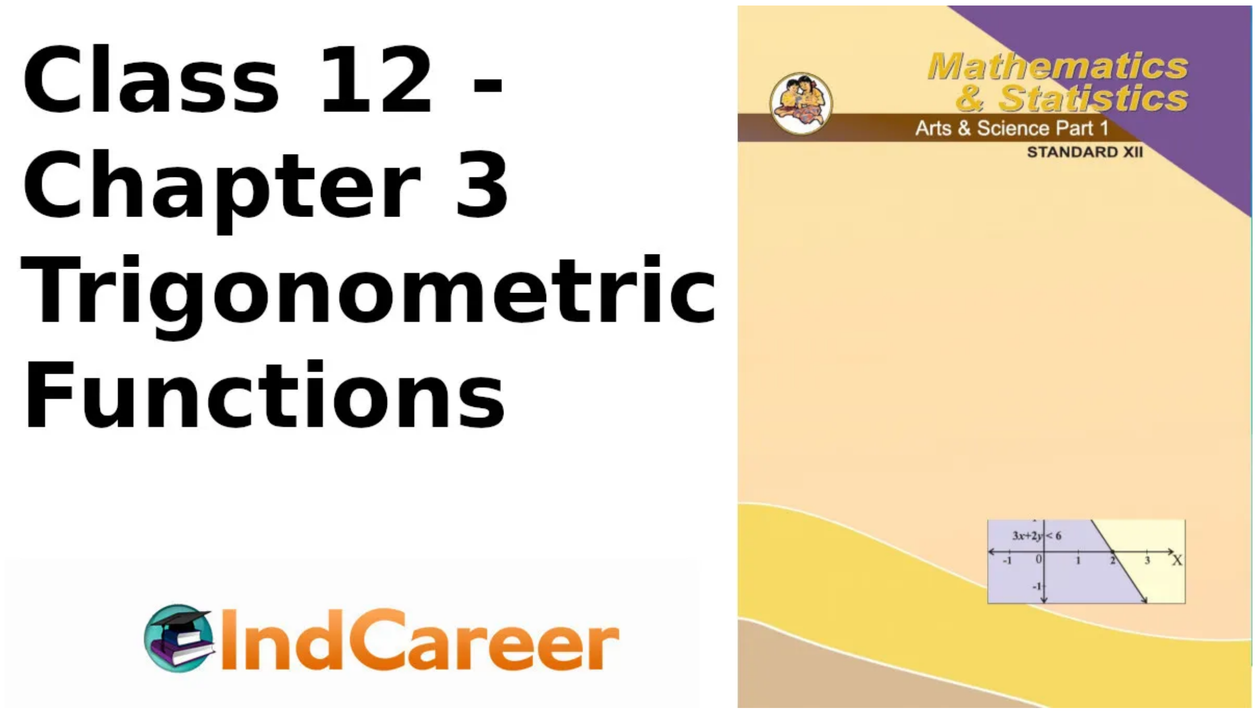
Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 3- Trigonometric Functions



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Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 3- Trigonometric Functions

Class 12: Maths Chapter 3 solutions. Complete Class 12 Maths Chapter 3 Notes.

Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 3- Trigonometric Functions

Maharashtra Board 12th Maths Chapter 3, Class 12 Maths Chapter 3 solutions

Ex 3.1

Question 1.

Find the principal solutions of the following equations :

(i) cos θ= $\frac{1}{2}$

Solution:

We know that, cos$\frac{\pi}{3}$ = $\frac{1}{2}$ and cos (2π – θ) = cos θ

∴ cos$\frac{\pi}{3}$ = cos(2π – $\frac{\pi}{3}$) = cos$\frac{5\pi}{3}$

∴ cos$\frac{\pi}{3}$ = cos$\frac{5\pi}{3}$ = $\frac{1}{2}$, where

0 < $\frac{\pi}{3}$ < 2π and 0 < $\frac{5\pi}{3}$ < 2π

∴ cos θ = $\frac{1}{2}$ gives cos θ = cos$\frac{\pi}{3}$ = cos$\frac{5\pi}{3}$

∴ θ = $\frac{\pi}{3}$ and θ = $\frac{5\pi}{3}$

Hence, the required principal solutions are

θ = $\frac{\pi}{3}$ and θ = $\frac{5\pi}{3}$

(ii) sec θ = $\frac{2}{\sqrt{3}}$

Solution:

(iii) cot θ = $\sqrt{3}$

Solution:

The given equation is cot θ = $\sqrt{3}$ which is same as tan θ = $\frac{1}{\sqrt{3}}$.

We know that,

$$\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}} \text{ and } \tan(\pi+\theta)=\tan\theta$$

$$\therefore \tan\frac{\pi}{6}=\tan\left(\pi+\frac{\pi}{6}\right)=\tan\frac{7\pi}{6}$$

$$\therefore \tan\frac{\pi}{6}=\tan\frac{7\pi}{6}=\frac{1}{\sqrt{3}}, \text{ where}$$

$$0<\frac{\pi}{6}<2\pi \text{ and } 0<\frac{7\pi}{6}<2\pi$$

$$\therefore \cot\theta=\sqrt{3}, \text{ i.e. } \tan\theta=\frac{1}{\sqrt{3}} \text{ gives}$$

$$\tan\theta=\tan\frac{\pi}{6}=\tan\frac{7\pi}{6}$$

$$\therefore \theta=\frac{\pi}{6} \text{ and } \theta=\frac{7\pi}{6}$$

Hence, the required principal solution are

θ = $\frac{\pi}{6}$ and θ = $\frac{7\pi}{6}$.

(iv) cot θ = 0.
Solution:

Question 2.
Find the principal solutions of the following equations:
(i) sinθ = $-\frac{1}{2}$
Solution:
We know that,
sin$\frac{\pi}{6}$ = $\frac{1}{2}$ and sin (π + θ) = -sinθ,
sin(2π – θ) = -sinθ

$$\therefore \sin\left(\pi+\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$$

$$\text{and } \sin\left(2\pi-\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$$

$$\therefore \sin\frac{7\pi}{6}=\sin\frac{11\pi}{6}=-\frac{1}{2}, \text{ where}$$

$$0<\frac{7\pi}{6}<2\pi \text{ and } 0<\frac{11\pi}{6}<2\pi$$

$$\therefore \sin\theta=-\frac{1}{2} \text{ gives,}$$

$$\sin\theta=\sin\frac{7\pi}{6}=\sin\frac{11\pi}{6}$$

$$\therefore \theta=\frac{7\pi}{6} \text{ and } \theta=\frac{11\pi}{6}$$

Hence, the required principal solutions are
θ = $\frac{7\pi}{6}$ and θ = $\frac{11\pi}{6}$.

(ii) tanθ = -1

Solution:

We know that,

tan$\frac{\pi}{4}$ = 1 and tan(π – θ) = -tanθ,

tan(2π – θ) = -tanθ

$$\therefore \tan\left(\pi-\frac{\pi}{4}\right)=-\tan\frac{\pi}{4}=-1$$

$$\text{and } \tan\left(2\pi-\frac{\pi}{4}\right)=-\tan\frac{\pi}{4}=-1$$

$$\therefore \tan\frac{3\pi}{4}=\tan\frac{7\pi}{4}=-1, \text{ where}$$

$$0<\frac{3\pi}{4}<2\pi \text{ and } 0<\frac{7\pi}{4}<2\pi$$

$$\therefore \tan\theta=-1 \text{ gives,}$$

$$\tan\theta=\tan\frac{3\pi}{4}=\tan\frac{7\pi}{4}$$

$$\therefore \theta=\frac{3\pi}{4} \text{ and } \theta=\frac{7\pi}{4}$$

Hence, the required principal solutions are

θ = $\frac{3\pi}{4}$ and θ = $\frac{7\pi}{4}$.

(iii) $\sqrt{3}$ cosecθ + 2 = 0.

Solution:

Question 3.
Find the general solutions of the following equations :
(i) $\sin\theta = \frac{1}{2}$
Solution:
(i) The general solution of $\sin\theta = \sin\alpha$ is
$\theta = n\pi + (-1)^n\alpha, n \in Z$
Now, $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$ ...[$\because \sin\frac{\pi}{6} = \frac{1}{2}$]
∴ the required general solution is
$\theta = n\pi + (-1)^n\frac{\pi}{6}, n \in Z.$

(ii) $\cos\theta = \frac{\sqrt{3}}{2}$
Solution:
The general solution of $\cos\theta = \cos\alpha$ is
$\theta = 2n\pi \pm \alpha, n \in Z$
Now, $\cos\theta = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$ ...[$\because \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$]
∴ the required general solution is
$\theta = 2n\pi \pm \frac{\pi}{6}, n \in Z.$

(iii) $\tan\theta = \frac{1}{\sqrt{3}}$
Solution:
The general solution of $\tan\theta = \tan\alpha$ is
$\theta = n\pi + \alpha, n \in Z$
Now, $\tan\theta = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$ ...[$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$]

Now, tan θ = $\frac{1}{\sqrt{3}}$ = tan $\frac{\pi}{6}$ ...[tan $\frac{\pi}{6}$ = $\frac{1}{\sqrt{3}}$]
∴ the required general solution is
θ = nπ + $\frac{\pi}{6}$, n ∈ Z.

(iv) cotθ = 0.
Solution:
The general solution of tan θ = tan ∝ is
θ = nπ + ∝, n ∈ Z
Now, cot θ = 0 ∴ tan θ does not exist
∴ tanθ = tan $\frac{\pi}{2}$ [∵ tan $\frac{\pi}{2}$ does not exist]
∴ the required general solution is
θ = nπ + $\frac{\pi}{2}$, n ∈ Z.

Question 4.
Find the general solutions of the following equations:
(i) secθ = $\sqrt{2}$
Solution:
The general solution of cos θ = cos ∝ is
θ = nπ ± ∝, n ∈ Z.
Now, secθ = $\sqrt{2}$ ∴ cosθ = $\frac{1}{\sqrt{2}}$
∴ cosθ = cos $\frac{\pi}{4}$ ....[cos $\frac{\pi}{4}$ = $\frac{1}{\sqrt{2}}$]
∴ the required general solution is
θ = 2nπ ± $\frac{\pi}{4}$, n ∈ Z.

(ii) cosecθ = $-\sqrt{2}$

Solution:

The general solution of sinθ = sin∝ is

$\theta = n\pi + (-1)^n\alpha,\ n \in Z$

Now, $\operatorname{cosec}\theta = -\sqrt{2}$

$\therefore\ \sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore\ \sin\theta = -\sin\frac{\pi}{4}$ ... $\left[\because\ \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$

$\therefore\ \sin\theta = \sin\left(\pi + \frac{\pi}{4}\right)$ ... $[\because\ \sin(\pi + \theta) = -\sin\theta]$

$\therefore\ \sin\theta = \sin\frac{5\pi}{4}$

$\therefore$ the required general solution is

$\theta = n\pi + (-1)^n\left(\frac{5\pi}{4}\right),\ n \in Z.$

(iii) tanθ = -1

Solution:

The general solution of tanθ = tan∝ is

$\theta = n\pi + \alpha,\ n \in Z$

Now, $\tan\theta = -1$

$\therefore\ \tan\theta = -\tan\frac{\pi}{4}$ ... $\left[\because\ \tan\frac{\pi}{4} = 1\right]$

$\therefore\ \tan\theta = \tan\left(\pi - \frac{\pi}{4}\right)$ ... $[\because\ \tan(\pi - \theta) = -\tan\theta]$

$\therefore \tan\theta = \tan\frac{3\pi}{4}$

$\therefore$ the required general solution is

$\theta = n\pi + \frac{3\pi}{4}, n \in Z.$

Question 5.
Find the general solutions of the following equations :
(i) $\sin 2\theta = \frac{1}{2}$
Solution:
The general solution of $\sin\theta = \sin\propto$ is
$\theta = n\pi + (-1)^n \propto, n \in Z$

Now, $\sin 2\theta = \frac{1}{2}$

$\therefore \sin 2\theta = \sin\frac{\pi}{6}$ ... $\left[\because \sin\frac{\pi}{6} = \frac{1}{2}\right]$

$\therefore$ the required general solution is given by

$2\theta = n\pi + (-1)^n\left(\frac{\pi}{6}\right), n \in Z$

i.e. $\theta = \frac{n\pi}{2} + (-1)^n\left(\frac{\pi}{12}\right), n \in Z.$

(ii) $\tan\frac{2\theta}{3} = \sqrt{3}$
Solution:
The general solution of $\tan\theta = \tan\propto$ is

$\theta = n\pi + \alpha,\ n \in Z$

Now, $\tan\frac{2\theta}{3} = \sqrt{3}$

$\therefore\ \tan\frac{2\theta}{3} = \tan\frac{\pi}{3}$ ... $\left[\because\ \tan\frac{\pi}{3} = \sqrt{3}\right]$

$\therefore$ the required general solution is given by

$\frac{2\theta}{3} = n\pi + \frac{\pi}{3},\ n \in Z$

i.e. $\theta = \frac{3n\pi}{2} + \frac{\pi}{2},\ n \in Z.$

(iii) cot 4θ = -1

Solution:

The general solution of tan θ = tan ∝ is

$\theta = n\pi + \alpha,\ n \in Z$

Now, $\cot 4\theta = -1$

$\therefore\ \tan 4\theta = -1$

$\therefore\ \tan 4\theta = -\tan\frac{\pi}{4}$ ... $\left[\because\ \tan\frac{\pi}{4} = 1\right]$

$\therefore\ \tan 4\theta = \tan\left(\pi - \frac{\pi}{4}\right)$ ... $[\because\ \tan(\pi - \theta) = -\tan\theta]$

$\therefore\ \tan 4\theta = \tan\frac{3\pi}{4}$

$\therefore$ the required general solution is given by

∴ the required general solution is given by

$$4\theta = n\pi + \frac{3\pi}{4},\ n \in Z$$

i.e. $\theta = \frac{n\pi}{4} + \frac{3\pi}{16},\ n \in Z.$

Question 6.
Find the general solutions of the following equations :
(i) $4\cos^2\theta = 3$
Solution:
The general solution of $\cos^2\theta = \cos^2\alpha$ is
$\theta = n\pi \pm \alpha,\ n \in Z$
Now, $4\cos^2\theta = 3$

$$\therefore\ \cos^2\theta = \frac{3}{4} = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\therefore\ \cos^2\theta = \left(\cos\frac{\pi}{6}\right)^2 \qquad \ldots\left[\because\ \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\right]$$

$$\therefore\ \cos^2\theta = \cos^2\frac{\pi}{6}$$

∴ the required general solution is given by

$$\theta = n\pi \pm \frac{\pi}{6},\ n \in Z.$$

(ii) $4\sin^2\theta = 1$
Solution:
The general solution of $\sin^2\theta = \sin^2\alpha$ is

θ = nπ ± ∝, n ∈ Z

Now, $4 \sin^2\theta = 3$

$$\therefore \sin^2\theta = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

$$\therefore \sin^2\theta = \left(\sin\frac{\pi}{6}\right)^2 \qquad \ldots \left[\because \sin\frac{\pi}{6} = \frac{1}{2}\right]$$

$$\therefore \sin^2\theta = \sin^2\frac{\pi}{6}$$

$\therefore$ the required general solution is $\theta = n\pi \pm \frac{\pi}{6}, n \in Z.$

(iii) cos 4θ = cos 2θ

Solution:

The general solution of cos θ = cos ∝ is

θ = 2nπ ± ∝, n ∈ Z

∴ the general solution of cos 4θ = cos 2θ is given by

4θ = 2nπ ± 2θ, n ∈ Z

Taking positive sign, we get

4θ = 2nπ + 2θ, n ∈ Z

∴ 2θ = 2nπ, n ∈ Z

∴ θ = nπ, n ∈ Z

Taking negative sign, we get

4θ = 2nπ – 2θ, n ∈ Z

∴ 6θ = 2nπ, n ∈ Z

∴ θ = $\frac{n\pi}{3}$, n ∈ Z

Hence, the required general solution is

Hence, the required general solution is

θ = $\frac{n\pi}{3}$, n ∈ Z or ∴ θ = nπ, n ∈ Z.

Alternative Method:

cos 4θ = cos 2θ

∴ cos4θ – cos 20 = 0

∴ -2sin$\left(\frac{4\theta+2\theta}{2}\right)$·sin$\left(\frac{4\theta-2\theta}{2}\right)$ = 0

∴ sin3θ·sinθ = 0

∴ either sin3θ = 0 or sin θ = 0

The general solution of sin θ = 0 is

θ = nπ, n ∈ Z.

∴ the required general solution is given by

3θ = nπ, n ∈ Z or θ = nπ, n ∈ Z

i.e. θ = $\frac{n\pi}{3}$, n ∈ Z or θ = nπ, n ∈ Z.

Question 7.

Find the general solutions of the following equations :

(i) sinθ = tanθ

Solution:

sin θ = tan θ

∴ sin θ = $\frac{\sin\theta}{\cos\theta}$

∴ sin θ cos θ = sin θ

∴ sin θ cos θ – sinθ = 0

∴ sin θ (cos θ – 1) = θ

∴ either sinθ = 0 or cosθ – 1 = 0

∴ either sin θ = 0 or cos θ = 1

∴ either sinθ = 0 or cosθ = cosθ …[∵ cos0 = 1]

∴ either $\sin\theta = 0$ or $\cos\theta = \cos\theta$ …[∵ $\cos 0 = 1$]
The general solution of $\sin\theta = 0$ is $\theta = n\pi, n \in Z$ and $\cos\theta = \cos\alpha$ is $\theta = 2n\pi \pm \alpha$, where $n \in Z$.
∴ the required general solution is given by
$\theta = n\pi, n \in Z$ or $\theta = 2n\pi \pm 0, n \in Z$
∴ $\theta = n\pi, n \in Z$ or $\theta = 2n\pi, n \in Z$.

(ii) $\tan^3\theta = 3\tan\theta$
Solution:
$\tan^3\theta = 3\tan\theta$
∴ $\tan^3\theta - 3\tan\theta = 0$
∴ $\tan\theta(\tan^2\theta - 3) = 0$
∴ either $\tan\theta = 0$ or $\tan^2\theta - 3 = 0$
∴ either $\tan\theta = 0$ or $\tan^2\theta = 3$
∴ either $\tan\theta = 0$ or $\tan^2\theta = (\sqrt{3})^3$
∴ either $\tan\theta = 0$ or $\tan^2\theta = (\tan\frac{\pi}{3})^3$ …[$\tan\frac{\pi}{3} = \sqrt{3}$]
∴ either $\tan\theta = 0$ or $\tan^2\theta = \tan^2\frac{\pi}{3}$
The general solution of
$\tan\theta = 0$ is $\theta = n\pi, n \in Z$ and
$\tan^2\theta = \tan^2\alpha$ is $\theta = n\pi \pm \alpha, n \in Z$.
∴ the required general solution is given by
$\theta = n\pi, n \in Z$ or $\theta = n\pi \pm \frac{\pi}{3}, n \in Z$.

(iii) $\cos\theta + \sin\theta = 1$.
Solution:
$\cos\theta + \sin\theta = 1$

Dividing both sides by $\sqrt{(1)^2+(1)^2}=\sqrt{2}$, we get

$$\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta=\frac{1}{\sqrt{2}}$$

$$\therefore\ \cos\frac{\pi}{4}\cos\theta+\sin\frac{\pi}{4}\sin\theta=\cos\frac{\pi}{4}$$

$$\therefore\ \cos\left(\theta-\frac{\pi}{4}\right)=\cos\frac{\pi}{4} \quad \ldots (1)$$

The general solution of

$\cos\theta=\cos\alpha$ is $\theta=2n\pi\pm\alpha$, $n\in Z$.

$\therefore$ the general solution of (1) is given by

$$\theta-\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4},\ n\in Z$$

Taking positive sign, we get

$$\theta-\frac{\pi}{4}=2n\pi+\frac{\pi}{4},\ n\in Z$$

$$\therefore\ \theta=2n\pi+\frac{\pi}{2},\ n\in Z$$

Taking negative sign, we get

$$\theta-\frac{\pi}{4}=2n\pi-\frac{\pi}{4},\ n\in Z$$

$$\therefore\ \theta=2n\pi,\ n\in Z$$

$\therefore$ the required general solution is

$$\theta=2n\pi+\frac{\pi}{2},\ n\in Z \text{ or } \theta=2n\pi,\ n\in Z.$$

Alternative Method :

$\cos\theta + \sin\theta = 1$

$\therefore\ \sin\theta = 1 - \cos\theta$

$\therefore\ 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}$

$\therefore\ 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - 2\sin^2\frac{\theta}{2} = 0$

$\therefore\ 2\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right) = 0$

$\therefore\ 2\sin\frac{\theta}{2} = 0$ or $\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = 0$

$\therefore\ \sin\frac{\theta}{2} = 0$ or $\sin\frac{\theta}{2} = \cos\frac{\theta}{2}$

$\therefore\ \sin\frac{\theta}{2} = 0$ or $\tan\frac{\theta}{2} = 1$ ... $\left[\because\ \cos\frac{\theta}{2} \neq 0\right]$

$\therefore\ \sin\frac{\theta}{2} = 0$ or $\tan\frac{\theta}{2} = \tan\frac{\pi}{4}$ ... $\left[\because\ \tan\frac{\pi}{4} = 1\right]$

The general solution of $\sin\theta = 0$ is $\theta = n\pi,\ n \in Z$ and

$\tan\theta = \tan\alpha$ is $\theta = n\pi + \alpha,\ n \in Z$

$\therefore$ the required general solution is

$\frac{\theta}{2} = n\pi,\ n \in Z$ or $\frac{\theta}{2} = n\pi + \frac{\pi}{4},\ n \in Z$

i.e. $\theta = 2n\pi,\ n \in Z$ or $\theta = 2n\pi + \frac{\pi}{2},\ n \in Z$.

Question 8.
Which of the following equations have solutions ?
(i) $\cos 2\theta = -1$
Solution:
$\cos 2\theta = -1$
Since $-1 \le \cos\theta \le 1$ for any θ,
$\cos 2\theta = -1$ has solution.

(ii) $\cos^2\theta = -1$
Solution:
$\cos^2\theta = -1$
This is not possible because $\cos^2\theta \ge 0$ for any θ.
$\therefore \cos^2\theta = -1$ does not have any solution.

(iii) $2\sin\theta = 3$
Solution:
$2\sin\theta = 3 \therefore \sin\theta = \frac{3}{2}$
This is not possible because $-1 \le \sin\theta \le 1$ for any θ.
$\therefore 2\sin\theta = 3$ does not have any solution.

(iv) $3\tan\theta = 5$
Solution:
$3\tan\theta = 5 \therefore \tan\theta = \frac{5}{3}$
This is possible because $\tan\theta$ is any real number.
$\therefore 3\tan\theta = 5$ has solution.

Ex 3.2

Question 1.
Find the Cartesian co-ordinates of the point whose polar co-ordinates are :
(i) $\left(\sqrt{2}, \frac{\pi}{4}\right)$
Solution:
Here, r = $\sqrt{2}$ and θ = $\frac{\pi}{4}$
Let the cartesian coordinates be (x, y)
Then, x = rcosθ = $\sqrt{2}\cos\frac{\pi}{4}$ = $\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$ = 1
y = rsinθ = $\sqrt{2}\sin\frac{\pi}{4}$ = $\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$ = 1
∴ the cartesian coordinates of the given point are (1, 1).

(ii) $\left(4, \frac{\pi}{2}\right)$
Solution:

(iii) $\left(\frac{3}{4}, \frac{3\pi}{4}\right)$
Solution:
Here, r = $\frac{3}{4}$ and θ = $\frac{3\pi}{4}$
Let the cartesian coordinates be (x, y)

$$\text{Then, } x = r\cos\theta = \frac{3}{4}\cos\frac{3\pi}{4} = \frac{3}{4}\cos\left(\pi - \frac{\pi}{4}\right)$$

$$= -\frac{3}{4}\cos\frac{\pi}{4} = -\frac{3}{4} \times \frac{1}{\sqrt{2}} = -\frac{3}{4\sqrt{2}}$$

$$y = r\sin\theta = \frac{3}{4}\sin\frac{3\pi}{4} = \frac{3}{4}\sin\left(\pi - \frac{\pi}{4}\right)$$

$$= \frac{3}{4}\sin\frac{\pi}{4} = \frac{3}{4} \times \frac{1}{\sqrt{2}} = \frac{3}{4\sqrt{2}}$$

∴ the cartesian coordinates of the given point are

$$\left(-\frac{3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right).$$

(iv) $\left(\frac{1}{2}, \frac{7\pi}{3}\right)$
Solution:

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Here, r = $\frac{1}{2}$ and θ = $\frac{7\pi}{4}$
Let the cartesian coordinates be (x, y)

$$\text{Then, } x = r\cos\theta = \frac{1}{2}\cos\frac{7\pi}{3} = \frac{1}{2}\cos\left(2\pi + \frac{\pi}{3}\right)$$

$$= \frac{1}{2}\cos\frac{\pi}{3} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$y = r\sin\theta = \frac{1}{2}\sin\frac{7\pi}{3} = \frac{1}{2}\sin\left(2\pi + \frac{\pi}{3}\right)$$

$$= \frac{1}{2}\sin\frac{\pi}{3} = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

∴ the cartesian coordinates of the given point are $\left(\frac{1}{4}, \frac{\sqrt{3}}{4}\right)$

Question 2.
Find the of the polar co-ordinates point whose Cartesian co-ordinates are.
(i) $(\sqrt{2}, \sqrt{2})$
Solution:
Here x = $\sqrt{2}$ and y = $\sqrt{2}$
∴ the point lies in the first quadrant.
Let the polar coordinates be (r, θ)
Then, $r^2 = x^2 + y^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 2 + 2 = 4$
∴ r = 2 … [∵ r > 0]
cos θ = $\frac{x}{r} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
and sin θ = $\frac{y}{r} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
∴ tan θ = 1

Since the point lies in the first quadrant and
$0 \le \theta \le 2\pi$, tan θ = 1 = $\tan\frac{\pi}{4}$
∴ θ = $\frac{\pi}{4}$
∴ the polar coordinates of the given point are $\left(2, \frac{\pi}{4}\right)$.

(ii) $\left(0, \frac{1}{2}\right)$
Solution:
Here x = 0 and y = $\frac{1}{2}$
the point lies on the positive side of Y-axis. Let the polar coordinates be (r, θ)
Then, $r^2 = x^2 + y^2 = (0)^2 + \left(\frac{1}{2}\right)^2 = 0 + \frac{1}{4} = \frac{1}{4}$
∴ r = $\frac{1}{2}$ ...[∵ r > 0]
cosθ = $\frac{x}{r} = \frac{0}{(1/2)}$ = 0
and sin θ = $\frac{y}{r} = \frac{(1/2)}{(1/2)}$ = 1
Since, the point lies on the positive side of Y-axis and $0 \le \theta \le 2\pi$
cosθ = 0 = $\cos\frac{\pi}{2}$ and sinθ = 1 = $\sin\frac{\pi}{2}$
∴ θ = $\frac{\pi}{2}$
∴ the polar coordinates of the given point are $\left(\frac{1}{2}, \frac{\pi}{2}\right)$.

(iii) $(1, -\sqrt{3})$
Solution:
Here x = 1 and y = $-\sqrt{3}$
∴ the point lies in the fourth quadrant.

∴ the point lies in the fourth quadrant.
Let the polar coordinates be (r, θ).
Then, $r^2 = x^2 + y^2 = (1)^2 + (-\sqrt{3})^2 = 1 + 3 = 4$
∴ r = 2 … [∵ r > 0]

$\cos\theta = \frac{x}{r} = \frac{1}{2}$

and $\sin\theta = \frac{y}{r} = -\frac{\sqrt{3}}{2}$

$\therefore \tan\theta = -\sqrt{3}$

Since, the point lies in the fourth quadrant and $0 \leqslant \theta < 2\pi$.

$$\tan\theta = -\sqrt{3} = -\tan\frac{\pi}{3}$$

$$= \tan\left(2\pi - \frac{\pi}{3}\right) \quad \ldots [\because \tan(2\pi - \theta) = -\tan\theta]$$

$$= \tan\frac{5\pi}{3}$$

$$\therefore \theta = \frac{5\pi}{3}$$

∴ the polar coordinates of the given point are $\left(2, \frac{5\pi}{3}\right)$.

(iv) $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$
Solution:

(iv) $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$
Solution:

Question 3.
In ΔABC, if ∠A = 45°, ∠B = 60° then find the ratio of its sides.
Solution:
By the sine rule,
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
$\therefore \frac{a}{b} = \frac{\sin A}{\sin B}$ and $\frac{b}{c} = \frac{\sin B}{\sin C}$
∴ a : b : c = sinA : sinB : sinC
Given ∠A = 45° and ∠B = 60°
∵ ∠A + ∠B + ∠C = 180°
∴ 45° + 60° + ∠C = 180°
∴ ∠C = 180° – 105° = 75°

$$\text{Now, } \sin A = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin B = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{and } \sin C = \sin 75^\circ = \sin(45^\circ + 30^\circ)$$
$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$\therefore$ the ratio of the sides of $\triangle ABC$

$$= a : b : c = \sin A : \sin B : \sin C$$

$$= \frac{1}{\sqrt{2}} : \frac{\sqrt{3}}{2} : \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\therefore\ a : b : c = 2 : \sqrt{6} : (\sqrt{3}+1)$$

Question 4.

In ΔABC, prove that $\sin\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right)\cos\frac{A}{2}$.

Solution:

By the sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\therefore\ a = k\sin A,\ b = k\sin B,\ c = k\sin C$$

$$\text{RHS} = \left(\frac{b-c}{a}\right)\cos\frac{A}{2}$$

$$= \left(\frac{k\sin B - k\sin C}{k\sin A}\right)\cos\frac{A}{2}$$

$$= \left(\frac{\sin B - \sin C}{\sin A}\right)\cos\frac{A}{2}$$

$$= \frac{2\cos\left(\frac{B+C}{2}\right)\cdot\sin\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cdot\cos\frac{A}{2}}\cdot\cos\frac{A}{2}$$

$$= \frac{\cos\left(\frac{B+C}{2}\right)\cdot\sin\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}}$$

$$= \frac{\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\cdot\sin\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}} \quad \ldots [\because\ A+B+C=\pi]$$

$$= \frac{\sin\frac{A}{2}\cdot\sin\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}}$$

$$= \sin\left(\frac{B-C}{2}\right) = \text{LHS.}$$

Question 5.
With usual notations prove that 2 $\left\{a\sin^2\frac{C}{2}+c\sin^2\frac{A}{2}\right\}$ = a – b + c.
Solution:

$$\text{LHS} = 2\left\{a\sin^2\frac{C}{2}+c\sin^2\frac{A}{2}\right\}$$

$$= a\left(2\sin^2\frac{C}{2}\right)+c\left(2\sin^2\frac{A}{2}\right)$$

$$= a(1-\cos C)+c(1-\cos A)$$

$$= a\left[1-\frac{a^2+b^2-c^2}{2ab}\right]+c\left[1-\frac{b^2+c^2-a^2}{2bc}\right]$$

… [By cosine rule]

$$= a\left[\frac{2ab - a^2 - b^2 + c^2}{2ab}\right] + c\left[\frac{2bc - b^2 - c^2 + a^2}{2bc}\right]$$

$$= \frac{2ab - a^2 - b^2 + c^2}{2b} + \frac{2bc - b^2 - c^2 + a^2}{2b}$$

$$= \frac{2ab - a^2 - b^2 + c^2 + 2bc - b^2 - c^2 + a^2}{2b}$$

$$= \frac{2ab - 2b^2 + 2bc}{2b}$$

$$= a - b + c = \text{RHS.}$$

Question 6.

In ΔABC, prove that $a^3\sin(B - C) + b^3\sin(C - A) + c^3\sin(A - B) = 0$

Solution:

By the sine rule,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

∴ a = k sin A, b = k sin B, c = k sin C

LHS = $a^3\sin(B - C) + b^3\sin(C - A) + c^3\sin(A - B)$

= $a^3(\sin B \cos C - \cos B \sin C) + b^3(\sin C\cos A - \cos C \sin A) + c^3(\sin A\cos B - \cos A \sin B)$

$$= a^3\left(\frac{b}{k}\cos C - \frac{c}{k}\cos B\right) + b^3\left(\frac{c}{k}\cos A - \frac{a}{k}\cos C\right) + c^3\left(\frac{a}{k}\cos B - \frac{b}{k}\cos A\right)$$

$$= \frac{1}{k}[a^3b\cos C - a^3c\cos B + b^3c\cos A - b^3a\cos C +$$

$$c^3a\cos B - c^3b\cos A]$$

$$= \frac{1}{k}\left[a^3b\left(\frac{a^2+b^2-c^2}{2ab}\right) - a^3c\left(\frac{c^2+a^2-b^2}{2ca}\right) + b^3c\left(\frac{b^2+c^2-a^2}{2bc}\right) - ab^3\left(\frac{a^2+b^2-c^2}{2ab}\right) + ac^3\left(\frac{c^2+a^2-b^2}{2ca}\right) - bc^3\left(\frac{b^2+c^2-a^2}{2bc}\right)\right]$$

… [By cosine rule]

$= \frac{1}{2k}$ $[a^2(a^2 + b^2 – c^2) – a^2(a^2 + c^2 – b^2) + b^2(b^2 + c^2 – a^2) – b^2(a^2 + b^2 – c^2) + c^2(c^2 + a^2 – b^2) – c^2(b^2 + c^2 – a^2)]$

$= \frac{1}{2k}$ $[a^4 + a^2b^2 – a^2c^2 – a^4 – a^2c^2 + a^2b^2 + b^4 + b^2c^2 – a^2b^2 – a^2b^2 – b^4 + b^2c^2 + c^4 + a^2c^2 – b^2c^2 – b^2c^2 – c^4 + a^2c^2]$

$= \frac{1}{2k}(0) = 0$ = RHS.

Question 7.

In ΔABC, if cot A, cot B, cot C are in A.P. then show that a^2, b^2, c^2 are also in A.P

Solution:

By the sine rule,

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$

∴ sin A = ka, sin B = kb, sin C = kc …(1)

Now, cot A, cotB, cotC are in A.P.

∴ cotC – cotB = cotB – cot A

∴ cotA + cotC = 2cotB

cos A   cos C

$\therefore \frac{\cos A}{\sin A}+\frac{\cos C}{\sin C}=2\cot B$

$\therefore \frac{\sin C\cos A+\sin A\cos C}{\sin A\cdot\sin C}=2\cot B$

$\therefore \frac{\sin(A+C)}{\sin A\cdot\sin C}=2\cot B$

$\therefore \frac{\sin(\pi-B)}{\sin A\cdot\sin C}=2\cot B$ ... $[\because A+B+C=\pi]$

$\therefore \frac{\sin B}{\sin A\cdot\sin C}=\frac{2\cos B}{\sin B}$

$\therefore \frac{\sin^2 B}{\sin A\cdot\sin C}=2\cos B$

$\therefore \frac{k^2b^2}{(ka)(kc)}=2\left(\frac{a^2+c^2-b^2}{2ac}\right)$

$\therefore \frac{b^2}{ac}=\frac{a^2+c^2-b^2}{ac}$

$\therefore b^2=a^2+c^2-b^2$ $\quad\therefore 2b^2=a^2+c^2$

Hence, a^2, b^2, c^2 are in A.P.

Question 8.

In ΔABC, if a cos A = b cos B then prove that the triangle is right angled or an isosceles traingle.

Solution:

By the sine rule,

$\frac{a}{\sin A} = \frac{b}{\sin B} = k$
a = k sin A and b = k sin B
∴ a cos A = b cos B gives
k sin A cos A = k sin B cos B
∴ 2 sin A cos A = 2 sin B cos B
∴ sin 2A = sin 2B ∴ sin 2A – sin 2B = 0
∴ 2 cos (A + B)·sin (A -B) = 0
∴ 2cos (π – C)·sin(A – B) = 0 … [∵ A + B + C = π]
∴ -2 cos C·sin (A – B) = 0
∴ cos C = 0 OR sin(A -B) = 0
∴ C = 90° OR A – B = 0
∴ C = 90° OR A = B
∴ the triangle is either rightangled or an isosceles triangle.

Question 9.
With usual notations prove that 2(bc cos A + ac cos B + ab cos C) = $a^2 + b^2 + c^2$.
Solution:
LHS = 2 (bc cos A + ac cos B + ab cos C)
= 2bc cos A + 2ac cos B + 2ab cos C
= $2bc\left(\frac{b^2+c^2-a^2}{2bc}\right) + 2ac\left(\frac{c^2+a^2-b^2}{2ca}\right) + 2ab\left(\frac{a^2+b^2-c^2}{2ab}\right)$ …(By cosine rule]
= $b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2 = a^2 + b^2 + c^2$ = RHS.

Question 10.

Question 10.
In △ABC, if a = 18, b = 24, c = 30 then find the values of
(i) cos A
Solution:
Given : a = 18, b = 24 and c = 30
∴ 2s = a + b + c = 18 + 24 + 30 = 72 ∴ s = 36

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(24)^2 + (30)^2 - (18)^2}{2(24)(30)}$$

$$= \frac{576 + 900 - 324}{1440} = \frac{1152}{1440} = \frac{4}{5}.$$

(ii) $\sin\frac{A}{2}$
Solution:

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} = \sqrt{\frac{(36-24)(36-30)}{(24)(30)}}$$

$$= \sqrt{\frac{12 \times 6}{24 \times 30}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}}.$$

(iii) $\cos\frac{A}{2}$
Solution:

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} = \sqrt{\frac{36(36-18)}{(24)(30)}}$$

$$= \sqrt{\frac{36 \times 18}{24 \times 30}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}.$$

(iv) $\tan\frac{A}{2}$
Solution:

$$\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \frac{1/\sqrt{10}}{3/\sqrt{10}} = \frac{1}{3}.$$

(v) A(△ABC)
Solution:

$$\begin{aligned} A(\triangle ABC) &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{36(36-18)(36-24)(36-30)} \\ &= \sqrt{36 \times 18 \times 12 \times 6} \\ &= \sqrt{36 \times 18 \times 4 \times 18} \\ &= 6 \times 18 \times 2 = 216 \text{ sq units.} \end{aligned}$$

(iv) sin A.
Solution:

$$A(\triangle ABC) = \tfrac{1}{2} bc \sin A$$

$$\therefore\ 216 = \tfrac{1}{2}(24)(30) \sin A$$

$$\therefore\ \sin A = \frac{216}{12 \times 30} = \frac{216}{360} = \frac{3}{5}.$$

Question 11.
In △ABC prove that $(b + c - a) \tan\frac{A}{2} = (c + a - b) \tan\frac{B}{2} = (a + b - c) \tan\frac{C}{2}$.
Solution:

(b + c – a) tan $\frac{A}{2}$

$$= (a+b+c-2a)\cdot\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$= (2s-2a)\cdot\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$= 2\sqrt{\frac{(s-a)(s-b)(s-c)}{s}} \quad \ldots (1)$$

$$(c+a-b)\tan\frac{B}{2} = (a+b+c-2b)\cdot\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$= (2s-2b)\cdot\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$= 2\sqrt{\frac{(s-a)(s-b)(s-c)}{s}} \quad \ldots (2)$$

$$(a+b-c)\tan\frac{C}{2} = (a+b+c-2c)\cdot\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= (2s-2c)\cdot\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= 2\sqrt{\frac{(s-a)(s-b)(s-c)}{s}} \quad \ldots (3)$$

From (1), (2) and (3), we get

$$(b+c-a)\tan\frac{A}{2} = (c+a-b)\tan\frac{B}{2} = (a+b-c)\tan\frac{C}{2}.$$

Question 12.

In $\triangle$ABC prove that sin $\frac{A}{2}$·sin $\frac{A}{2}$·sin $\frac{A}{2}$ = $\frac{[A(\triangle ABC)]^2}{abcs}$

Solution:

LHS = sin $\frac{A}{2}$·sin $\frac{B}{2}$·sin $\frac{C}{2}$

$$=\sqrt{\frac{(s-b)(s-c)}{bc}}\cdot\sqrt{\frac{(s-a)(s-c)}{ac}}\cdot\sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$=\sqrt{\frac{(s-a)^2(s-b)^2(s-c)^2}{a^2b^2c^2}}$$

$$=\frac{(s-a)(s-b)(s-c)}{abc}$$

$$=\frac{s(s-a)(s-b)(s-c)}{abcs}$$

$$=\frac{[A(\triangle ABC)]^2}{abcs} \quad \ldots \left[\because A(\triangle ABC)=\sqrt{s(s-a)(s-b)(s-c)}\right]$$

$= \text{RHS.}$

Ex 3.3

Question 1.
Find the principal values of the following :
(i) $\sin^{-1}\left(\frac{1}{2}\right)$
Solution:
The principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
Let $\sin^{-1}\left(\frac{1}{2}\right) = \propto$, where $\frac{-\pi}{2} \leq \propto \leq \frac{\pi}{2}$
$\therefore \sin\propto = \frac{1}{2} = \sin\frac{\pi}{6}$
$\therefore \propto = \frac{\pi}{6}$ ...$[\because -\frac{\pi}{2} \leq \frac{\pi}{6} \leq \frac{\pi}{2}]$
$\therefore$ the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is $\frac{\pi}{6}$.

(ii) $\text{cosec}^{-1}(2)$

Solution:

The principal value branch of $\text{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Let $\text{cosec}^{-1}(2) = \propto$, where $\frac{-\pi}{2} \leq \propto \leq \frac{\pi}{2}$, $\propto \neq 0$

$\therefore \text{cosec}^{-1} \propto = 2 = \text{cosec}\frac{\pi}{6}$

$\therefore \propto = \frac{\pi}{6}$ ...[$\because -\frac{\pi}{2} \leq \frac{\pi}{6} \leq \frac{\pi}{2}$]

$\therefore$ the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

(iii) $\tan^{-1}(-1)$

Solution:

The principal value branch of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Let $\tan^{-1}(-1) = \propto$, where $\frac{-\pi}{2} < \propto < \frac{\pi}{2}$

$\therefore \tan\propto = -1 = -\tan\frac{\pi}{4}$

$\therefore \tan\propto = \tan\left(-\frac{\pi}{4}\right)$ ...[$\because \tan(-\theta) = -\tan\theta$]

$\therefore \propto = -\frac{\pi}{4}$ ...[$\because -\frac{\pi}{2} < \frac{-\pi}{4} < \frac{\pi}{2}$]

$\therefore$ the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

(iv) $\tan^{-1}(-\sqrt{3})$

Solution:

The principal value branch of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Let $\tan^{-1}(-\sqrt{3}) = \propto$, where $\frac{-\pi}{2} < \propto < \frac{\pi}{2}$

$\therefore \tan\propto = -\sqrt{3} = -\tan\frac{\pi}{3}$

∴ $\tan\propto = \tan\left(-\frac{\pi}{3}\right)$ ...[∵ tan(-θ) = -tanθ]
∴ $\propto = -\frac{\pi}{3}$ ...[∵ $-\frac{\pi}{2} < \frac{-\pi}{3} < \frac{\pi}{2}$]
∴ the principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

(v) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$
Solution:
The principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \propto$, where $\frac{-\pi}{2} < \propto < \frac{\pi}{2}$
∴ $\sin\propto = \left(\frac{1}{\sqrt{2}}\right) = \sin\frac{\pi}{4}$
∴ $\propto = \frac{\pi}{4}$ ...[∵ $-\frac{\pi}{2} \leq \frac{\pi}{4} \leq \frac{\pi}{2}$]
∴ the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$.

(vi) $\cos^{-1}\left(-\frac{1}{2}\right)$
Solution:
The principal value branch of $\cos^{-1}x$ is (0, π).
Let $\cos^{-1}\left(-\frac{1}{2}\right) = \propto$, where $0 \leq \propto \leq \pi$
∴ $\cos\propto = -\frac{1}{2} = -\cos\frac{\pi}{3}$
∴ $\cos\propto = \cos\left(\pi - \frac{\pi}{3}\right)$ ...[∵ cos(π – θ) = -cosθ)
∴ $\cos\propto = \cos\frac{2\pi}{3}$
∴ $\propto = \frac{2\pi}{3}$ ...[∵ $0 \leq \frac{2\pi}{3} \leq \pi$]
∴ the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Question 2.

Evaluate the following :

(i) $\tan^{-1}(1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

Solution:

Let $\tan^{-1}(1) = \alpha$, where $\frac{-\pi}{2} < \alpha < \frac{\pi}{2}$

$\therefore \tan\alpha = 1 = \tan\frac{\pi}{4}$

$\therefore \alpha = \frac{\pi}{4}$ ... $\left[\because \frac{-\pi}{2} < \frac{\pi}{4} < \frac{\pi}{2}\right]$

$\therefore \tan^{-1}(1) = \frac{\pi}{4}$ ... (1)

Let $\cos^{-1}\left(\frac{1}{2}\right) = \beta$, where $0 \leqslant \beta \leqslant \pi$

$\therefore \cos\beta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore \beta = \frac{\pi}{3}$ ... $[\because 0 < \frac{\pi}{3} < \pi]$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ ... (2)

Let $\sin^{-1}\left(\frac{1}{2}\right) = \gamma$, where $\frac{-\pi}{2} \leqslant \gamma \leqslant \frac{\pi}{2}$

$\therefore \sin\gamma = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \gamma = \frac{\pi}{6}$ ... $\left[\because \frac{-\pi}{2} \leqslant \frac{\pi}{6} \leqslant \frac{\pi}{2}\right]$

$\therefore\ \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$ ... (3)

$\therefore\ \tan^{-1}(1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$

$=\frac{\pi}{4}+\frac{\pi}{3}+\frac{\pi}{6}$ ... [By (1), (2) and (3)]

$=\frac{3\pi+4\pi+2\pi}{12}=\frac{9\pi}{12}=\frac{3\pi}{4}$.

(ii) $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Solution:

Let $\cos^{-1}\left(\frac{1}{2}\right)=\alpha$, where $0\le\alpha\le\pi$

$\therefore \cos\alpha=\frac{1}{2}=\cos\frac{\pi}{3}$

$\therefore \alpha=\frac{\pi}{3}$ ... $\left[\because 0<\frac{\pi}{3}<\pi\right]$

$\therefore \cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$ ...(1)

Let $\sin^{-1}\left(\frac{1}{2}\right)=\beta$, where $\frac{-\pi}{2}\le\beta\le\frac{\pi}{2}$

$\therefore \sin\beta=\frac{1}{2}=\sin\frac{\pi}{6}$

$\therefore \beta=\frac{\pi}{6}$ ... $\left[\because \frac{-\pi}{2}\le\frac{\pi}{6}\le\frac{\pi}{2}\right]$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ ...(2)

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ and $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3} + 2\left(\frac{\pi}{6}\right)$

$= \frac{\pi}{3} + \frac{\pi}{3}$

$= \frac{2\pi}{3}.$

(iii) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

Solution:

Let $\tan^{-1}(\sqrt{3}) = \alpha$, where $\frac{-\pi}{2} < \alpha < \frac{\pi}{2}$

$\therefore \tan\alpha = \sqrt{3} = \tan\frac{\pi}{3}$

$\therefore \alpha = \frac{\pi}{3}$ ... $\left[\because \frac{-\pi}{2} < \frac{\pi}{3} < \frac{\pi}{2}\right]$

$\therefore \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ ... (1)

Let $\sec^{-1}(-2) = \beta$, where $0 \leqslant \beta \leqslant \pi$, $\beta \neq \frac{\pi}{2}$

$\therefore \sec\beta = -2 = -\sec\frac{\pi}{3}$

$\therefore \sec\beta = \sec\left(\pi - \frac{\pi}{3}\right)$ ... $[\because \sec(\pi-\theta) = -\sec\theta]$

$\therefore \sec\beta = \sec\frac{2\pi}{3}$

$\therefore \beta = \frac{2\pi}{3}$ ... $[\because 0 \leqslant \frac{2\pi}{3} \leqslant \pi]$

$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$ ... (2)

$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2)$
$= \frac{\pi}{3} - \frac{2\pi}{3}$ ...[By (1) and (2)]
$= -\frac{\pi}{3}$.

(iv) $\text{cosec}^{-1}(-\sqrt{2}) + \cot^{-1}(\sqrt{3})$
Solution:

Let $\text{cosec}^{-1}(-\sqrt{2}) = \alpha$, where $\frac{-\pi}{2} \leqslant y \leqslant \frac{\pi}{2}, y \neq 0$

$\therefore \text{cosec}\,\alpha = -\sqrt{2} = -\text{cosec}\frac{\pi}{4}$

$\therefore \text{cosec}\,\alpha = \text{cosec}\left(-\frac{\pi}{4}\right)$

... $[\because \text{cosec}(-\theta) = -\text{cosec}\,\theta]$

$\therefore \alpha = -\frac{\pi}{4}$ ... $\left[\frac{-\pi}{2} \leqslant \frac{-\pi}{4} \leqslant \frac{\pi}{2}\right]$

$\therefore\ \operatorname{cosec}^{-1}(-\sqrt{2}) = -\frac{\pi}{4}$ ... (1)

Let $\cot^{-1}(\sqrt{3}) = \beta$, where $0 < \beta < \pi$

$\therefore\ \cot\beta = \sqrt{3} = \cot\frac{\pi}{6}$

$\therefore\ \beta = \frac{\pi}{6}$ ... $\left[\because\ 0 < \frac{\pi}{6} < \pi\right]$

$\therefore\ \cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$ ... (2)

$\therefore\ \operatorname{cosec}^{-1}(-\sqrt{2}) + \cot^{-1}(\sqrt{3})$

$= -\frac{\pi}{4} + \frac{\pi}{6}$ ... [By (1) and (2)]

$= \frac{-3\pi + 2\pi}{12} = -\frac{\pi}{12}$.

Question 3.
Prove the following :
(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - 3\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{3\pi}{4}$
Question is modified.
$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - 3\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{3\pi}{4}$
Solution:

Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \alpha$, where $-\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2}$

$\therefore\ \sin\alpha = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\therefore\ \alpha = \frac{\pi}{4}$ ... $\left[\because\ -\frac{\pi}{2} \leqslant \frac{\pi}{4} \leqslant \frac{\pi}{2}\right]$

$\therefore\ \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ ... (1)

Let $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \beta$, where $-\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2}$

$\therefore\ \sin\beta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$

$\therefore\ \beta = \frac{\pi}{3}$ ... $\left[\because\ -\frac{\pi}{2} \leqslant \frac{\pi}{3} \leqslant \frac{\pi}{2}\right]$

$\therefore\ \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ ... (2)

$\text{LHS} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - 3\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{4} - 3\left(\frac{\pi}{3}\right)$ ... [By (1) and (2)]

$= \frac{\pi}{4} - \pi = -\frac{3\pi}{4} = \text{RHS}.$

(ii) $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \cos^{-1}\left(-\frac{1}{2}\right)$

Solution:

Let $\sin^{-1}\left(-\frac{1}{2}\right) = \alpha$, where $-\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2}$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=\alpha$, where $-\frac{\pi}{2}\leqslant\alpha\leqslant\frac{\pi}{2}$

$\therefore\ \sin\alpha=-\frac{1}{2}=-\sin\frac{\pi}{6}$

$\therefore\ \sin\alpha=\sin\left(-\frac{\pi}{6}\right)$ ... $[\because\ \sin(-\theta)=-\sin\theta]$

$\therefore\ \alpha=-\frac{\pi}{6}$ ... $\left[\because\ -\frac{\pi}{2}\leqslant-\frac{\pi}{6}\leqslant\frac{\pi}{2}\right]$

$\therefore\ \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$ ... (1)

Let $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\beta$, where $0\leqslant\beta\leqslant\pi$

$\therefore\ \cos\beta=-\frac{\sqrt{3}}{2}=-\cos\frac{\pi}{6}$

$\therefore\ \cos\beta=\cos\left(\pi-\frac{\pi}{6}\right)$ ... $[\because\ \cos(\pi-\theta)=-\cos\theta]$

$\therefore\ \cos\beta=\cos\frac{5\pi}{6}$

$\therefore\ \beta=\frac{5\pi}{6}$ ... $\left[\because\ 0\leqslant\frac{5\pi}{6}\leqslant\pi\right]$

$\therefore\ \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{6}$ ... (2)

Let $\cos^{-1}\left(-\frac{1}{2}\right)=\gamma$, where $0\leqslant\gamma\leqslant\pi$

$\therefore\ \cos\gamma=-\frac{1}{2}=-\cos\frac{\pi}{3}$

$\therefore \cos\gamma = -\frac{1}{2} = -\cos\frac{\pi}{3}$

$\therefore \cos\gamma = \cos\left(\pi - \frac{\pi}{3}\right)$ ... $[\because \cos(\pi - \theta) = -\cos\theta]$

$\therefore \cos\gamma = \cos\frac{2\pi}{3}$

$\therefore \gamma = \frac{2\pi}{3}$ ... $\left[\because 0 \leqslant \frac{2\pi}{3} \leqslant \pi\right]$

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ ... (3)

$\text{LHS} = \sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= -\frac{\pi}{6} + \frac{5\pi}{6}$ ... [By (1) and (2)]

$= \frac{4\pi}{6} = \frac{2\pi}{3}$

$= \cos^{-1}\left(-\frac{1}{2}\right)$ ... [By (3)]

$= \text{RHS}.$

(iii) $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Solution:

Let $\sin^{-1}\left(\frac{3}{5}\right) = x$, $\cos^{-1}\left(\frac{12}{13}\right) = y$ and $\sin^{-1}\left(\frac{56}{65}\right) = z$.

Then $\sin x=\frac{3}{5}$, where $0<x<\frac{\pi}{2}$

$\cos y=\frac{12}{13}$, where $0<y<\frac{\pi}{2}$

and $\sin z=\frac{56}{65}$, where $0<z<\frac{\pi}{2}$

$\therefore\ \cos x>0, \sin y>0$

Now, $\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

and $\sin y=\sqrt{1-\cos^2 y}=\sqrt{1-\frac{144}{169}}=\sqrt{\frac{25}{169}}=\frac{5}{13}$

We have to prove that, $x+y=z$

Now, $\sin(x+y)=\sin x\cos y+\cos x\sin y$

$$=\left(\frac{3}{5}\right)\left(\frac{12}{13}\right)+\left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$$

$$=\frac{36}{65}+\frac{20}{65}=\frac{56}{65}$$

$\therefore\ \sin(x+y)=\sin z \qquad \therefore\ x+y=z$

Hence, $\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\sin^{-1}\left(\frac{56}{65}\right)$.

(iv) $\cos^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{4}{5}\right)=\frac{\pi}{2}$

Solution:

Solution:

Let $\cos^{-1}\left(\frac{3}{5}\right) = x$

$\therefore \cos x = \left(\frac{3}{5}\right)$, where $0 < x < \frac{\pi}{2}$ $\therefore \sin x > 0$

Now, $\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore x = \sin^{-1}\left(\frac{4}{5}\right)$

$\therefore \cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{4}{5}\right)$ ... (1)

$\text{LHS} = \cos^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right)$

$= \sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right)$ ... [By (1)]

$= \frac{\pi}{2}$ ... $\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$= \text{RHS}.$

(v) $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

Solution:

LHS = $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)$

$= \tan^{-1}\left(\frac{3+2}{6-1}\right) = \tan^{-1}(1)$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$

= RHS.

(vi) $2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

Solution:

$$\text{LHS} = 2\tan^{-1}\left(\frac{1}{3}\right)$$

$$= \tan^{-1}\left[\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right]$$

$$\ldots\left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= \tan^{-1}\left[\frac{\left(\frac{2}{3}\right)}{1-\frac{1}{9}}\right] = \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right)$$

$$= \text{RHS.}$$

Alternative Method :

$$\text{LHS} = 2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{3}\right)$$

$$= \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{3}}{1-\frac{1}{3}\times\frac{1}{3}}\right]$$

$$= \tan^{-1}\left(\frac{3+3}{9-1}\right) = \tan^{-1}\left(\frac{6}{8}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right)$$

$$= \text{RHS.}$$

(vii) $\tan^{-1}\left[\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right] = \frac{\pi}{4} + \theta$ if $\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

Solution:

$$\text{LHS} = \tan^{-1}\left[\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}\right]$$

$$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$$

$$= \frac{\pi}{4}+\theta \quad \ldots [\because \tan^{-1}(\tan\theta)=\theta]$$

= RHS.

(viii) $\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\theta}{2}$, if θ ∈ (0, π)

Solution:

$$\frac{1-\cos\theta}{1+\cos\theta} = \frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}$$

$$= \tan^2\left(\frac{\theta}{2}\right)$$

$$\therefore \sqrt{\frac{1-\cos\theta}{1+\text{co}\,\theta}} = \sqrt{\tan^2\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$$

$$\therefore \text{LHS} = \tan^{-1}\left[\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\theta}{2}\right)\right]$$

$= \frac{\theta}{2}$ ...[∵ $\tan^{-1}(\tan\theta) = \theta$]

= RHS.

—— X ——

Maharashtra Board Solutions Class 12 Arts & Science Maths (Part 1)

About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

FAQs

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How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

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