Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 6- Line and Plane

Class 12 - Chapter 6 Line and Plane

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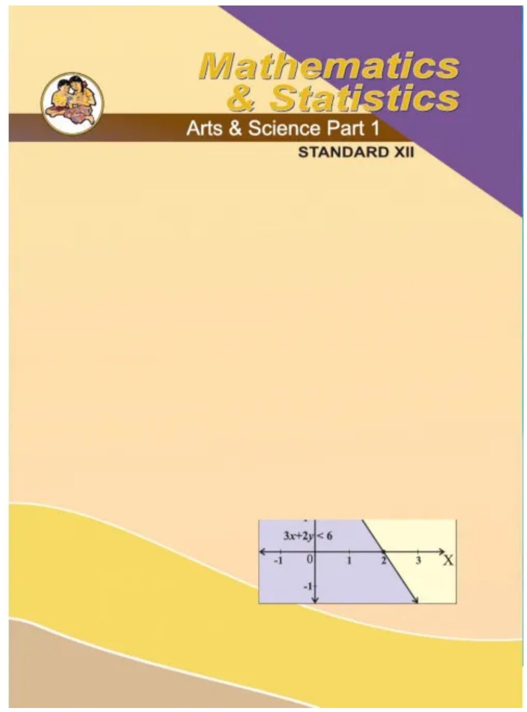

For any clarifications or questions you can write to **info@indcareer.com**

Postal Address

IndCareer.com, 52, Shilpa Nagar, Somalwada Nagpur - 440015
Maharashtra, India

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Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 6- Line and Plane

Class 12: Maths Chapter 6 solutions. Complete Class 12 Maths Chapter 6 Notes.

Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 6- Line and Plane

Maharashtra Board 12th Maths Chapter 6, Class 12 Maths Chapter 6 solutions

Ex 6.1

Question 1.
Find the vector equation of the line passing through the point having position vector $-2\hat{i} + \hat{j} + \hat{k}$ and parallel to vector $4\hat{i} - \hat{j} + 2\hat{k}$.
Solution:
The vector equation of the line passing through A ($\bar{a}$) and parallel to the vector $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.
∴ the vector equation of the line passing through the point having position vector $-2\hat{i} + \hat{j} + \hat{k}$ and parallel to the vector $4\hat{i} - \hat{j} + 2\hat{k}$ is
$\bar{r} = (-2\hat{i} + \hat{j} + \hat{k}) + \lambda(4\hat{i} - \hat{j} + 2\hat{k})$.

Question 2.
Find the vector equation of the line passing through points having position vectors $3\hat{i} + 4\hat{j} - 7\hat{k}$ and $6\hat{i} - \hat{j} + \hat{k}$.
Solution:
The vector equation of the line passing through the A ($\bar{a}$) and B($\bar{b}$) is $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$, λ is a scalar
∴ the vector equation of the line passing through the points having position vectors $3\hat{i} + 4\hat{j} - 7\hat{k}$ and $6\hat{i} - \hat{j} + \hat{k}$ is
is $\bar{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda[(6\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} + 4\hat{j} - 7\hat{k})]$
i.e. $\bar{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(3\hat{i} - 5\hat{j} + 8\hat{k})$.

Question 3.
Find the vector equation of line passing through the point having position vector $5\hat{i} + 4\hat{j} + 3\hat{k}$ and having direction ratios -3, 4, 2.
Solution:
Let A be the point whose position vector is $\bar{a} = 5\hat{i} + 4\hat{j} + 3\hat{k}$.
Let $\bar{b}$ be the vector parallel to the line having direction ratios -3, 4, 2
Then, $\bar{b} = -3\hat{i} + 4\hat{j} + 2\hat{k}$
The vector equation of the line passing through A ($\bar{a}$) and parallel to $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.
∴ the required vector equation of the line is
$\bar{r} = 5\hat{i} + 4\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 4\hat{j} + 2\hat{k})$.

Question 4.
Find the vector equation of the line passing through the point having position vector $\hat{i} + 2\hat{j} + 3\hat{k}$ and perpendicular to vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$.
Solution:

Let $\bar{b} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{c} = 2\hat{i} - \hat{j} + \hat{k}$

The vector perpendicular to the vectors $\bar{b}$ and $\bar{c}$ is given by

$$\bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(1+1) - \hat{j}(1-2) + \hat{k}(-1-2)$$
$$= 2\hat{i} + \hat{j} - 3\hat{k}$$

Since the line is perpendicular to the vector $\bar{b}$ and $\bar{c}$, it is parallel to $\bar{b} \times \bar{c}$. The vector equation of the line passing through A ($\bar{a}$) and parallel to $\bar{b} \times \bar{c}$ is
$\bar{r} = \bar{a} + \lambda(\bar{b} \times \bar{c})$, where λ is a scalar.
Here, $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$
Hence, the vector equation of the required line is
$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$

Question 5.
Find the vector equation of the line passing through the point having position vector $-\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the line $\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$.
Solution:

Let A be point having position vector $\bar{a} = -\hat{i} - \hat{j} + 2\hat{k}$
The required line is parallel to the line

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$$

$\therefore$ it is parallel to the vector

$$\bar{b} = 3\hat{i} + 2\hat{j} + \hat{k}$$

The vector equation of the line passing through A($\bar{a}$) and parallel to $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$ where λ is a scalar.
∴ the required vector equation of the line is

$$\bar{r} = (-\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + \hat{k}).$$

Question 6.
Find the Cartesian equations of the line passing through A(-1, 2, 1) and having direction ratios 2, 3, 1.
Solution:
The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

∴ the cartesian equations of the line passing through the point (-1, 2, 1) and having direction ratios 2, 3, 1 are

$$\frac{x - (-1)}{2} = \frac{y - 2}{3} = \frac{z - 1}{1}$$

i.e. $\frac{x+1}{2}=\frac{y-2}{3}=\frac{z-1}{1}$.

Question 7.
Find the Cartesian equations of the line passing through A(2, 2, 1) and B(1, 3, 0).
Solution:
The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Here, $(x_1, y_1, z_1) = (2, 2, 1)$ and $(x_2, y_2, z_2) = (1, 3, 0)$
∴ the required cartesian equations are

$$\frac{x-2}{1-2}=\frac{y-2}{3-2}=\frac{z-1}{0-1}$$

i.e. $\frac{x-2}{-1}=\frac{y-2}{1}=\frac{z-1}{-1}$.

Question 8.
A(-2, 3, 4), B(1, 1, 2) and C(4, -1, 0) are three points. Find the Cartesian equations of the line AB and show that points A, B, C are collinear.
Solution:
We find the cartesian equations of the line AB. The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Here, $(x_1, y_1, z_1) = (-2, 3, 4)$ and $(x_2, y_2, z_2) = (4, -1, 0)$

Here, $(x_1, y_1, z_1) = (-2, 3, 4)$ and $(x_2, y_2, z_2) = (4, -1, 0)$
∴ the required cartesian equations of the line AB are

$$\frac{x-(-2)}{4-(-2)}=\frac{y-3}{-1-3}=\frac{z-4}{0-4}$$

$$\therefore \frac{x+2}{6}=\frac{y-3}{-4}=\frac{z-4}{-4}$$

$$\therefore \frac{x+2}{3}=\frac{y-3}{-2}=\frac{z-4}{-2}$$

$C=(4, -1, 0)$

For $x=4, \frac{x+2}{3}=\frac{4+2}{3}=2$

For $y=-1, \frac{y-3}{-2}=\frac{-1-3}{-2}=2$

For $z=0, \frac{z-4}{-2}=\frac{0-4}{-2}=2$

∴ coordinates of C satisfy the equations of the line AB.
∴ C lies on the line passing through A and B.
Hence, A, B, C are collinear.

Question 9.
Show that lines $\frac{x+1}{-10}=\frac{y+3}{-1}=\frac{z-4}{1}$ and $\frac{x+10}{-1}=\frac{y+1}{-3}=\frac{z-1}{4}$ intersect each other. Find the co-ordinates of their point of intersection.
Solution:
The equations of the lines are

$$\frac{x+1}{-10}=\frac{y+3}{-1}=\frac{z-4}{1}=\lambda \quad \text{... (Say)} \quad \text{... (1)}$$

and $\frac{x+10}{-1}=\frac{y+1}{-3}=\frac{z-1}{4}=\mu$ ... (Say) ... (2)

From (1), x = -1 -10λ, y = -3 – 2, z = 4 + λ
∴ the coordinates of any point on the line (1) are
(-1 – 10λ, – 3 – λ, 4 + λ)
From (2), x = -10 – u, y = -1 – 3u, z = 1 + 4u
∴ the coordinates of any point on the line (2) are
(-10 – u, -1 – 3u, 1 + 4u)
Lines (1) and (2) intersect, if
(- 1 – 10λ, – 3 – λ, 4 + 2) = (- 10 – u, -1 – 3u, 1 + 4u)
∴ the equations -1 – 10λ = -10 – u, -3 – 2= – 1 – 3u
and 4 + λ = 1 + 4u are simultaneously true.
Solving the first two equations, we get, λ = 1 and u = 1. These values of λ and u satisfy the third equation also.
∴ the lines intersect.
Putting λ = 1 in (-1 – 10λ, -3 – 2, 4 + 2) or u = 1 in (-10 – u, -1 – 3u, 1 + 4u), we get
the point of intersection (-11, -4, 5).

Question 10.
A line passes through (3, -1, 2) and is perpendicular to lines
$\bar{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\bar{r}=(2\hat{i}+\hat{j}-3\hat{k})+\mu(\hat{i}-2\hat{j}+2\hat{k})$. Find its equation.
Solution:
The line $\bar{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ is

parallel to the vector $\bar{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and the line

$\bar{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$ is parallel to the vector

$\bar{c} = \hat{i} - 2\hat{j} + 2\hat{k}$.

The vector perpendicular to the vectors $\bar{b}$ and $\bar{c}$ is given by

$$\bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-4+2) - \hat{j}(4-1) + \hat{k}(-4+2)$$

$$= -2\hat{i} - 3\hat{j} - 2\hat{k}$$

Since the required line is perpendicular to the given lines, it is perpendicular to both $\bar{b}$ and $\bar{c}$.
$\therefore$ it is parallel to $\bar{b} \times \bar{c}$
The equation of the line passing through A($\bar{a}$) and parallel to $\bar{b} \times \bar{c}$ is
$\bar{r} = \bar{a} + \lambda(\bar{b} \times \bar{c})$, where λ is a scalar.
Here, $\bar{a} = 3\hat{i} - \hat{j} + 2\hat{k}$
$\therefore$ the equation of the required line is

$\bar{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-2\hat{i} - 3\hat{j} - 2\hat{k})$ or

$\bar{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 2\hat{k})$, where $\mu = -\lambda$.

Question 11.
Show that the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+4}{-2}$ passes through the origin.
Solution:
The equation of the line is

$\frac{x-2}{1}=\frac{y-4}{2}=\frac{z+4}{-2}$

The coordinates of the origin O are (0, 0, 0)

For $x=0, \frac{x-2}{1}=\frac{0-2}{1}=-2$

For $y=0, \frac{y-4}{2}=\frac{0-4}{2}=-2$

For $z=0, \frac{z+4}{-2}=\frac{0+4}{-2}=-2$

∴ coordinates of the origin O satisfy the equation of the line.

Hence, the line passes through the origin.

Ex 6.2

Question 1.
Find the length of the perpendicular from (2, -3, 1) to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z+1}{-1}$
Solution:
Let PM be the perpendicular drawn from the point P (2, -3, 1) to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z+1}{-1}$ = λ …(Say)
The coordinates of any point on the line are given by x = -1 + 2λ, y = 3 + 3λ, z = -1 – λ
Let the coordinates of M be
(-1 + 2λ, 3 + 3λ, -1 – λ) … (1)
The direction ratios of PM are
-1 + 2λ – 2, 3 + 3λ + 3, -1 – λ – 1
i.e. 2λ – 3, 3λ + 6, -λ – 2
The direction ratios of the given line are 2, 3, -1.
Since PM is perpendicular to the given line, we get
2(2λ – 3) + 3(3λ + 6) – 1(-λ – 2) = 0
∴ 4λ – 6 + 9λ + 18 + λ + 2 = 0
∴ 14λ + 14 = 0 ∴ λ = -1.
Put λ = -1 in (1), the coordinats of M are
(-1 – 2, 3 – 3, -1 + 1) i.e. (-3, 0,0).
∴ length of perpendicular from P to the given line

$$= \text{PM} = \sqrt{(-3-2)^2+(0+3)^2+(0-1)^2}$$
$$= \sqrt{25+9+1}$$
$$= \sqrt{35} \text{ units.}$$

Alternative Method:
We know that the perpendicular distance from the point P$|\bar{\alpha}|$ to the line $\bar{r}=\bar{a}+\lambda\vec{b}$ is given by

$$\sqrt{|\bar{\alpha}-\bar{a}|^2-\left[\frac{(\bar{\alpha}-\bar{a})\cdot\bar{b}}{|\bar{b}|}\right]^2} \quad \ldots (1)$$

Here, $\bar{\alpha}=2\hat{i}-3\hat{j}+\hat{k}$, $\bar{a}=-\hat{i}+3\hat{j}-\hat{k}$, $\bar{b}=2\hat{i}+3\hat{j}-\hat{k}$

$\therefore\ \bar{\alpha}-\bar{a}=(2\hat{i}-3\hat{j}+\hat{k})-(-\hat{i}+3\hat{j}-\hat{k})$

$=3\hat{i}-6\hat{j}+2\hat{k}$

$\therefore\ |\bar{\alpha}-\bar{a}|^2=3^2+(-6)^2+2^2=9+36+4=49$

Also, $(\bar{\alpha}-\bar{a})\cdot\bar{b}=(3\hat{i}-6\hat{j}+2\hat{k})\cdot(2\hat{i}+3\hat{j}-\hat{k})$

$=(3)(2)+(-6)(3)+(2)(-1)$

$=6-18-2=-14$

$|\bar{b}|=\sqrt{2^2+3^2+(-1)^2}=\sqrt{14}$

Substituting these values in (1), we get
length of perpendicular from P to given line

$=\text{PM}=\sqrt{49-\left(\frac{-14}{\sqrt{14}}\right)^2}$

$=\sqrt{49-14}=\sqrt{35}$ units.

Question 2.
Find the co-ordinates of the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\bar{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.
Solution:
Let M be the foot of perpendicular drawn from the point P $(2\hat{i} - \hat{j} + 5\hat{k})$ on the line
$\bar{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$.
Let the position vector of the point M be

$$(11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$$
$$= (11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k}.$$

Then $\overline{PM}$ = Position vector of M – Position vector of P
= $[(11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + -8 - 11\lambda)\hat{k}] - (2\hat{i} - \hat{j} + 5\hat{k})$
= $(9 + 10\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (-13 - 11\lambda)\hat{k}$
Since PM is perpendicular to the given line which is parallel to $\bar{b} = 10\hat{i} - 4\hat{j} - 11\hat{k}$,
$\overline{PM} \perp^r \bar{b} \therefore \overline{PM} \cdot \bar{b} = 0$
$\therefore [(9 + 10\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (-13 - 11\lambda)\hat{k}] \cdot (10\hat{i} - 4\hat{j} - 11\hat{k}) = 0$
$\therefore 10(9 + 10\lambda) - 4(-1 - 4\lambda) - 11(-13 - 11\lambda) = 0$
$\therefore 90 + 100\lambda + 4 + 16\lambda + 143 + 121\lambda = 0$
$\therefore 237\lambda + 237 = 0$
$\therefore \lambda = -1$
Putting this value of λ, we get the position vector of M as $\hat{i} + 2\hat{j} + 3\hat{k}$.
∴ coordinates of the foot of perpendicular M are (1, 2, 3).

$$\text{Now, } \overline{PM} = (\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} - \hat{j} + 5\hat{k})$$
$$= -\hat{i} + 3\hat{j} - 2\hat{k}$$
$$\therefore |\overline{PM}| = \sqrt{(-1)^2 + (3)^2 + (-2)^2}$$

$$\therefore\ |\overline{PM}| = \sqrt{(-1)^2 + (3)^2 + (-2)^2}$$
$$= \sqrt{1+9+4} = \sqrt{14}$$

Hence, the coordinates of the foot of perpendicular are (1,2, 3) and length of perpendicular = $\sqrt{14}$ units.

Question 3.

Find the shortest distance between the lines $\bar{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$

Solution:

We know that the shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\bar{r} = \overline{a_2} + \mu\overline{b_2}$ is given by

$$d = \left|\frac{(\overline{a_2} - \overline{a_1})\cdot(\overline{b_1} \times \overline{b_2})}{|\overline{b_1} \times \overline{b_2}|}\right|.$$

Here, $\overline{a_1} = 4\hat{i} - \hat{j}$, $\overline{a_2} = \hat{i} - \hat{j} + 2\hat{k}$,

$\overline{b_1} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\overline{b_2} = \hat{i} + 4\hat{j} - 5\hat{k}$.

$$\therefore\ \overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}$$
$$= (-10+12)\hat{i} - (-5+3)\hat{j} + (4-2)\hat{k}$$
$$= 2\hat{i} + 2\hat{j} + 2\hat{k}$$

and $\overline{a_2} - \overline{a_1} = (\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} - \hat{j})$
$$= -3\hat{i} + 2\hat{k}$$

$\therefore\ (\overline{a_2}-\overline{a_1})\cdot(\overline{b_1}\times\overline{b_2}) = (-3\hat{i}+2\hat{k})\cdot(2\hat{i}+2\hat{j}+2\hat{k})$

$$= -3(2)+0(2)+2(2)$$

$$= -6+0+4 = -2$$

and $|\overline{b_1}\times\overline{b_2}| = \sqrt{2^2+2^2+2^2}$

$$= \sqrt{4+4+4} = 2\sqrt{3}$$

$\therefore$ required shortest distance between the given lines

$$= \left|\frac{-2}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}} \text{ units.}$$

Question 4.

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Solution:

The shortest distance between the lines

$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ is

given by

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2+(l_2n_1-l_1n_2)^2+(l_1m_2-l_2m_1)^2}}$$

The equations of the given lines are

$\frac{x+1}{} = \frac{y+1}{} = \frac{z+1}{}$ and $\frac{x-3}{} = \frac{y-5}{} = \frac{z-7}{}$

$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

$\therefore x_1=-1, y_1=-1, z_1=-1, x_2=3, y_2=5, z_2=7,$

$l_1=7, m_1=-6, n_1=1, l_2=1, m_2=-2, n_2=1$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

= 4(-6 + 2) – 6(7 – 1) + 8(-14 + 6)
= -16 – 36 – 64 = -116
and $(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2$
$= (-6 + 2)^2 + (1 - 7)^2 + (-14 + 6)^2$
= 16 + 36 + 64 = 116
Hence, the required shortest distance between the given lines = $\left|\frac{-116}{\sqrt{116}}\right| = \sqrt{116} = 2\sqrt{29}$ units

Question 5.
Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ Also find the co-ordinates of the foot of the perpendicular.
Solution:
Let PM be the perpendicular drawn from the point (1, 0, 0) to the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ = λ …(Say)
The coordinates of any point on the line are given by x = -1 + 2λ, y = 3 + 2λ, z = 8 – λ
Let the coordinates of M be

(-1 + 2λ, 3 + 3λ, -1 – λ) …..(1)
The direction ratios of PM are
-1 + 2λ – 2, 3 + 3λ + 3, -1 – λ – 1
i.e. 2λ – 3, 3λ = 6, -λ – 2
The direction ratios of the given line are 2, 3, 8.
Since PM is perpendicular to the given line, we get
2(2λ – 3) + 3(3λ + 6) – 1(-λ – 2) = O
∴ 4λ – 6 + 9λ + 18 + λ + 2 = 0
∴ 14λ + 14 = 0
∴ λ = -1
Put λ in (1), the coordinates of M are
(-1 – 2, 3 – 3, -1 + 1) i.e. (-3, 0, 0).
∴ length of perpendicular from P to the given line
= PM
= $\sqrt{(-3-2)^2+(0+3)^2+(0-1)^2}$
= $\sqrt{(25+9+1)}$
= $\sqrt{35}$units.
Alternative Method :
We know that the perpendicular distance from the point

$$\sqrt{\left|\overline{\infty}-\bar{a}\right|^2-\left[\frac{\left(\overline{oo}-\bar{a}\right).\bar{b}}{\left|\bar{b}\right|}\right]^2} \quad ...(1)$$

Here, $\overline{\infty}=2\hat{i}-3\hat{j}+\hat{k}, \bar{a}=-\hat{i}+3\hat{j}-\hat{k}, \bar{b}=2\hat{i}+3\hat{j}-\hat{k}$
$\therefore \overline{\infty}-\bar{a}=\left(2\hat{i}-3\hat{j}+\hat{k}\right)-\left(-\hat{i}+3\hat{j}-\hat{k}\right)$

$= 3\hat{i} - 6\hat{j} + 2\hat{k}$

$\therefore \left|\overline{\infty} - \bar{a}\right|^2$ = 3^2 + (– 6)2 + 2^2 = 9 + 36 + 4 = 49

Also, $\left(\overline{\infty} - \bar{a}\right).\bar{b} = \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right).\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

= (3)(2) + (– 6)(3) + (2)(– 1)

= 6 – 18 – 2

= –14

$\left|\bar{b}\right| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$

Substitutng tese values in (1), w get
length of perpendicular from P to given line
= PM

$= \sqrt{49 - \left(-\frac{14}{\sqrt{14}}\right)^2}$

$= \sqrt{49 - 14}$

$= \sqrt{35}$units

or

$2\sqrt{6}$units, $(3, -4, 2)$.

Question 6.
A(1, 0, 4), B(0, -11, 13), C(2, -3, 1) are three points and D is the foot of the perpendicular from A to BC. Find the co-ordinates of D.
Solution:
Equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

∴ the equation of the line BC passing through the points B (0, – 11, 13) and C (2, – 3, 1) is

$$\frac{x-0}{2-0}=\frac{y+11}{-3+11}=\frac{z-13}{1-13}$$

i.e. $\frac{x}{2}=\frac{y+11}{8}=\frac{z-13}{-12}=\lambda$ … (Say)

AD is the perpendicular from the point A (1, 0, 4) to the line BC.

The coordinates of any point on the line BC are given by x = 2λ, y = -11 + 8λ, z = 13 – 12λ

Let the coordinates of D be (2λ, -11 + 8λ, 13 – 12λ) … (1)

∴ the direction ratios of AD are

2λ – 1, -1λ + 8λ – 0, 13 – 12λ – 4 i.e.

2λ – 1, -11 + 8λ, 9 – 12λ

The direction ratios of the line BC are 2, 8, -12.

Since AD is perpendicular to BC, we get

2(2λ – 1) + 8(-11 + 8λ) – 12(9 – 12λ) = 0

∴ 42λ – 2 – 88 + 64λ – 108 + 144λ = 0

∴ 212λ – 198 = 0

$$\therefore\ \lambda=\frac{198}{212}=\frac{99}{106}$$

Putting $\lambda=\frac{99}{106}$ in (1), the coordinates of D are

$$\left(\frac{198}{106},\ -11+\frac{792}{106},\ 13-\frac{1188}{106}\right)$$

$$\left(\frac{198}{106}, -11+\frac{792}{106}, 13-\frac{1188}{106}\right)$$

i.e. $\left(\frac{198}{106}, \frac{-374}{106}, \frac{190}{106}\right)$, i.e. $\left(\frac{99}{53}, \frac{-187}{53}, \frac{95}{53}\right)$.

Question 7.
By computing the shortest distance, determine whether following lines intersect each other.
(i) $\bar{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\bar{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$
Solution:
The shortest distance between the lines

$\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\bar{r} = \overline{a_2} + \mu\overline{b_2}$ is given by

$$d = \left|\frac{(\overline{a_2} - \overline{a_1})\cdot(\overline{b_1} \times \overline{b_2})}{|\overline{b_1} \times \overline{b_2}|}\right|.$$

Here, $\overline{a_1} = \hat{i} - \hat{j}$, $\overline{a_2} = 2\hat{i} - \hat{j}$, $\overline{b_1} = 2\hat{i} + \hat{k}$, $\overline{b_2} = \hat{i} + \hat{j} - \hat{k}$.

$$\therefore\ \overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= (0-1)\hat{i} - (-2-1)\hat{j} + (2-0)\hat{k}$$

$$= -\hat{i} + 3\hat{j} + 2\hat{k}$$

and $\overline{a_2} - \overline{a_1} = (2\hat{i} - \hat{j}) - (\hat{i} - \hat{j}) = \hat{i}$

$$\therefore\ (\overline{a_2} - \overline{a_1})\cdot(\overline{b_1} \times \overline{b_2}) = \hat{i}\cdot(-\hat{i} + 3\hat{j} + 2\hat{k})$$

$$= 1(-1) + 0(3) + 0(2) = -1$$

and $|\overline{b}_1 \times \overline{b}_2| = \sqrt{(-1)^2 + 3^2 + 2^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

$\therefore$ the shortest distance between the given lines

$= \left|\frac{-1}{\sqrt{14}}\right| = \frac{1}{\sqrt{14}}$ unit

Hence, the given lines do not intersect.

(ii) $\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$

Solution:

The shortest distance between the lines

$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$ is give n by

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2 + (l_2n_1 - 1_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}$$

The equation of the given lines are

$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$

$\therefore x_1 = -1, y_1 = -1, z_1 = -1, x_2 = 3, y_2 = 5, z_2 = 7,$

$l_1 = 7, m_1 = -6, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

= 4(- 6 + 2) – 6(7 – 1) + 8(-14 + 6)

= -16 – 36 – 64

= -116

and

$(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2$

$= (-6 + 2)^2 + (1 - 7)^2 + (-14 + 6)^2$

= 16 + 36 + 64

= 116

Hence, the required shortest distance between the given lines

$= \left|\frac{-116}{\sqrt{116}}\right|$

$= \sqrt{116}$

$=2\sqrt{29}$ units

or

The shortest distance between the lines

$= \frac{282}{\sqrt{3830}}$ units

Hence, the gives lines do not intersect.

Question 8.

If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other then find k.

Solution:

The lines

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$$

intersect, if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

The equations of the given lines are

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4} \text{ and } \frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$$

$\therefore x_1 = 1, y_1 = -1, z_1 = 1, x_2 = 3, y_2 = k, z_2 = 0,$
$l_1 = 2, m_1 = 3, n_1 = 4, l_2 = 1, m_2 = 2, n_2 = 1.$
Since these lines intersect, we get

$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\therefore 2(3-8)-(k+1)(2-4)-1(4-3)=0$
$\therefore -10+2(k+1)-1=0$
$\therefore 2(k+1)=11$
$\therefore k+1=\frac{11}{2}$
$\therefore k=\frac{9}{2}$

Ex 6.3

Question 1.
In each of the following examples verify that the given expression is a solution of the corresponding differential equation.
(i) xy = log y + c; $\frac{dy}{dx} = \frac{y^2}{1-xy}$
Solution:
xy = log y + c
Differentiating w.r.t. x, we get

$$x \cdot \frac{dy}{dx} + y \times 1 = \frac{1}{y} \cdot \frac{dy}{dx} + 0$$

$$\therefore x\frac{dy}{dx} + y = \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\left(x - \frac{1}{y}\right)\frac{dy}{dx} = -y$$

$$\therefore \left(\frac{xy-1}{y}\right)\frac{dy}{dx} = -y$$

$$\therefore \frac{dy}{dx} = \frac{-y^2}{xy-1} = \frac{y^2}{1-xy}, \text{ if } xy \neq 1$$

Hence, xy = log y + c is a solution of the D.E.
$\frac{dy}{dx} = \frac{y^2}{1-xy'}, xy \neq 1$

(ii) y = $(\sin^{-1}x)^2$ + c; $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

Solution:

y = $(\sin^{-1} x)^2$ + c ....(1)

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}x)^2 + 0$$

$$\therefore \frac{dy}{dx} = 2(\sin^{-1}x)\cdot\frac{d}{dx}(\sin^{-1}x)$$

$$= 2\sin^{-1}x \times \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1}x$$

$$\therefore (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4(\sin^{-1}x)^2$$

$$\therefore (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4(y-c) \quad \text{....... [By (1)]}$$

Differentiating again w.r.t. x, we get

$$(1-x^2)\cdot\frac{d}{dx}\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2\cdot\frac{d}{dx}(1-x^2) = 4\frac{d}{dx}(y-c)$$

$$\therefore (1-x^2)\cdot 2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = 4\left(\frac{dy}{dx} - 0\right)$$

Cancelling $2\frac{dy}{dx}$ throughout, we get

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2.$$

Hence, $y=(\sin^{-1}x)^2+c$ is a solution of the D.E.

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2.$$

(iii) $y = e^{-x} + Ax + B$; $e^x\frac{d^2y}{dx^2}=1$

Solution:

$y = e^{-x} + Ax + B$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=e^{-x}\times(-1)+A\times 1+0$$

$$\therefore\ \frac{dy}{dx}=-e^{-x}+A$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2}=-e^{-x}\times(-1)+0$$

$$\therefore\ \frac{d^2y}{dx^2}=\frac{1}{e^x}$$

$$\therefore e^x\frac{d^2y}{dx^2}=1$$

Hence, $y = e^{-x} + Ax + B$ is a solution of the D.E.

$$e^x\frac{d^2y}{dx^2}=1$$

(iv) y = x^m; $x^2\frac{d^2y}{dx^2} - mx\frac{dy}{dx} + my = 0$

Solution:

y = x^m

Differentiating twice w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^m) = mx^{m-1}$$

$$\text{and } \frac{d^2y}{dx^2} = \frac{d}{dx}(mx^{m-1}) = m\frac{d}{dx}(x^{m-1}) = m(m-1)x^{m-2}$$

$$\therefore\ x^2\frac{d^2y}{dx^2} - mx\frac{dy}{dx} + my$$

$$= x^2 \cdot m(m-1)x^{m-2} - mx \cdot mx^{m-1} + m \cdot x^m$$

$$= m(m-1)x^m - m^2x^m + mx^m$$

$$= (m^2 - m - m^2 + m)x^m = 0$$

This shows that y = x^m is a solution of the D.E.

$x^2\frac{d^2y}{dx^2} - mx\frac{dy}{dx} + my = 0$

(v) y = a + $\frac{b}{x}$; $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

Solution:

y = a + $\frac{b}{x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 0 + b\left(-\frac{1}{x^2}\right) = -\frac{b}{x^2}$$

$$\therefore x^2 \frac{dy}{dx} = -b$$

Differentiating again w.r.t. x, we get

$$x^2 \cdot \frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx} \cdot \frac{d}{dx}(x^2) = 0$$

$$\therefore x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} \times 2x = 0$$

$$\therefore x \frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

Hence, y = a + $\frac{b}{x}$ is a solution of the D.E.

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

(vi) $y = e^{ax}$; $x \frac{dy}{dx} = y \log y$

Solution:

$y = e^{ax}$

$\log y = \log e^{ax} = ax \log e$

log y = ax …….(1) ……..[∵ log e = 1]

Differentiating w.r.t. x, we get

$\frac{1}{y} \cdot \frac{dy}{dx} = a \times 1$

$\therefore \frac{dy}{dx} = ay$

$\therefore x \frac{dy}{dx} = (ax)y$

$\therefore x \frac{dy}{dx} = y \log y$ …….. [By (1)]

Hence, $y = e^{ax}$ is a solution of the D.E.
$x \frac{dy}{dx} = y \log y$.

Question 2.
Solve the following differential equations.
(i) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$
Solution:

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

$$\therefore \frac{1}{1+y^2} dy = \frac{1}{1+x^2} dx$$

Integrating both sides, we get

$$\int \frac{1}{1+y^2} dy = \int \frac{1}{1+x^2} dx$$

$$\therefore \tan^{-1} y = \tan^{-1} x + c$$

This is the general solution.

(ii) $\log(\frac{dy}{dx}) = 2x + 3y$
Solution:

$$\log\left(\frac{dy}{dx}\right) = 2x + 3y$$

$$\therefore \frac{dy}{dx} = e^{2x+3y} = e^{2x} \cdot e^{3y}$$

$$\therefore \frac{1}{e^{3y}} dy = e^{2x} dx$$

Integrating both sides, we get

$\int e^{-3y}\, dy = \int e^{2x} dx$

$\therefore\ \frac{e^{-3y}}{-3} = \frac{e^{2x}}{2} + c_1$

$\therefore\ 2e^{-3y} = -3e^{2x} + 6c_1$

$\therefore\ 2e^{-3y} + 3e^{2x} = c$, where $c = 6c_1$

This is the general solution.

(iii) $y - x\frac{dy}{dx} = 0$

Solution:

$y - x\frac{dy}{dx} = 0$

$\therefore x\frac{dy}{dx} = y$

$\therefore \frac{1}{x}dx = \frac{1}{y}dy$

Integrating both sides, we get

$\int \frac{1}{x}dx = \int \frac{1}{y}dy$

∴ log |x| = log |y| + log c

∴ log |x| = log |cy|

∴ x = cy

This is the general solution.

(iv) $\sec^2 x \cdot \tan y\, dx + \sec^2 y \cdot \tan x\, dy = 0$

Solution:

$\sec^2 x \cdot \tan y\, dx + \sec^2 y \cdot \tan x\, dy = 0$

$\sec^2 x \cdot \tan y\, dx + \sec^2 y \cdot \tan x\, dy = 0$

$\therefore \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$

Integrating both sides, we get

$\int \frac{\sec^2 x}{\tan x} dx + \int \frac{\sec^2 y}{\tan y} dy = c_1$

Each of these integrals is of the type

$\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c$

∴ the general solution is

∴ log|tan x| + log|tan y | = log c, where c_1 = log c

∴ log |tan x . tan y| = log c

∴ tan x . tan y = c

This is the general solution.

(v) cos x . cos y dy – sin x . sin y dx = 0

Solution:

cos x . cos y dy – sin x . sin y dx = 0

$\frac{\cos y}{\sin y} dy - \frac{\sin x}{\cos x} dx = 0$

Integrating both sides, we get

∫cot y dy – ∫tan x dx = c_1

∴ log|sin y| – [-log|cos x|] = log c, where c_1 = log c

∴ log |sin y| + log|cos x| = log c

∴ log|sin y . cos x| = log c

∴ sin y . cos x = c

This is the general solution.

(vi) $\frac{dy}{dx}$ = -k, where k is a constant.

(vi) $\frac{dy}{dx}$ = -k, where k is a constant.
Solution:
$\frac{dy}{dx}$ = -k
∴ dy = -k dx
Integrating both sides, we get
∫dy = -k∫dx
∴ y = -kx + c
This is the general solution.

(vii) $\frac{\cos^2 y}{x}dy + \frac{\cos^2 x}{y}dx = 0$
Solution:
$\frac{\cos^2 y}{x}dy + \frac{\cos^2 x}{y}dx = 0$
∴ y $\cos^2$y dy + x $\cos^2$x dx = 0
$\therefore x\left(\frac{1+\cos 2x}{2}\right)dx + y\left(1+\frac{\cos 2y}{2}\right)dy = 0$
∴ x(1 + cos 2x) dx + y(1 + cos 2y) dy = 0
∴ x dx + x cos 2x dx + y dy+ y cos 2y dy = 0
Integrating both sides, we get
∫x dx + ∫y dy + ∫x cos 2x dx + ∫y cos 2y dy = c_1 ……..(1)
Using integration by parts

$$\int x\cos 2x\,dx = x\int\cos 2x\,dx - \int\left[\frac{d}{dx}(x)\int\cos 2x\,dx\right]dx$$

$$= x\left(\frac{\sin 2x}{2}\right) - \int 1\cdot\frac{\sin 2x}{2}\,dx$$

$$=\frac{x\sin 2x}{2}+\frac{1}{2}\cdot\frac{\cos 2x}{2}=\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}$$

Similarly,

$$\int y\cos 2y\,dy=\frac{y\sin 2y}{2}+\frac{\cos 2y}{4}$$

$\therefore$ from (1), we get

$$\frac{x^2}{2}+\frac{y^2}{2}+\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}+\frac{y\sin 2y}{2}+\frac{\cos 2y}{4}=c_1$$

Multiplying throughout by 4, this becomes

$2x^2 + 2y^2 + 2x \sin 2x + \cos 2x + 2y \sin 2y + \cos 2y = 4c_1$

$\therefore 2(x^2 + y^2) + 2(x \sin 2x + y \sin 2y) + \cos 2y + \cos 2x + c = 0$, where $c = -4c_1$

This is the general solution.

(viii) $y^3 - \frac{dy}{dx} = x^2\frac{dy}{dx}$

Solution:

$$y^3-\frac{dy}{dx}=x^2\frac{dy}{dx}$$

$$\therefore\ y^3=\frac{dy}{dx}+x^2\frac{dy}{dx}$$

$$\therefore\ y^3=(1+x^2)\frac{dy}{dx}\qquad\therefore\ \frac{1}{1+x^2}dx=\frac{1}{y^3}dy$$

Integrating both sides, we get

$$\int\frac{1}{1+x^2}dx=\int y^{-3}dy$$

$$\therefore\ \tan^{-1}x=\frac{y^{-2}}{-2}+c_1$$

$$\therefore\ \tan^{-1}x = -\frac{1}{2y^2} + c_1$$

$$\therefore\ 2y^2\tan^{-1}x = -1 + 2c_1y^2$$

$$\therefore\ 2y^2\tan^{-1}x + 1 = cy^2, \text{ where } c = 2c_1$$

This is the general solution.

(ix) $2e^{x+2y}\,dx - 3\,dy = 0$

Solution:

$$2e^{x+2y}\,dx - 3dy = 0$$

$$\therefore\ 2e^x \cdot e^{2y}\,dx - 3dy = 0$$

$$\therefore\ 2e^x\,dx - \frac{3}{e^{2y}}\,dy = 0$$

Integrating both sides, we get

$$2\int e^x\,dx - 3\int e^{-2y}\,dy = c_1$$

$$\therefore\ 2e^x - 3\cdot\frac{e^{-2y}}{(-2)} = c_1$$

$$\therefore\ 4e^x + 3e^{-2y} = 2c_1$$

$$\therefore\ 4e^x + 3e^{-2y} = c, \text{ where } c = 2c_1$$

This is the general solution.

(x) $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

Solution:

$$\frac{dy}{dx} = e^{x+y} + x^2e^y$$

$\therefore \frac{dy}{dx} = e^x \cdot e^y + x^2 e^y = e^y (e^x + x^2)$

$\therefore \frac{1}{e^y} dy = (e^x + x^2)\, dx$

Integrating both sides, we get

$\int e^{-y} dy = \int (e^x + x^2)\, dx$

$\therefore \frac{e^{-y}}{-1} = e^x + \frac{x^3}{3} + c_1$

$\therefore e^x + e^{-y} + \frac{x^3}{3} = -c_1$

$\therefore 3e^x + 3e^{-y} + x^3 = -3c_1$

$\therefore 3e^x + 3e^{-y} + x^3 = c$, where $c = -3c_1$

This is the general solution.

Question 3.

For each of the following differential equations, find the particular solution satisfying the given condition:

(i) $3e^x \tan y\, dx + (1 + e^x) \sec^2 y\, dy = 0$, when $x = 0$, $y = \pi$

Solution:

$3e^x \tan y\, dx + (1 + e^x) \sec^2 y\, dy = 0$

$\therefore \frac{3e^x}{1+e^x} dx + \frac{\sec^2 y}{\tan y} dy = 0$

Integrating both sides, we get

$3\int \frac{e^x}{1+e^x} dx + \int \frac{\sec^2 y}{\tan y} dy = c_1$

Each of these integrals is of the type

Each of these integrals is of the type

$$\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$$

$\therefore$ the general solution is

$3 \log|1+e^x| + \log|\tan y| = \log c$, where $c_1 = \log c$

$\therefore \log|(1+e^x)^3 \cdot \tan y| = \log c$

$\therefore (1+e^x)^3 \tan y = c$

When $x = 0$, $y = \pi$, we have

$(1+e^0)^3 \tan \pi = c$

$\therefore c = 0$

$\therefore$ the particular solution is $(1+e^x)^3 \tan y = 0$.

(ii) $(x - y^2x)\, dx - (y + x^2y)\, dy = 0$, when $x = 2$, $y = 0$

Solution:

$(x - y^2x)\, dx - (y + x^2y)\, dy = 0$

$\therefore x(1 - y^2)\, dx - y(1 + x^2)\, dy = 0$

$$\therefore \frac{x}{1+x^2} dx - \frac{y}{1-y^2} dy = 0$$

$$\therefore \frac{2x}{1+x^2} - \frac{2y}{1-y^2} dy = 0$$

Integrating both sides, we get

$$\int \frac{2x}{1+x^2} dx + \int \frac{-2y}{1-y^2} dy = c_1$$

Each of these integrals is of the type

$$\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$$

$\therefore$ the general solution is

$\log|1+x^2| + \log|1-y^2| = \log c$, where $c_1 = \log c$

$\therefore \log|(1+x^2)(1-y^2)| = \log c$

$\therefore (1+x^2)(1-y^2) = c$

When x = 2, y = 0, we have

(1 + 4)(1 – 0) = c

$\therefore$ c = 5

$\therefore$ the particular solution is $(1 + x^2)(1 - y^2) = 5$.

(iii) $y(1 + \log x) \frac{dx}{dy} - x \log x = 0$, $y = e^2$, when x = e

Solution:

$y(1 + \log x) \frac{dx}{dy} - x \log x = 0$

$$\therefore \frac{1+\log x}{x \log x} dx - \frac{dy}{y} = 0$$

Integrating both sides, we get

$$\therefore \int \frac{1+\log x}{x \log x} dx - \int \frac{dy}{y} = c_1 \quad \text{... (1)}$$

Put $x \log x = t$.

$$\text{Then } \left[x \cdot \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)\right] dx = dt$$

$$\therefore \left[\frac{x}{x} + (\log x)(1)\right] dx = dt \quad \therefore (1 + \log x)\, dx = dt$$

$\therefore \left[\frac{x}{x}+(\log x)(1)\right] dx = dt \quad \therefore (1+\log x)\, dx = dt$

$\therefore \int \frac{1+\log x}{x \log x} dx = \int \frac{dt}{t} = \log |t| = \log |x \log x|$

$\therefore$ from (1), the general solution is

$\log |x \log x| - \log |y| = \log c$, where $c_1 = \log c$

$\therefore \log \left|\frac{x \log x}{y}\right| = \log c \quad \therefore \frac{x \log x}{y} = c$

$\therefore x \log x = cy$

This is the general solution.

Now, $y = e^2$, when $x = e$

$\therefore e \log e = c \cdot e^2 \qquad 1 = c \cdot e \qquad \ldots [\because \log e = 1]$

$\therefore c = \frac{1}{e}$

$\therefore$ the particular solution is $x \log x = \left(\frac{1}{e}\right) y$

$\therefore y = ex \log x.$

(iv) $(e^y + 1) \cos x + e^y \sin x \frac{dy}{dx} = 0$, when $x = \frac{\pi}{6}$, $y = 0$

Solution:

$(e^y + 1) \cos x + e^y \sin x \frac{dy}{dx} = 0$

$\therefore \frac{\cos x}{\sin x} dx + \frac{e^y}{e^y + 1} dy = 0$

Integrating both sides, we get

$$\int \frac{\cos\ x}{\sin\ x}\,dx + \int \frac{e^y}{e^y+1}\,dy = c_1 \qquad \dots (1)$$

Now, $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(e^y+1) = e^y$ and

$\int \frac{f'(x)}{f(x)}dx$ = log|f(x)| + c

∴ from (1), the general solution is

log|sin x| + log|e^y + 1| = log c, where c_1 = log c

∴ log|sin x . (e^y + 1)| = log c

∴ sin x . (e^y + 1) = c

When x = $\frac{\pi}{4}$, y = 0, we get

$\left(\sin\frac{\pi}{4}\right)\left(e^0+1\right) = c$

∴ c = $\frac{1}{\sqrt{2}}$ (1 + 1) = √2

∴ the particular solution is sin x . (e^y + 1) = √2

(v) (x + 1) $\frac{dy}{dx}$ – 1 = $2e^{-y}$, y = 0, when x = 1

Solution:

$$(x+1)\frac{dy}{dx} - 1 = 2e^{-y}$$

$$\therefore\ (x+1)\frac{dy}{dx} = \frac{2}{e^y} + 1 = \frac{2+e^y}{e^y}$$

$$\therefore\ \frac{e^y}{2+e^y}dy = \frac{1}{x+1}dx$$

Integrating both sides, we get

$$\int \frac{e^y}{2+e^y} dy = \int \frac{1}{x+1} dx$$

$\therefore \log|2+e^y| = \log|x+1| + \log c$

$$\dots \left[\because \frac{d}{dy}(2+e^y) = e^y \text{ and } \int \frac{f'(y)}{f(y)} dy = \log|f(y)| + c\right]$$

$\therefore \log|2+e^y| = \log|c(x+1)|$

$\therefore 2+e^y = c(x+1)$

This is the general solution.

Now, y = 0, when x = 1

∴ $2 + e^0 = c(1 + 1)$

∴ 3 = 2c

∴ $c = \frac{3}{2}$

∴ the particular solution is $2 + e^y = \frac{3}{2}(x + 1)$

∴ $2(2 + e^y) = 3(x + 1)$.

(vi) $\cos\left(\frac{dy}{dx}\right) = a$, $a \in R$, $y(0) = 2$

Solution:

$\cos\left(\frac{dy}{dx}\right) = a$

∴ $\frac{dy}{dx} = \cos^{-1} a$

∴ $dy = (\cos^{-1} a)\, dx$

Integrating both sides, we get

$\int dy = (\cos^{-1} a) \int dx$

∴ $y = (\cos^{-1} a)\, x + c$

∴ $y = x \cos^{-1} a + c$

$\therefore y = x \cos^{-1} a + c$
This is the general solution.
Now, y(0) = 2, i.e. y = 2,
when x = 0, 2 = 0 + c
$\therefore c = 2$
$\therefore$ the particular solution is
$\therefore y = x \cos^{-1} a + 2$
$\therefore y - 2 = x \cos^{-1} a$
$\therefore \frac{y-2}{x} = \cos^{-1} a$
$\therefore \cos(\frac{y-2}{x}) = a$

Question 4.
Reduce each of the following differential equations to the variable separable form and hence solve:
(i) $\frac{dy}{dx} = \cos(x + y)$
Solution:

$$\frac{dy}{dx} = \cos(x+y) \quad \ldots (1)$$

Put $x + y = u$. Then $1 + \frac{dy}{dx} = \frac{du}{dx}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} - 1$$

$\therefore$ (1) becomes, $\frac{du}{dx} - 1 = \cos u$

$$\therefore \frac{du}{dx} = 1 + \cos u$$

$$\therefore \frac{1}{1+\cos u}\, du = dx$$

Integrating both sides, we get

$$\int \frac{1}{1+\cos u}\, du = \int dx$$

$$\therefore \int \frac{1}{2\cos^2\left(\frac{u}{2}\right)}\, du = \int dx$$

$$\therefore \frac{1}{2}\int \sec^2\left(\frac{u}{2}\right) du = \int dx$$

$$\therefore \frac{1}{2} \cdot \frac{\tan\left(\frac{u}{2}\right)}{\left(\frac{1}{2}\right)} = x + c$$

$$\therefore \tan\left(\frac{x+y}{2}\right) = x + c$$

This is the general solution.

(ii) $(x - y)^2 \frac{dy}{dx} = a^2$

Solution:

$$(x-y)^2 \frac{dy}{dx} = a^2 \qquad \ldots (1)$$

Put $x - y = u \quad \therefore x - u = y \quad \therefore 1 - \frac{du}{dx} = \frac{dy}{dx}$

$\therefore$ (1) becomes, $u^2\left(1-\frac{du}{dx}\right)=a^2$

$\therefore\ u^2-u^2\frac{du}{dx}=a^2$

$\therefore\ u^2-a^2=u^2\frac{du}{dx} \quad \therefore\ dx=\frac{u^2}{u^2-a^2}du$

Integrating both sides, we get

$\int dx=\int\frac{(u^2-a^2)+a^2}{u^2-a^2}du$

$\therefore\ x=\int 1\,du+a^2\int\frac{du}{u^2-a^2}+c_1$

$=u+a^2\cdot\frac{1}{2a}\log\left|\frac{u-a}{u+a}\right|+c_1$

$\therefore\ x=x-y+\frac{a}{2}\log\left|\frac{x-y-a}{x-y+a}\right|+c_1$

$\therefore\ -c_1+y=\frac{a}{2}\log\left|\frac{x-y-a}{x-y+a}\right|$

$\therefore\ -2c_1+2y=a\log\left|\frac{x-y-a}{x-y+a}\right|$

$\therefore\ c+2y=a\log\left|\frac{x-y-a}{x-y+a}\right|$, where $c=-2c_1$

This is the general solution.

(iii) $x+y\frac{dy}{dx}=\sec(x^2+y^2)$

Solution:

$$x+y\frac{dy}{dx}=\sec(x^2+y^2) \quad \ldots (1)$$

Put $x^2+y^2=u$ $\quad \therefore\ 2x+2y\frac{dy}{dx}=\frac{du}{dx}$

$$\therefore\ x+y\frac{dy}{dx}=\frac{1}{2}\cdot\frac{du}{dx}$$

$\therefore$ (1) becomes, $\frac{1}{2}\cdot\frac{du}{dx}=\sec u$ $\quad \therefore\ \frac{1}{\sec u}du=2\cdot dx$

Integrating both sides, we get
$\int\cos u\ du = 2\int dx$
$\therefore \sin u = 2x + c$
$\therefore \sin(x^2 + y^2) = 2x + c$
This is the general solution.

(iv) $\cos^2(x - 2y) = 1 - 2\frac{dy}{dx}$
Solution:

$$\cos^2(x-2y)=1-2\frac{dy}{dx} \quad \ldots (1)$$

Put $x-2y=u$. Then $1-2\frac{dy}{dx}=\frac{du}{dx}$

$\therefore$ (1) becomes, $\cos^2 u=\frac{du}{dx}$

$$\therefore\ dx=\frac{1}{\cos^2 u}du$$

Integrating both sides, we get

Integrating both sides, we get
$\int dx = \int \sec^2 u \, du$
$\therefore x = \tan u + c$
$\therefore x = \tan(x - 2y) + c$
This is the general solution.

(v) (2x – 2y + 3) dx – (x – y + 1) dy = 0, when x = 0, y = 1
Solution:
(2x – 2y + 3) dx – (x – y + 1) dy = 0
$\therefore$ (x – y + 1) dy = (2x – 2y + 3) dx
$\therefore \frac{dy}{dx} = \frac{2(x-y)+3}{(x-y)+1}$ ………(1)
Put x – y = u, Then $1 - \frac{dy}{dx} = \frac{du}{dx}$

$$\therefore \frac{dy}{dx} = 1 - \frac{du}{dx}$$

$$\therefore \text{(1) becomes, } 1 - \frac{du}{dx} = \frac{2u+3}{u+1}$$

$$\therefore \frac{du}{dx} = 1 - \frac{2u+3}{u+1} = \frac{u+1-2u-3}{u+1}$$

$$\therefore \frac{du}{dx} = \frac{-u-2}{u+1} = -\left(\frac{u+2}{u+1}\right)$$

$$\therefore \frac{u+1}{u+2} du = -dx$$

Integrating both sides, we get

$$\int \frac{u+1}{u+2}\,du = -\int 1dx$$

$$\therefore \int \frac{(u+2)-1}{u+2}\,du = -\int 1dx$$

$$\therefore \int \left(1 - \frac{1}{u+2}\right) du = -\int 1dx$$

∴ u – log|u + 2| = -x + c
∴ x – y – log|x – y + 2| = -x + c
∴ (2x – y) – log|x – y + 2| = c
This is the general solution.
Now, y = 1, when x = 0.
∴ (0 – 1) – log|0 – 1 + 2| = c
∴ -1 – o = c
∴ c = -1
∴ the particular solution is
(2x – y) – log|x – y + 2| = -1
∴ (2x – y) – log|x – y + 2| + 1 = 0

Ex 6.4

I. Solve the following differential equations:

Question 1.
$x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x\right] dx$
Solution:

$$x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x\right] dx$$

$$\therefore \frac{dy}{dx} = \frac{y \sin\left(\frac{y}{x}\right) - x}{x \sin\left(\frac{y}{x}\right)} \quad \ldots (1)$$

Put $y = vx \quad \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$ and $\frac{y}{x} = v$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = \frac{vx \sin v - x}{x \sin v}$

$$\therefore x\frac{dv}{dx} = \frac{v \sin v - 1}{\sin v} - v$$

$$\therefore x\frac{dv}{dx} = \frac{v \sin v - 1 - v \sin v}{\sin v} = \frac{-1}{\sin v}$$

$$\therefore \sin v \, dv = -\frac{1}{x} dx$$

Integrating both sides, we get

$$\int \sin v \, dv = -\int \frac{1}{x} dx$$

$$\therefore -\cos v = -\log x - c$$

$$\therefore \cos\left(\frac{y}{x}\right) = \log x + c$$

This is the general solution.

Question 2.
$(x^2 + y^2)\ dx - 2xy\ .\ dy = 0$
Solution:
$(x^2 + y^2)\ dx - 2xy\ dy = 0$
$\therefore 2xy\ dy = (x^2 + y^2)\ dx$
$\therefore \frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ ....(1)

Put $y = vx \quad \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x(vx)}$

$$\therefore v + x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\therefore x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1+v^2-2v^2}{2v}$$

$$\therefore x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\therefore \frac{2v}{1-v^2}\,dv = \frac{1}{x}\,dx$$

Integrating both sides, we get

$$\int \frac{2v}{1-v^2}\,dv = \int \frac{1}{x}\,dx$$

$$-\int \frac{-2v}{1-v^2}\,dv = \int \frac{1}{x}\,dx$$

$$\therefore -\log|1-v^2| = \log x + \log c_1$$

$$\dots\left[\because \frac{d}{dv}(1-v^2) = -2v \text{ and } \int\frac{f'(x)}{f(x)}dx = \log|f(x)|+c\right]$$

$$\therefore \log\left|\frac{1}{1-v^2}\right| = \log c_1 x$$

$$\therefore \log\left|\frac{1}{1-\left(\frac{y^2}{x^2}\right)}\right| = \log c_1 x$$

$$\therefore \log\left|\frac{x^2}{x^2-y^2}\right| = \log c_1 x$$

$$\therefore \frac{x^2}{x^2-y^2} = c_1 x$$

$$\therefore x^2 - y^2 = \frac{1}{c_1}x$$

$$\therefore x^2 - y^2 = cx, \text{ where } c = \frac{1}{c_1}$$

This is the general solution.

Question 3.

$$\left(1+2e^{\frac{x}{y}}\right) + 2e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)\frac{dy}{dx} = 0$$

Solution:

$$(1+2e^{\frac{x}{y}}) + 2e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)\frac{dy}{dx} = 0$$

$$\therefore (1+2e^{\frac{x}{y}}) + 2e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)\cdot\frac{1}{\left(\frac{dx}{dy}\right)} = 0$$

$$\therefore\ (1+2e^{\frac{x}{y}})+2e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)\cdot\frac{1}{\left(\frac{dx}{dy}\right)}=0$$

$$\therefore\ (1+2e^{\frac{x}{y}})\frac{dx}{dy}+2e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)=0 \qquad \ldots (1)$$

Put $\frac{x}{y}=u \qquad \therefore\ x=uy$

$$\therefore\ \frac{dx}{dy}=u+y\frac{du}{dy}$$

$\therefore$ (1) becomes, $(1+2e^{u})\left(u+y\frac{du}{dy}\right)+2e^{u}(1-u)=0$

$$\therefore\ u+2ue^{u}+y\left(1+2e^{u}\right)\frac{du}{dy}+2e^{u}-2ue^{u}=0$$

$$\therefore\ (u+2e^{u})+y(1+2e^{u})\frac{du}{dy}=0$$

$$\therefore\ \frac{dy}{y}+\frac{1+2e^{u}}{u+2e^{u}}\,du=0$$

Integrating both sides, we get

$$\int\frac{1}{y}\,dy+\int\frac{1+2e^{u}}{u+2e^{u}}\,du=c_1$$

$\therefore\ \log|y|+\log|u+2e^{u}|=\log c$, where $c_1=\log c$

$$\ldots\left[\because\ \frac{d}{du}(u+2e^{u})=1+2e^{u} \text{ and } \int\frac{f'(u)}{f(u)}\,du=\log|f(u)|+c\right]$$

$\therefore \log|y(u+2e^u)| = \log c$

$\therefore y(u+2e^u) = c$

$\therefore y\left(\frac{x}{y}+2e^{\frac{x}{y}}\right) = c$

$\therefore x+2ye^{\frac{x}{y}} = c$

This is the general solution.

Question 4.

$y^2\,dx + (xy + x^2)\,dy = 0$

Solution:

$y^2\,dx + (xy + x^2)\,dy = 0$

$\therefore (xy + x^2)\,dy = -y^2\,dx$

$\therefore \frac{dy}{dx} = \frac{-y^2}{xy+x^2}$ ……..(1)

Put y = vx

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

Substituting these values in (1), we get

$$v + x\frac{dv}{dx} = \frac{-v^2x^2}{x\cdot vx + x^2} = \frac{-v^2}{v+1}$$

$$\therefore x\frac{dv}{dx} = \frac{-v^2}{v+1} - v = \frac{-v^2-v^2-v}{v+1}$$

$$\therefore x\frac{dv}{dx} = \frac{-2v^2-v}{v+1} = -\left(\frac{2v^2+v}{v+1}\right)$$

$$\therefore \frac{v+1}{2v^2+v}dv = -\frac{1}{x}dx$$

Integrating both sides, we get

$$\int \frac{v+1}{2v^2+v}dv = -\int \frac{1}{x}dx$$

$$\therefore \int \frac{v+1}{v(2v+1)}dv = -\int \frac{1}{x}dx$$

$$\therefore \int \frac{(2v+1)-v}{v(2v+1)}dv = -\int \frac{1}{x}dx$$

$$\therefore \int \left(\frac{1}{v} - \frac{1}{2v+1}\right)dv = -\int \frac{1}{x}dx$$

$$\therefore \int \frac{1}{v}dv - \int \frac{1}{2v+1}dv = -\int \frac{1}{x}dx$$

$$\therefore \log|v| - \frac{1}{2}\log|2v+1| = -\log|x| + \log c$$

$$\therefore 2\log|v| - \log|2v+1| = -2\log|x| + 2\log c$$

$$\therefore \log|v^2| - \log|2v+1| = -\log|x^2| + \log c^2$$

$$\therefore \log\left|\frac{v^2}{2v+1}\right| = \log\left|\frac{c^2}{x^2}\right|$$

$$\therefore \frac{v^2}{2v+1} = \frac{c^2}{x^2}$$

$$\therefore \frac{\left(\frac{y^2}{x^2}\right)}{2\left(\frac{y}{x}\right)+1} = \frac{c^2}{x^2} \qquad \therefore \frac{y^2}{x(2y+x)} = \frac{c^2}{x^2}$$

$\therefore\ xy^2 = c^2(x + 2y)$

This is the general solution.

Question 5.

$(x^2 - y^2)\ dx + 2xy\ dy = 0$

Solution:

$(x^2 - y^2)dx + 2xy\ dy = 0$

$\therefore - 2xy\ dy = (x^2 - y^2)dx$

$\therefore \frac{dy}{dx} = \frac{x^2 - y^2}{-} 2xy \quad ....(1)$

put y = vx

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{-2x(vx)}$

$\therefore v + x\frac{dv}{dx} = \frac{1 - v^2}{-2v}$

$\therefore x\frac{dv}{dx} = \frac{1 - v^2}{-2v} - v = \frac{1 - v^2 + 2v^2}{-2v}$

$$\therefore x\frac{dv}{dx} = \frac{1+v^2}{-2v}$$

$$\therefore \frac{-2v}{1+v^2}dv = \frac{1}{x}dx$$

Integrating both sides, we get

$$\therefore \int \frac{-2v}{1+v^2}dv = \int \frac{1}{x}dx$$

$$\therefore \log\left|1+v^2\right| = \log x + \log c_1$$

$$\ldots\left[\because \frac{d}{dx}\left(1+v^2\right) = 2v \text{ and } \int\left[\frac{f\prime(x)}{f(x)}dx = \log|f(x)| + c\right]\right.$$

$$\therefore \log\left|\frac{1}{1+v^2}\right| = \log c_1 x$$

$$\therefore \log\left|\frac{1}{1+\left(\frac{y^2}{x^2}\right)}\right| = \log c_1 x$$

$$\therefore \log\left|\frac{x^2}{x^2+y^2}\right| = \log c_1 x$$

$$\therefore \frac{x^2}{x^2+y^2} = c_1 x$$

$\therefore x^2 + y^2 = \frac{1}{c_1}x$

$\therefore x^2 + y^2 = cx$ where $c = \frac{1}{c_1}$

This is the general solution.

Question 6.
$\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$
Solution:

$$\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$$

$$\therefore \frac{dy}{dx} = -\left(\frac{x-2y}{2x-y}\right) \quad \text{.........(1)}$$

Put $y = vx$. $\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = -\left(\frac{x-2vx}{2x-vx}\right)$

$$\therefore v + x\frac{dv}{dx} = -\left(\frac{1-2v}{2-v}\right)$$

$$\therefore x\frac{dv}{dx} = -\left(\frac{1-2v}{2-v}\right) - v$$

$$\therefore x\frac{dv}{dx} = \frac{-1+2v-2v+v^2}{2-v}$$

$\therefore\ x\frac{dv}{dx}=\frac{v^2-1}{2-v}$

$\therefore\ \frac{2-v}{v^2-1}dv=\frac{1}{x}dx$

Integrating both sides, we get

$\int\frac{2-v}{v^2-1}dv=\int\frac{1}{x}dx$

$\therefore\ 2\int\frac{1}{v^2-1}dv-\frac{1}{2}\int\frac{2v}{v^2-1}dv=\int\frac{1}{x}dx$

$\therefore\ 2\times\frac{1}{2}\log\left|\frac{v-1}{v+1}\right|-\frac{1}{2}\log|v^2-1|=\log|x|+\log c_1$

$\ldots\left[\because\ \frac{d}{dv}(v^2-1)=2v \text{ and } \int\frac{f'(x)}{f(x)}dx=\log|f(x)|+c\right]$

$\therefore\ \log\left|\frac{v-1}{v+1}\right|-\log|(v^2-1)^{\frac{1}{2}}|=\log|c_1x|$

$\therefore\ \log\left|\frac{v-1}{v+1}\cdot\frac{1}{\sqrt{v^2-1}}\right|=\log|c_1x|$

$\therefore\ \frac{v-1}{v+1}\cdot\frac{1}{\sqrt{v^2-1}}=c_1x$

$\therefore\ \frac{\frac{y}{x}-1}{\frac{y}{x}+1}\cdot\frac{1}{\sqrt{\frac{y^2}{x^2}-1}}=c_1x$

$$\therefore \frac{y-x}{y+x} = c_1\sqrt{y^2-x^2}$$

$$\therefore \frac{y-x}{y+x} = c_1\sqrt{y-x}\cdot\sqrt{y+x}$$

$$\therefore \sqrt{y-x} = c_1(y+x)^{\frac{3}{2}}$$

$$\therefore y-x = c_1{}^2(x+y)^3$$

$$\therefore y-x = c(x+y)^3, \text{ where } c = c_1{}^2$$

$$\therefore y = c(x+y)^3 + x$$

This is the general solution.

Question 7.

$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$

Solution:

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0 \qquad (1)$$

Put $y = vx$ $\quad\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$ and $\frac{y}{x} = v$

$$\therefore \text{ (1) becomes, } x\left(v + x\frac{dv}{dx}\right) - vx + x\sin v = 0$$

$$\therefore vx + x^2\frac{dv}{dx} - vx + x\sin v = 0$$

$$\therefore x^2\frac{dv}{dx} + x\sin v = 0$$

$$\therefore \frac{1}{\sin v} dv + \frac{1}{x} dx = 0$$

Integrating, we get

$$\int \operatorname{cosec} v \, dv + \int \frac{1}{x} dx = c_1$$

$$\therefore \log|\operatorname{cosec} v - \cot v| + \log|x| = \log c, \text{ where } c_1 = \log c$$

$$\therefore \log|x(\operatorname{cosec} v - \cot v)| = \log c$$

$$\therefore x\left(\frac{1}{\sin v} - \frac{\cos v}{\sin v}\right) = c$$

$$\therefore x(1 - \cos v) = c \sin v$$

$$\therefore x\left[1 - \cos\left(\frac{y}{x}\right)\right] = c \sin\left(\frac{y}{x}\right)$$

This is the general solution.

Question 8.

$$\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{X}{y}\right) dy = 0$$

Solution:

$$(1 + e^{x/y})\, dx + e^{x/y}\left(1 - \frac{x}{y}\right) dy = 0$$

$$\therefore (1 + e^{x/y}) \frac{dx}{dy} + e^{x/y}\left(1 - \frac{x}{y}\right) = 0 \qquad \text{... (1)}$$

Put $\frac{x}{y} = u \quad \therefore x = uy \quad \therefore \frac{dx}{dy} = u + y\frac{du}{dy}$

$\therefore$ (1) becomes, $(1+e^u)\left(u+y\frac{du}{dy}\right)+e^u(1-u)=0$

$\therefore\ u+ue^u+y(1+e^u)\frac{du}{dy}+e^u-ue^u=0$

$\therefore\ (u+e^u)+y(1+e^u)\frac{du}{dy}=0$

$\therefore\ \frac{dy}{y}+\frac{1+e^u}{u+e^u}du=0$

$\therefore\ \int\frac{dy}{y}+\int\frac{1+e^u}{u+e^u}du=c_1$ ... (2)

$\frac{d}{du}(u+e^u)=1+e^u$ and $\int\frac{f'(u)}{f(u)}du=\log|f(u)|+c$

$\therefore$ from (2), the general solution is

$\log|y|+\log|u+e^u|=\log c$, where $c_1=\log c$

$\therefore\ \log|y(u+e^u)|=\log c$ $\therefore\ y(u+e^u)=c$

$\therefore\ y\left(\frac{x}{y}+e^{x/y}\right)=c$ $\therefore\ x+ye^{x/y}=c$

This is the general solution.

Question 9.

$y^2-x^2\frac{dy}{dx}=xy\frac{dy}{dx}$

Solution:

$$y^2 - x^2\frac{dy}{dx} = xy\frac{dy}{dx}$$

$$\therefore\ x^2\frac{dy}{dx} + xy\frac{dy}{dx} = y^2$$

$$\therefore\ (x^2 + xy)\frac{dy}{dx} = y^2$$

$$\therefore\ \frac{dy}{dx} = \frac{y^2}{x^2 + xy} \qquad \ldots (1)$$

Put $y = vx$ $\quad \therefore\ \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = \frac{v^2x^2}{x^2 + x \cdot vx} = \frac{v^2}{1+v}$

$$\therefore\ x\frac{dv}{dx} = \frac{v^2}{1+v} - v = \frac{v^2 - v - v^2}{1+v}$$

$$\therefore\ x\frac{dv}{dx} = \frac{-v}{1+v} \qquad \therefore\ \frac{1+v}{v}dv = -\frac{1}{x}dx$$

Integrating, we get

$$\int\left(\frac{1+v}{v}\right)dv = -\int\frac{1}{x}dx$$

$$\int\left(\frac{1}{v} + 1\right)dv = -\int\frac{1}{x}dx$$

$$\therefore\ \int\frac{1}{v}dv + \int 1\,dv = -\int\frac{1}{x}dx$$

$$\therefore\ \log|v| + v = -\log|x| + c$$

$$\therefore \log|y| - \log|x| + \frac{y}{x} = -\log|x| + c$$

$$\therefore \frac{y}{x} + \log|y| = c$$

This is the general solution.

Question 10.

xy $\frac{dy}{dx}$ = $x^2 + 2y^2$, y(1) = 0

Solution:

$$xy\frac{dy}{dx} = x^2 + 2y^2$$

$$\therefore \frac{dy}{dx} = \frac{x^2 + 2y^2}{xy} \quad \ldots (1)$$

Put $y = vx$. Then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \text{(1) becomes, } v + x\frac{dv}{dx} = \frac{x^2 + 2v^2x^2}{x \cdot vx} = \frac{1 + 2v^2}{v}$$

$$\therefore x\frac{dv}{dx} = \frac{1 + 2v^2}{v} - v = \frac{1 + 2v^2 - v^2}{v}$$

$$\therefore x\frac{dv}{dx} = \frac{1 + v^2}{v}$$

$$\therefore \frac{v}{1 + v^2}dv = \frac{1}{x}dx$$

Integrating, we get

$$\int \frac{v}{1+v^2}dv = \int \frac{1}{x}dx$$

$$\therefore \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{1}{x}dx + \log c_1$$

$$\therefore \frac{1}{2}\log|1+v^2| = \log|x| + \log c_1$$

$$\therefore \log|1+v^2| = 2\log|x| + 2\log c_1$$

$$\therefore \log|1+v^2| = \log|x^2| + \log {c_1}^2$$

$$\therefore \log|1+v^2| = \log|cx^2|, \text{ where } c = {c_1}^2$$

$$\therefore 1+v^2 = cx^2$$

$$\therefore 1+\frac{y^2}{x^2} = cx^2$$

$$\therefore \frac{x^2+y^2}{x^2} = cx^2$$

$$\therefore x^2+y^2 = cx^4$$

This is the general solution.

Now, $y(1) = 0$, i.e. when $x = 1$, $y = 0$, we get

$1 + 0 = c(1)$ $\quad \therefore c = 1$

$\therefore$ the particular solution is $x^2 + y^2 = x^4$.

Question 11.

x dy + 2y · dx = 0, when x = 2, y = 1

Solution:

∴ x dy + 2y · dx = 0

∴ x dy = -2y dx

∴ $\frac{1}{y}dy = \frac{-2}{x}dx$

Integrating, we get

$$\int \frac{1}{y}dy = -2\int \frac{1}{x}dx$$

$$\therefore \log|y| = -2\log|x| + \log c$$

$$\therefore \log|y| = -\log|x^2| + \log c$$

$$\therefore \log|y| = \log\left|\frac{c}{x^2}\right|$$

$$\therefore y = \frac{c}{x^2} \qquad \therefore x^2y = c$$

This is the general solution.

When x = 2, y = 1, we get

4(1) = c

∴ c = 4

∴ the particular solution is $x^2y = 4$.

Question 12.

$x^2 \frac{dy}{dx} = x^2 + xy + y^2$

Solution:

$x^2 \frac{dy}{dx} = x^2 + xy + y^2$

∴ $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$ ....(1)

$\therefore \frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$ ....(1)

Put y = vx

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = \frac{x^2 + x \cdot vx + v^2x^2}{x^2}$

$\therefore v + x\frac{dv}{dx} = 1 + v + v^2$

$\therefore x\frac{dv}{dx} = 1 + v^2$

$\therefore \frac{1}{1+v^2}dv = \frac{1}{x}dx$

Integrating, we get

$\int \frac{1}{1+v^2}dv = \int \frac{1}{x}dx + c$

$\therefore \tan^{-1} v = \log|x| + c$

$\therefore \tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$

This is the general solution.

Question 13.

(9x + 5y) dy + (15x + 11y) dx = 0

Solution:

(9x + 5y) dy + (15x + 11y) dx = 0

(9x + 5y) dy + (15x + 11y) dx = 0

∴ (9x + 5y) dy = -(15x + 11y) dx

$\therefore \frac{dy}{dx} = \frac{-(15x+11y)}{9x+5y}$ ....(1)

Put y = vx

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = \frac{-(15x+11vx)}{9x+5vx}$

$$\therefore v + x\frac{dv}{dx} = \frac{-(15+11v)}{9+5v}$$

$$\therefore x\frac{dv}{dx} = \frac{-(15+11v)}{9+5v} - v = \frac{-15-11v-9v-5v^2}{9+5v}$$

$$\therefore x\frac{dv}{dx} = \frac{-5v^2-20v-15}{9+5v} = -\left(\frac{5v^2+20v+15}{5v+9}\right)$$

$$\therefore \frac{5v+9}{5v^2+20v+15}dv = -\frac{1}{x}dx \quad \ldots (2)$$

Integrating, we get

$$\frac{1}{5}\int \frac{5v+9}{v^2+4v+3}dv = -\int \frac{1}{x}dx$$

Let $\frac{5v+9}{v^2+4v+3} = \frac{5v+9}{(v+3)(v+1)} = \frac{A}{v+3} + \frac{B}{v+1}$

$\therefore 5v+9 = A(v+1) + B(v+3)$

Put $v+3=0$, i.e. $v=-3$, we get

$-15+9 = A(-2) + B(0)$

$\therefore -6 = -2A \qquad \therefore A = 3$

Put $v+1=0$, i.e. $v=-1$, we get

$-5+9=A(0)+B(2)$

$\therefore\ 4=2B \qquad \therefore\ B=2$

$$\therefore\ \frac{5v+9}{v^2+4v+3}=\frac{3}{v+3}+\frac{2}{v+1}$$

$\therefore$ (2) becomes,

$$\frac{1}{5}\int\left(\frac{3}{v+3}+\frac{2}{v+1}\right)dv=-\int\frac{1}{x}dx$$

$$\therefore\ \frac{3}{5}\int\frac{1}{v+3}dv+\frac{2}{5}\int\frac{1}{v+1}dv=-\int\frac{1}{x}dx$$

$$\therefore\ \frac{3}{5}\log|v+3|+\frac{2}{5}\log|v+1|=-\log|x|+c_1$$

$$\therefore\ 3\log|v+3|+2\log|v+1|=-5\log x+5c_1$$

$$\therefore\ \log|(v+3)^3|+\log|(v+1)^2|=-\log|x^5|+\log c,$$

where $5c_1=\log c$

$$\therefore\ \log|(v+3)^3(v+1)^2|=\log\left|\frac{c}{x^5}\right|$$

$$\therefore\ (v+3)^3(v+1)^2=\frac{c}{x^5}$$

$$\therefore\ \left(\frac{y}{x}+3\right)^3\left(\frac{y}{x}+1\right)^2=\frac{c}{x^5}$$

$$\therefore\ \frac{(y+3x)^3}{x^3}\times\frac{(y+x)^2}{x^2}=\frac{c}{x^5}$$

This is the general solution.

Question 14.
$(x^2 + 3xy + y^2)\, dx - x^2\, dy = 0$
Solution:
$(x^2 + 3xy + y^2)\, dx - x^2\, dy = 0$
$\therefore x^2\, dy = (x^2 + 3xy + y^2)\, dx$
$\therefore \frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2}$ ....(1)

Put $y = vx$ $\quad \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = \frac{x^2 + 3x \cdot vx + v^2x^2}{x^2}$

$\therefore v + x\frac{dv}{dx} = 1 + 3v + v^2$

$x\frac{dv}{dx} = v^2 + 2v + 1 = (v+1)^2$

$\therefore \frac{1}{(v+1)^2}dv = \frac{1}{x}dx$

Integrating, we get

$\int (v+1)^{-2}dv = \int \frac{1}{x}dx$

$\therefore \frac{(v+1)^{-1}}{-1} = \log|x| + c_1$

$\therefore -\frac{1}{v+1} = \log|x| + c_1$

$\therefore \ -\frac{1}{v+1} = \log|x| + c_1$

$\therefore \ -\frac{1}{\frac{y}{x}+1} = \log|x| + c_1$

$\therefore \ -\frac{x}{y+x} = \log|x| + c_1$

$\therefore \ \log|x| + \frac{x}{x+y} = -c_1$

$\therefore \ \log|x| + \frac{x}{x+y} = c$, where $c = -c_1$

This is the general solution.

Question 15.
$(x^2 + y^2)\,dx - 2xy\,dy = 0.$
Solution:

$(x^2 + y^2)dx - 2xy\,dy = 0$

$\therefore 2xy\,dy = (x^2 + y^2)dx$

$\therefore \frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ ....(1)

Put $y = vx$

Put y = vx

$\therefore \frac{dy}{dx} = v + x\,du/dx$

$\therefore$ (1) becomes, $v + x\frac{du}{dx} = \frac{x^2 + v^2x^2}{2x(vx)}$

$\therefore v + x\frac{du}{dx} = \frac{1 + v^2}{2v}$

$\therefore x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v = \frac{1 + v^2 - 2v^2}{2v}$

$\therefore x\frac{dv}{dx} = \frac{1 - v^2}{2v}$

$\therefore \frac{2v}{1 - v^2}dv = \frac{1}{x}dx$

Integrating both sides, we get

$\int \frac{2v}{1 - v^2}dv = \int \frac{1}{x}dx$

$-\int \frac{-2v}{1 - v^2}dv = \int \frac{1}{x}dx$

$\therefore -\log|1 - v^2| = \log x + \log c_1$ .... $\left[\because \frac{d}{dv}\left(1 - v^2\right) = -2v \text{ and } \int \frac{f\prime(x)}{f(x)}dx = \log|f(x)| + c\right]$

$$\therefore \log\left|\frac{1}{1-v^2}\right| = \log c_1 x$$

$$\therefore \log\left|\frac{1}{1-\left(\frac{y^2}{x^2}\right)}\right| = \log c_1 x$$

$$\therefore \log\left|\frac{x^2}{x^2-y^2}\right| = \log c_1 x$$

$$\therefore \frac{x^2}{x^2-y^2} = c_1 x$$

$$\therefore x^2 - y^2 = \frac{1}{c_1} x$$

$\therefore x^2 - y^2 = cx$, where $c = \frac{1}{c_1}$

This is the general solution.

Maharashtra Board Solutions Class 12 Arts & Science Maths (Part 1)

About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

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