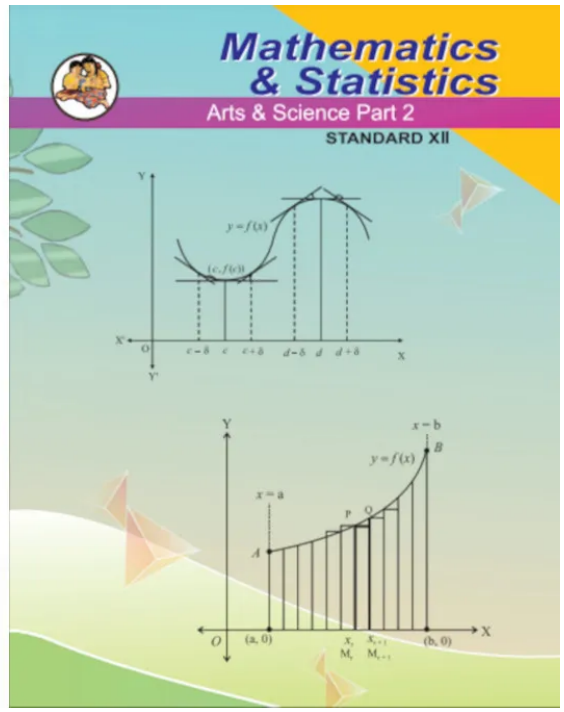
Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 4- Definite Integration

Class 12 - Chapter 4 Definite Integration



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Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 4- Definite Integration

Class 12: Maths Chapter 4 solutions. Complete Class 12 Maths Chapter 4 Notes.

Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 4- Definite Integration

Maharashtra Board 12th Maths Chapter 4, Class 12 Maths Chapter 4 solutions

I. Evaluate the following integrals as a limit of a sum.

Question 1.

$\int_1^3 (3x - 4) \cdot dx$

Solution:

Let f(x) = 3x – 4, for 1 ≤ x ≤ 3

Divide the closed interval [1, 3] into n subintervals each of length h at the points

1, 1 + h, 1 + 2h, 1 + rh, ….., 1 + nh = 3

∴ nh = 2

∴ h = $\frac{2}{n}$ and as n → ∞, h → 0

Here, a = 1

∴ f(a + rh) = f(1 + rh)

= 3(1 + rh) – 4

= 3rh – 1

$$\because \int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{r=1}^{n} f(a+rh) \cdot h$$

$$\therefore \int_1^3 (3x-4)\,dx = \lim_{n \to \infty} \sum_{r=1}^{n} (3rh-1)h$$

$$= \lim_{n \to \infty} \sum_{r=1}^{n} \left(3r \cdot \frac{2}{n} - 1\right) \cdot \frac{2}{n} \quad \ldots \left[\because h = \frac{2}{n}\right]$$

$$= \lim_{n \to \infty} \sum_{r=1}^{n} \left(\frac{12r}{n^2} - \frac{2}{n}\right)$$

$$= \lim_{n \to \infty} \left[\frac{12}{n^2} \sum_{r=1}^{n} r - \frac{2}{n} \sum_{r=1}^{n} 1\right]$$

$$= \lim_{n \to \infty} \left[\frac{12}{n^2} \cdot \frac{n(n+1)}{2} - \frac{2}{n} \cdot n\right]$$

$$= \lim_{n \to \infty} \left[6\left(\frac{n+1}{n}\right) - 2\right]$$

$$= \lim_{n \to \infty} \left[6\left(1+\frac{1}{n}\right)-2\right]$$

$$= 6(1+0)-2 \qquad \ldots \left[\because \lim_{n \to \infty} \frac{1}{n} = 0\right]$$

$$= 4.$$

Question 2.

$\int_0^4 x^2 dx$

Solution:

Let $f(x) = x^2$, for $0 \le x \le 4$

Divide the closed interval [0, 4] into n subintervals each of length h at the points

0, 0 + h, 0 + 2h, ….., 0 + rh, ….., 0 + nh = 4

i.e. 0, h, 2h, ….., rh, ….., nh = 4

$\therefore h = \frac{4}{n}$ as $n \to \infty, h \to 0$

Here, a = 0

$$\therefore f(a+rh) = f(0+rh) = f(rh) = r^2h^2$$

$$\because \int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{r=1}^{n} f(a+rh) \cdot h$$

$$\therefore \int_0^4 x^2\,dx = \lim_{n \to \infty} \sum_{r=1}^{n} r^2h^2 \cdot h$$

$$= \lim_{n \to \infty} \sum_{r=1}^{n} r^2 \frac{64}{n^3} \qquad \ldots \left[\because h = \frac{4}{n}\right]$$

$$= \lim_{n \to \infty} \left[\frac{64}{n^3} \sum_{r=1}^{n} r^2\right]$$

$$= \lim_{n \to \infty} \left[\frac{64}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n \to \infty} \left[\frac{64}{6}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)\right]$$

$$= \lim_{n \to \infty} \left[\frac{64}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]$$

$$= \frac{64}{6}(1+0)(2+0) \quad \ldots \left[\because \lim_{n \to \infty} \frac{1}{n} = 0\right]$$

$$= \frac{64}{3}.$$

Question 3.

$\int_0^2 e^x dx$

Solution:

Let f(x) = e^x, for $0 \le x \le 2$

Divide the closed interval [0, 2] into n equal subntervals each of length h at the points

0, 0 + h, 0 + 2h, ….., 0 + rh, ….., 0 + nh = 2

i.e. 0, h, 2h, ….., rh, ….., nh = 2

∴ h = $\frac{2}{n}$ and as n → ∞, h → 0

Here, a = 0

$$\therefore f(a+rh) = f(0+rh) = f(rh) = e^{rh}$$

$$\because \int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{r=1}^{n} f(a+rh)h$$

$$\therefore \int_0^2 e^x\,dx = \lim_{n \to \infty} \sum_{r=1}^{n} e^{rh} \cdot h$$

$$= \lim_{h \to 0} \left[h \sum_{r=1}^{n} e^{rh}\right] \quad \ldots [\text{as } n \to \infty, h \to 0]$$

Now, $\sum_{r=1} e^{rh} = e^h + e^{2h} + \dots + e^{nh}$

$$= \frac{e^h[(e^h)^n - 1]}{e^h - 1} = \frac{e^h[e^{nh} - 1]}{e^h - 1}$$

$$= \frac{e^h \cdot (e^2 - 1)}{e^h - 1} \qquad \dots \left[\because h = \frac{2}{n} \quad \therefore nh = 2\right]$$

$$= (e^2 - 1)\frac{e^h}{e^h - 1}$$

$$\therefore \int_0^2 e^x\,dx = \lim_{h \to 0} \frac{h(e^2 - 1)\,e^h}{e^h - 1}$$

$$= (e^2 - 1)\lim_{h \to 0} \frac{e^h}{\left(\frac{e^h - 1}{h}\right)}$$

$$= (e^2 - 1)\frac{\lim\limits_{h \to 0} e^h}{\lim\limits_{h \to 0}\left(\frac{e^h - 1}{h}\right)} = (e^2 - 1)\frac{\lim\limits_{h \to 0} e^h}{\lim\limits_{h \to 0}\left(\frac{e^h - 1}{h}\right)}$$

$$= (e^2 - 1)\cdot\frac{e^0}{1} \qquad \dots \left[\because \lim_{h \to 0} \frac{e^{h-1}}{h} = 1\right]$$

$$= (e^2 - 1).$$

Question 4.

$\int_0^2 (3x^2 - 1)\,dx$

Solution:

Let f(x) = $3x^2 - 1$, for $0 \le x \le 2$

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Divide the closed interval [0, 2] into n subintervals each of length h at the points.

0, 0 + h, 0 + 2h, ….., 0 + rh, ……, 0 + nh = 2

i.e. 0, h, 2h, ….., rh, ….., nh = 2

$\therefore h = \frac{2}{n}$ and as $n \to \infty$, $h \to 0$

Here, a = 0

$\therefore$ f(a + rh) = f(0 + rh)

= f(rh)

= $3(rh)^2 - 1$

= $3r^2h^2 - 1$

$$\because \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} f(a+rh)\cdot h$$

$$\therefore \int_0^2 (3x^2-1)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} (3r^2h^2-1)\cdot h$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(3r^2 \times \frac{4}{n^2} - 1\right)\cdot\frac{2}{n} \qquad \ldots \left[\because h = \frac{2}{n}\right]$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{24r^2}{n^3} - \frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \left[\frac{24}{n^3} \sum_{r=1}^{n} r^2 - \frac{2}{n} \sum_{r=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \left[\frac{24}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{2}{n}\cdot n\right]$$

$$= \lim_{n\to\infty} \left[4\cdot\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) - 2\right]$$

$$= \lim_{n\to\infty} \left[4\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 2\right]$$

$$= 4(1+0)(2+0)-2 \qquad \ldots \left[\because \lim_{n\to\infty} \frac{1}{n} = 0\right]$$

$$= 8 - 2 = 6.$$

Question 5.

$\int_1^3 x^3 dx$

Solution:

Let f(x) = x^3, for 1 ≤ x ≤ 3.

Divide the closed interval [1, 3] into n equal su bintervals each of length h at the points

1, 1 + h, 1 + 2h, ……, 1 + rh, ……, 1 + nh = 3

∴ nh = 2

∴ h = $\frac{2}{n}$ and as n → ∞, h → 0

Here a = 1

$$\therefore f(a+rh) = f(1+rh) = (1+rh)^3$$
$$= 1 + 3rh + 3r^2h^2 + r^3h^3$$

$$\because \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} f(a+rh)\cdot h$$

$$\therefore \int_1^3 x^3\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} (1+3rh+3r^2h^2+r^3h^3)\cdot h$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} (h+3rh^2+3r^2h^3+r^3h^4)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left[\frac{2}{n} + 3r\left(\frac{2}{n}\right)^2 + 3r^2\left(\frac{2}{n}\right)^3 + r^3\left(\frac{2}{n}\right)^4\right]$$

$$\dots \left[\because h = \frac{2}{n}\right]$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left[\frac{2}{n} + \frac{12r}{n^2} + \frac{24r^2}{n^3} + \frac{16r^3}{n^4}\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\sum_{r=1}^{n} 1 + \frac{12}{n^2}\sum_{r=1}^{n} r + \frac{24}{n^3}\sum_{r=1}^{n} r^2 + \frac{16}{n^4}\sum_{r=1}^{n} r^3\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\cdot n + \frac{12}{n^2}\cdot\frac{n(n+1)}{2} + \frac{24}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{16}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right]$$

$$= \lim_{n\to\infty} \left[2 + 6\left(\frac{n+1}{n}\right) + 4\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) + 4\left(\frac{n+1}{n}\right)^2\right]$$

$$= \lim_{n\to\infty} \left[2 + 6\left(1+\frac{1}{n}\right) + 4\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 4\left(1+\frac{1}{n}\right)^2\right]$$

$$= [2 + 6(1+0) + 4(1+0)(2+0) + 4(1+0)^2]$$

$$\dots \left[\because \lim_{n\to\infty} \frac{1}{n} = 0\right]$$

$$= 2 + 6 + 8 + 4 = 20.$$

Ex 4.2

I. Evaluate:

Question 1.
$\int_{1}^{9} \frac{x+1}{\sqrt{x}} \cdot dx$
Solution:

$$\int_{1}^{9} \frac{x+1}{\sqrt{x}} dx = \int_{1}^{9} \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx$$

$$= \int_{1}^{9} x^{\frac{1}{2}} dx + \int_{1}^{9} x^{-\frac{1}{2}} dx$$

$$= \left[\frac{x^{3/2}}{3/2}\right]_{1}^{9} + \left[\frac{x^{1/2}}{1/2}\right]_{1}^{9}$$

$$= \frac{2}{3}[9^{\frac{3}{2}} - 1^{\frac{3}{2}}] + 2[9^{\frac{1}{2}} - 1^{\frac{1}{2}}]$$

$$= \frac{2}{3}[(3^2)^{\frac{3}{2}} - 1] + 2[3-1]$$

$$= \frac{2}{3}[27-1] + 4$$

$$= \frac{52}{3} + 4 = \frac{64}{3}.$$

Question 2.
$\int_{2}^{3} \frac{1}{x^2+5x+6} \cdot dx$
Solution:

$$\int_{2}^{3} \frac{1}{x^2+5x+6} dx = \int_{2}^{3} \frac{1}{(x+2)(x+3)} dx$$

$$= \int_2^3 \frac{(x+3)-(x+2)}{(x+2)(x+3)} dx$$

$$= \int_2^3 \left[\frac{1}{x+2} - \frac{1}{x+3}\right] dx$$

$$= [\log(x+2) - \log(x+3)]_2^3$$

$$= \left[\log\left|\frac{x+2}{x+3}\right|\right]_2^3$$

$$= \log\left(\frac{3+2}{3+3}\right) - \log\left(\frac{2+2}{2+3}\right) = \log\frac{5}{6} - \log\frac{4}{5}$$

$$= \log\left(\frac{5}{6} \times \frac{5}{4}\right) = \log\left(\frac{25}{24}\right).$$

Question 3.

$\int_0^{\pi/4} \cot^2 \cdot dx$

Solution:

$$\int_0^{\pi/4} \cot^2 x \, dx = \int_0^{\pi/4} (\operatorname{cosec}^2 x - 1) \, dx$$

$$= \int_0^{\pi/4} \operatorname{cosec}^2 x \, dx - \int_0^{\pi/4} dx$$

$$= [-\cot x]_0^{\pi/4} - [x]_0^{\pi/4}$$

$$= \left[\left(-\cot\frac{\pi}{4}\right) - (-\cot 0)\right] - \left[\frac{\pi}{4} - 0\right]$$

$$= -1 + \cot 0 - \frac{\pi}{4}$$

The integral does not exist since cot 0 is not defined.

Question 4.

$\int_{-\pi/4}^{\pi/4} \frac{1}{1-\sin x} \cdot dx$

Solution:

$$\int_{-\pi/4}^{\pi/4} \frac{1}{1-\sin x} dx$$

$$= \int_{-\pi/4}^{\pi/4} \frac{1}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} dx$$

$$= \int_{-\pi/4}^{\pi/4} \frac{1+\sin x}{1-\sin^2 x} dx = \int_{-\pi/4}^{\pi/4} \frac{1+\sin x}{\cos^2 x} dx$$

$$= \int_{-\pi/4}^{\pi/4} \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right) dx$$

$$= \int_{-\pi/4}^{\pi/4} (\sec^2 x + \sec x \tan x)\, dx$$

$$= [\tan x + \sec x]_{-\pi/4}^{\pi/4}$$

$$= \left(\tan\frac{\pi}{4} + \sec\frac{\pi}{4}\right) - \left[\tan\left(-\frac{\pi}{4}\right) + \sec\left(-\frac{\pi}{4}\right)\right]$$

$$= (1 + \sqrt{2}) - \left(-\tan\frac{\pi}{4} + \sec\frac{\pi}{4}\right)$$

$$= (1 + \sqrt{2}) - (-1 + \sqrt{2})$$

$$= 1 + \sqrt{2} + 1 - \sqrt{2} = 2.$$

Question 5.

$\int_3^5 \frac{1}{\sqrt{2x+3} - \sqrt{2x-3}} \cdot dx$

Solution:

$$\int_3^5 \frac{1}{\sqrt{2x+3} - \sqrt{2x-3}} dx$$

$$= \int_3^5 \frac{1}{\sqrt{2x+3} - \sqrt{2x-3}} \times \frac{\sqrt{2x+3} + \sqrt{2x-3}}{\sqrt{2x+3} + \sqrt{2x-3}} dx$$

$$= \int_3^5 \frac{\sqrt{2x+3} + \sqrt{2x-3}}{(2x+3) - (2x-3)} dx$$

$$= \frac{1}{6}\int_3^5 (2x+3)^{\frac{1}{2}} dx + \frac{1}{6}\int_3^5 (2x-3)^{\frac{1}{2}} dx$$

$$= \frac{1}{6}\left[\frac{(2x+3)^{3/2}}{2\left(\frac{3}{2}\right)}\right]_3^5 + \frac{1}{6}\left[\frac{(2x-3)^{3/2}}{2\left(\frac{3}{2}\right)}\right]_3^5$$

$$= \frac{1}{18}\left[(10+3)^{3/2} - (6+3)^{3/2}\right] + \frac{1}{18}\left[(10-3)^{3/2} - (6-3)^{3/2}\right]$$

$$= \frac{1}{18}[13\sqrt{13} - 9\sqrt{9}] + \frac{1}{18}[7\sqrt{7} - 3\sqrt{3}]$$

$$= \frac{1}{18}(13\sqrt{13} - 27 + 7\sqrt{7} - 3\sqrt{3})$$

$$= \frac{1}{18}(13\sqrt{13} + 7\sqrt{7} - 3\sqrt{3} - 27).$$

Question 6.
$\int_0^1 \frac{x^2-2}{x^2+1} \cdot dx$
Solution:

$$\int_0^1 \frac{x^2-2}{x^2+1} dx = \int_0^1 \frac{(x^2+1)-3}{x^2+1} dx$$

$$= \int_0^1 \left(1 - \frac{3}{x^2+1}\right) dx$$

$$= [x - 3\tan^{-1} x]_0^1$$

$$= (1 - 3\tan^{-1} 1) - (0 - 3\tan^{-1} 0)$$

$$= 1 - 3\left(\frac{\pi}{4}\right) - 0 = 1 - \frac{3\pi}{4}.$$

Question 7.

Question 7.
$\int_0^{\pi/4} \sin 4x \sin 3x \cdot dx$
Solution:

$$\int_0^{\pi/4} \sin 4x \sin 3x\, dx$$

$$=\frac{1}{2}\int_0^{\pi/4} 2 \sin 4x \sin 3x\, dx$$

$$=\frac{1}{2}\int_0^{\pi/4} [\cos(4x-3x)-\cos(4x+3x)]\, dx$$

$$=\frac{1}{2}\int_0^{\pi/4} \cos x\, dx-\frac{1}{2}\int_0^{\pi/4} \cos 7x\, dx$$

$$=\frac{1}{2}[\sin x]_0^{\pi/4}-\frac{1}{2}\left[\frac{\sin 7x}{7}\right]_0^{\pi/4}$$

$$=\frac{1}{2}\left[\sin\frac{\pi}{4}-\sin 0\right]-\frac{1}{14}\left[\sin\frac{7\pi}{4}-\sin 0\right]$$

$$=\frac{1}{2}\left[\frac{1}{\sqrt{2}}-0\right]-\frac{1}{14}\left[\sin\left(2\pi-\frac{\pi}{4}\right)-0\right]$$

$$=\frac{1}{2\sqrt{2}}-\frac{1}{14}\left(-\sin\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}+\frac{1}{14\sqrt{2}}=\frac{7+1}{14\sqrt{2}}$$

$$=\frac{4}{7\sqrt{2}}.$$

Question 8.

$\int_0^{\pi/4} \sqrt{1+\sin 2x} \cdot dx$

Solution:

$$\int_0^{\pi/4} \sqrt{1+\sin\ 2x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\sin^2 x + \cos^2 x + 2\ \sin\ x\ \cos\ x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{(\sin\ x + \cos\ x)^2}\, dx$$

$$= \int_0^{\pi/4} (\sin\ x + \cos\ x)\, dx = \int_0^{\pi/4} \sin\ x\ dx + \int_0^{\pi/4} \cos\ x\ dx$$

$$= [-\cos\ x]_0^{\pi/4} + [\sin\ x]_0^{\pi/4}$$

$$= \left[-\cos\frac{\pi}{4} - (-\cos\ 0)\right] + \left[\sin\frac{\pi}{4} - \sin\ 0\right]$$

$$= -\frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} - 0 = 1.$$

Question 9.

$\int_0^{\pi/4} \sin^4 x \cdot dx$

Solution:

$$\text{Consider } \sin^4 x = (\sin^2 x)^2 = \left(\frac{1-\cos\ 2x}{2}\right)^2$$

$=\frac{1}{4}[1-2\cos 2x+\cos^2 2x]$

$=\frac{1}{4}\left[1-2\cos 2x+\frac{1+\cos 4x}{2}\right]$

$=\frac{1}{4}\left[\frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x\right]$

$\therefore \int_0^{\pi/4} \sin^4 x\, dx$

$=\frac{1}{4}\int_0^{\pi/4}\left[\frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x\right]dx$

$=\frac{3}{8}\int_0^{\pi/4} 1dx-\frac{1}{2}\int_0^{\pi/4}\cos 2x\,dx+\frac{1}{8}\int_0^{\pi/4}\cos 4x\,dx$

$=\frac{3}{8}[x]_0^{\pi/4}-\frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi/4}+\frac{1}{8}\left[\frac{\sin 4x}{4}\right]_0^{\pi/4}$

$=\frac{3}{8}\left[\frac{\pi}{4}-0\right]-\frac{1}{4}\left[\sin\frac{\pi}{2}-\sin 0\right]+\frac{1}{32}[\sin\pi-\sin 0]$

$=\frac{3\pi}{32}-\frac{1}{4}[1-0]+\frac{1}{32}(0-0)$

$=\frac{3\pi}{32}-\frac{1}{4}=\frac{3\pi-8}{32}.$

Question 10.

$\int_{-4}^{2} \frac{1}{x^2+4x+13} \cdot dx$

Solution:

$$\int_{-4}^{2} \frac{1}{x^2+4x+13} dx$$

$$= \int_{-4}^{2} \frac{1}{x^2+4x+4+9} dx = \int_{-4}^{2} \frac{1}{(x+2)^2+3^2} dx$$

$$= \left[\frac{1}{3} \tan^{-1}\left(\frac{x+2}{3}\right)\right]_{-4}^{2}$$

$$= \frac{1}{3} \tan^{-1}\left(\frac{2+2}{3}\right) - \frac{1}{3} \tan^{-1}\left(\frac{-4+2}{3}\right)$$

$$= \frac{1}{3} \tan^{-1}\left(\frac{4}{3}\right) - \frac{1}{3} \tan^{-1}\left(-\frac{2}{3}\right)$$

$$= \frac{1}{3}\left[\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{2}{3}\right] \quad \ldots [\because \tan^{-1}(-x) = -\tan^{-1}x]$$

Question 11.

$\int_{0}^{4} \frac{1}{\sqrt{4x-x^2}} \cdot dx$

Solution:

$$\int_{0}^{4} \frac{1}{\sqrt{4x-x^2}} dx$$

$$= \int_{0}^{4} \frac{1}{\sqrt{4-(x^2-4x+4)}} dx$$

$$= \int_0^4 \frac{1}{\sqrt{2^2 - (x-2)^2}} \, dx$$

$$= \left[\sin^{-1}\left(\frac{x-2}{2}\right) \right]_0^4 = \sin^{-1}\left(\frac{4-2}{2}\right) - \sin^{-1}\left(\frac{0-2}{2}\right)$$

$$= \sin^{-1} 1 - \sin^{-1}(-1)$$

$$= 2 \sin^{-1} 1 \qquad \ldots [\because \sin^{-1}(-x) = -\sin^{-1} x]$$

$$= 2\left(\frac{\pi}{2}\right) = \pi.$$

Question 12.

$\int_0^1 \frac{1}{\sqrt{3+2x-x^2}} \cdot dx$

Solution:

$$\int_0^1 \frac{1}{\sqrt{3+2x-x^2}} \, dx$$

$$= \int_0^1 \frac{1}{\sqrt{3-(x^2-2x+1)+1}} \, dx$$

$$= \int_0^1 \frac{1}{\sqrt{(2)^2-(x-1)^2}} \, dx$$

$$= \left[\sin^{-1}\left(\frac{x-1}{2}\right) \right]_0^1$$

$$= \sin^{-1}(0) - \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= 0 - \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$$

$$= -\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right] = -\left(-\frac{\pi}{6}\right) = \frac{\pi}{6}.$$

Question 13.

$\int_0^{\pi/2} x \cdot \sin x \cdot dx$

Solution:

$$\int_0^{\pi/2} x \sin x \, dx$$

$$= \left[x \int \sin x \, dx\right]_0^{\pi/2} - \int_0^{\pi/2} \left[\frac{d}{dx}(x) \int \sin x \, dx\right] dx$$

$$= \left[x(-\cos x)\right]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot (-\cos x) \, dx$$

$$= -\left[x \cos x\right]_0^{\pi/2} + \int_0^{\pi/2} \cos x \, dx$$

$$= -\left[\frac{\pi}{2}\cos\frac{\pi}{2} - 0\right] + \left[\sin x\right]_0^{\pi/2}$$

$$= 0 + \left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$= 1.$$

Question 14.
$\int_0^1 x \cdot \tan^{-1} x \cdot dx$
Solution:

$$\text{Let } I=\int_0^1 x \tan^{-1} x\, dx$$

$$=\int_0^1 (\tan^{-1} x)(x)\, dx$$

$$=\left[(\tan^{-1} x)\int x\, dx\right]_0^1-\int_0^1\left[\frac{d}{dx}(\tan^{-1} x)\cdot\int x\, dx\right] dx$$

$$=\left[\frac{x^2 \tan^{-1} x}{2}\right]_0^1-\int_0^1 \frac{1}{1+x^2}\cdot\frac{x^2}{2} dx$$

$$=\left(\frac{1^2 \tan^{-1} 1}{2}-0\right)-\frac{1}{2}\int_0^1 \frac{1+x^2-1}{1+x^2}\, dx$$

$$=\frac{\pi/4}{2}-\frac{1}{2}\int_0^1\left(1-\frac{1}{1+x^2}\right) dx$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[x-\tan^{-1}(x)\right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}[(1 - \tan^{-1} 1) - 0]$$

$$= \frac{\pi}{8} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right) = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \frac{\pi}{4} - \frac{1}{2}.$$

Question 15.

$\int_0^\infty x \cdot e^{-x} \cdot dx$

Solution:

$$\int_0^\infty xe^{-x}\, dx$$

$$= [x \int e^{-x}\, dx]_0^\infty - \int_0^\infty \left[\frac{d}{dx}(x) \int e^{-x}\, dx\right] dx$$

$$= \left[x\left(\frac{e^{-x}}{-1}\right)\right]_0^\infty - \int_0^\infty 1 \cdot \frac{e^{-x}}{(-1)}\, dx$$

$$= \left[-\frac{x}{e^x}\right]_0^\infty + \int_0^\infty e^{-x} dx$$

$$= \left[-\frac{x}{e^x}\right]_0^\infty + [-e^x]_0^\infty$$

$$= [0 - (-0)] + [0 - (-1)]$$

$$= 1. \qquad \ldots [\because e^0 = 1, e^{-x} = 0, \text{ when } x = \infty]$$

II. Evaluate:

Question 1.

$\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} \cdot dx$

Solution:

$$\text{Let } I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1}x}{(1-x^2)^{3/2}}\, dx = \int_0^{1/\sqrt{2}} \frac{\sin^{-1}x}{(1-x^2)\sqrt{1-x^2}}\, dx$$

$$\text{Put } \sin^{-1}x = t \qquad \therefore \frac{1}{\sqrt{1-x^2}}\, dx = dt$$

Also, $x = \sin t$

$$\text{When } x = \frac{1}{\sqrt{2}},\ t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

When $x = 0$, $t = \sin^{-1}0 = 0$

$$\therefore\ I = \int_0^{\pi/4} \frac{t}{1-\sin^2 t}\, dt = \int_0^{\pi/4} \frac{t}{\cos^2 t}\, dt = \int_0^{\pi/4} t \sec^2 t\, dt$$

$$= \left[t \int \sec^2 t\, dt\right]_0^{\pi/4} - \int_0^{\pi/4} \left[\frac{d}{dt}(t) \int \sec^2 t\, dt\right] dt$$

$$= [t \tan t]_0^{\pi/4} - \int_0^{\pi/4} 1 \cdot \tan t\, dt$$

$$= \left[\frac{\pi}{4} \tan \frac{\pi}{4} - 0\right] - [\log |\sec t|]_0^{\pi/4}$$

$$= \frac{\pi}{4} - \left[\log \left(\sec \frac{\pi}{4}\right) - \log (\sec 0)\right]$$

$$= \frac{\pi}{4} - [\log \sqrt{2} - \log 1]$$

$$= \frac{\pi}{4} - \frac{1}{2} \log 2. \qquad \ldots [\because \log 1 = 0]$$

Question 2.

$\int_0^{\pi/4} \frac{\sec^2 x}{3 \tan^2 x + 4 \tan x + 1} \cdot dx$

Solution:

$$\text{Let } I = \int_0^{\pi/4} \frac{\sec^2 x}{3 \tan^2 x + 4 \tan x + 1} dx$$

Put $\tan x = t \quad \therefore \sec^2 x \, dx = dt$

When $x = 0,\ t = \tan 0 = 0$

When $x = \frac{\pi}{4},\ t = \tan \frac{\pi}{4} = 1$

$$\therefore I = \int_0^1 \frac{dt}{3t^2 + 4t + 1} = \frac{1}{3} \int_0^1 \frac{dt}{t^2 + \frac{4}{3}t + \frac{1}{3}}$$

$$= \frac{1}{3}\int_0^1 \frac{dt}{t^2 + \frac{4t}{3} + \frac{4}{9} - \frac{4}{9} + \frac{1}{3}} = \frac{1}{3}\int_0^1 \frac{dt}{\left(t + \frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2}$$

$$= \frac{1}{3} \frac{1}{2\left(\frac{1}{3}\right)} \left[\log \left|\frac{t + \frac{2}{3} - \frac{1}{3}}{t + \frac{2}{3} + \frac{1}{3}}\right|\right]_0^1$$

$$= \frac{1}{2}\left[\log\left(\frac{1 + \frac{1}{3}}{1 + 1}\right) - \log\left(\frac{0 + \frac{1}{3}}{0 + 1}\right)\right]$$

$$= \frac{1}{2}\left[\log\left(\frac{2}{3}\right) - \log\left(\frac{1}{3}\right)\right] = \frac{1}{2}\log 2.$$

Question 3.

$\int_0^{4\pi} \frac{\sin 2x}{\sin^4 x + \cos^4 x} \cdot dx$

Solution:

$$\text{Let } I = \int_0^{\pi/4} \frac{\sin\ 2x}{\sin^4 x + \cos^4 x}\, dx$$

$$= \int_0^{\pi/4} \frac{2\ \sin x\ \cos x}{\sin^4 x + \cos^4 x}\, dx$$

Dividing each term by $\cos^4 x$, we get

$$I = \int_0^{\pi/4} \frac{2\frac{\sin x}{\cos x}\cdot\frac{1}{\cos^2 x}}{\frac{\sin^4 x}{\cos^4 x}+1}\,dx$$

$$= \int_0^{\pi/4} \frac{2\tan x\sec^2 x}{(\tan^2 x)^2+1}\,dx$$

Put $\tan^2 x = t \quad \therefore\ 2\tan x\sec^2 x\,dx = dt$

When $x = 0,\ t = \tan^2 0 = 0$

When $x = \frac{\pi}{4},\ t = \tan^2\frac{\pi}{4} = 1$

$$\therefore\ I = \int_0^1 \frac{dt}{1+t^2} = [\tan^{-1} t]_0^1$$

$$= \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}.$$

Question 4.
$\int_0^{2\pi} \sqrt{\cos x} \cdot \sin^3 x \cdot dx$
Solution:

$$\text{Let } I = \int_0^{2\pi} \sqrt{\cos x}\,\sin^3 x\,dx$$

$$= \int_0^{2\pi} \sqrt{\cos x}\,\sin^2 x\,\sin x\,dx$$

$$= \int_0^{2\pi} \sqrt{\cos x}\,(1-\cos^2 x)\sin x\,dx$$

Put $\cos x = t \quad \therefore \; -\sin x\,dx = dt$

$\therefore \; \sin x\,dx = -dt$

When $x = 0,\ t = \cos 0 = 1$

When $x = 2\pi,\ t = \cos 2\pi = 1$

$\therefore \; I = \int_1^1 \sqrt{t}(1-t^2)(-dt) = 0. \qquad \ldots \left[\because \int_a^a f(x)\,dx = 0\right]$

Question 5.

$\int_0^{\pi/2} \frac{1}{5+4\cos x} \cdot dx$

Solution:

$$\text{Let } I = \int_0^{\pi/2} \frac{1}{5+4\cos x}\,dx$$

Put $\tan\left(\frac{x}{2}\right) = t \quad \therefore \; x = 2\tan^{-1} t \quad \therefore \; dx = \frac{2dt}{1+t}$

and $\cos x = \frac{1-t^2}{1+t^2}$

When $x = \frac{\pi}{2},\ t = \tan\left(\frac{\pi}{4}\right) = 1$

When $x = 0,\ t = \tan 0 = 0$

$$\therefore\ I=\int_0^1 \frac{\left(\frac{2\,dt}{1+t^2}\right)}{5+4\left(\frac{1-t^2}{1+t^2}\right)}=\int_0^1 \frac{2dt}{5(1+t^2)+4(1-t^2)}$$

$$=2\int_0^1 \frac{1}{t^2+9}\,dt$$

Question 6.

$\int_0^{\pi/4} \frac{\cos x}{4-\sin^2 x} \cdot dx$

Solution:

$$\text{Let } I=\int_0^{\pi/4} \frac{\cos\ x}{4-\sin^2 x}\,dx$$

Put $\sin x=t \quad \therefore\ \cos x\,dx=dt$

When $x=\frac{\pi}{4},\ t=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

When $x=0,\ t=\sin 0=0.$

$$\therefore\ I=\int_0^{1/\sqrt{2}} \frac{dt}{2^2-t^2}$$

$$=\left[\frac{1}{2(2)}\log\left|\frac{2+t}{2-t}\right|\right]_0^{1/\sqrt{2}}$$

$$=\frac{1}{4}\left[\log\left(\frac{2+\frac{1}{\sqrt{2}}}{2-\frac{1}{\sqrt{2}}}\right)-\log\left(\frac{2+0}{2-0}\right)\right]$$

$$=\frac{1}{4}\left[\log\left(\frac{2\sqrt{2}+1}{2\sqrt{2}-1}\right)-\log 1\right]$$

$$=\frac{1}{4}\log\left(\frac{2\sqrt{2}+1}{2\sqrt{2}-1}\right). \qquad \ldots [\because \log 1=0]$$

Question 7.

$\int_0^{\pi/2} \frac{\cos X}{(1+\sin x)(2+\sin x)} \cdot dx$

Solution:

$$\text{Let } I=\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)}\, dx$$

Put $\sin x=t \quad \therefore \cos x\, dx=dt$

When $x=\frac{\pi}{2}$, $t=\sin\frac{\pi}{2}=1$

When $x=0$, $t=\sin 0=0$

$$\therefore I=\int_0^1 \frac{dt}{(1+t)(2+t)}=\int_0^1 \frac{(2+t)-(1+t)}{(1+t)(2+t)}\, dt$$

$$=\int_0^1\left[\frac{1}{1+t}-\frac{1}{2+t}\right] dt$$

$$= \int_0^1 \frac{1}{1+t}\,dt - \int_0^1 \frac{1}{2+t}\,dt$$

$$= [\log|1+t|]_0^1 - [\log|2+t|]_0^1$$

$$= [\log(1+1) - \log(1+0)] - [\log(2+1) - \log(2+0)]$$

$$= \log 2 - \log 3 + \log 2 \qquad \ldots [\because \log 1 = 0]$$

$$= \log\left(\frac{2\times 2}{3}\right) = \log\left(\frac{4}{3}\right).$$

Question 8.

$\int_{-1}^{1} \frac{1}{a^2e^x+b^2e^{-x}} \cdot dx$

Solution:

$$\text{Let } I = \int_{-1}^{1} \frac{1}{a^2e^x + b^2e^{-x}}\,dx$$

Multiplying each term by e^x, we get

$$I = \int_{-1}^{1} \frac{e^x}{a^2(e^x)^2 + b^2}\,dx$$

Put $e^x = t \quad \therefore\ e^x\,dx = dt$

When $x = 1$, $t = e$

When $x = -1$, $t = e^{-1} = \frac{1}{e}$

$$\therefore\ I=\int_{1/e}^{e}\frac{dt}{a^2t^2+b^2}=\int_{1/e}^{e}\frac{dt}{(at)^2+b^2}$$

$$=\left[\frac{1}{a}\cdot\frac{1}{b}\tan^{-1}\left(\frac{at}{b}\right)\right]_{1/e}^{e}$$

$$=\frac{1}{ab}\tan^{-1}\left(\frac{ae}{b}\right)-\frac{1}{ab}\tan^{-1}\left(\frac{a}{be}\right)$$

$$=\frac{1}{ab}\left[\tan^{-1}\left(\frac{ae}{b}\right)-\tan^{-1}\left(\frac{a}{be}\right)\right].$$

Question 9.

$\int_0^{\pi}\frac{1}{3+2\sin x+\cos x}\cdot dx$

Solution:

$$\text{Let } I=\int_0^{\pi}\frac{1}{3+2\sin x+\cos x}\,dx$$

$$\text{Put } \tan\frac{x}{2}=t\quad\therefore\ x=2\tan^{-1}t$$

$$\therefore\ dx=\frac{2dt}{1+t^2}\ \text{ and }\ \sin x=\frac{2t}{1+t^2},\ \cos x=\frac{1-t^2}{1+t^2}$$

$$\text{When } x=0,\ t=\tan 0=0\qquad \text{When } x=\pi,\ t=\tan\frac{\pi}{2}=\infty$$

$$\therefore\ I=\int_0^{\infty}\frac{1}{3+2\left(\frac{2t}{1+t^2}\right)+\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$$

$$=\int_0^{\infty}\frac{1+t^2}{3+3t^2+4t+1-t^2}\cdot\frac{2dt}{1+t^2}$$

$$=2\int_0^{\infty}\frac{1}{2t^2+4t+4}\,dt$$

$$=\frac{2}{2}\int_0^{\infty}\frac{1}{t^2+2t+2}\,dt=\int_0^{\infty}\frac{1}{(t^2+2t+1)+1}\,dt$$

$$=\int_0^{\infty}\frac{1}{(t+1)^2+(1)^2}\,dt=\frac{1}{1}\left[\tan^{-1}\left(\frac{t+1}{1}\right)\right]_0^{\infty}$$

$$=\left[\tan^{-1}(t+1)\right]_0^{\infty}$$

$$=\tan^{-1}\infty-\tan^{-1}1\qquad=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}.$$

Question 10.
$\int_0^{\pi/4}\sec^4 x\cdot dx$
Solution:

$$\text{Let } I=\int_0^{\pi/4}\sec^4 x\,dx$$

$$=\int_0^{\pi/4}\sec^2 x\cdot\sec^2 x\,dx$$

$$= \int_0^{\pi/4} (1+\tan^2 x)\sec^2 x\, dx$$

Put $\tan x = t \quad \therefore \sec^2 x\, dx = dt$

When $x = 0$, $t = \tan 0 = 0$

When $x = \frac{\pi}{4}$, $t = \tan\frac{\pi}{4} = 1$

$$\therefore I = \int_0^1 (1+t^2)\, dt = \left[t + \frac{t^3}{3}\right]_0^1$$

$$= 1 + \frac{1}{3} - 0 = \frac{4}{3}.$$

Question 11.

$\int_0^1 \sqrt{\frac{1-x}{1+x}} \cdot dx$

Solution:

$$\text{Let } I = \int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx$$

Put $x = \cos\theta \quad \therefore dx = -\sin\theta\, d\theta$

When $x = 0$, $\cos\theta = 0 = \cos\frac{\pi}{2} \quad \therefore \theta = \frac{\pi}{2}$

When $x = 1$, $\cos\theta = 1 = \cos 0 \quad \therefore \theta = 0$

$$\therefore I = \int_{\pi/2}^{0} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \cdot (-\sin\theta)\, d\theta$$

$$= \int_{\pi/2}^{0} \sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}} \left(-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) d\theta$$

$$= \int_{\pi/2}^{0} \frac{\sin(\theta/2)}{\cos(\theta/2)} \left[-2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] d\theta$$

$$= \int_{\pi/2}^{0} -2\sin^2\left(\frac{\theta}{2}\right) d\theta$$

$$= -\int_{\pi/2}^{0} (1-\cos\theta)\, d\theta$$

$$= -[\theta - \sin\theta]_{\pi/2}^{0}$$

$$= -\left[(0-\sin 0) - \left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right)\right]$$

$$= -\left[0 - \frac{\pi}{2} + 1\right] = \frac{\pi}{2} - 1.$$

Question 12.
$\int_0^{\pi} \sin^3 x(1+2\cos x)(1+\cos x)^2 \cdot dx$
Solution:

$$\text{Let } I = \int_0^{\pi} \sin^3 x(1+2\cos x)(1+\cos x)^2\, dx$$

$$= \int_0^{\pi} \sin^2 x(1+2\cos x)(1+\cos x)^2 \cdot \sin x\, dx$$

$$= \int_0^{\pi} (1-\cos^2 x)(1+2\cos x)(1+\cos x)^2 \cdot \sin x\, dx$$

Put $\cos x = t \quad \therefore \quad -\sin x\, dx = dt$.

$\therefore \ \sin x\, dx = -dt$

When $x = 0$, $t = \cos 0 = 1$

When $x = \pi$, $t = \cos \pi = -1$

$$\therefore \ I = \int_{1}^{-1} (1 - t^2)(1 + 2t)(1 + t)^2(-dt)$$

$$= -\int_{1}^{-1} (1 + 2t - t^2 - 2t^3)(1 + 2t + t^2)\, dt$$

$$= -\int_{1}^{-1} (1 + 2t - t^2 - 2t^3 + 2t + 4t^2 - 2t^3 - 4t^4 + t^2 + 2t^3 - t^4 - 2t^5)\, dt$$

$$= -\int_{1}^{-1} (1 + 4t + 4t^2 - 2t^3 - 5t^4 - 2t^5)\, dt$$

$$= -\left[t + 4\left(\frac{t^2}{2}\right) + 4\left(\frac{t^3}{3}\right) - 2\left(\frac{t^4}{4}\right) - 5\left(\frac{t^5}{5}\right) - 2\left(\frac{t^6}{6}\right)\right]_{1}^{-1}$$

$$= -\left[t + 2t^2 + \frac{4}{3}t^3 - \frac{1}{2}t^4 - t^5 - \frac{1}{3}t^6\right]_{1}^{-1}$$

$$= -\left[\left(-1 + 2 - \frac{4}{3} - \frac{1}{2} + 1 - \frac{1}{3}\right) - \left(1 + 2 + \frac{4}{3} - \frac{1}{2} - 1 - \frac{1}{3}\right)\right]$$

$$= -\left[-1+2-\frac{4}{3}-\frac{1}{2}+1-\frac{1}{3}-1-2-\frac{4}{3}+\frac{1}{2}+1+\frac{1}{3}\right]$$

$$= -\left[-\frac{8}{3}\right] = \frac{8}{3}.$$

Question 13.

$\int_0^{\pi/2} \sin 2x \cdot \tan^{-1}(\sin x) \cdot dx$

Solution:

$$\text{Let } I = \int_0^{\pi/2} \sin 2x \cdot \tan^{-1}(\sin x)\, dx$$

$$= \int_0^{\pi/2} \tan^{-1}(\sin x) \cdot (2 \sin x \cos x)\, dx$$

Put $\sin x = t \quad \therefore \cos x\, dx = dt$

When $x = 0,\ t = \sin 0 = 0.$ When $x = \frac{\pi}{2},\ t = \sin\frac{\pi}{2} = 1$

$$\therefore\ I = \int_0^1 (\tan^{-1} t)(2t)\, dt$$

$$= \left[\tan^{-1} t \int 2t\, dt\right]_0^1 - \int_0^1 \left(\frac{d}{dt}(\tan^{-1} t) \int 2t\, dt\right) dt$$

$$= \left[(\tan^{-1} t)(t^2)\right]_0^1 - \int_0^1 \frac{1}{1+t^2} \cdot t^2\, dt$$

$$= \left[t^2 \tan^{-1} t\right]_0^1 - \int_0^1 \frac{(1+t^2)-1}{1+t^2}\, dt$$

$$= [1 \cdot \tan^{-1} 1 - 0] - \int_0^1 \left(1 - \frac{1}{1+t^2}\right) dt$$

$$= \frac{\pi}{4} - [t - \tan^{-1} t]_0^1$$

$$= \frac{\pi}{4} - [(1 - \tan^{-1} 1) - 0]$$

$$= \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1.$$

Question 14.

$$\int_{\frac{1}{\sqrt{2}}}^{1} \frac{\left(e^{\cos^{-1} x}\right)\left(\sin^{-1} x\right)}{\sqrt{1-x^2}} \cdot dx$$

Solution:

$$\text{Let } I = \int_{1/\sqrt{2}}^{1} \frac{e^{\cos^{-1} x} \sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$\text{Put } \sin^{-1} x = t \qquad \therefore \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\text{When } x = 1,\ t = \sin^{-1} 1 = \frac{\pi}{2}$$

$$\text{When } x = \frac{1}{\sqrt{2}},\ t = \sin^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

Also, $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x = \frac{\pi}{2} - t$

$$\therefore\ I = \int_{\pi/4}^{\pi/2} e^{\frac{\pi}{2}-t} \cdot t\, dt$$

$$= e^{\frac{\pi}{2}} \int_{\pi/4}^{\pi/2} te^{-t}\, dt$$

$$= e^{\frac{\pi}{2}} \left\{ [t \int e^{-t}\, dt]_{\pi/4}^{\pi/2} - \int_{\pi/4}^{\pi/2} \left[\frac{d}{dt}(t) \int e^{-t} dt \right] dt \right\}$$

$$= e^{\frac{\pi}{2}} \{ [-te^{-t}]_{\pi/4}^{\pi/2} - \int_{\pi/4}^{\pi/2} (1)(-e^{-t})\, dt \}$$

$$= e^{\frac{\pi}{2}} \left\{ \frac{-\pi}{2} e^{-\frac{\pi}{2}} + \frac{\pi}{4} e^{-\frac{\pi}{4}} + \int_{\pi/4}^{\pi/2} e^{-t}\, dt \right\}$$

$$= -\frac{\pi}{2} e^0 + \frac{\pi}{4} e^{\frac{\pi}{2}-\frac{\pi}{4}} + e^{\frac{\pi}{2}} [-e^{-t}]^{\frac{\pi}{2}}$$

$$= -\frac{\pi}{2} + \frac{\pi}{4} e^{\frac{\pi}{4}} + e^{\frac{\pi}{2}} [-e^{-\frac{\pi}{2}} + e^{\frac{-\pi}{4}}]$$

$$= -\frac{\pi}{2} + e^{\frac{\pi}{4}} \frac{\pi}{4} - e^0 + e^{\frac{\pi}{2}-\frac{\pi}{4}}$$

$$= -\frac{\pi}{2} + e^{\frac{\pi}{4}} \frac{\pi}{4} - 1 + e^{\frac{\pi}{4}}$$

$$= e^{\frac{\pi}{4}} \left(\frac{\pi}{4} + 1 \right) - \left(\frac{\pi}{2} + 1 \right).$$

Question 15.
$\int_2^3 \frac{\cos(\log x)}{x} \cdot dx$
Solution:

$$\text{Let } I=\int_1^3 \frac{\cos(\log x)}{x}\, dx$$

$$=\int_1^3 \cos(\log x)\cdot\frac{1}{x}\, dx$$

Put $\log x = t \quad \therefore \frac{1}{x}\,dx = dt$

When $x=1$, $t=\log 1=0$

When $x=3$, $t=\log 3$

$$\therefore\ I=\int_0^{\log 3} \cos t\, dt = [\sin t]_0^{\log 3}$$

$$=\sin(\log 3)-\sin 0=\sin(\log 3).$$

III. Evaluate:

Question 1.
$\int_0^a \frac{1}{x+\sqrt{a^2-x^2}} \cdot dx$
Solution:

$$\text{Let } I=\int_0^a \frac{1}{x+\sqrt{a^2-x^2}}\, dx$$

Put $x = a\sin\theta \quad \therefore\ dx = a\cos\theta\, d\theta$

and $\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2\sin^2\theta} = \sqrt{a^2(1-\sin^2\theta)}$

$= \sqrt{a^2\cos^2\theta} = a\cos\theta$

When $x = 0$, $a\sin\theta = 0 \quad \therefore\ \theta = 0$

When $x = a$, $a\sin\theta = a \quad \therefore\ \theta = \frac{\pi}{2}$

$$\therefore\ I = \int_0^{\pi/2} \frac{a\cos\theta\, d\theta}{a\sin\theta + a\cos\theta}$$

$$\therefore\ I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta + \cos\theta}\, d\theta \qquad \ldots (1)$$

We use the property, $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

Hence in I, we change θ by $(\pi/2) - \theta$.

$$\therefore\ I = \int_0^{\pi/2} \frac{\cos\ [(\pi/2)-\theta]}{\sin\ [(\pi/2)-\theta] + \cos\ [(\pi/2)-\theta]}\, d\theta$$

$$= \int_0^{\pi/2} \frac{\sin\ \theta}{\cos\ \theta + \sin\ \theta}\, d\theta \qquad \ldots (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\pi/2} \frac{\cos\ \theta}{\sin\ \theta + \cos\ \theta} d\theta + \int_0^{\pi/2} \frac{\sin\ \theta}{\cos\ \theta + \sin\ \theta} d\theta$$

$$= \int_0^{\pi/2} \frac{\cos\ \theta + \sin\ \theta}{\cos\ \theta + \sin\ \theta} d\theta = \int_0^{\pi/2} 1\ d\theta = [\theta]_0^{\pi/2}$$

$$= (\pi/2) - 0 = \pi/2$$

$$\therefore\ I = \pi/4.$$

Question 2.

$\int_0^{\pi/2} \log \tan x \cdot dx$

Solution:

$$\text{Let } I = \int_0^{\pi/2} \log(\tan x) dx$$

We use the property, $\int_0^a f(x) dx = \int_0^a f(a-x) dx$.

Here, $a = \frac{\pi}{2}$. Hence changing x by $\frac{\pi}{2} - x$, we get

$$I = \int_0^{\pi/2} \log\left[\tan\left(\frac{\pi}{2} - x\right)\right] dx$$

$$= \int_0^{\pi/2} \log(\cot x)\,dx = \int_0^{\pi/2} \log\left(\frac{1}{\tan x}\right)dx$$

$$= \int_0^{\pi/2} \log(\tan x)^{-1}\,dx = \int_0^{\pi/2} -\log(\tan x)\,dx$$

$$= -\int_0^{\pi/2} \log(\tan x)\,dx = -I$$

$\therefore\ 2I = 0 \qquad \therefore\ I = 0.$

Question 3.

$\int_0^1 \log\left(\frac{1}{x} - 1\right) \cdot dx$

Solution:

$$\text{Let } I = \int_0^1 \log\left(\frac{1}{x} - 1\right)dx = \int_0^1 \log\left(\frac{1-x}{x}\right)dx$$

$$= \int_0^1 [\log(1-x) - \log x]\,dx \qquad \ldots (1)$$

We use the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Here, $a = 1$

Hence in I, changing x to $1-x$, we get

$$I = \int_0^1 [\log|1-(1-x)| - \log(1-x)]\,dx$$

$$= \int_0^1 [\log x - \log(1-x)]\,dx$$

$$= -\int_0^1 [\log(1-x) - \log x]\,dx = -I \qquad \ldots \text{[By (1)]}$$

$\therefore\ 2I = 0 \qquad \therefore\ I = 0.$

Question 4.
$\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cdot \cos x} \cdot dx$
Solution:

$$\text{Let } I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

We use the property, $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx.$

Here $a = \frac{\pi}{2}$. Hence In I, we change x by $\frac{\pi}{2} - x$.

$$\therefore\ I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)}$$

$$= \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$$

$$= -\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx = -I$$

$\therefore\ 2I = 0 \qquad \therefore\ I = 0.$

Question 5.
$\int_0^3 x^2 (3 - x)^{\frac{5}{2}} \cdot dx$
Solution:

$$\text{Let } I = \int^3 x^2 (3 - x)^{5/2} dx$$

We use the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Here, $a = 3$

Hence in I, changing x to $3-x$, we get

$$I = \int_0^3 (3-x)^2 [3-(3-x)]^{5/2} dx = \int_0^3 (9-6x+x^2)\, x^{5/2}\, dx$$

$$= \int_0^3 [9x^{5/2} - 6x^{7/2} + x^{9/2}]\, dx$$

$$= 9\int_0^3 x^{5/2} dx - 6\int_0^3 x^{7/2} dx + \int_0^3 x^{9/2} dx$$

$$= 9\left[\frac{x^{7/2}}{7/2}\right]_0^3 - 6\left[\frac{x^{9/2}}{9/2}\right]_0^3 + 9\left[\frac{x^{11/2}}{11/2}\right]_0^3$$

$$= 9\left[\frac{2\cdot 3^{7/2}}{7} - 0\right] - 6\left[\frac{2\cdot 3^{9/2}}{9} - 0\right] + \left[\frac{2}{11}\cdot 3^{11/2} - 0\right]$$

$$= \frac{18}{7} 3^{7/2} - \frac{2\cdot 6}{9}\cdot 3^{\frac{7}{2}}\cdot 3 + \frac{2}{11}\cdot 3^{\frac{7}{2}}\cdot 3^2$$

$$= 2(3)^{7/2}\left[\frac{9}{7} - 2 + \frac{9}{11}\right] = 2(3)^{\frac{7}{2}}\left[\frac{99-154+63}{77}\right]$$

$$= 2(3)^{7/2} \times \frac{8}{77} = \frac{16}{77}(3)^{7/2}$$

Question 6.

$\int_{-3}^{3} \frac{x^3}{9-x^2} \cdot dx$

Solution:

Let $I = \int_{-3}^{3} \frac{x^3}{9-x^2}\, dx$

Let $f(x) = \frac{x^3}{9-x^2}$

$\therefore f(-x) = \frac{(-x)^3}{9-(-x)^2} = \frac{-x^3}{9-x^2} = -f(x)$

$\therefore$ f is an odd function.

$\therefore \int_{-3}^{3} f(x)\,dx = 0$, i.e. $\int_{-3}^{3} \frac{x^3}{9-x^2}\,dx = 0.$

Question 7.

$\int_{-\pi/2}^{\pi/2} \log\left(\frac{2+\sin x}{2-\sin x}\right) \cdot dx$

Solution:

Let $I = \int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin x}{2+\sin x}\right) dx$

Let $f(x) = \log\left(\frac{2-\sin x}{2+\sin x}\right)$

$\therefore f(-x) = \log\left[\frac{2-\sin(-x)}{2+\sin(-x)}\right] = \log\left(\frac{2+\sin x}{2-\sin x}\right)$

$= -\log\left(\frac{2-\sin x}{2+\sin x}\right) = -f(x)$

$\therefore$ f is an odd function.

$\therefore \int_{-\pi/2}^{\pi/2} f(x)\,dx = 0$

$$\therefore \int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin x}{2+\sin x}\right) dx = 0.$$

Question 8.

$\int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x} \cdot dx$

Solution:

$$\text{Let } I = \int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$$

$$= \int_{-\pi/4}^{\pi/4} \left[\frac{x}{2-\cos 2x} + \frac{\pi/4}{2-\cos 2x}\right]$$

$$= \int_{-\pi/4}^{\pi/4} \frac{x}{2-\cos 2x} dx + \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{1}{2-\cos 2x} dx$$

$$= I_1 + \frac{\pi}{4} I_2 \qquad \ldots (1)$$

$$\text{Let } f(x) = \frac{x}{2-\cos 2x}$$

$$\therefore f(-x) = \frac{-x}{2-\cos[2(-x)]} = \frac{-x}{2-\cos 2x} = -f(x)$$

$\therefore$ f is an odd function

$$\therefore \int_{-\pi/4}^{\pi/4} f(x)\,dx = 0$$

$$\text{i.e. } \int_{-\pi/4}^{\pi/4} \frac{x}{2-\cos 2x} dx = 0, \text{ i.e. } I_1 = 0 \qquad \ldots (2)$$

In I_2, put $\tan x = t$

$\therefore\ x = \tan^{-1} t \qquad \therefore\ dx = \dfrac{1}{1+t^2}\,dt$

and $\cos 2x = \dfrac{1-t^2}{1+t^2}$

When $x = -\dfrac{\pi}{4}$, $t = \tan\left(-\dfrac{\pi}{4}\right) = -1$

When $x = \dfrac{\pi}{4}$, $t = \tan\dfrac{\pi}{4} = 1.$

$$\therefore\ I_2 = \int_{-1}^{1} \frac{1}{2-\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{1}{(1+t^2)}\,dt$$

$$= \int_{-1}^{1} \frac{1}{2(1+t^2)-(1-t^2)}\,dt$$

$$= \int_{-1}^{1} \frac{1}{3t^2+1}\,dt = \int_{-1}^{1} \frac{1}{(\sqrt{3}\ t)^2+1}$$

$$= \left[\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}t}{1}\right)\right]_{-1}^{1}$$

$$= \frac{1}{\sqrt{3}}[\tan^{-1}\sqrt{3} - \tan^{-1}(-\sqrt{3})]$$

$$= \frac{1}{\sqrt{3}}[\tan^{-1}\sqrt{3} + \tan^{-1}\sqrt{3}]$$

$$= \frac{1}{\sqrt{3}}\left[\frac{\pi}{3}+\frac{\pi}{3}\right] = \frac{2\pi}{3\sqrt{3}} \quad \ldots (3)$$

From (1), (2) and (3), we get

$$I = 0 + \frac{\pi}{4}\left[\frac{2\pi}{3\sqrt{3}}\right] = \frac{\pi^2}{6\sqrt{3}}.$$

Question 9.

$\int_{-\pi/4}^{\pi/4} x^3 \cdot \sin^4 x \cdot dx$

Solution:

Let I = $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^4 x \cdot dx$

Let f(x) = $x^3 \sin^4 x$

$\therefore$ f(−x) = $(-x)^3 \sin^4(-x)$

= $-x^3 \sin^4 x$

= $-f(x)$

$\therefore$ f is an odd function.

$\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} f(x) \cdot dx = 0$, i.e. $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^4 x \cdot dx$ = 0.

Question 10.

$\int_0^1 \frac{\log(x+1)}{x^2+1} \cdot dx$

Solution:

$$\text{Let } I=\int_0^1 \frac{\log\ (x+1)}{x^2+1}\,dx$$

Put $x=\tan\theta$. $\therefore\ dx=\sec^2\theta\,d\theta$ and

$x^2+1=\tan^2\theta+1=\sec^2\theta$

When $x=0$, $\tan\theta=0$ $\quad\therefore\ \theta=0$

When $x=1$, $\tan\theta=1$ $\quad\therefore\ \theta=\pi/4$

$$\therefore\ I=\int_0^{\pi/4} \frac{\log\ (\tan\ \theta+1)}{\sec^2\theta}\cdot\sec^2\theta\,d\theta$$

$$=\int_0^{\pi/4} \log\ (1+\tan\ \theta)\,d\theta \qquad \ldots (1)$$

We use the property, $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$.

Here, $a=\frac{\pi}{4}$. Hence changing θ by $\frac{\pi}{4}-\theta$, we have,

$$I=\int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$=\int_0^{\pi/4} \log\left(1+\frac{1-\tan\ \theta}{1+\tan\ \theta}\right)d\theta$$

$$= \int_0^{\pi/4} \log\left(\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right) d\theta$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$$

$$= \int_0^{\pi/4} [\log 2 - \log(1+\tan\theta)]\, d\theta$$

$$= \log 2 \int_0^{\pi/4} 1\, d\theta - \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$$

$$= (\log 2)\,[\theta]_0^{\pi/4} - I = \frac{\pi}{4}\log 2 - I$$

$$\therefore\ 2I = \frac{\pi}{4}\log 2 \qquad \therefore\ I = \frac{\pi}{8}\log 2.$$

Question 11.

$\int_{-1}^{1} \frac{x^3+2}{\sqrt{x^2+4}} \cdot dx$

Solution:

$$\text{Let } I = \int_{-1}^{1} \frac{x^3+2}{\sqrt{x^2+4}}\, dx$$

$$= \int_{-1}^{1} \left[\frac{x^3}{\sqrt{x^2+4}} + \frac{2}{\sqrt{x^2+4}}\right] dx$$

$$= \int_{-1}^{1} \frac{x^3}{\sqrt{x^2+4}}\,dx + 2\int \frac{1}{\sqrt{x^2+4}}\,dx$$

$$= I_1 + 2I_2 \qquad \ldots (1)$$

Let $f(x) = \dfrac{x^3}{\sqrt{x^2+4}}$

$\therefore f(-x) = \dfrac{(-x)^3}{\sqrt{(-x)^2+4}} = \dfrac{-x^3}{\sqrt{x^2+4}} = -f(x)$

$\therefore$ f is an odd function.

$$\therefore \int_{-1}^{1} f(x)\,dx = 0, \text{ i.e. } I_1 = \int_{-1}^{1} \frac{x^3}{\sqrt{x^2+4}}\,dx = 0 \qquad \ldots (2)$$

$\because (-x)^2 = x^2$

$\therefore \dfrac{1}{\sqrt{x^2+4}}$ is an even function.

$$\therefore \int_{-1}^{1} f(x)dx = 2\int_{0}^{1} f(x)dx$$

$$\therefore I_2 = 2\int_{0}^{1} \frac{1}{\sqrt{x^2+4}}\,dx$$

$$= 2\left[\log\left(x + \sqrt{x^2+4}\right)\right]_0^1$$

$$= 2\left[\log\left(1+\sqrt{1+4}\right) - \log\left(0+\sqrt{0+4}\right)\right]$$

$$= 2\left[\log\left(\sqrt{5}+1\right) - \log 2\right]$$

$$= 2\log\left(\frac{\sqrt{5}+1}{2}\right) \qquad \ldots (3)$$

From (1), (2) and (3), we get

$$I = 0 + 2\left[2 \log\left(\frac{\sqrt{5}+1}{2}\right)\right] = 4 \log\left(\frac{\sqrt{5}+1}{2}\right).$$

Question 12.

$\int_{-a}^{a} \frac{x+x^3}{16-x^2} \cdot dx$

Solution:

Let I = $\int_{-a}^{a} \frac{x+x^3}{16-x^2} \cdot dx$

Let f(x) = $\frac{x+x^3}{16-x^2}$

$\therefore$ f(−x) = $\frac{(-x)+(-x)^3}{16-(-x)^2}$

= $\frac{-(x+x^3)}{16-x^2}$

= $-f(x)$

$\therefore$ f is an odd function.

$\therefore \int_{-a}^{a} f(x) \cdot dx = 0$, i.e. $\int_{a}^{a} \frac{x+x^3}{16-x^2} \cdot dx$ = 0.

Question 13.

$\int_0^1 t^2 \sqrt{1-t} \cdot dt$

$\int_0^1 t^2 \sqrt{1-t} \cdot dt$

Solution:

We use the property,

$$\int_0^a f(t)\,dt = \int_0^a f(a-t)\,dt$$

$$\therefore \int_0^1 t^2\sqrt{1-t}\,dt = \int_0^1 (1-t)^2\sqrt{1-1+t}\,dt$$

$$= \int_0^1 (1-2t+t^2)\sqrt{t}\,dt$$

$$= \int_0^1 \left(t^{\frac{1}{2}} - 2t^{\frac{3}{2}} + t^{\frac{5}{2}}\right) dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{3/2} - 2\cdot\frac{t^{\frac{5}{2}}}{5/2} + \frac{t^{\frac{7}{2}}}{7/2}\right]_0^1$$

$$= \frac{2}{3}(1)^{\frac{3}{2}} - \frac{4}{5}(1)^{\frac{5}{2}} + \frac{2}{7}(1)^{\frac{7}{2}} - 0$$

$$= \frac{2}{3} - \frac{4}{5} + \frac{2}{7} = \frac{70-84+30}{105} = \frac{16}{105}.$$

Question 14.

$\int_0^\pi x \cdot \sin x \cdot \cos^2 x \cdot dx$

Solution:

Let $I = \int_0^\pi x \sin x \cos^2 x\,dx$

$$=\frac{1}{2}\int_0^\pi x(2\sin x\cos x)\cos x\,dx$$

$$=\frac{1}{2}\int_0^\pi x(\sin 2x\cos x)\,dx$$

$$=\frac{1}{4}\int_0^\pi x(2\sin 2x\cos x)\,dx$$

$$=\frac{1}{4}\int_0^\pi x\,[\sin(2x+x)+\sin(2x-x)]\,dx$$

$$=\frac{1}{4}\left[\int_0^\pi x\sin 3x\,dx+\int_0^\pi x\sin x\,dx\right]$$

$$=\frac{1}{4}[I_1+I_2] \quad \ldots (1)$$

$$I_1=\int_0^\pi x\sin 3x\,dx$$

$$=[x\int\sin 3x\,dx]_0^\pi-\int\left[\left\{\frac{d}{dx}(x)\int\sin 3x\,dx\right\}\right]dx$$

$$=\left[x\left(\frac{-\cos 3x}{3}\right)\right]_0^\pi-\int_0^\pi 1\left(\frac{-\cos 3x}{3}\right)dx$$

$$=\left[-\frac{\pi\cos 3\pi}{3}+0\right]+\frac{1}{3}\int_0^\pi\cos 3x\,dx$$

$$=-\frac{\pi}{3}(-1)+\frac{1}{3}\left[\frac{\sin 3x}{3}\right]_0^\pi$$

$$=\frac{\pi}{3}+\frac{1}{3}\left[\frac{\sin 3\pi}{3}-\frac{\sin 0}{3}\right]$$

$$= \frac{\pi}{3} + \frac{1}{3}[0 - 0] = \frac{\pi}{3} \quad \ldots (2)$$

$$I_2 = \int_0^{\pi} x \sin x \, dx$$

$$= [x \int \sin x \, dx]_0^{\pi} - \int_0^{\pi} \left[\left\{\frac{d}{dx}(x) \int \sin x \, dx\right\}\right] dx$$

$$= [x(-\cos x)]_0^{\pi} - \int_0^{\pi} 1 \cdot (-\cos x) \, dx$$

$$= [-\pi \cos\pi + 0] + \int_0^{\pi} \cos x \, dx$$

$$= -\pi(-1) + [\sin x]_0^{\pi}$$

$$= \pi + [\sin \pi - \sin 0] = \pi + (0 - 0) = \pi \quad \ldots (3)$$

From (1), (2) and (3), we get

$$I = \frac{1}{4}\left[\frac{\pi}{3} + \pi\right] = \frac{1}{4}\left(\frac{4\pi}{3}\right) = \frac{\pi}{3}.$$

Question 15.

$\int_0^1 \frac{\log x}{\sqrt{1-x^2}} \cdot dx$

Solution:

$$\text{Let } I = \int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx$$

Put $x=\sin\theta \quad \therefore\ dx=\cos\theta\, d\theta$ and

$\sqrt{1-x^2}=\sqrt{1-\sin^2\theta}=\sqrt{\cos^2\theta}=\cos\theta$

When $x=0$, $\sin\theta=0 \quad \therefore\ \theta=0$

When $x=1$, $\sin\theta=1 \quad \therefore\ \theta=\pi/2$

$$\therefore\ I=\int_0^{\pi/2}\frac{\log\sin\theta}{\cos\theta}\cdot\cos\theta\, d\theta$$

$$=\int_0^{\pi/2}\log\sin\theta\, d\theta$$

Using the property, $\int_0^{2a} f(x)\,dx=\int_0^{a}[f(x)+f(2a-x)]\,dx$,

we get,

$$I=\int_0^{\pi/4}\left[\log\sin\theta+\log\sin\left(\frac{\pi}{2}-\theta\right)\right]d\theta$$

$$=\int_0^{\pi/4}(\log\sin\theta+\log\cos\theta)\,d\theta$$

$$=\int_0^{\pi/4}\log\sin\theta\cos\theta\, d\theta$$

$$=\int_0^{\pi/4}\log\left(\frac{2\sin\theta\cos\theta}{2}\right)d\theta$$

$$=\int_0^{\pi/4}(\log\sin 2\theta-\log 2)\,d\theta$$

$$= \int_0^{\pi/4} \log \sin 2\theta \, d\theta - \int_0^{\pi/4} \log 2 \, d\theta$$

$$= I_1 - I_2 \quad \text{....... (Say)}$$

$$I_2 = \int_0^{\pi/4} \log 2 \, d\theta = \log 2 \int_0^{\pi/4} 1 \, d\theta$$

$$= \log 2 \, [\theta]_0^{\pi/4} = (\log 2)\left[\frac{\pi}{4} - 0\right]$$

$$= \frac{\pi}{4} \log 2$$

$$I_1 = \int_0^{\pi/4} \log \sin 2\theta \, d\theta$$

Put $2\theta = t$. Then $d\theta = \frac{dt}{2}$

When $\theta = 0$, $t = 0$

When $\theta = \pi/4$, $t = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$

$$\therefore I_1 = \int_0^{\pi/2} \log \sin t \times \frac{dt}{2}$$

$$= \frac{1}{2} \int_0^{\pi/2} \log \sin\theta \, d\theta = \frac{1}{2} I \quad \ldots \left[\because \int_a^b f(x)\,dx = \int_a^b f(t)\,dt\right]$$

$$\therefore I = \frac{1}{2} I - \frac{\pi}{4} \log 2$$

$$\therefore I_1 = \int_0^{\pi/2} \log \sin t \times \frac{dt}{2}$$

$$= \frac{1}{2} \int_0^{\pi/2} \log \sin\theta \, d\theta = \frac{1}{2} I \quad \ldots \left[\because \int_a^b f(x)\,dx = \int_a^b f(t)\,dt\right]$$

$$\therefore I = \frac{1}{2} I - \frac{\pi}{4} \log 2$$

$$\therefore \frac{1}{2} I = -\frac{\pi}{4} \log 2$$

$$\therefore I = -\frac{\pi}{2} \log 2 = \frac{\pi}{2} \log \left(\frac{1}{2}\right).$$

Maharashtra Board Solutions Class 12 Arts & Science Maths (Part 2)

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The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

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