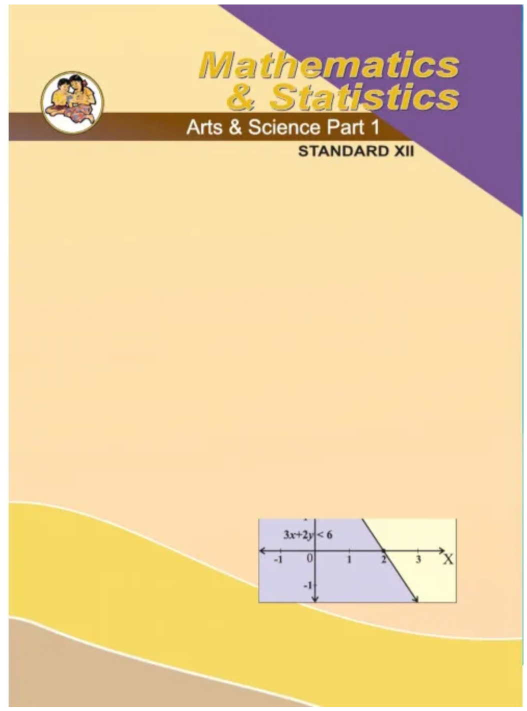
Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 2- Matrices

Class 12 - Chapter 2 Matrices



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Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 2- Matrices

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Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 1): Chapter 2- Matrices

Maharashtra Board 12th Maths Chapter 2, Class 12 Maths Chapter 2 solutions

Ex 2.1

Question 1.
Apply the given elementary transformation on each of the following matrices.

A = $\begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}$, $R_1 \leftrightarrow R_2$

Solution:

A = $\begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}$

By $R_1 \leftrightarrow R_2$, we get,

A ~ $\begin{bmatrix} -1 & 3 \\ 1 & 0 \end{bmatrix}$

Question 2.

B = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 5 & 4 \end{bmatrix}$, $R_1 \rightarrow R_1 \rightarrow R_2$

Solution:

B = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 5 & 4 \end{bmatrix}$,

$R_1 \rightarrow R_1 \rightarrow R_2$ gives,

B ~ $\begin{bmatrix} -1 & -6 & -1 \\ 2 & 5 & 4 \end{bmatrix}$

Question 3.

A = $\begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$, $C_1 \leftrightarrow C_2$; B = $\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, $R_1 \leftrightarrow R_2$. What do you observe?

Solution:

A = $\begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$

By $C_1 \leftrightarrow C_2$, we get,

A ~ $\begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$ ...(1)

B = $\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$

By $R_1 \leftrightarrow R_2$, we get,

B ~ $\begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$ ...(2)

From (1) and (2), we observe that the new matrices are equal.

Question 4.

A = $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \end{bmatrix}$, $2C_2$

B = $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 4 & 5 \end{bmatrix}$, $-3R_1$

Find the addition of the two new matrices.

Solution:

A = $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \end{bmatrix}$

By $2C_2$, we get,

A ~ $\begin{bmatrix} 1 & 4 & -1 \\ 0 & 2 & 3 \end{bmatrix}$

B = $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 4 & 5 \end{bmatrix}$

By $-3R_1$, we get,

B ~ $\begin{bmatrix} -3 & 0 & -6 \\ 2 & 4 & 5 \end{bmatrix}$

Now, addition of the two new matrices

$$= \begin{bmatrix} 1 & 4 & -1 \\ 0 & 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 0 & -6 \\ 2 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 & -7 \\ 2 & 6 & 8 \end{bmatrix}.$$

Question 5.

A = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$, $3R_3$ and then $C_3 + 2C_2$.

Solution:

A = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$

By $3R_3$, we get

A ~ $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 9 & 9 & 3 \end{bmatrix}$

By $C_3 + 2C_2$, we get,

A ~ $\begin{pmatrix} 1 & -1 & 3+2(-1) \\ 2 & 1 & 0+2(1) \\ 9 & 9 & 3+2(9) \end{pmatrix}$

∴ A ~ $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 9 & 9 & 21 \end{pmatrix}$

Question 6.

A = $\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{pmatrix}$, $C_3 + 2C_2$ and then $3R_3$. What do you conclude from Ex. 5 and Ex. 6 ?

Solution:

A = $\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{pmatrix}$

By $C_3 + 2C_2$, we get,

A ~ $\begin{pmatrix} 1 & -1 & 3+2(-1) \\ 2 & 1 & 0+2(1) \\ 3 & 3 & 1+2(3) \end{pmatrix}$

∴ A ~ $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{pmatrix}$

By $3R_3$, we get

A ~ $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 9 & 9 & 21 \end{pmatrix}$

We conclude from Ex. 5 and Ex. 6 that the matrix remains same by interchanging the order of the elementary transformations. Hence, the transformations are commutative.

Question 7.

Use suitable transformation on $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ into an upper triangular matrix.

Solution:

Let A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

By $R_2 - 3R_1$, we get,

A ~ $\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$

This is an upper triangular matrix.

Question 8.

Convert $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ into an identity matrix by suitable row transformations.

Solution:

Let A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

By $R_2 - 2R_1$, we get,

$$A \sim \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}$$

By $\left(\frac{1}{5}\right)R_2$, we get,

$$A \sim \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

By $R_1 + R_2$, we get,

$$A \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This is an identity matrix.

Question 9.

Transform $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$ into an upper triangular matrix by suitable row transformations.

Solution:

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get

$$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 5 & -2 \end{bmatrix}$$

By $R_3 - \left(\frac{5}{3}\right)R_2$, we get,

$$A \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

This is an upper triangular matrix.

Ex 2.2

Question 1.
Find the co-factors of the elements of the following matrices

(i) $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$

Here, a_{11} = -11, M_{11} = 4

$\therefore A_{11} = (-1)^{1+1}(4) = 4$

a_{12} = 2, M_{12} = -3

$\therefore A_{12} = (-1)^{1+2}(-3) = 3$

a_{21} = – 3, M_{21} = -2

$\therefore A_{21} = (-1)^{2+1}(2) = -2$

a_{22} = 4, M_{22} = -1

$\therefore A_{22} = (-1)^{2+2}(-1) = -1$.

(ii) $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

The co-factor of a_{ij} is given by $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix} = -3 - 0 = -3$

$\therefore A_{11} = (-1)^{1+1}(-3) = -3$

$M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix} = 2 + 10 = 12$

$\therefore A_{12} = (-1)^{1+2}(12) = -12$

$M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$

$\therefore A_{13} = (-1)^{1+3}(6) = 6$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1 - 0 = 1$

$\therefore A_{21} = (-1)^{2+1}(1) = -1$

$$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -1 + 4 = 3$$

$\therefore A_{22} = (-1)^{2+2}(3) = 3$

$$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 0 - 2 = -2$$

$\therefore A_{23} = (-1)^{2+3}(-2) = 2$

$$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$$

$\therefore A_{31} = (-1)^{3+1}(-11) = -11$

$$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} = 5 + 4 = 9$$

$\therefore A_{32} = (-1)^{3+2}(9) = -9$

$$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = 3 - 2 = 1$$

$\therefore A_{33} = (-1)^{3+3}(1) = 1.$

Question 2.

Find the matrix of co-factors for the following matrices

(i) $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$

Here, $a_{11} = 1, M_{11} = -1$

$\therefore A_{11} = (-1)^{1+1}(-1) = -1$

$a_{12} = 3, M_{12} = 4$

$\therefore A_{12} = (-1)^{1+2}(4) = -4$

$a_{21} = 4, M_{21} = 3$

$\therefore A_{21} = (-1)^{2+1}(3) = -3$

$a_{22} = -1, M_{22} = 1$

$\therefore A_{22} = (-1)^{2+1}(1) = 1$

$\therefore$ the co-factor matrix = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

= $\begin{pmatrix} -1 & -4 \\ -3 & 1 \end{pmatrix}$

(ii) $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \\ 0 & 3 & -5 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \\ 0 & 3 & -5 \end{bmatrix}$

Here, $a_{11} = 1$,

$A_{11} = \begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix} = -14$

$a_{12} = 0$,

$A_{12} = \begin{vmatrix} -2 & 0 \\ 3 & -5 \end{vmatrix} = -10$

$a_{13} = 2$,

$A_{13} = \begin{vmatrix} -2 & 1 \\ 0 & 3 \end{vmatrix} = -6$

$a_{21} = -2$

$A_{21} = \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} = 6$

$a_{22} = 1$

$A_{22} = \begin{vmatrix} 1 & 0 \\ 2 & -5 \end{vmatrix} = -5$

$a_{23} = 3$

$A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = -3$

$a_{31} = 0$

$A_{31} = \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = -2$

$a_{32} = 3$

$A_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} = -7$

$a_{33} = -5$

$A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix}$

$A_{11} = -14$, $A_{12} = -10$, $A_{13} = -6$,
$A_{21} = 6$, $A_{22} = -5$, $A_{23} = -3$,
$A_{31} = -2$, $A_{32} = -7$, $A_{33} = 1$.
∴ the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -14 & -10 & -6 \\ 6 & -5 & -3 \\ -2 & -7 & 1 \end{bmatrix}$$

Question 3.
Find the adjoint of the following matrices.

(i) $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$

Here, $a_{11} = 2$, $M_{11} = 5$

$\therefore A_{11} = (-1)^{1+1}(5) = 5$

$a_{12} = -3$, $M_{12} = 3$

$\therefore A_{12} = (-1)^{1+2}(3) = -3$

$a_{21} = 3$, $M_{21} = -3$

$\therefore A\ A_{21} = (-1)^{2+1}(-3) = 3$

$a_{22} = 5$, $M_{22} = 2$

$\therefore A_{22} = (-1)^{2+1} = 2$

$\therefore$ the co-factor matrix = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

= $\begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$

$\therefore$ adj A = $\begin{pmatrix} 5 & 3 \\ -3 & 2 \end{pmatrix}$

(ii) $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

Solution:

Now, $M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix} = -3 - 0 = -3$

$\therefore A_{11} = (-1)^{1+1}(-3) = -3$

$M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix} = 2 + 10 = 12$

$\therefore A_{12} = (-1)^{1+2}(12) = -12$

$M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$

$\therefore A_{13} = (-1)^{1+3}(6) = 6$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1 - 0 = 1$

$\therefore A_{21} = (-1)^{2+1}(1) = -1$

$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -1 + 4 = 3$

$\therefore A_{22} = (-1)^{2+2}(3) = 3$

$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 0 - 2 = -2$

$\therefore A_{23} = (-1)^{2+3}(-2) = 2$

$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5-6 = -11$

$\therefore A_{31} = (-1)^{3+1}(-11) = -11$

$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} = 5+4 = 9$

$\therefore A_{32} = (-1)^{3+2}(9) = -9$

$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = 3-2 = 1$

$\therefore A_{33} = (-1)^{3+3}(1) = 1.$

A_{11} = -3, A_{12} = -12, A_{13} = 6,
A_{21} = -1, A_{22} = 3, A_{23} = 2,
A_{31} = -11, A_{32} = -9, A_{33} = 1

∴ the co-factor matrix = $\begin{bmatrix} A_{11} & A_{12} & A_{15} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

= $\begin{bmatrix} -3 & -12 & 6 \\ -1 & 3 & 2 \\ -11 & -9 & 1 \end{bmatrix}$

∴ adj A = $\begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$

Question 4.

If A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, verify that A (adj A) = (adj A) A = | A | · I

Solution:

A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$

$= 1(0+0) + 1(9+2) + 2(0-0)$

$= 0 + 11 + 0 = 11$

First we have to find the co-factor matrix $= [A_{ij}]_{3 \times 3}$,
where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 0 + 0 = 0$

$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -(9+2) = -11$

$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$

$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -(-3-0) = 3$

$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3 - 2 = 1$

$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(0+1) = -1$

$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2 - 0 = 2$

$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -(-2-6) = 8$

$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 0 + 3 = 3$

Hence the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0-0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \ldots (1)$$

$(\text{adj } A)A$

$$= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0-0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \ldots (2)$$

$$|A| \cdot I = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \ldots (3)$$

From (1), (2) and (3), we get,
A(adj A) = (adj A)A = |A|·I.
Note: This relation is valid for any non-singular matrix A.

Question 5.
Find the inverse of the following matrices by the adjoint method
(i) $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$
Solution:

Let A = $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix} = -2 + 15 = 13 \neq 0$

$\therefore A^{-1}$ exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{2\times2}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = 2$

$A_{12} = (-1)^{1+2}M_{12} = -(-3) = 3$

$A_{21} = (-1)^{2+1}M_{21} = -5$

$A_{22} = (-1)^{2+2}M_{22} = -1$

Hence, the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

(ii) $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Solution:

Solution:

Let A = $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

|A| = = 6 + 8 = 14 ≠ 0

$\therefore A^{-1}$ exist

First we have to find the co-factor matrix

$= [A_{ij}]_{2\times2}$ where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = 3$

$A_{12} = (-1)^{1+2}M = -4$

$A_{21} = (-2)^{2+1}M_{21} = (-2) = 2$

$A_{22} = (-1)^{2+2}M_{22} = 2$

Hence the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{14}\begin{pmatrix} 3 & 2 \\ -4 & 2 \end{pmatrix}$$

(iii) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$

$= 1(-3-0)-0+0$

$= -3 \neq 0$

$\therefore A^{-1}$ exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{3\times 3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3-0 = -3$

$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = -(-3-0) = 3$

$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = 6-15 = -9$

$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = -(0-0) = 0$

$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = -1-0 = -1$

$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = 0-0 = 0$

$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = -(0-0) = 0$

$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 3-0 = 3$

$\therefore$ the co-factor matrix

$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$

$\therefore \text{adj } A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$= \frac{1}{-3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ -3 & 1 & 0 \\ 9 & 2 & -3 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

∴ |A| = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

= 1(10 – 0) – 0 + 0

= 1(10) – 0 + 0

= 10 ≠ 0

∴ A^{-1} exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{3\times 3}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = 10 - 0 = 10$

$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = -0 - 0 = 0$

$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -10 - 0 = -10$

$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5 - 0 = 5$

$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -0 - 0 = 0$

$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 8 - 6 = 2$

$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -4 - 0 = -4$

$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

∴ the co-factor matrix

$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$

∴ adj A = $\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

∴ $A^{-1} = \frac{1}{|A|}$ (adj A)

$= \frac{1}{10}\begin{pmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{pmatrix}$

$$\therefore A^{-1} = \frac{1}{10}\begin{pmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{pmatrix}$$

Question 6.

Find the inverse of the following matrices

(i) $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$\therefore \ |A| = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5 \neq 0$

$\therefore \ A^{-1}$ exists.

Consider $AA^{-1} = I$

$\therefore \ \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $R_2 - 2R_1$, we get,

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

By $\left(-\frac{1}{5}\right)R_2$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 2/5 & -1/5 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.$$

The answer can be checked by finding the product AA^{-1}.

$$AA^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$$

$$= \begin{bmatrix} 1\left(\frac{1}{5}\right)+2\left(\frac{2}{5}\right) & 1\left(\frac{2}{5}\right)+2\left(-\frac{1}{5}\right) \\ 2\left(\frac{1}{5}\right)-1\left(\frac{2}{5}\right) & 2\left(\frac{2}{5}\right)-1\left(-\frac{1}{5}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5}+\frac{4}{5} & \frac{2}{5}-\frac{2}{5} \\ \frac{2}{5}-\frac{2}{5} & \frac{4}{5}+\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, A^{-1} is the required answer.

(ii) $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$\therefore \ |A| = \begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} = 4 - 3 = 1 \neq 0$

$\therefore \ A^{-1}$ exists.

Consider $AA^{-1} = I$

$\therefore \ \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

By $R_1 + R_2$, we get,

$\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

By $R_2 + R_1$, we get,

$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

By $R_1 + R_2$, we get,

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$

(iii) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$\therefore \; |A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$

$= 0(2-3) - 1(1-9) + 2(1-6)$

$= 0 + 8 - 10$

$= -2 \neq 0.$

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$\therefore \; \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $R_1 \leftrightarrow R_2$, we get,

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $R_3 - 3R_1$, we get,

$$\begin{bmatrix}1 & 2 & 3\\0 & 1 & 2\\0 & -5 & -8\end{bmatrix}A^{-1}=\begin{bmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & -3 & 1\end{bmatrix}$$

By R_1-2R_2 and R_3+5R_2, we get,

$$\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 2\end{bmatrix}A^{-1}=\begin{bmatrix}-2 & 1 & 0\\1 & 0 & 0\\5 & -3 & 1\end{bmatrix}$$

By $\left(\frac{1}{2}\right)R_3$, we get,

$$\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}-2 & 1 & 0\\1 & 0 & 0\\5/2 & -3/2 & 1/2\end{bmatrix}$$

By R_1+R_3 and R_2-2R_3, we get,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}1/2 & -1/2 & 1/2\\-4 & 3 & -1\\5/2 & -3/2 & 1/2\end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{2}\begin{bmatrix}1 & -1 & 1\\-8 & 6 & -2\\5 & -3 & 1\end{bmatrix}$$

(iv) $\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$

Solution:

Solution:

Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$

$= 2(3-0) - 0(15-0) - 1(5-0)$

$= 6 - 0 - 5 = 1 \neq 0.$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $3R_1$, we get,

$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $R_1 - R_2$, we get,

$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $R_2 - 5R_1$, we get,

$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \end{bmatrix}$

By $R_2 - 5R_3$, we get,

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 + R_2$ and $R_3 - R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_3$, we get,

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

By $R_1 + 3R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$

Ex 2.3

Question 1.
Solve the following equations by inversion method.
(i) x + 2y = 2, 2x + 3y = 3
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$$

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

By $(-1)R_2$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$\therefore A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Now, premultiply AX = B by A^{-1}, we get,

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = = \begin{bmatrix} -6+6 \\ 4-3 \end{bmatrix} = = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

By equality of matrices,

x = 0, y = 1 is the required solution.

(ii) x + y = 4, 2x – y = 5

Solution:

x + y = 4, 2x – y = 5

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

This is of the form AX = B ⇒ X ⇒ $A^{-1}B$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

|A| = -1 – 2 = -3 ≠ 0

$$\text{Adj } A = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{1}{-3} \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 8 & -5 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

By equality of matrices.

x = 3, y = 1

(iii) 2x + 6y = 8, x + 3y = 5

Solution:

The given equations can be written in the matrix form as :

$$\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$$

This is of the form AX = B, where

A = $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B = $\begin{bmatrix} 8 \\ 5 \end{bmatrix}$

Let us find A^{-1}.

|A| = $\begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix}$ = 6 − 6 = 0

∴ A^{-1} does not exist.

Hence, x and y do not exist.

Question 2.

Solve the following equations by reduction method.

(i) 2x + y = 5, 3x + 5y = -3

Solution:

The given equations can be written in the matrix form as :

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

By $2R_2$, we get,

$$\begin{bmatrix} 2 & 1 \\ 6 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

By $R_2 - 3R_1$, we get,

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x+y \\ 0+7y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

By equality of matrices,

2x + y = 5 …(1)

7y = -21 …(2)

From (2), y = -3

Substituting y = -3 in (1), we get,

2x – 3 = 5

∴ 2x = 8 ∴ x = 4

Hence, x = 4, y = -3 is the required solution.

(ii) x + 3y = 2, 3x + 5y = 4.

Solution:

The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

By $R_2 - 3R_1$, we get

$$\begin{bmatrix} 1 & 3 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\therefore \begin{bmatrix} x+3 \\ 0-4y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

By equality of matrices,

x + 3y = 2 …(1)

-4y = -2

From (2), y = $\frac{1}{2}$

Substituting y = $\frac{1}{2}$ in (1), we get,

x + $\frac{3}{2}$ = 2

$\therefore$ x = 2 − $\frac{3}{2} = \frac{1}{2}$

Hence, x = $\frac{1}{2}$, y = $\frac{1}{2}$ is the required solution.

(iii) 3x – y = 1, 4x + y = 6

Solution:

The given equations can be written in the matrix form as :

$$\begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

By $4R_1$ and $3R_2$, we get,

$$\begin{bmatrix} 12 & -4 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 18 \end{bmatrix}$$

By $R_2 - R_1$, we get,

$$\begin{bmatrix} 12 & -4 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 12x - 4y \\ 0 + 7y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

By equality of matrices,

12x – 4y = 4 … (1)

From (2), y = 2
Substituting y = 2 in (1), we get,
12x – 8 = 4
∴ 12x = 12 ∴ x = 1
Hence, x = 1, y = 2 is the required solution.

(iv) 5x + 2y = 4, 7x + 3y = 5
Solution:
5x + 2y = 4 ....(1)
7x + 3y = 5 ....(2)
Multiplying Eq. (1) with 7 and Eq. (2) with 5

$$\begin{array}{r} 35x + 14y = 28 \\ 35x + 15y = 25 \\ - \quad\quad - \quad\quad \\ \hline -1y = 3 \\ y = -3 \end{array}$$

Put y = -3 into Eq. (1)
5x + 2y = 4
5x + 2(-3) = 4
5x – 6 = 4
5x = 4 + 6
5x = 10
x = $\frac{10}{5}$
x = 2
Hence, x = 2, y = -3 is the required solution.

Question 3.
The cost of 4 pencils, 3 pens and 2 erasers is ₹ 60. The cost of 2 pencils, 4 pens and 6 erasers is ₹ 90, whereas the cost of 6 pencils, 2 pens and 3 erasers is ₹ 70. Find the cost of each item by using matrices.
Solution:
Let the cost of 1 pencil, 1 pen and 1 eraser be ₹ x, ₹ y and ₹ z respectively.
Then, from the given conditions,
$4x + 3y + 2z = 60$
$2x + 4y + 6z = 90$, i.e., $x + 2y + 3z = 45$
$6x + 2y + 3z = 70$
These equations can be written in the matrix form as :

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 60 \\ 70 \end{bmatrix}$$

By $R_2 - 4R_1$ and $R_3 - 6R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -10 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \\ -200 \end{bmatrix}$$

By $R_3 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+2y+3z \\ 0-5y-10z \\ 0+0+5z \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \\ 40 \end{bmatrix}$$

By equality of matrices,

x + 2y + 3z = 45 …….(1)

– 5y – 10z = – 120 …….(2)

5z = 40

From (3), z = 8

Substituting z = 8 in (2), we get,

– 5y – 80 = -120

∴ – 5y = -40 ∴ y = 8

Substituting y = 8, z = 8 in (1), we get,

x + 16 + 24 = 45

∴ x + 40 = 45 ∴ x = 5

∴ x = 5, y = 8, z = 8

Hence, the cost is ₹ 5 for a pencil, ₹ 8 for a pen and ₹ 8 for an eraser.

Question 4.

If three numbers are added, their sum is 2. If 2 times the second number is subtracted from the sum of first and third numbers, we get 8 and if three times the first number is added to the sum of second and third numbers, we get 4. Find the numbers using matrices.

Solution:

Let the three numbers be x, y and z. According to the given conditions,

x + y + z = 2

x + z – 2y = 8, i.e., x – 2y + 2 = 8

x + z – 2y = 8, i.e., x – 2y + 2 = 8
and y + z + 3x = 4, i.e., 3x + y + z = 4
Hence, the system of linear equations is
x + y + z = 2
x – 2y + z = 8
3x + y + z = 4
These equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + y + z \\ 0 - 3y + 0 \\ 0 - 2y - 2z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}$$

By equality of matrices,
x + y + z = 2 ……(1)
-3y = 6 ……(2)
– 2y – 2z = -2 ……..(3)
From (2), y = -2
Substituting y = -2 in (3), we get,
-2(-2) – 2z = -2
∴ -2z = -6 ∴ z = 3
Substituting y = -2, z = 3 in (1), we get,

Question 5.

The total cost of 3 T.V. sets and 2 V.C.R.s is ₹ 35000. The shop-keeper wants profit of ₹ 1000 per television and ₹ 500 per V.C.R. He can sell 2 T. V. sets and 1 V.C.R. and get the total revenue as ₹ 21,500. Find the cost price and the selling price of a T.V. sets and a V.C.R.

Solution:

Let the cost of each T.V. set be ₹ x and each V.C.R. be ₹ y. Then the total cost of 3 T.V. sets and 2 V.C.R.'s is ₹ (3x + 2y) which is given to be ₹ 35,000.

∴ 3x + 2y = 35000

The shopkeeper wants profit of ₹ 1000 per T.V. set and of ₹ 500 per V.C.R.

∴ the selling price of each T.V. set is ₹ (x + 1000) and of each V.C.R. is ₹ (y + 500).

∴ selling price of 2 T.V. set and 1 V.C.R. is

₹ [2(x + 1000) + (y + 500)] which is given to be ₹ 21,500.

∴ 2(x + 1000) + (y + 500) = 21500

∴ 2x + 2000 + y + 500 = 21500

∴ 2x + y = 19000

Hence, the system of linear equations is

3x + 2y = 35000

2x + y = 19000

These equations can be written in the matrix form as :

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35000 \\ 19000 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19000 \\ 35000 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19000 \\ -3000 \end{bmatrix}$$

IndCareer

$$\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19000 \\ -3000 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x+y \\ -x+0 \end{bmatrix} = \begin{bmatrix} 19000 \\ -3000 \end{bmatrix}$$

By equality of matrices,

2x + y = 19000 ……….(1)

-x = -3000 ……….(2)

From (2), x = 3000

Substituting x = 3000 in (1), we get,

2(3000) + y = 19000

∴ y = 13000

∴ the cost price of one T.V. set is ₹ 3000 and of one V.C.R. is ₹ 13000 and the selling price of one T.V. set is ₹ 4000 and of one V.C.R. is ₹ 13500.

Maharashtra Board Solutions Class 12 Arts & Science Maths (Part 1)

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Step 3: From the "Classes" section, select your class.

Step 4: From "Medium", select the medium suitable to you.

Step 5: All Maharashtra board books for class 11th will now be displayed on the right side.

Step 6: Click on the "Download" option to download the PDF book.

Who developed the Maharashtra State board books?

As of now, the MSCERT and Balbharti are responsible for the syllabus and textbooks of Classes 1 to 8, while Classes 9 and 10 are under the Maharashtra State Board of Secondary and Higher Secondary Education (MSBSHSE).

How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

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