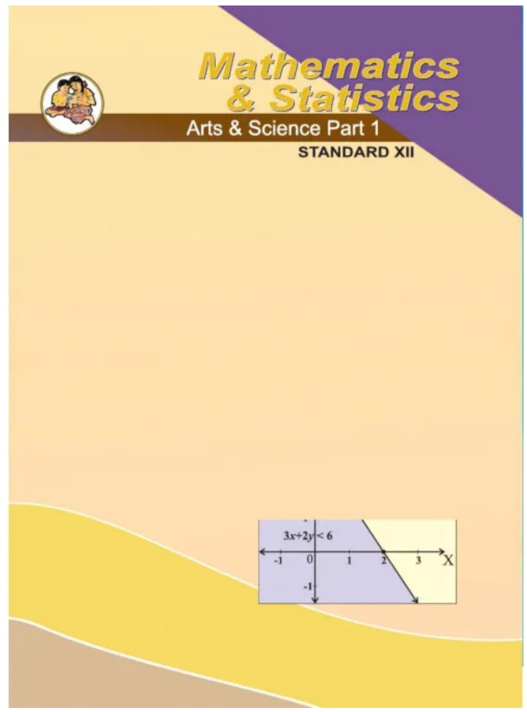
Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 4- Pair of Straight Lines

Class 12 - Chapter 4 Pair of Straight Lines



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Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 4- Pair of Straight Lines

Class 12: Maths Chapter 4 solutions. Complete Class 12 Maths Chapter 4 Notes.

Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 4- Pair of Straight Lines

Maharashtra Board 12th Maths Chapter 4, Class 12 Maths Chapter 4 solutions

Ex 4.1

Question 1.
Find the combined equation of the following pairs of lines:
(i) $2x + y = 0$ and $3x - y = 0$
Solution:
The combined equation of the lines $2x + y = 0$ and $3x - y = 0$ is
$(2x + y)(3x - y) = 0$
$\therefore 6x^2 - 2xy + 3xy - y^2 = 0$
$\therefore 6x^2 - xy - y^2 = 0.$

(ii) $x + 2y - 1 = 0$ and $x - 3y + 2 = 0$
Solution:
The combined equation of the lines $x + 2y - 1 = 0$ and $x - 3y + 2 = 0$ is
$(x + 2y - 1)(x - 3y + 2) = 0$
$\therefore x^2 - 3xy + 2x + 2xy - 6y^2 + 4y - x + 3y - 2 = 0$
$\therefore x^2 - xy - 6y^2 + x + 7y - 2 = 0.$

(iii) Passing through (2, 3) and parallel to the co-ordinate axes.
Solution:
Equations of the coordinate axes are $x = 0$ and $y = 0$.
$\therefore$ the equations of the lines passing through (2, 3) and parallel to the coordinate axes are $x = 2$ and
i.e. $x - 2 = 0$ and $y - 3 = 0$.
$\therefore$ their combined equation is
$(x - 2)(y - 3) = 0.$
$\therefore xy - 3x - 2y + 6 = 0.$

(iv) Passing through (2, 3) and perpendicular to lines $3x + 2y - 1 = 0$ and $x - 3y + 2 = 0$
Solution:
Let L_1 and L_2 be the lines passing through the point (2, 3) and perpendicular to the lines $3x + 2y - 1 = 0$ and $x - 3y + 2 = 0$ respectively.
Slopes of the lines $3x + 2y - 1 = 0$ and $x - 3y + 2 = 0$ are $\frac{-3}{2}$ and $\frac{-1}{-3} = \frac{1}{3}$ respectively.
$\therefore$ slopes of the lines L_1 and L_2 are $\frac{2}{3}$ and -3 respectively.
Since the lines L_1 and L_2 pass through the point (2, 3), their equations are
$y - 3 = \frac{2}{3}(x - 2)$ and $y - 3 = -3(x - 2)$

$y - 3 = \frac{2}{3}(x - 2)$ and $y - 3 = -3(x - 2)$

∴ $3y - 9 = 2x - 4$ and $y - 3 = -3x + 6$

∴ $2x - 3y + 5 = 0$ and $3x - y - 9 = 0$

∴ their combined equation is

$(2x - 3y + 5)(3x + y - 9) = 0$

∴ $6x^2 + 2xy - 18x - 9xy - 3y^2 + 27y + 15x + 5y - 45 = 0$

∴ $6x^2 - 7xy - 3y^2 - 3x + 32y - 45 = 0$.

(v) Passsing through (-1, 2),one is parallel to $x + 3y - 1 = 0$ and the other is perpendicular to $2x - 3y - 1 = 0$.

Solution:

Let L_1 be the line passing through (-1, 2) and parallel to the line $x + 3y - 1 = 0$ whose slope is $-\frac{1}{3}$.

∴ slope of the line L_1 is $-\frac{1}{3}$

∴ equation of the line L_1 is

$y - 2 = -\frac{1}{3}(x + 1)$

∴ $3y - 6 = -x - 1$

∴ $x + 3y - 5 = 0$

Let L_2 be the line passing through (-1, 2) and perpendicular to the line $2x - 3y - 1 = 0$ whose slope is $\frac{-2}{-3} = \frac{2}{3}$.

∴ slope of the line L_2 is $-\frac{3}{2}$

∴ equation of the line L_2 is

$y - 2 = -\frac{3}{2}(x + 1)$

∴ $2y - 4 = -3x - 3$

∴ $3x + 2y - 1 = 0$

Hence, the equations of the required lines are

$x + 3y - 5 = 0$ and $3x + 2y - 1 = 0$

∴ their combined equation is

∴ their combined equation is
$(x + 3y - 5)(3x + 2y - 1) = 0$
$\therefore 3x^2 + 2xy - x + 9xy + 6y^2 - 3y - 15x - 10y + 5 = 0$
$\therefore 3x^2 + 11xy + 6y^2 - 16x - 13y + 5 = 0$

Question 2.
Find the separate equations of the lines represented by following equations:
(i) $3y^2 + 7xy = 0$
Solution:
$3y^2 + 7xy = 0$
$\therefore y(3y + 7x) = 0$
∴ the separate equations of the lines are $y = 0$ and $7x + 3y = 0$.

(ii) $5x^2 - 9y^2 = 0$
Solution:
$5x^2 - 9y^2 = 0$
$\therefore (\sqrt{5}x)^2 - (3y)^2 = 0$
$\therefore (\sqrt{5}x + 3y)(\sqrt{5}x - 3y) = 0$
∴ the separate equations of the lines are
$\sqrt{5}x + 3y = 0$ and $\sqrt{5}x - 3y = 0$.

(iii) $x^2 - 4xy = 0$
Solution:

$x^2 - 4xy = 0$
$\therefore x(x - 4y) = 0$
$\therefore$ the separate equations of the lines are $x = 0$ and $x - 4y = 0$

(iv) $3x^2 - 10xy - 8y^2 = 0$
Solution:
$3x^2 - 10xy - 8y^2 = 0$
$\therefore 3x^2 - 12xy + 2xy - 8y^2 = 0$
$\therefore 3x(x - 4y) + 2y(x - 4y) = 0$
$\therefore (x - 4y)(3x + 2y) = 0$
$\therefore$ the separate equations of the lines are $x - 4y = 0$ and $3x + 2y = 0$.

(v) $3x^2 - 2\sqrt{3}\,xy - 3y^2 = 0$
Solution:
$3x^2 - 2\sqrt{3}xy - 3y2 = 0$
$\therefore 3x^2 - 3\sqrt{3}xy + \sqrt{3}xy - 3y^2 = 0$
$\therefore 3x(x - \sqrt{3}y) + \sqrt{3}y(x - \sqrt{3}y) = 0$
$\therefore (x - \sqrt{3}y)(3x + \sqrt{3}y) = 0$
$\therefore$ the separate equations of the lines are
$\therefore x - \sqrt{3}y = 0$ and $3x + \sqrt{3}y = 0$.

(vi) $x^2 + 2(\text{cosec } \propto)xy + y^2 = 0$
Solution:
$x^2 + 2(\text{cosec } \propto)xy - y^2 = 0$

i.e. $y^2 + 2(\operatorname{cosec}\alpha)xy + x^2 = 0$

Dividing by x^2, we get,

$$\left(\frac{y}{x}\right)^2 + 2\operatorname{cosec}\alpha\cdot\left(\frac{y}{x}\right) + 1 = 0$$

$$\therefore \frac{y}{x} = \frac{-2\operatorname{cosec}\alpha \pm \sqrt{4\operatorname{cosec}^2\alpha - 4\times1\times1}}{2\times1}$$

$$= \frac{-2\operatorname{cosec}\alpha \pm 2\sqrt{\operatorname{cosec}^2\alpha - 1}}{2}$$

$$= -\operatorname{cosec}\alpha \pm \cot\alpha$$

$$\therefore \frac{y}{x} = (\cot\alpha - \operatorname{cosec}\alpha) \quad \text{and}$$

$$\frac{y}{x} = -(\operatorname{cosec}\alpha + \cot\alpha)$$

∴ the separate equations of the lines are

$(\operatorname{cosec}\alpha - \cot\alpha)x + y = 0$ and $(\operatorname{cosec}\alpha + \cot\alpha)x + y = 0$.

(vii) $x^2 + 2xy\tan\alpha - y^2 = 0$

Solution:

$x^2 + 2xy\tan\alpha - y^2 = 0$

Dividind by y^2

$$\left(\frac{x}{y}\right)^2 + 2\left(\frac{x}{y}\right)\tan\alpha - 1 = 0$$

$$\therefore \frac{x}{y} = \frac{-2\tan\alpha \pm \sqrt{4\tan^2\alpha - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{-2\tan\alpha \pm 2\sqrt{\tan^2\alpha - 1}}{2}$$

= - tan α ± sec α

$$\left(\frac{x}{y}\right) = (\sec\alpha - \tan\alpha) \text{ and}$$

$$\left(\frac{x}{y}\right) = -(\tan\alpha + \sec\alpha)$$

The separate equations of the lines are
(sec∝ – tan ∝)x + y = 0 and (sec ∝ + tan ∝)x – y = 0

Question 3.
Find the combined equation of a pair of lines passing through the origin and perpendicular to the lines represented by following equations :
(i) $5x^2 - 8xy + 3y^2 = 0$
Solution:
Comparing the equation $5x^2 - 8xy + 3y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get,
a = 5, 2h = -8, b = 3
Let m_1 and m_2 be the slopes of the lines represented by $5x^2 - 8xy + 3y^2 = 0$.
$\therefore m_1 + m_2 = \frac{-2h}{b} = \frac{8}{3}$
amd $m_1m_2 = \frac{a}{b} = \frac{5}{3}$ ...(1)

amd $m_1m_2 = \frac{a}{b} = \frac{5}{3}$ …(1)
Now required lines are perpendicular to these lines
∴ their slopes are $-1/m_1$ and $-1/m_2$ Since these lines are passing through the origin, their separate equations are
y = $\frac{-1}{m_1}$x and y = $\frac{-1}{m_2}$x
i.e. $m_1y = -x$ and $m_2y = -x$
i.e. $x + m_1y = 0$ and $x + m_2y = 0$
∴ their combined equation is
$(x + m_1y)(x + m_2y) = 0$
∴ $x^2 + (m_1 + m_2)xy + m_1m_2y^2 = 0$
∴ $x^2 + \frac{8}{3}xy + \frac{5}{3}y^2 = 0$ … [By (1)]
∴ $x^2 + 8xy + 5y\frac{8}{3} = 0$

(ii) $5x^2 + 2xy – 3y^2 = 0$
Solution:
Comparing the equation $5x^2 + 2xy – 3y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get,
a = 5, 2h = 2, b = -3
Let m_1 and m_2 be the slopes of the lines represented by $5x^2 + 2xy – 3y^2 = 0$
∴ $m_1 + m_2 = \frac{-2h}{b} = \frac{-2}{-3} = \frac{2}{3}$ and $m_1m_2 = \frac{a}{b} = \frac{5}{-3}$ ..(1)
Now required lines are perpendicular to these lines
∴ their slopes are $\frac{-1}{m_1}$ and $\frac{-1}{m_2}$
Since these lines are passing through the origin, their separate equations are
y = $\frac{-1}{m_1}$x and y = $\frac{-1}{m_2}$x
i.e. $m_1y = -x$ amd $m_2y = -x$
i.e. $x + m_1y = 0$ and $x + m_2y = 0$
∴ their combined equation is

∴ $(x + m_1y)(x + m_2y) = 0$

$x^2 + (m_1 + m_2)xy + m_1m_2y^2 = 0$

∴ $x^2 + \frac{2}{3}xy - \frac{5}{3}y = 0$ …[By (1)]

∴ $3x^2 + 2xy - 5y^2 = 0$

(iii) $xy + y^2 = 0$

Solution:

Comparing the equation $xy + y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get,

$a = 0, 2h = 1, b = 1$

Let m_1 and m_2 be the slopes of the lines represented by $xy + y^2 = 0$

$$\left.\begin{aligned} \therefore\ m_1 + m_2 &= \frac{-2h}{b} = \frac{-1}{1} = -1 \\ \text{and } m_1m_2 &= \frac{a}{b} = \frac{0}{1} = 0 \end{aligned}\right\} \quad \ldots (1)$$

Now required lines are perpendicular to these lines

∴ their slopes are $\frac{-1}{m_1}$ and $\frac{-1}{m_2}$.

Since these lines are passing through the origin, their separate equations are

$y = \frac{-1}{m_1}x$ and $y = \frac{-1}{m_2}x$

i.e. $m_1y = -x$ and $m_2y = -x$

i.e. $x + m_1y = 0$ and $x + m_2y = 0$

∴ their combined equation is

$(x + m_1y)(x + m_2y) = 0$

∴ $x^2 + (m_1 + m_2)xy + m_1m_2y^2 = 0$

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∴ their combined equation is

$(x + m_1y)(x + m_2y) = 0$

$\therefore x^2 + (m_1 + m_2)xy + m_1m_2y^2 = 0$

$\therefore x^2 - xy = 0.y^2 = 0$ … [By (1)]

$\therefore x^2 - xy = 0.$

Alternative Method :

Consider $xy + y^2 = 0$

$\therefore y(x + y) = 0$

∴ separate equations of the lines are y = 0 and

$3x^2 + 8xy + 5y^2 = 0.$

$x + y = 0.$

Let m_1 and m_2 be the slopes of these lines.

Then $m_1 = 0$ and $m_2 = -1$

Now, required lines are perpendicular to these lines.

∴ their slopes are $-\frac{1}{m_1}$ and $-\frac{1}{m_2}$

Since, $m_1 = 0$, $-\frac{1}{m_1}$ does not exist.

Also, $m_2 = -1$, $-\frac{1}{m_2} = 1$

Since these lines are passing through the origin, their separate equations are x = 0 and y = x,

i.e. $x - y = 0$

∴ their combined equation is

$x(x - y) = 0$

$x^2 - xy = 0.$

(iv) $3x^2 - 4xy = 0$

Solution:

Consider $3x^2 - 4xy = 0$

$\therefore x(3x - 4y) = 0$

Consider $3x^2 - 4xy = 0$
$\therefore x(3x - 4y) = 0$
$\therefore$ separate equations of the lines are $x = 0$ and $3x - 4y = 0$.
Let m_1 and m_2 be the slopes of these lines.
Then m_1 does not exist and and $m_1 = \frac{3}{4}$.
Now, required lines are perpendicular to these lines.
$\therefore$ their slopes are $-\frac{1}{m_1}$ and $-\frac{1}{m_2}$.
Since m_1 does not exist, $-\frac{1}{m_1} = 0$
Also $m_2 = \frac{3}{4'} - \frac{1}{m_2} = -\frac{4}{3}$
Since these lines are passing through the origin, their separate equations are $y = 0$ and $y = -\frac{4}{3}x$,
i.e. $4x + 3y = 0$
$\therefore$ their combined equation is
$y(4x + 3y) = 0$
$\therefore 4xy + 3y^2 = 0$.

Question 4.
Find k if,
(i) the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product.
Solution:
Comparing the equation $x^2 + kxy - 3y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get, $a = 1$, $2h = k$, $b = -3$.
Let m_1 and m_2 be the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$.
$\therefore m_1 + m_2 = \frac{-2h}{b} = -\frac{k}{(-3)} = \frac{k}{3}$
and $m_1m_2 = \frac{a}{b} = \frac{1}{(-3)} = -\frac{1}{3}$
Now, $m_1 + m_2 = 2(m_1m_2)$ ..(Given)

Now, $m_1 + m_2 = 2(m_1m_2)$ ..(Given)
$\therefore \frac{k}{3} = 2\left(-\frac{1}{3}\right)$ ∴ k = -2

(ii) slopes of lines represent by $3x^2 + kxy - y^2 = 0$ differ by 4.
Solution:
(ii) Comparing the equation $3x^2 + kxy - y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get, a = 3, 2h = k, b = -1.
Let m_1 and m_2 be the slopes of the lines represented by $3x^2 + kxy - y^2 = 0$.
$\therefore m_1 + m_2 = \frac{-2h}{b} = -\frac{k}{-1} = k$
and $m_{12} = \frac{a}{b} = \frac{3}{-1} = -3$
$\therefore (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$
$= k^2 - 4(-3)$
$= k^2 + 12$ … (1)
But $|m_1 - m_2| = 4$
$\therefore (m_1 - m_2)^2 = 16$ … (2)
∴ from (1) and (2), $k^2 + 12 = 16$
$\therefore k^2 = 4 \therefore k = \pm 2$.

(iii) slope of one of the lines given by $kx^2 + 4xy - y^2 = 0$ exceeds the slope of the other by 8.
Solution:
Comparing the equation $kx^2 + 4xy - y^2 = 0$ with $^2 + 2hxy + by^2 = 0$, we get, a = k, 2h = 4, b = -1. Let m_1 and m_2 be the slopes of the lines represented by $kx^2 + 4xy - y^2 = 0$.
$\therefore m_1 + m_2 = \frac{-2h}{b} = \frac{-4}{-1} = 4$
and $m_1m_2 = \frac{a}{b} = \frac{k}{-1} = -k$
We are given that $m_2 = m_1 + 8$
$m_1 + m_1 + 8 = 4$
$\therefore 2m_1 = -4 \therefore m_1 = -2$ … (1)

Also, $m_1(m_1 + 8) = -k$
$(-2)(-2 + 8) = -k$ … [By(1)]
$\therefore (-2)(6) = -k$
$\therefore -12 = -k \therefore k = 12$.

Question 5.
Find the condition that :
(i) the line $4x + 5y = 0$ coincides with one of the lines given by $ax^2 + 2hxy + by^2 = 0$.
Solution:
The auxiliary equation of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is $bm^2 + 2hm + a = 0$.
Given that $4x + 5y = 0$ is one of the lines represented by $ax^2 + 2hxy + by^2 = 0$.
The slope of the line $4x + 5y = 0$ is $-\frac{4}{5}$.
$\therefore m = -\frac{4}{5}$ is a root of the auxiliary equation $bm^2 + 2hm + a = 0$.
$\therefore b\left(-\frac{4}{5}\right)^2 + 2h\left(-\frac{4}{5}\right) + a = 0$
$\therefore \frac{16b}{25} - \frac{8h}{5} + a = 0$
$\therefore 16b - 40h + 25a = 0$
$\therefore 25a + 16b = 40k$.
This is the required condition.

(ii) the line $3x + y = 0$ may be perpendicular to one of the lines given by $ax^2 + 2hxy + by^2 = 0$.
Solution:
The auxiliary equation of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is $bm^2 + 2hm + a = 0$.
Since one line is perpendicular to the line $3x + y = 0$

whose slope is $-\frac{3}{1}$ = -3

∴ slope of that line = m = $\frac{1}{3}$

∴ m = $\frac{1}{3}$ is the root of the auxiliary equation $bm^2 + 2hm + a = 0$.

∴ $b\left(\frac{1}{3}\right)^2 + 2h\left(\frac{1}{3}\right) + a = 0$

∴ $\frac{b}{9} + \frac{2h}{3} + a = 0$

∴ b + 6h + 9a = 0

∴ 9a + b + 6h = 0

This is the required condition.

Question 6.

If one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is perpendicular to px + qy = 0 then show that $ap^2 + 2hpq + bq^2 = 0$.

Solution:

To prove $ap^2 + 2hpq + bq^2 = 0$.

Let the slope of the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ be m_1 and m_2

Then, $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

Slope of the line px + qy = 0 is $\frac{-p}{q}$

But one of the lines of $ax^2 + 2hxy + by^2 = 0$ is perpendicular to px + qy = 0

$\Rightarrow m_1 = \frac{q}{p}$

Now, $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

$\Rightarrow \frac{q}{p} + m_2 = \frac{-2h}{b}$ and $\left(\frac{q}{p}\right) m_2 = \frac{a}{b}$

$\Rightarrow \frac{q}{p} + m_2 = \frac{-2h}{b}$ and $m_2 = \frac{ap}{bq}$

$\Rightarrow \frac{q}{p} + \frac{ap}{bq} = \frac{-2h}{b}$

$\Rightarrow \frac{bq^2 + ap^2}{pq} = -2h$

$\Rightarrow bq^2 + ap^2 = -2hpq$

$\Rightarrow ap^2 + 2hpq + bq^2 = 0$

Question 7.

Find the combined equation of the pair of lines passing through the origin and making an equilateral triangle with the line y = 3.

Solution:

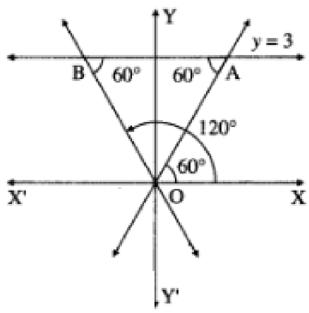
Let OA and OB be the lines through the origin making.an angle of 60° with the line y = 3.

∴ OA and OB make an angle of 60° and 120° with the positive direction of X-axis.

∴ slope of OA = tan60° = $\sqrt{3}$

∴ equation of the line OA is

$y = \sqrt{3}x$, i.e. $\sqrt{3}x - y = 0$



Slope of OB = tan 120° = tan (180° – 60°)
= -tan 60°= $-\sqrt{3}$
∴ equation of the line OB is
y = $-\sqrt{3}$ x, i.e. $\sqrt{3}$ x + y = 0
∴ required joint equation of the lines is
$(\sqrt{3}x - y)(\sqrt{3}x + y) = 0$
i.e. $3x^2 - y^2 = 0$.

Question 8.
If slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is four times the other then show that $16h^2 = 25ab$.
Solution:
Let m_1 and m_2 be the slopes of the lines given by $ax^2 + 2hxy + by^2 = 0$.
∴ $m_1 + m_2 = -\frac{2h}{b}$
and $m_1m_2 = \frac{a}{b}$
We are given that $m_2 = 4m_1$

$$\therefore \; m_1 + 4m_1 = -\frac{2h}{b}$$

$\therefore\ 5m_1 = \frac{-2h}{b}$

$\therefore\ m_1 = -\frac{2h}{5b}$ ... (1)

Also, $m_1(4m_1) = \frac{a}{b}$

$\therefore\ 4m_1^2 = \frac{a}{b}$

$\therefore\ m_1^2 = \frac{a}{4b}$

$\therefore\ \left(\frac{-2h}{5b}\right)^2 = \frac{a}{4b}$ ... [By (1)]

$\therefore\ \frac{4h^2}{25b^2} = \frac{a}{4b}$

$\therefore\ \frac{4h^2}{25b} = \frac{a}{4}$, as $b \neq 0$

$\therefore 16h^2 = 25ab$

This is the required condition.

Question 9.

If one of the lines given by $ax^2 + 2hxy + by^2 = 0$ bisects an angle between co-ordinate axes then show that $(a + b)^2 = 4h^2$.

Solution:

The auxiliary equation of the lines given by $ax^2 + 2hxy + by^2 = 0$ is $bm^2 + 2hm + a = 0$.

Since one of the line bisects an angle between the coordinate axes, that line makes an angle of 45° or 135° with the positive direction of X-axis.

$\therefore$ slope of that line = tan45° or tan 135°

$\therefore m = \tan 45° = 1$

or $m = \tan 135° = \tan(180° – 45°)$

$= -\tan 45° = -1$

$\therefore m = \pm 1$ are the roots of the auxiliary equation $bm^2 + 2hm + a = 0$.

$\therefore b(\pm 1)^2 + 2h(\pm 1) + a = 0$

$\therefore b \pm 2h + a = 0$

$\therefore a + b = \pm 2h$

$\therefore (a + b)^2 = 4h^2$

This is the required condition.

Ex 4.2

Question 1.
Show that lines represented by $3x^2 – 4xy – 3y^2 = 0$ are perpendicular to each other.
Solution:
Comparing the equation $3x^2 – 4 xy – 3y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get, a = 3, 2h = -4, b = -3
Since a + b = 3 + (-3) = 0, the lines represented by $3x^2 – 4xy – 3y^2 = 0$ are perpendicular to each other.

Question 2.
Show that lines represented by $x^2 + 6xy + gy^2= 0$ are coincident.
Question is modified.
Show that lines represented by $x^2 + 6xy + 9y^2= 0$ are coincident.
Solution:
Comparing the equation $x^2 + 6xy + 9y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get,
a = 1, 2h = 6, i.e. h = 3 and b = 9
Since $h^2 – ab = (3)^2 – 1(9)$
= 9 – 9 = 0, .
the lines represented by $x^2 + 6xy + 9y^2 = 0$ are coincident.

Question 3.
Find the value of k if lines represented by $kx^2 + 4xy – 4y^2 = 0$ are perpendicular to each other.
Solution:
Comparing the equation $kx^2 + 4xy – 4y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get,
a = k, 2h = 4, b = -4
Since lines represented by $kx^2 + 4xy – 4y^2 = 0$ are perpendicular to each other,
a + b = 0
∴ k – 4 = 0 ∴ k = 4.

Question 4.
Find the measure of the acute angle between the lines represented by:
(i) $3x^2 – 4\sqrt{3}xy + 3y^2 = 0$
Solution:
Comparing the equation $3x^2 – 4\sqrt{3}xy + 3y^2 = 0$ with
$ax^2 + 2hxy + by^2 = 0$, we get,

a = 3, 2h = -4$\sqrt{3}$, i.e. h = -24$\sqrt{3}$ and b = 3

Let θ be the acute angle between the lines.

$$\therefore \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$= \left|\frac{2\sqrt{(-2\sqrt{3})^2 - 3(3)}}{3+3}\right|$$

$$= \left|\frac{2\sqrt{12-9}}{6}\right| = \left|\frac{2\sqrt{3}}{6}\right|$$

$$\therefore \tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$$

∴ θ = 30°.

(ii) $4x^2 + 5xy + y^2 = 0$

Solution:

Comparing the equation $4x^2 + 5xy + y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get,

a = 4, 2h = 5, i.e. h = $\frac{5}{2}$ and b = 1.

Let θ be the acute angle between the lines.

$$\therefore \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$= \left| \frac{2\sqrt{\left(\frac{5}{2}\right)^2 - 4(1)}}{4+1} \right|$$

$$= \left| \frac{2\sqrt{\frac{25}{4} - 4}}{5} \right| = \left| \frac{2 \times \frac{3}{2}}{5} \right|$$

$$\therefore \tan\theta = \frac{3}{5}$$

$$\therefore \theta = \tan^{-1}\left(\frac{3}{5}\right).$$

(iii) $2x^2 + 7xy + 3y^2 = 0$
Solution:
Comparing the equation
$2x^2 + 7xy + 3y^2 = 0$ with
$ax^2 + 2hxy + by^2 = 0$, we get,
$a = 2$, $2h = 7$ i.e. $h = \frac{7}{2}$ and $b = 3$
Let θ be the acute angle between the lines.

$$\therefore \tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$$

$$= \left| \frac{2\sqrt{\left(\frac{7}{2}\right)^2 - 2(3)}}{2+3} \right|$$

$= \left|\frac{2\sqrt{\left(\frac{49}{4}\right)-6}}{5}\right|$

$= \left|\frac{2\sqrt{\left(\frac{49-24}{4}\right)}}{5}\right|$

$= \left|\frac{2\sqrt{\left(\frac{25}{4}\right)}}{5}\right|$

$= \frac{2\times\left(\frac{5}{2}\right)}{5}$

$= \frac{5}{5}$

tanθ = 1
∴ θ = tan 1 = 45°
∴ θ = 45°

(iv) $(a^2 – 3b^2)x^2 + 8abxy + (b^2 – 3a^2)y^2 = 0$
Solution:
Comparing the equation
$(a^2 – 3b^2)x^2 + 8abxy + (b^2 – 3a^2)y^2 = 0$, with
$Ax^2 + 2Hxy + By^2 = 0$, we have,

$A = a^2 - 3b^2$, $H = 4ab$, $B = b^2 - 3a^2$.
$\therefore H^2 - AB = 16a^2b^2 - (a^2 - 3b^2)(b^2 - 3a^2)$
$= 16a^2b^2 + (a^2 - 3b^2)(3a^2 - b^2)$
$= 16a^2b^2 + 3a^4 - 10a^2b^2 + 3b^4$
$= 3a^4 + 6a^2b^2 + 3b^4$
$= 3(a^4 + 2a^2b^2 + b^4)$
$= 3(a^2 + b^2)^2$
$\therefore \sqrt{H^2 - AB} = \sqrt{3}(a^2 + b^2)$
Also, $A + B = (a^2 - 3b^2) + (b^2 - 3a^2)$
$= -2(a^2 + b^2)$
If θ is the acute angle between the lines, then
$\tan\theta = \left|\frac{2\sqrt{H^2-AB}}{A+B}\right| = \left|\frac{2\sqrt{3}(a^2+b^2)}{-2(a^2+b^2)}\right|$
$= \sqrt{3} = \tan 60°$
$\therefore \theta = 60°$

Question 5.
Find the combined equation of lines passing through the origin each of which making an angle of 30° with the line $3x + 2y - 11 = 0$
Solution:
The slope of the line $3x + 2y - 11 = 0$ is $m_1 = -\frac{3}{2}$.
Let m be the slope of one of the lines making an angle of 30° with the line $3x + 2y - 11 = 0$.
The angle between the lines having slopes m and m1 is 30°.

$$\therefore \tan 30° = \left|\frac{m - m_1}{1 + m \cdot m_1}\right|, \text{ where } \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{1}{\sqrt{3}} = \left|\frac{m-\left(-\frac{3}{2}\right)}{1+m\left(-\frac{3}{2}\right)}\right|$$

$$\therefore \frac{1}{\sqrt{3}} = \left|\frac{2m+3}{2-3m}\right|$$

On squaring both sides, we get,

$\frac{1}{3} = \frac{(2m+3)^2}{(2-3m)^2}$

$\therefore (2 - 3m)^2 = 3(2m + 3)^2$

$\therefore 4 - 12m + 9m^2 = 3(4m^2 + 12m + 9)$

$\therefore 4 - 12m + 9m^2 = 12m^2 + 36m + 27$

$3m^2 + 48m + 23 = 0$

This is the auxiliary equation of the two lines and their joint equation is obtained by putting m = $\frac{y}{x}$.

∴ the combined equation of the two lines is

$3\left(\frac{y}{x}\right)^2 + 48\left(\frac{y}{x}\right) + 23 = 0$

$\therefore \frac{3y^2}{x^2} + \frac{48y}{x} + 23 = 0$

$\therefore 3y^2 + 48xy + 23x^2 = 0$

$\therefore 23x^2 + 48xy + 3y^2 = 0.$

Question 6.

If the angle between lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between lines represented by $2x^2 - 5xy + 3y^2 = 0$ then show that $100(h^2 - ab) = (a + b)^2$.

Solution:

The acute angle θ between the lines $ax^2 + 2hxy + by^2 = 0$ is given by

$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$ ..(1)

Comparing the equation $2x^2 - 5xy + 3y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get,

a = 2, 2h= -5, i.e. $h = -\frac{5}{2}$ and b = 3

Let ∝ be the acute angle between the lines $2x^2 - 5xy + 3y^2 = 0$.

$$\therefore\ \tan\alpha = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

$$= \left|\frac{2\sqrt{\left(\frac{5}{2}\right)^2 - 2(3)}}{2+3}\right|$$

$$= \left|\frac{2\sqrt{\frac{25}{4}-6}}{5}\right| = \left|\frac{2\times\frac{1}{2}}{5}\right|$$

$$\therefore\ \tan\alpha = \frac{1}{5} \qquad \ldots (2)$$

If $\theta = \alpha$, then $\tan\theta = \tan\alpha$

$$\therefore\ \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \frac{1}{5} \qquad \ldots \text{[By (1) and (2)]}$$

$$\therefore\ \frac{4(h^2-ab)}{(a+b)^2} = \frac{1}{25} \quad \therefore\ 100(h^2-ab) = (a+b)^2$$

This is the required condition.

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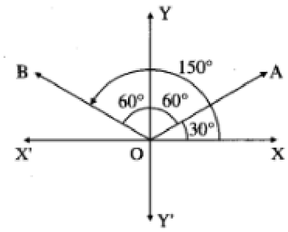
Question 7.
Find the combined equation of lines passing through the origin and each of which making angle 60° with the Y- axis.
Solution:



Let OA and OB be the lines through the origin making an angle of 60° with the Y-axis.
Then OA and OB make an angle of 30° and 150° with the positive direction of X-axis.
∴ slope of OA = tan 30° = $\frac{1}{\sqrt{3}}$
∴ equation of the line OA is
y = $\frac{1}{\sqrt{3}}$ = x, i.e. $x - \sqrt{3}y = 0$
Slope of OB = tan 150° = tan (180° – 30°)
= tan 30° = $-\frac{1}{\sqrt{3}}$
∴ equation of the line OB is
y = $-\frac{1}{\sqrt{3}}x$, i.e. $x + \sqrt{3}\,y = 0$
∴ required combined equation is
$(x - \sqrt{3}y)(x + \sqrt{3}y) = 0$
i.e. $x^2 - 3y^2 = 0$.

Ex 4.3

Question 1.

Find the joint equation of the pair of lines:

(i) Through the point (2, -1) and parallel to lines represented by $2x^2 + 3xy - 9y^2 = 0$

Solution:

The combined equation of the given lines is

$2x^2 + 3xy - 9y^2 = 0$

i.e. $2x^2 + 6xy - 3xy - 9y^2 = 0$

i.e. $2x(x + 3y) - 3y(x + 3y) = 0$

i.e. $(x + 3y)(2x - 3y) = 0$

∴ their separate equations are

$x + 3y = 0$ and $2x - 3y = 0$

∴ their slopes are $m_1 = \frac{-1}{3}$ and $m_2 = \frac{-2}{-3} = \frac{2}{3}$.

The slopes of the lines parallel to these lines are m_1 and m_2, i.e. $-\frac{1}{3}$ and $\frac{2}{3}$.

∴ the equations of the lines with these slopes and through the point (2, -1) are

$y + 1 = -\frac{1}{3}(x - 2)$ and $y + 1 = \frac{2}{3}(x - 2)$

i.e. $3y + 3 = -x + 2$ and $3y + 3 = 2x - 4$

i.e. $x + 3y + 1 = 0$ and $2x - 3y - 7 = 0$

∴ the joint equation of these lines is

$(x + 3y + 1)(2x - 3y - 7) = 0$

∴ $2x^2 - 3xy - 7x + 6xy - 9y^2 - 21y + 2x - 3y - 7 = 0$

∴ $2x^2 + 3xy - 9y^2 - 5x - 24y - 7 = 0$.

(ii) Through the point (2, -3) and parallel to lines represented by $x^2 + xy - y^2 = 0$

Solution:

Comparing the equation

$x^2 + xy - y^2 = 0$ … (1)

with $ax^2 + 2hxy + by^2 = 0$, we get,

a = 1, 2h = 1, b = -1

Let m_1 and m_2 be the slopes of the lines represented by (1).

$$\left.\begin{aligned} \text{Then } m_1 + m_2 &= -\frac{2h}{b} = \frac{-1}{-1} = 1 \\ \text{and } m_1 m_2 &= \frac{a}{b} = \frac{1}{-1} = -1 \end{aligned}\right\} \quad \ldots (2)$$

The slopes of the lines parallel to these lines are m_1 and m_2.

∴ the equations of the lines with these slopes and through the point (2, -3) are

$y + 3 = m_1(x - 2)$ and $y + 3 = m_2(x - 2)$

i.e. $m_1(x - 2) - (y + 3) = 0$ and $m_2(x - 2) - (y + 3) = 0$

∴ the joint equation of these lines is

$[m_1(x - 2) - (y + 3)][m_2(x - 2) - (y + 3)] = 0$

∴ $m_1m_2(x - 2)^2 - m_1(x - 2)(y + 3) - m_2(x - 2)(y + 3) + (y + 3)^2 = o$

∴ $m_1m_2(x - 2)^2 - (m_1 + m_2)(x - 2)(y + 3) + (y + 3)^3 = 0$

∴ $-(x - 2)^2 - (x - 2)(y + 3) + (y + 3)^2 = 0$ …… [By (2)]

∴ $(x - 2)^2 + (x - 2)(y + 3) - (y + 3)^2 = 0$

∴ $(x^2 - 4x + 4) + (xy + 3x - 2y - 6) - (y^2 + 6y + 9) = 0$

∴ $x^2 - 4x + 4 + xy + 3x - 2y - 6 - y^2 - 6y - 9 = 0$

∴ $x^2 + xy - y^2 - x - 8y - 11 = 0$.

Question 2.
Show that equation $x^2 + 2xy + 2y^2 + 2x + 2y + 1 = 0$ does not represent a pair of lines.
Solution:
Comparing the equation
$x^2 + 2xy + 2y^2 + 2x + 2y + 1 = 0$ with
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get,
a = 1, h = 1, b = 2, g = 1, f = 1, c = 1.
The given equation represents a pair of lines, if

D = $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ = 0 and $h^2 - ab \geq 0$

Now, D = $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

= 1 (2 – 1) – 1(1 – 1) + 1 (1 – 2)
= 1 – 0 – 1 = 0
and $h^2 - ab = (1)^2 - 1(2) = -1 < 0$
∴ given equation does not represent a pair of lines.

Question 3.
Show that equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of lines.
Solution:
Comparing the equation
$2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$
with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get,
a = 2, h = $-\frac{1}{2}$, b = -3, g = -3, f = $\frac{19}{2}$, c = -20.

$\therefore D = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 2 & -\frac{1}{2} & -3 \\ -\frac{1}{2} & -3 & \frac{19}{2} \\ -3 & \frac{19}{2} & -20 \end{vmatrix}$

Taking $\frac{1}{2}$ common from each row, we get,

$D = \frac{1}{8}\begin{vmatrix} 4 & -1 & -6 \\ -1 & -6 & 19 \\ -6 & 19 & -40 \end{vmatrix}$

$= \frac{1}{8}[4(240 - 361) + 1(40 + 114) - 6(-19 - 36)]$

$= \frac{1}{8}[4(-121) + 154 - 6(-55)]$

$= \frac{11}{8}[4(-11) + 14 - 6(-5)]$

$= \frac{1}{8}(-44 + 14 + 30) = 0$

Also $h^2 - ab = \left(-\frac{1}{2}\right)^2 - 2(-3) = \frac{1}{4} + 6 = \frac{25}{4} > 0$

$\therefore$ the given equation represents a pair of lines.

Question 4.

Show the equation $2x^2 + xy - y^2 + x + 4y - 3 = 0$ represents a pair of lines. Also find the acute angle between them.

Solution:

Comparing the equation

$2x^2 + xy — y^2 + x + 4y — 3 = 0$ with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c — 0$, we get,

$a = 2, h = \frac{1}{2}, b = -1, g = \frac{1}{2}, f = 2, c = -3$.

$\therefore$ D = $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 2 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -1 & 2 \\ \frac{1}{2} & 2 & -3 \end{vmatrix}$

Taking $\frac{1}{2}$ common from each row, we get,

D = $\frac{1}{8}\begin{vmatrix} 4 & 1 & 1 \\ 1 & -2 & 4 \\ 1 & 4 & -6 \end{vmatrix}$

= $\frac{1}{8}$[4(12 —16) — 1(—6 — 4) + 1(4 + 2)]

= $\frac{1}{8}$[4(– 4) – 1(-10) + 1(6)]

= $\frac{1}{8}$(—16 + 10 + 6) = 0

Also, $h^2 - ab = \left(\frac{1}{2}\right)^2 - 2(-1) = \frac{1}{4} + 2 = \frac{9}{4} > 0$

$\therefore$ the given equation represents a pair of lines. Let θ be the acute angle between the lines

$\therefore \tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

$$= \left|\frac{2\sqrt{\left(\frac{1}{2}\right)^2 - 2(-1)}}{2-1}\right|$$

$$= \left|\frac{2\sqrt{\frac{1}{4}+2}}{1}\right| = 2 \times \frac{3}{2} = 3$$

$\therefore \theta = \tan^{-1}(3).$

Question 5.

Find the separate equation of the lines represented by the following equations :

(i) $(x-2)^2 - 3(x-2)(y+1) + 2(y+1)^2 = 0$

Solution:

$(x-2)^2 - 3(x-2)(y+1) + 2(y+1)2 = 0$

$\therefore (x-2)^2 - 2(x-2)(y+1) - (x-2)(y+1) + 2(y+1)^2 = 0$

$\therefore (x-2)\,[(x-2) - 2(y+1)] - (y+1)[(x-2) - 2(y+1)] = 0$

$\therefore (x-2)(x-2-2y-2) - (y+1)(x-2-2y-2) = 0$

$\therefore (x-2)(x-2y-4) - (y+1)(x-2y-4) = 0$

$\therefore (x-2y-4)(x-2-y-1) = 0$

$\therefore (x-2y-4)(x-y-3) = 0$

$\therefore$ the separate equations of the lines are

$x - 2y - 4 = 0$ and $x - y - 3 = 0$.

Alternative Method :

$(x-2)^2 - 3(x-2)(y+1) + 2(y+1)^2 = 0$ … (1)

Put $x - 2 = X$ and $y + 1 = Y$

$\therefore$ (1) becomes,

$X^2 - 3XY + 2Y^2 = 0$

$\therefore X^2 - 2XY - XY + 2Y^2 = 0$

$\therefore X(X-2Y) - Y(X-2Y) = 0$

$\therefore (X-2Y)(X-Y) = 0$

$\therefore$ the separate equations of the lines are

$\therefore X - 2Y = 0$ and $X - Y = 0$

$\therefore (x-2) - 2(y+1) = 0$ and $(x-2) - (y+1) = 0$

$\therefore x - 2y - 4 = 0$ and $x - y - 3 = 0$.

(ii) $10(x + 1)^2 + (x + 1)(y - 2) - 3(y - 2)^2 = 0$
Solution:
$10(x + 1)^2 + (x + 1)(y - 2) - 3(y - 2)^2 = 0$ ...(1)
Put $x + 1 = X$ and $y - 2 = Y$
$\therefore$ (1) becomes
$10x^2 + xy - 3y^2 = 0$
$10x^2 + 6xy - 5xy - 3y^2 = 0$
$2x(5x + 3y) - y(5x + 3y) = 0$
$(2x - y)(5x + 3y) = 0$
$5x + 3y = 0$ and $2x - y = 0$
$5x + 3y = 0$
$5(x + 1) + 3(y - 2) = 0$
$5x + 5 + 3y - 6 = 0$
$\therefore 5x + 3y - 1 = 0$
$2x - y = 0$
$2(x + 1) - (y - 2) = 0$
$2x + 2 - y + 2 = 0$
$\therefore 2x - y + 4 = 0$

Question 6.
Find the value of k if the following equations represent a pair of lines :
(i) $3x^2 + 10xy + 3y^2 + 16y + k = 0$
Solution:
Comparing the given equation with
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$,
we get, $a = 3$, $h = 5$, $b = 3$, $g = 0$, $f = 8$, $c = k$.
Now, given equation represents a pair of lines.
$\therefore abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

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$\therefore$ (3)(3)(k) + 2(8)(0)(5) – 3(8)^2 – 3(0)^2 – k(5)^2 = 0
$\therefore$ 9k + 0 – 192 – 0 – 25k = 0
$\therefore$ -16k – 192 = 0
$\therefore$ – 16k = 192
$\therefore$ k= -12.

(ii) kxy + 10x + 6y + 4 = 0
Solution:
Comparing the given equation with
ax^2 + 2 hxy + by^2 + 2gx + 2fy + c = 0,
we get, a = 0, h = $\frac{k}{2}$, b = 0, g = 5, f = 3, c = 4
Now, given equation represents a pair of lines.
$\therefore$ abc + 2fgh – af^2 – bg^2 – ch^2 = 0
$\therefore$ (0)(0)(4) + 2(3)(5)$\left(\frac{k}{2}\right)$ – 0(3)^2 – 0(5)^2 – 4$\left(\frac{k}{2}\right)^2$ = 0
$\therefore$ 0 + 15k – 0 – 0 – k^2 = 0
$\therefore$ 15k – k^2 = 0
$\therefore$ -k(k – 15) = 0
$\therefore$ k = 0 or k = 15.
If k = 0, then the given equation becomes
10x + 6y + 4 = 0 which does not represent a pair of lines.
$\therefore$ k ≠ o
Hence, k = 15.

(iii) x^2 + 3xy + 2y^2 + x – y + k = 0

Comparing the given equation with
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$,
we get, $a = 1$, $h = \frac{3}{2}$, $b = 2$, $g = \frac{1}{2}$, $f = -\frac{1}{2}$, $c = k$.
Now, given equation represents a pair of lines.

$$\therefore \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$$\text{i.e.} \begin{vmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ \frac{3}{2} & 2 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & k \end{vmatrix} = 0$$

Taking out $\frac{1}{2}$ common from each row, we get,

$$\frac{1}{8} \begin{vmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 1 & -1 & 2k \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 1 & -1 & 2k \end{vmatrix} = 0$$

$\therefore 2(8k - 1) - 3(6k + 1) + 1(-3 - 4) = 0$
$\therefore 16k - 2 - 18k - 3 - 7 = 0$

∴ -2k – 12 = 0
∴ -2k = 12 ∴ k = -6.

Question 7.
Find p and q if the equation $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$ represents a pair of perpendicular lines.
Solution:
The given equation represents a pair of lines perpendicular to each other
∴ (coefficient of x^2) + (coefficient of y^2) = 0
∴ p + 3 = 0 p = -3
With this value of p, the given equation is
$-3x^2 - 8xy + 3y^2 + 14x + 2y + q = 0$.
Comparing this equation with
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we have,
a = -3, h = -4, b = 3, g = 7, f = 1 and c = q.

$$D = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} -3 & -4 & 7 \\ -4 & 3 & 1 \\ 7 & 1 & q \end{vmatrix}$$

= -3(3q – 1) + 4(-4q – 7) + 7(-4 – 21)
= -9q + 3 – 16q – 28 – 175
= -25q – 200= -25(q + 8)
Since the given equation represents a pair of lines, D = 0
∴ -25(q + 8) = 0 ∴ q= -8.
Hence, p = -3 and q = -8.

Question 8.
Find p and q if the equation $2x^2 + 8xy + py^2 + qx + 2y - 15 = 0$ represents a pair of parallel lines.
Solution:
The given equation is
$2x^2 + 8xy + py^2 + qx + 2y - 15 = 0$
Comparing it with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get,
$a = 2, h = 4, b = p, g = \frac{q}{2}, f = 1, c = -15$
Since the lines are parallel, $h^2 = ab$
$\therefore (4)^2 = 2p \therefore P = 8$
Since the given equation represents a pair of lines

$$D=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0, \text{ where } b=p=8$$

$$\text{i.e.} \begin{vmatrix} 2 & 4 & q/2 \\ 4 & 8 & 1 \\ q/2 & 1 & -15 \end{vmatrix}=0$$

$$\text{i.e, } 2(-120-1)-4\left(-60-\frac{q}{2}\right)+\frac{q}{2}(4-4q)=0$$

i.e. $-242 + 240 + 2q + 2q - 2q^2 = 0$
i.e. $-2q^2 + 4q - 2 = 0$
i.e. $q^2 - 2q + 1 = 0$
i.e. $(q - 1)^2 = 0 \therefore q - 1 = 0 \therefore q = 1$.
Hence, p = 8 and q = 1.

Question 9.
Equations of pairs of opposite sides of a parallelogram are $x^2 - 7x + 6 = 0$ and $y^2 - 14y + 40 = 0$. Find the joint equation of its diagonals.
Solution:

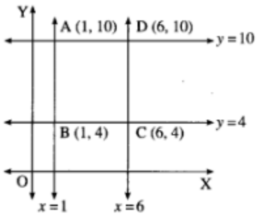
Let ABCD be the parallelogram such that the combined equation of sides AB and CD is $x^2 - 7x + 6 = 0$ and the combined equation of sides BC and AD is $y^2 - 14y + 40 = 0$.

The separate equations of the lines represented by $x^2 - 7x + 6 = 0$, i.e. $(x - 1)(x - 6) = 0$ are $x - 1 = 0$ and $x - 6 = 0$.

Let equation of the side AB be $x - 1 = 0$ and equation of side CD be $x - 6 = 0$.

The separate equations of the lines represented by $y^2 - 14y + 40 = 0$, i.e. $(y - 4)(y - 10) = 0$ are $y - 4 = 0$ and $y - 10 = 0$.

Let equation of the side BC be $y - 4 = 0$ and equation of side AD be $y - 10 = 0$.



Coordinates of the vertices of the parallelogram are A(1, 10), B(1, 4), C(6, 4) and D(6, 10).

∴ equation of the diagonal AC is

$\frac{y-10}{x-1} = \frac{10-4}{1-6} = \frac{6}{-5}$

∴ $-5y + 50 = 6x - 6$

∴ $6x + 5y - 56 = 0$

and equation of the diagonal BD is

$\frac{y-4}{x-1} = \frac{4-10}{1-6} = \frac{-6}{-5} = \frac{6}{5}$

∴ $5y - 20 = 6x - 6$

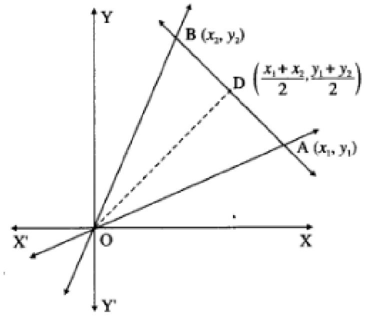
∴ $6x - 5y + 14 = 0$

Hence, the equations of the diagonals are $6x + 5y - 56 = 0$ and $6x - 5y + 14 = 0$.

∴ the joint equation of the diagonals is $(6x + 5y - 56)(6x - 5y + 14) = 0$

∴ $36x^2 - 30xy + 84x + 30xy - 25y^2 + 70y - 336x + 280y - 784 = 0$

Question 10.
ΔOAB is formed by lines $x^2 - 4xy + y^2 = 0$ and the line $2x + 3y - 1 = 0$. Find the equation of the median of the triangle drawn from O.
Solution:



Let D be the midpoint of seg AB where A is (x_1, y_1) and B is (x_2, y_2).
Then D has coordinates $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
The joint (combined) equation of the lines OA and OB is $x^2 - 4xy + y^2 = 0$ and the equation of the line AB is $2x + 3y - 1 = 0$.
$\therefore$ points A and B satisfy the equations $2x + 3y - 1 = 0$
and $x^2 - 4xy + y^2 = 0$ simultaneously.
We eliminate x from the above equations, i.e.,
put $x = \frac{1-3y}{2}$ in the equation $x^2 - 4xy + y^2 = 0$, we get,
$\therefore \left(\frac{1-3y}{2}\right)^2 - 4\left(\frac{1-3y}{2}\right)y + y^2 = 0$
$\therefore (1 - 3y)^2 - 8(1 - 3y)y + 4y^2 = 0$
$\therefore 1 - 6y + 9y^2 - 8y + 24y^2 + 4y^2 = 0$

∴ $37y^2 - 14y + 1 = 0$
The roots y_1 and y_2 of the above quadratic equation are the y-coordinates of the points A and B.
∴ $y_1 + y_2 = \frac{-b}{a} = \frac{14}{37}$
∴ y-coordinate of D = $\frac{y_1+y_2}{2} = \frac{7}{37}$.
Since D lies on the line AB, we can find the x-coordinate of D as
$2x + 3\left(\frac{7}{37}\right) - 1 = 0$
∴ $2x = 1 - \frac{21}{37} = \frac{16}{37}$
∴ $x = \frac{8}{37}$
∴ D is (8/37, 7/37)
∴ equation of the median OD is $\frac{x}{8/37} = \frac{y}{7/37}$,
i.e., $7x - 8y = 0$.

Question 11.
Find the co-ordinates of the points of intersection of the lines represented by $x^2 - y^2 - 2x + 1 = 0$.
Solution:
Consider, $x^2 - y^2 - 2x + 1 = 0$
∴ $(x^2 - 2x + 1) - y^2 = 0$
∴ $(x - 1)^2 - y^2 = 0$
∴ $(x - 1 + y)(x - 1 - y) = 0$
∴ $(x + y - 1)(x - y - 1) = 0$
∴ separate equations of the lines are
$x + y - 1 = 0$ and $x - y + 1 = 0$.
To find the point of intersection of the lines, we have to solve

$x + y - 1 = 0$ … (1)
and $x - y + 1 = 0$ … (2)
Adding (1) and (2), we get,
$2x = 0$ ∴ $x = 0$
Substituting x = 0 in (1), we get,
$0 + y - 1 = 0$ ∴ $y = 1$
∴ coordinates of the point of intersection of the lines are (0, 1).

Maharashtra Board Solutions Class 12 Arts & Science Maths (Part 1)

About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

FAQs

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