and B(4, 16)
Required area = area of the region OABCO = (area of the region ODBCO) – (area of the region ODBAO)
Now, area of the region ODBCO = area under the line y = 4x between x = 0 and x = 4
= $\int_0^4 y dx$, where y = 4x
= $\int_0^4 4x dx$
= 4$\int_0^4 x dx$
= 4$[latex] \int_0^4 x dx$[/latex]
= 2(16 – 0)

= 32
Area of the region ODBAO = area under the parabola $y = x^2$ between x = 0 and x = 4
= $\int_0^4 y dx$, where y = x2
= $\int_0^4 x^2 dx$
= $\left[\frac{x^3}{3}\right]_0^4$
= $\frac{1}{3}$ (64 – 0)
= $\frac{64}{3}$
∴ required area = $32 - \frac{64}{3} = \frac{32}{3}$ sq units.

(iv) $y^2 = 4ax$ and y = x.
Solution:
The vertex of the parabola $y^2 = 4ax$ is at the origin O = (0, 0).



To find the points of intersection of the line and the parabola, equaling the values of 4ax from both the equations we get,

$\therefore y^2 = y$

$\therefore y^2 - y = 0$

$\therefore y(y - 1) = 0$

$\therefore y = 0$ or $y = 1$

When $y = 0$, $x = \frac{0}{2} = 0$

When $y = 1$, $x = \frac{1}{2}$

$\therefore$ the points of intersection are O(0, 0) and B($\frac{1}{2}$, 1)

Required area = area of the region OABCO = area of the region OABDO – area of the region

OCBDO
Now, area of the region OABDO
= area under the parabola $y^2 = 4ax$ between x = 0 and x = $\frac{1}{2}$

= $\int_0^{\frac{1}{2}} y \cdot dx$, where $y = \sqrt{2}x$

= $\int_0^{\frac{1}{2}} \sqrt{2}x dx$

= $\sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{1}{2}}$

= $\sqrt{2}\left[\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} - 0\right]$

= $\sqrt{2}\left[\frac{2}{3} \cdot \frac{1}{2\sqrt{2}}\right]$

= $\frac{1}{3}$

Area of the region OCBDO
= area under the line y
= 4ax between x = 0 and x = $\frac{1}{4ax}$

= $\int_0^{\frac{1}{2}} y \cdot dx$, where $y = x$

$= \int_0^{\frac{1}{2}} 2x \cdot dx$

$= \left[\frac{2x^2}{2}\right]_0^{\frac{1}{2}}$

$= \frac{4}{3} - 0$

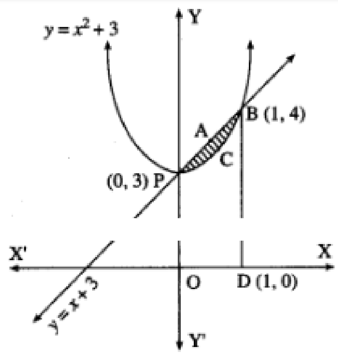
$= \frac{2a^2}{1}$

∴ required area

$= \frac{4}{3} = \frac{2a^2}{1}$

$= \frac{8a^2}{3}$ sq units.

(v) $y = x^2 + 3$ and $y = x + 3$.
Solution:
The given parabola is $y = x^2 + 3$, i.e. $(x - 0)^2 = y - 3$
∴ its vertex is P(0, 3).



To find the points of intersection of the line and the parabola.
Equating the values of y from both the equations, we get
$x^2 + 3 = x + 3$
$\therefore x^2 - x = 0$
$\therefore x(x - 1) = 0$
$\therefore x = 0$ or $x = 1$
When $x = 0$, $y = 0 + 3 = 3$
When $x = 1$, $y = 1 + 3 = 4$
$\therefore$ the points of intersection are P(0, 3) and B(1, 4)
Required area = area of the region PABCP = area of the region OPABDO – area of the region OPCBDO
Now, area of the region OPABDO
= area under the line $y = x + 3$ between $x = 0$ and $x = 1$

$$= \int_0^1 y\,dx, \quad \text{where } y = x + 3$$

$$= \int_0^1 (x + 3)\,dx$$

$$= \int_0^1 x\,dx + 3\int_0^1 1dx = \left[\frac{x^2}{2}\right]_0^1 + 3[x]_0^1$$

$$= \left(\frac{1}{2} - 0\right) + 3(1 - 0) = \frac{7}{2}$$

Area of the region OPCBDO = area under the parabola $y = x^2 + 3$ between $x = 0$ and $x = 1$

$$= \int_0^1 y\,dx, \quad \text{where } y = x^2 + 3$$

$$= \int_0^1 (x^2 + 3)dx = \int_0^1 x^2\,dx + 3\int_0^1 1dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + 3[x]_0^1 = \left(\frac{1}{3} - 0\right) + 3(1 - 0) = \frac{10}{3}$$

$$\therefore \text{ required area} = \frac{7}{2} - \frac{10}{3} = \frac{21 - 20}{6} = \frac{1}{6} \text{ sq unit.}$$

Maharashtra Board Solutions Class 12 Arts & Science Maths (Part 2)

About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

FAQs

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As of now, the MSCERT and Balbharti are responsible for the syllabus and textbooks of Classes 1 to 8, while Classes 9 and 10 are under the Maharashtra State Board of Secondary and Higher Secondary Education (MSBSHSE).

How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

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