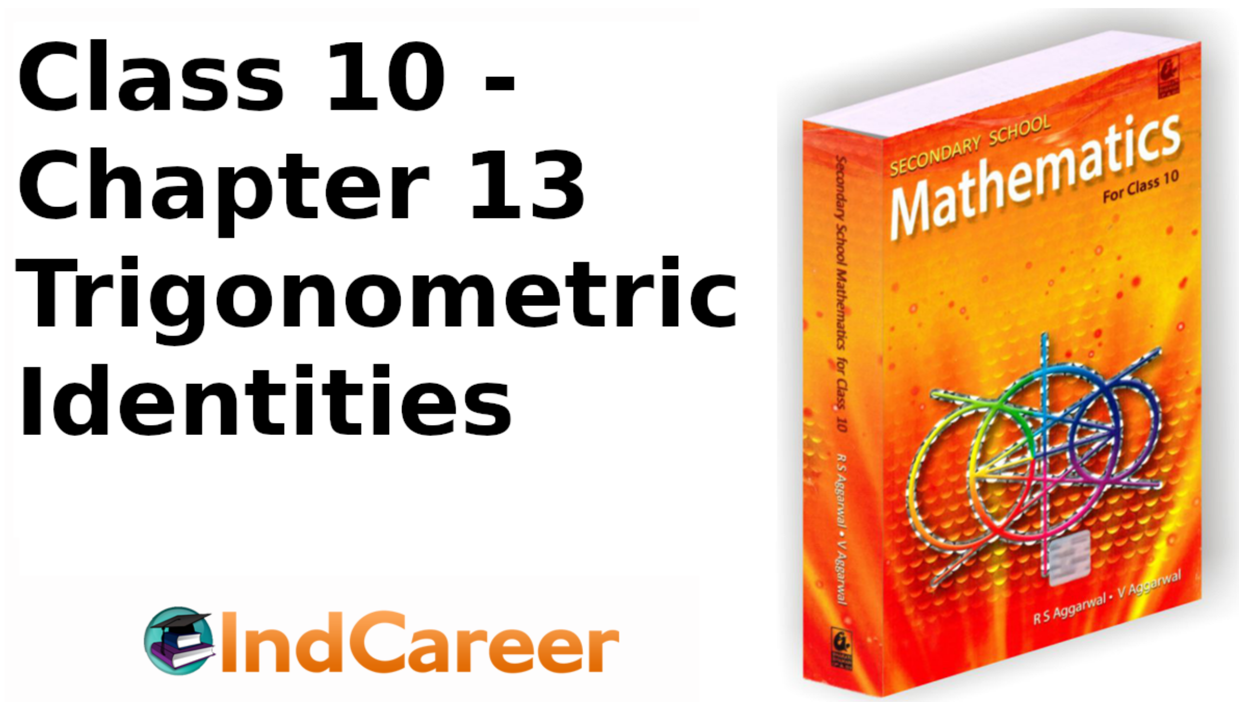
RS Aggarwal Solutions for Class 10 Maths Chapter 13–Trigonometric Identities



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RS Aggarwal Solutions for Class 10 Maths Chapter 13–Trigonometric Identities

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RS Aggarwal Solutions for Class 10 Maths Chapter 13–Trigonometric Identities

RS Aggarwal 10th Maths Chapter 13, Class 10 Maths Chapter 13 solutions

Ex 8a

Question 1.

Solution:

(i) $LHS = (1 - \cos^2\theta)\operatorname{cosec}^2\theta$

$= \sin^2\theta \times \operatorname{cosec}^2\theta \quad [\because (1 - \cos^2\theta) = \sin^2\theta]$

$= \sin^2\theta \times \frac{1}{\sin^2\theta} = 1 = RHS$

$\therefore$ LHS = RHS

(ii) $LHS = (1 + \cot^2\theta)\sin^2\theta$

$= \operatorname{cosec}^2\theta \times \sin^2\theta \quad [\because (1 + \cot^2\theta) = \operatorname{cosec}^2\theta]$

$= \frac{1}{\sin^2\theta} \times \sin^2\theta = 1 = RHS$

$\therefore$ LHS = RHS

Question 2.

Solution:

(i) $LHS = (\sec^2\theta - 1)\cot^2\theta \quad [\because (\sec^2\theta - 1) = \tan^2\theta]$

$= \tan^2\theta \times \cot^2\theta$

$= \tan^2\theta \times \frac{1}{\tan^2\theta} = 1 = RHS$

$\therefore LHS = RHS$

(ii) $LHS = (\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$

$= \tan^2\theta \times \cot^2\theta \quad \left[\begin{array}{l} \because (\sec^2\theta - 1) = \tan^2\theta \\ \text{and } (\operatorname{cosec}^2\theta - 1) = \cot^2\theta \end{array}\right]$

$= \tan^2\theta \times \frac{1}{\tan^2\theta} = 1 = RHS$

$\therefore LHS = RHS$

(iii) $(1 - \cos^2\theta)\sec^2\theta$

$= \sin^2\theta \times \sec^2\theta \quad [\because (1 - \cos^2\theta) = \sin^2\theta]$

$= \sin^2\theta \times \frac{1}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta = RHS$

$\therefore LHS = RHS$

Question 3.

Solution:

(i) $LHS = \sin^2\theta + \frac{1}{1+\tan^2\theta}$ $\left[\because \left(1+\tan^2\theta\right) = \sec^2\theta\right]$

$= \sin^2\theta + \frac{1}{\sec^2\theta}$

$= \sin^2\theta + \cos^2\theta = 1 = RHS$

$\therefore$ LHS = RHS

(ii) $LHS = \frac{1}{\left(1+\tan^2\theta\right)} + \frac{1}{\left(1+\cot^2\theta\right)}$

$= \frac{1}{\sec^2\theta} + \frac{1}{\operatorname{cosec}^2\theta}$ $\left[\because \left(1+\tan^2\theta\right) = \sec^2 \text{ or } \left(1+\cot^2\theta\right) = \operatorname{cosec}^2\theta\right]$

$= \cos^2\theta + \sin^2\theta = 1 = RHS$

$\therefore$ LHS = RHS

Question 4.

Solution:

(i)

$$= \left(1 - \cos^2\theta\right)\left(1 + \cot^2\theta\right)$$

$$= \sin^2\theta \times \operatorname{cosec}^2\theta$$

$$\left[\because \left(1 - \cos^2\theta\right) = \sin^2\theta \text{ and } \left(1 + \cot^2\theta\right) = \operatorname{cosec}^2\theta\right]$$

$$= \sin^2\theta \times \frac{1}{\sin^2\theta} = 1 = \text{RHS}$$

(ii)

$$= \left(\operatorname{cosec}\theta + \frac{\cos\theta}{\sin\theta}\right)\left(\operatorname{cosec}\theta - \cot\theta\right)$$

$$= \left(\operatorname{cosec}\theta + \cot\theta\right)\left(\operatorname{cosec}\theta - \cot\theta\right)$$

$$= \left(\operatorname{cosec}^2\theta - \cot^2\theta\right) = 1 = \text{RHS}$$

$\therefore$ LHS = RHS

Question 5.

Solution:

(i)

$$\text{L.H.S.} = \cot^2\theta - \frac{1}{\sin^2\theta}$$
$$= \cot^2\theta - \text{cosec}^2\theta$$
$$= -1 \quad \ldots.(\text{since } 1+\cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta - \text{cosec}^2\theta = -1)$$
$$= \text{R.H.S.}$$

(ii)

$$\text{L.H.S.} = \tan^2\theta - \frac{1}{\cos^2\theta}$$
$$= \tan^2\theta - \sec^2\theta$$
$$= -1 \quad \ldots.(\text{since } 1+\tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta - \sec^2\theta = -1)$$
$$= \text{R.H.S.}$$

(iii)

$$\text{L.H.S.} = \cos^2\theta + \frac{1}{(1+\cot^2\theta)}$$
$$= \cos^2\theta + \frac{1}{\text{cosec}^2\theta}$$
$$= \cos^2\theta + \sin^2\theta$$
$$= 1$$
$$= \text{R.H.S.}$$

Question 6.

Solution:

$$\text{L.H.S.} = \frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)}$$
$$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$
$$= \frac{2}{1-\sin^2\theta}$$
$$= \frac{2}{\cos^2\theta}$$
$$= 2\sec^2\theta$$
$$= \text{R.H.S.}$$

Question 7.

Solution:

(i) $LHS = \sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta)$

$$= \left[\sec\theta - \frac{\sin\theta}{\cos\theta}\right] \times (\sec\theta + \tan\theta)$$

$$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$$

$$= (\sec^2\theta - \tan^2\theta) = 1 = RHS$$

$\therefore$ LHS = RHS

(ii) $LHS = \sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta)$

$$= \left(\sin\theta + \frac{\sin^2\theta}{\cos\theta}\right) + \cos\theta\left(1 + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{\sin\theta\cos\theta + \sin^2\theta}{\cos\theta}\right) + \left(\cos\theta + \frac{\cos^2\theta}{\sin\theta}\right)$$

$$= \left(\frac{\sin\theta\cos\theta + \sin^2\theta}{\cos\theta}\right) + \left(\frac{\cos\theta\sin\theta + \cos^2\theta}{\sin\theta}\right)$$

$$= \frac{\sin\theta\cos\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta\sin\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{\sin\theta\cos\theta}{\cos\theta} + \frac{\cos\theta\sin\theta}{\sin\theta} + \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta}$$

$$= \sin\theta\cos\theta\left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right) + \frac{\sin^3\theta + \cos^3\theta}{\cos\theta\sin\theta}$$

$$= \sin\theta\cos\theta\left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right) + \frac{(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{\cos\theta\sin\theta}$$

$$\left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$$

$$= (\sin\theta + \cos\theta)\left[\frac{\sin\theta\cos\theta}{\cos\theta\sin\theta} + \frac{(1 - \sin\theta\cos\theta)}{\cos\theta\sin\theta}\right]$$

$$= (\sin\theta + \cos\theta)\left[\frac{\sin\cos\theta + 1 - \sin\theta\cos\theta}{\cos\theta\sin\theta}\right]$$

$$= (\sin\theta + \cos\theta)\left(\frac{1}{\cos\theta\sin\theta}\right)$$

$$= \frac{\sin}{\cos\theta\sin\theta} + \frac{\cos\theta}{\cos\theta\sin\theta} = \sec\theta + \operatorname{cosec}\theta = RHS$$

Question 8.

Solution:

(i) LHS = $1+\frac{\cot^2\theta}{(1+\operatorname{cosec}\theta)}$

$=1+\frac{\operatorname{cosec}^2\theta-1}{(1+\operatorname{cosec}\theta)}$

$=\frac{1+\operatorname{cosec}\theta+\operatorname{cosec}^2\theta-1^2}{(1+\operatorname{cosec}\theta)}$

$=\frac{(1+\operatorname{cosec}\theta)+(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)}{(1+\operatorname{cosec}\theta)}$

$=\frac{(1+\operatorname{cosec}\theta)[1+(\operatorname{cosec}\theta-1)]}{(1+\operatorname{cosec}\theta)}$

$=1+\operatorname{cosec}\theta-1$

$=\operatorname{cosec}\theta=\text{RHS}$

LHS = RHS

(ii) $\text{LHS}=1+\frac{\tan^2\theta}{(1+\sec\theta)}=\frac{1+\sec\theta+\tan^2\theta}{(1+\sec\theta)}$

$=\frac{\sec^2\theta+\sec\theta}{(1+\sec\theta)}\quad\left[\because(1+\tan^2\theta)=\sec^2\theta\right]$

$=\frac{\sec\theta(1+\sec\theta)}{(1+\sec\theta)}=\sec\theta=\text{RHS}$

Question 9.

Solution:

$\text{LHS}=\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta}=\frac{\sec^2\theta\cot\theta}{\operatorname{cosec}^2\theta}$

$\left[\because(1+\tan^2\theta)=\sec^2\theta\right]$

$=\frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}\times\sin^2\theta=\frac{\sin\theta}{\cos\theta}$

$=\tan\theta=\text{RHS}$

LHS = RHS

Question 10.

Solution:

$$\frac{\tan^2\theta}{(1+\tan^2\theta)}+\frac{\cot^2\theta}{(1+\cot^2\theta)}$$

$$\text{LHS}=\frac{\tan^2\theta}{\sec^2\theta}+\frac{\cot^2\theta}{\operatorname{cosec}^2\theta}$$
$$\left[\because(1+\tan^2\theta)=\sec^2\theta \text{ and } (1+\cot^2\theta)=\operatorname{cosec}^2\theta\right]$$
$$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}+\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$$
$$=\sin^2\theta+\cos^2\theta=1=\text{RHS}$$
Hence, LHS = RHS

Question 11.

Solution:

$$\text{LHS}=\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}+\frac{1+\cos\theta}{\sin\theta}$$
$$=\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}+\frac{1+\cos\theta}{\sin\theta}$$
$$=\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{(1-\cos\theta)}{\sin\theta}+\frac{1+\cos\theta}{\sin\theta}$$
$$=\frac{1-\cos\theta+1+\cos\theta}{\sin\theta}=\frac{2}{\sin\theta}=2\operatorname{cosec}\theta=\text{RHS}$$

Question 12.

Solution:

$$LHS = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$$

$$\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}+1 = 1+\sec\theta\operatorname{cosec}\theta = RHS$$

$\therefore$ RHS = LHS

Question 13.

Solution:

$$\text{LHS} = \frac{\cos^2\theta}{(1-\tan\theta)} + \frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$$

$$\frac{\cos^2\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)} + \frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$$

$$= \frac{\cos^3\theta}{(\cos\theta-\sin\theta)} + \frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$$

$$= \frac{\cos^3\theta}{(\cos\theta-\sin\theta)} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^3\theta-\sin^3\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta\sin\theta+\sin^2\theta)}{(\cos\theta-\sin\theta)}$$

$$\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$

$$= (1+\cos\theta\sin\theta) = \text{RHS}$$

$\therefore$ RHS = LHS

Question 14.

Solution:

$$\text{LHS} = \frac{\cos\theta}{(1-\tan\theta)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\cos\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\cos^2\theta}{(\cos\theta-\sin\theta)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)} = \frac{\cos^2\theta-\sin^2\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)} = (\cos\theta+\sin\theta) = \text{RHS}$$

$\therefore$ LHS = RHS

Question 15.

Solution:

$$LHS = (1 + \tan^2\theta)(1 + \cot^2\theta)$$

$$= \sec^2\theta \operatorname{cosec}^2\theta$$
$$= \frac{1}{\sin^2\theta\cos^2\theta} = \frac{1}{\sin^2\theta(1 - \sin^2\theta)}$$
$$= \frac{1}{\sin^2\theta - \sin^4\theta} = RHS$$

$$\therefore RHS = LHS$$

Question 16.

Solution:

$$\frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$$

$$= \frac{\tan\theta}{(\sec^2\theta)^2} + \frac{\cot\theta}{(\operatorname{cosec}^2\theta)^2}$$
$$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sec^4\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{1}{\operatorname{cosec}^4\theta}$$
$$= \frac{\sin\theta}{\cos\theta} \times \cos^4\theta + \frac{\cos\theta}{\sin\theta} \times \sin^4\theta$$
$$= \sin\theta\cos^3\theta + \cos\theta\sin^3\theta$$
$$= \sin\theta\cos\theta(\cos^2\theta + \sin^2\theta)$$
$$= \sin\theta\cos\theta = RHS$$
$$LHS = RHS$$

Question 17.

Solution:

(i)To prove $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

We know, $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

put $a = \sin^2\theta, b = \cos^2\theta$

$\therefore \sin^6\theta + \cos^6\theta = (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta \times (\sin^2\theta + \cos^2\theta)$

$= 1 - 3\sin^2\theta\cos^2\theta = RHS$

Therefore, LHS = RHS

(ii)$LHS = \sin^2\theta + \cos^4\theta = 1 - \cos^2\theta + \cos^4\theta$

$= 1 - \cos^2\theta(1 - \cos^2\theta)$

$= 1 - \cos^2\theta\sin^2\theta$

$RHS = \cos^2\theta + \sin^4\theta = 1 - \sin^2\theta + \sin^4\theta$

$= 1 - \sin^2\theta(1 - \sin^2\theta) = (1 - \sin^2\theta\cos^2\theta)$

Therefore, LHS = RHS

(iii) $\operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta$

$LHS = \operatorname{cosec}^2\theta(\operatorname{cosec}^2\theta - 1)$

$= (1 + \cot^2\theta)\cot^2\theta$

$= \cot^2\theta + \cot^4\theta = RHS$

Therefore, LHS = RHS

Question 18.

Solution:

(i) $\text{LHS} = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$$= \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\left(\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}\right)}{\left(\frac{\cos^2\theta + \sin^2\theta}{\cos^2}\right)} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\left(\cos^2\theta - \sin^2\theta\right)}{1} = \left(\cos^2\theta - \sin^2\theta\right) = \text{RHS}$$

$\therefore$ LHS = RHS

Question 19.

Solution:

(i) $\text{LHS} = \frac{\tan\theta}{(\sec\theta - 1)} + \frac{\tan\theta}{(\sec\theta + 1)}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1}{\cos\theta} - 1\right)} + \frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1}{\cos\theta} + 1\right)}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1 - \cos\theta}{\cos\theta}\right)} + \frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1 + \cos\theta}{\cos\theta}\right)}$

$= \frac{\sin\theta}{1 - \cos\theta} + \frac{\sin\theta}{1 + \cos\theta}$

$= \frac{\sin\theta(1 + \cos\theta) + \sin\theta(1 - \cos\theta)}{1 - \cos^2\theta}$

$= \frac{\sin\theta + \sin\theta\cos\theta + \sin\theta - \sin\theta\cos ec\theta}{\sin^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta} = \frac{2}{\sin\theta} = 2\cos ec\theta = \text{RHS}$

∴ LHS = RHS

(ii) LHS = $\dfrac{\cot\theta}{(\operatorname{cosec}\theta+1)}+\dfrac{(\operatorname{cosec}\theta+1)}{\cot\theta}$

$$=\frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{1}{\sin\theta}+1\right)}+\frac{\left(\frac{1}{\sin\theta}+1\right)}{\left(\frac{\cos\theta}{\sin\theta}\right)}$$

$$=\frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{1+\sin\theta}{\sin\theta}\right)}+\frac{\left(\frac{1+\sin\theta}{\sin\theta}\right)}{\left(\frac{\cos\theta}{\sin\theta}\right)}$$

$$=\frac{\cos\theta}{1+\sin\theta}+\frac{(1+\sin\theta)}{\cos\theta}=\frac{\cos^2\theta+(1+\sin\theta)^2}{\cos\theta(1+\sin\theta)}$$

$$=\frac{\cos^2\theta+1+\sin^2\theta+2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{1+1+2\sin\theta}{\cos\theta(1+\sin\theta)}=\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\frac{2}{\cos\theta}=2\sec\theta=\text{RHS}$$

Question 20.

Solution:

(i) LHS =

$$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\left(\frac{1}{\cos\theta} - 1\right)}{\left(\frac{1}{\cos\theta} + 1\right)} = \frac{1 - \cos\theta}{1 + \cos\theta}$$

$$= \frac{(1 - \cos\theta)}{(1 + \cos\theta)} \times \frac{(1 + \cos\theta)}{(1 + \cos\theta)} = \frac{1 - \cos^2\theta}{(1 + \cos\theta)^2}$$

$$= \frac{\sin^2\theta}{(1 + \cos\theta)^2}$$

Hence, LHS = RHS

(ii)

$$\text{LHS} = \frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{(1 - \sin\theta)}{(1 + \sin\theta)}$$

$$= \frac{(1 - \sin\theta)}{(1 + \sin\theta)} \times \frac{(1 + \sin\theta)}{(1 + \sin\theta)} = \frac{1 - \sin^2\theta}{(1 + \sin\theta)^2}$$

$$= \frac{\cos^2\theta}{(1 + \sin\theta)^2} = \text{RHS}$$

$\therefore$ LHS = RHS

Question 21.

Solution:

$$\text{L.H.S.} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$$
$$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$
$$= \frac{1+\sin\theta}{\cos\theta}$$
$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$
$$= \sec\theta + \tan\theta$$
$$= \text{R.H.S.}$$

$$\text{L.H.S.} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$
$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$$
$$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$
$$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$
$$= \frac{1-\cos\theta}{\sin\theta}$$
$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$
$$= \operatorname{cosec}\theta - \cot\theta$$
$$= \text{R.H.S.}$$

$$\text{L.H.S.} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$
$$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$$
$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$
$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$
$$= \frac{1+\cos\theta}{\sin\theta} + \frac{1-\cos\theta}{\sin\theta}$$
$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$
$$= \operatorname{cosec}\theta + \operatorname{cosec}\theta$$
$$= 2\operatorname{cosec}\theta$$
$$= \text{R.H.S.}$$

Question 22.

Solution:

$$LHS = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta - \cos\theta\sin\theta + \sin^2\theta)}{\cos\theta + \sin\theta} + \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \cos\theta\sin\theta + \sin^2\theta)}{\cos\theta - \sin\theta}$$

$$= \cos^2\theta - \cos\theta \times \sin\theta + \sin^2\theta + \cos^2\theta + \cos\theta\sin\theta + \sin^2\theta$$

$$= 2\left[\cos^2\theta + \sin^2\theta\right] = 2$$

$\therefore$ LHS = RHS

Question 23.

Solution:

$$LHS = \frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} - \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$$

$$= \frac{\sin\theta}{\operatorname{cosec}\theta + \cot\theta} + \frac{\sin\theta}{\operatorname{cosec}\theta - \cot\theta}$$

$$= \frac{\sin\theta(\operatorname{cosec}\theta - \cot\theta) + \sin\theta(\operatorname{cosec}\theta + \cot\theta)}{\operatorname{cosec}^2\theta - \cot^2\theta}$$

$$= \sin\theta(\operatorname{cosec}\theta - \cot\theta) + \sin\theta(\operatorname{cosec}\theta + \cot\theta)$$

$$\left[\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta \text{ and } \operatorname{cosec}^2\theta - \cot^2\theta = 1\right]$$

$$= 2\sin\theta\operatorname{cosec}\theta = 2\sin\theta \times \frac{1}{\sin\theta} = 2 = RHS$$

$\therefore$ LHS = RHS

Question 24.

Solution:

(i) $\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{1+1}{\sin^2\theta - (1 - \sin^2\theta)} = \frac{2}{(2\sin^2\theta - 1)} = \text{RHS}$$

$\therefore$ LHS = RHS

(ii) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$

$$\text{LHS} = \frac{(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta - 2\cos\theta\sin\theta}{1 - \cos^2\theta - \cos^2\theta}$$

$$= \frac{1+1}{1 - 2\cos^2\theta} = \frac{2}{(1 - 2\cos^2\theta)} = \text{RHS}$$

$\therefore$ LHS = RHS

Question 25.

Solution:

$$\text{LHS} = \frac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)} = \frac{1 + \cos\theta - (1 - \cos^2\theta)}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{1 + \cos\theta - 1 + \cos^2\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{\cos\theta(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)} = \frac{\cos\theta}{\sin\theta} = \cot\theta = \text{RHS}$$

$\therefore$ LHS = RHS

Question 26.

Solution:

(i)

$$LHS = \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\operatorname{cosec}\theta - \cot\theta)} \times \frac{(\operatorname{cosec}\theta + \operatorname{Cot}\theta)}{(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)^2}{(\operatorname{cosec}^2\theta - \cot^2\theta)} = (\operatorname{cosec}\theta + \cot\theta)^2$$

Further,

$$(\operatorname{cosec}\theta + \cot\theta)^2 = \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$$

$$= 1 + \cot^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$$

$$= 1 + 2\cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$$

$\therefore$ LHS = RHS

$$\text{(ii) } LHS = \frac{(\sec\theta + \tan\theta)}{(\sec\theta - \tan\theta)} \times \frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}$$

$$= \frac{(\sec\theta + \tan\theta)^2}{(\sec^2\theta - \tan^2\theta)} = (\sec\theta + \tan\theta)^2$$

Further,

$$(\sec\theta + \tan\theta)^2 = \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$$

$$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$$

$$= 1 + 2\tan^2\theta + 2\sec\theta\tan\theta = RHS$$

$\therefore$ LHS = RHS

Question 27.

Solution:

(i)

$$LHS = \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

On dividing the numerator and denominator of LHS by cosθ,We get

$$LHS = \frac{\sec\theta+1+\tan\theta}{\sec\theta+1-\tan\theta}$$

$$= \frac{(\sec\theta+\tan\theta)+(\sec^2\theta-\tan^2\theta)}{1+\sec\theta-\tan\theta}$$

$$\text{writing } 1 = (\sec^2\theta-\tan^2\theta)$$

$$= \frac{(\sec\theta+\tan\theta)+(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{(1+\sec\theta-\tan\theta)}$$

$$= \frac{(\sec\theta+\tan\theta)(1+\sec\theta-\tan\theta)}{(1+\sec\theta-\tan\theta)}$$

$$= (\sec\theta+\tan\theta) = \left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)$$

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right) = RHS$$

$$\therefore LHS = RHS$$

(ii)

$$LHS = \frac{\sin\theta + 1 - \cos\theta}{\cos\theta - 1 + \sin\theta}$$

On dividing the numerator and denominator of LHS by cosθ,We get

$$LHS = \frac{\tan\theta + \sec\theta - 1}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{(1 - \sec\theta + \tan\theta)}$$

$$(\text{writing } 1 = \sec^2\theta - \tan^2\theta)$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{(1 - \sec\theta + \tan\theta)}$$

$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{(1 - \sec\theta + \tan\theta)}$$

$$= \tan\theta + \sec\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = \frac{\sin\theta + 1}{\cos\theta} = RHS$$

$\therefore$ LHS = RHS

Question 28.

Solution:

$$\text{LHS} = \frac{\sin\theta}{(\sec\theta + \tan\theta - 1)} + \frac{\cos\theta}{(\operatorname{cosec}\theta + \cot\theta - 1)}$$
$$= \frac{\sin\theta}{\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} - 1\right)} + \frac{\cos\theta}{\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} - 1\right)}$$
$$= \frac{\sin\theta\cos\theta}{(1 + \sin\theta - \cos\theta)} + \frac{\cos\theta\sin\theta}{(1 + \cos\theta - \sin\theta)}$$
$$= \frac{\sin\theta\cos\theta(1 + \cos\theta - \sin\theta) + \cos\theta\sin\theta(1 + \sin\theta - \cos\theta)}{(1 + \sin\theta - \cos\theta)(1 + \cos\theta - \sin\theta)}$$
$$= \frac{\sin\theta\cos\theta + \sin\theta\cos^2\theta - \sin^2\theta\cos\theta + \cos\theta\sin\theta + \cos\theta\sin^2\theta - \cos^2\theta\sin\theta}{(1 + \sin\theta - \cos\theta)(1 + \cos\theta - \sin\theta)}$$
$$= \frac{2\sin\theta\cos\theta}{1 + \cos\theta - \sin\theta + \text{Sin}\theta + \sin\theta\cos\theta - \sin^2\theta - \cos\theta - \cos^2\theta + \cos\theta\sin\theta}$$
$$= \frac{2\sin\theta\cos\theta}{2\sin\theta\cos\theta} = 1 = \text{RHS}$$

$\therefore$ LHS = RHS

Question 29.

Solution:

$$\text{LHS} = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$$
$$= \frac{(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2}{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}$$
$$= \frac{(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta) + (\sin^2\theta + \cos^2 - 2\sin\theta\cos\theta)}{\sin^2\theta - \cos^2\theta}$$
$$= \frac{(1 + 2\sin\theta\cos\theta) + (1 - 2\sin\theta\cos\theta)}{\sin^2\theta - \cos^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{2}{\sin^2\theta - \cos^2\theta}$$

Also, $\frac{2}{\sin^2\theta - \cos^2\theta} = \frac{2}{\sin^2\theta - (1 - \sin^2\theta)} = \frac{2}{2\sin^2\theta - 1} = \text{RHS}$

$\therefore$ RHS = LHS

Question 30.

Solution:

$$LHS = \frac{\cos\theta \operatorname{cosec}\theta - \sin\theta \sec\theta}{\cos\theta + \sin\theta}$$

$$= \frac{\cos\theta \times \frac{1}{\sin\theta} - \sin\theta \times \frac{1}{\cos\theta}}{\cos\theta + \sin\theta} = \frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{(\cos\theta + \sin\theta)}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta(\cos\theta + \sin\theta)}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{\sin\theta\cos\theta(\cos\theta + \sin\theta)}$$

$$= \frac{\cos\theta}{\sin\theta\cos\theta} - \frac{\sin\theta}{\sin\theta\cos\theta}$$

$$= \operatorname{cosec}\theta - \sec\theta = RHS$$

$$\therefore LHS = RHS$$

Question 31.

Solution:

$$\text{LHS} = (1+\tan\theta+\cot\theta)(\sin\theta-\cos\theta)$$
$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)(\sin\theta-\cos\theta)$$
$$= \left(\frac{\cos\theta\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)(\sin\theta-\cos\theta)$$
$$= \frac{(\cos\theta\sin\theta+1)}{\cos\theta\sin\theta}(\sin\theta-\cos\theta)$$
$$\text{RHS} = \left(\frac{\sec\theta}{\operatorname{cosec}^2\theta}-\frac{\operatorname{cosec}\theta}{\sec^2\theta}\right) = \left(\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin^2\theta}}-\frac{\frac{1}{\sin\theta}}{\frac{1}{\cos^2\theta}}\right)$$
$$= \left(\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right) = \frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta}$$
$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\cos\theta\sin\theta)}{\cos\theta\sin\theta}$$
$$\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$
$$= \frac{(\sin\theta-\cos\theta)(1+\cos\theta\sin\theta)}{\cos\theta\sin\theta}$$
$$\therefore \text{RHS} = \text{LHS}$$

Question 32.

Solution:

$$\text{L.H.S.} = \frac{\cot^2\theta(\sec\theta - 1)}{(1+\sin\theta)} + \frac{\sec^2\theta(\sin\theta - 1)}{(1+\sec\theta)}$$

$$= \frac{\cot^2\theta(\sec\theta - 1)(1+\sec\theta) + \sec^2\theta(\sin\theta - 1)(1+\sin\theta)}{(1+\sin\theta)(1+\sec\theta)}$$

$$= \frac{\cot^2\theta(\sec^2\theta - 1) + \sec^2\theta(\sin^2\theta - 1)}{(1+\sin\theta)(1+\sec\theta)}$$

$$= \frac{\cot^2\theta\tan^2\theta + \sec^2\theta(-\cos^2\theta)}{(1+\sin\theta)(1+\sec\theta)}$$

$$= \frac{\cot^2\theta\tan^2\theta - \sec^2\theta\cos^2\theta}{(1+\sin\theta)(1+\sec\theta)}$$

$$= \frac{\cot^2\theta \times \frac{1}{\cot^2\theta} - \sec^2\theta \times \frac{1}{\sec^2\theta}}{(1+\sin\theta)(1+\sec\theta)}$$

$$= \frac{1 - 1}{(1+\sin\theta)(1+\sec\theta)}$$

$$= 0$$

$$= \text{R.H.S.}$$

Question 33.

Solution:

$$LHS = \left[\frac{1}{(\sec^2\theta - \cos^2\theta)} + \frac{1}{(\operatorname{cosec}^2\theta - \sin^2\theta)}\right] \times \sin^2\theta\cos^2\theta$$

$$= \left[\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta} +\right] \times \sin^2\theta\cos^2\theta$$

$$= \left[\frac{\sin^2\theta\cos^2\theta \times \cos^2\theta}{(1-\cos^4\theta)} + \frac{\sin^2\theta\cos^2\theta\sin^2\theta}{1-\sin^4\theta}\right]$$

$$= \left[\frac{\sin^2\theta \times \cos^4\theta}{(1+\cos^2\theta)(1-\cos^2\theta)} + \frac{\sin^4\theta\cos^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right]$$

$$= \left[\frac{\cos^4\theta}{(1+\cos^2\theta)} + \frac{\sin^4\theta}{1+\sin^2\theta}\right]$$

$$= \frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{\cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)}{1+\sin^2\theta + \cos^2\theta + \cos^2\theta\sin^2\theta}$$

$$\left[\because a^2 + b^2 = (a+b)^2 - 2ab\right]$$

$$= \frac{(\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta}{2 + \cos^2\theta\sin^2\theta}$$

$$= \frac{1 - \cos^2\theta\sin^2\theta}{2 + \cos^2\theta\sin^2\theta} = RHS$$

$\therefore$ LHS = RHS

Question 34.

Solution:

$$\text{LHS} = \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{(\sin A + \sin B)(\sin A - \sin B) + (\cos A + \cos B)(\cos A - \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 A + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)} = 0 = \text{RHS}$$

$\therefore$ LHS = RHS

Question 35.

Solution:

$$\text{LHS} = \frac{\tan A + \tan B}{\cot A + \cot B}$$

$$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}} = \frac{\frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B}}{\frac{\cos A \sin B + \cos B \sin A}{\sin A \sin B}}$$

$$= \frac{(\sin A \cos B + \sin B \cos A) \times \sin A \sin B}{\cos A \cos B \times (\cos A \sin B + \cos B \sin A)}$$

$$= \frac{\sin A \sin B}{\cos A \cos B} = \tan A \tan B = \text{RHS}$$

$\therefore$ LHS = RHS

Question 36.

Solution:

(i)

$\cos^2\theta + \cos\theta = 1$

Taking $\theta = 45°$, we have

$$\text{L.H.S.} = \cos^2 45° + \cos 45°$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{\sqrt{2}}$$
$$= \frac{1}{2} + \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{2}+1}{2\sqrt{2}}$$
$$\neq 1$$
$$\neq \text{R.H.S.}$$

(ii)

$\sin^2\theta + \sin\theta = 2$

Taking $\theta = 45°$, we have

$$\text{L.H.S.} = \sin^2 45° + \sin 45°$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{\sqrt{2}}$$
$$= \frac{1}{2} + \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{2}+1}{2\sqrt{2}}$$
$$\neq 2$$
$$\neq \text{R.H.S.}$$

(iii)

$\tan^2\theta + \sin\theta = \cos^2\theta$

Taking $\theta = 45°$, we have

$$\text{L.H.S.} = \tan^2 45° + \sin 45° = (1)^2 + \frac{1}{\sqrt{2}} = 1 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}}$$

$$\text{R.H.S.} = \cos^2 45 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

Question 37.

Solution:

$$\begin{aligned}\text{L.H.S.} &= (\sin\theta - 2\sin^3\theta)\\ &= \sin\theta(1 - 2\sin^2\theta)\\ &= \sin\theta(1 - 2\sin^2\theta)\end{aligned}$$

$$\begin{aligned}\text{R.H.S.} &= (2\cos^3\theta - \cos\theta)\tan\theta\\ &= \cos\theta(2\cos^2\theta - 1)\frac{\sin\theta}{\cos\theta}\\ &= [2(1 - \sin^2\theta) - 1]\sin\theta\\ &= (2 - 2\sin^2\theta - 1)\sin\theta\\ &= (1 - 2\sin^2\theta)\sin\theta\end{aligned}$$

$\Rightarrow$ L.H.S. = R.H.S.

$\therefore\ (\sin\theta - 2\sin^3\theta) = (2\cos^3\theta - \cos\theta)\tan\theta$

Ex 8b

Question 1.

Solution:

$m = a\cos\theta + b\sin\theta$ and $n = a\sin\theta - b\cos\theta$

$\therefore$ LHS $= m^2 + n^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$

$= \left(a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta\right)$

$+ \left(a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta\right)$

$= a^2\left(\cos^2\theta + \sin^2\theta\right) + b^2\left(\sin^2\theta + \cos^2\theta\right)$

$= a^2 + b^2 =$ RHS

$\therefore$ LHS = RHS

Question 2.

Solution:

$x = a\sec\theta + b\tan\theta,$ and $y = a\tan\theta + b\sec\theta$

LHS $= \left(x^2 - y^2\right) = (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$

$= \left(a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta\right)$

$- \left(a^2\tan^2\theta + b^2\sec^2\theta + 2ab\tan\theta\sec\theta\right)$

$= a^2\left(\sec^2\theta - \tan^2\theta\right) - b^2\left(\sec^2\theta - \tan^2\theta\right)$

$= a^2 - b^2 =$ RHS

LHS = RHS

Question 3.

Solution:

$\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1$ and $\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1$

Now, $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1$

(Squaring both sides, we get)

$\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - \frac{2xy}{ab}\sin\theta\cos\theta = 1 ------(1)$

$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1$

(Squaring both sides, we get)

$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{2xy}{ab}\sin\theta\cos = 1 ------(2)$

Adding (1) & (2), we get

$\frac{x^2}{a^2}\left(\sin^2\theta + \cos^2\theta\right) + \frac{y^2}{b^2}\left(\sin^2\theta + \cos^2\theta\right) = 2$

$\frac{x^2}{a} + \frac{y^2}{b} = 2$ (proved)

Question 4.

Solution:

$(\sec\theta + \tan\theta) = m, (\sec\theta - \tan\theta) = n$

$\text{LHS} = mn = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta = 1 = \text{RHS}$

$\therefore$ LHS = RHS

Question 5.

Solution:

$(\text{cosec}\,\theta + \cot\theta) = m, (\text{cosec}\,\theta - \cot\theta) = n$

$\text{LHS} = mn = (\text{cosec}\,\theta + \cot\theta) \times (\text{cosec}\,\theta - \cot\theta)$

$= \text{cosec}^2\theta - \cot^2\theta = 1 = \text{RHS}$

$\therefore$ LHS = RHS

Question 6.

Solution:

$x = a\cos^3\theta, y = b\sin^3\theta$

$$\text{LHS} = \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = \left(\frac{a\cos^3\theta}{a}\right)^{\frac{2}{3}} + \left(\frac{b\sin^3\theta}{b}\right)^{\frac{2}{3}}$$

$$= \left(\cos^3\theta\right)^{\frac{2}{3}} + \left(\sin^3\theta\right)^{\frac{2}{3}} = (\cos\theta)^{3\times\frac{2}{3}} + (\sin\theta)^{3\times\frac{2}{3}}$$

$$= \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$$

LHS = RHS

Question 7.

Solution:

$(\tan\theta + \sin\theta) = m$ and $(\tan\theta - \sin\theta) = n$

$$\text{LHS} = \left(m^2 - n^2\right)^2$$

$$= \left[(\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2\right]^2$$

$$= [4\tan\theta\sin\theta]^2 \quad \left[\because (a+b)^2 - (a-b)^2 = 4ab\right]$$

$$= 16\tan^2\theta\sin^2\theta \quad -----(1)$$

$$\text{RHS} = 16mn = 16(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$$

$$= 16\left(\tan^2\theta - \sin^2\theta\right) = 16\left(\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\right)$$

$$= 16\left(\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}\right)$$

$$= 16\,\frac{\sin^2\theta\left(1 - \cos^2\theta\right)}{\cos^2\theta} \quad \left[\because 1 - \cos^2\theta = \sin^2\theta\right]$$

$$= 16\,\frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta$$

$$\text{RHS} = 16\tan^2\sin^2\theta \quad ------(2)$$

$\therefore$ LHS = RHS

Question 8.

Solution:

$(\cot\theta + \tan\theta) = m$ and $(\sec\theta - \cos\theta) = n$

$$\Rightarrow \left(\frac{1}{\tan\theta} + \tan\theta\right) = m \text{ and } \left(\frac{1}{\cos\theta} - \cos\theta\right) = n$$

$$\Rightarrow \left(\frac{1+\tan^2\theta}{\tan\theta}\right) = m \text{ and } \frac{(1-\cos^2\theta)}{\cos\theta} = n$$

$$\Rightarrow \left(\frac{\sec^2\theta}{\tan\theta}\right) = m \text{ and } \frac{(1-\cos^2\theta)}{\cos\theta} = n$$

$$\Rightarrow \frac{1}{\cos^2\theta \times \frac{\sin\theta}{\cos\theta}} = m \text{ and } \frac{\sin^2\theta}{\cos\theta} = n$$

$$\Rightarrow \frac{1}{\cos\theta\sin\theta} = m \text{ and } \frac{\sin^2\theta}{\cos\theta} = n$$

$$\therefore (m^2n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}} = \left[\frac{1}{\cos^2\theta\sin^2\theta} \times \frac{\sin^2\theta}{\cos\theta}\right]^{\frac{2}{3}} - \left[\frac{1}{\cos\theta\sin\theta} \times \frac{\sin^4\theta}{\cos^2\theta}\right]^{\frac{2}{3}}$$

$$= \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \sec^2\theta - \tan^2\theta = 1 \quad [\because \sec^2\theta = 1 + \tan^2\theta]$$

Hence, $(m^2n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}} = 1$

Question 9.

Solution:

$\operatorname{cosec}\theta - \sin\theta = a^3$

$\Rightarrow a^3 = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$

$\Rightarrow a = \dfrac{\cos^{2/3}}{\sin^{1/3}}$

$\sec\theta - \cos\theta = b^3$

$\Rightarrow b^3 = \dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

$\Rightarrow b = \dfrac{\sin^{2/3}}{\cos^{1/3}}$

$\therefore$ L.H.S. $= a^2b^2(a^2+b^2)$

$= \quad = \ 1.$

Question 10.

Solution:

$(2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta)$

$= 4\sin^2\theta + 9\cos^2\theta + 12\sin\theta\cos\theta + 9\sin^2\theta + 4\cos^2\theta - 12\sin\theta\cos\theta$

$= 13\sin^2\theta + 12\cos^2\theta$

$= 13(\sin^2\theta + \cos^2\theta)$

$= 13$

Now,

$(2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta) = 13$

$\Rightarrow (2)^2 + (3\sin\theta - 2\cos\theta)^2 = 13$

$\Rightarrow 4 + (3\sin\theta - 2\cos\theta)^2 = 13$

$\Rightarrow (3\sin\theta - 2\cos\theta)^2 = 9$

$\Rightarrow 3\sin\theta - 2\cos\theta = \pm 3.$

Question 11.

Solution:

$\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow 1 + \cot\theta = \sqrt{2}\cot\theta$ ....(Dividing both sides by $\sin\theta$)

$\Rightarrow \left(\sqrt{2} - 1\right)\cot\theta = 1$

$$\Rightarrow \cot\theta = \frac{1}{\sqrt{2} - 1}$$

$$\Rightarrow \cot\theta = \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$\Rightarrow \cot\theta = \frac{\sqrt{2} + 1}{\left(\sqrt{2}\right)^2 - 1^2}$$

$$\Rightarrow \cot\theta = \frac{\sqrt{2} + 1}{2 - 1}$$

$\Rightarrow \cot\theta = \sqrt{2} + 1$

Question 12.

Solution:

$\cos\theta + \sin\theta = \sqrt{2}\sin\theta$

$\Rightarrow (\cos\theta + \sin\theta)^2 = \left(\sqrt{2}\sin\theta\right)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\sin^2\theta$

$\Rightarrow \sin^2\theta - 2\sin\theta\cos\theta = \cos^2\theta$

$\Rightarrow \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = \cos^2\theta + \cos^2\theta$

$\Rightarrow (\sin\theta - \cos\theta)^2 = 2\cos^2\theta$

$\Rightarrow \sin\theta - \cos\theta = \sqrt{2}\cos\theta$

Question 13.

Solution:

Given, $\sec\theta + \tan\theta = p$

$\Rightarrow \tan\theta = p - \sec\theta \quad ....(i)$

Now,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow p(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p} \quad ....(ii)$

From (i) and (ii),

$\sec\theta - p + \sec\theta = \frac{1}{p}$

$\Rightarrow 2\sec\theta - p = \frac{1}{p}$

$\Rightarrow 2\sec\theta = p + \frac{1}{p}$

$\Rightarrow \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

Given, $\sec\theta + \tan\theta = p$

$\Rightarrow \sec\theta = p - \tan\theta \quad ....(i)$

Now,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow p(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p} \quad ....(iii)$

From (i) and (ii),

$p - \tan\theta - \tan\theta = \frac{1}{p}$

$\Rightarrow p - 2\tan\theta = \frac{1}{p}$

$\Rightarrow 2\tan\theta = p - \frac{1}{p}$

$\Rightarrow \tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

Given, $\sec\theta + \tan\theta = p$

$\Rightarrow \sec\theta = p - \tan\theta \quad ....(i)$

$\Rightarrow \tan\theta = p - \sec\theta \quad ....(ii)$

Now,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow p(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p} \quad ....(iii)$

From (i) and (iii),

$p - \tan\theta - \tan\theta = \frac{1}{p}$

$\Rightarrow p - 2\tan\theta = \frac{1}{p}$

$\Rightarrow 2\tan\theta = p - \frac{1}{p}$

$\Rightarrow \tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

From (ii) and (iii),

$\sec\theta - p + \sec\theta = \frac{1}{p}$

$\Rightarrow 2\sec\theta - p = \frac{1}{p}$

$\Rightarrow 2\sec\theta = p + \frac{1}{p}$

$\Rightarrow \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

Now, $\sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)} = \frac{\left(\frac{p^2 - 1}{p}\right)}{\left(\frac{p^2 + 1}{p}\right)} = \frac{p^2 - 1}{p^2 + 1}$

Question 14.

Solution:

$\tan A = n\tan B$ and $\sin A = m\sin B$

$\Rightarrow \tan B = \frac{1}{n}\tan A$ and $\sin B = \frac{1}{m}\sin A$

$\Rightarrow \cot B = \frac{n}{\tan A}$ and $\operatorname{cosec} B = \frac{m}{\sin A}$

$\therefore \operatorname{cosec}^2 B - \cot^2 B = 1$

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1 \Rightarrow \frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow m^2 - n^2\cos^2 A = \sin^2 A \Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$\Rightarrow m^2 - 1 = n^2\cos^2 A - \cos^2 A$

$\Rightarrow m^2 - 1 = \cos^2 A\left(n^2 - 1\right)$

$\Rightarrow \cos^2 A = \frac{\left(m^2 - 1\right)}{\left(n^2 - 1\right)}$

Question 15.

Solution:

$m = (\cos\theta - \sin\theta)$ and $n = (\cos\theta + \sin\theta)$

$$\text{L.H.S.} = \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{m+n}{\sqrt{mn}}$$

Now,

$$m + n = (\cos\theta - \sin\theta) + (\cos\theta + \sin\theta) = 2\cos\theta$$

$$mn = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta) = \cos^2\theta - \sin^2\theta$$

$$\therefore \text{L.H.S.} = \frac{m+n}{\sqrt{mn}}$$

$$= \frac{2\cos\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$$

$$= \frac{2\cos\theta / \cos\theta}{\sqrt{\cos^2\theta / \cos^2\theta - \sin^2\theta / \cos^2\theta}}$$

$$= \frac{2}{\sqrt{1 - \tan^2\theta}}$$

$$= \text{R.H.S.}$$

Ex 8c

Question 1.

Solution:

$$(1 - \sin^2\theta)\sec^2\theta$$

$$= \cos^2\theta \times \frac{1}{\cos^2\theta}$$

$$= 1$$

Question 2.

Solution:

$$(1 - \cos^2\theta)\operatorname{cosec}^2\theta$$

$$= \sin^2\theta \times \frac{1}{\sin^2\theta}$$

$$= 1$$

Question 3.

Solution:

$$(1+\tan^2\theta)\cos^2\theta$$
$$=\sec^2\theta \times \frac{1}{\sec^2\theta}$$
$$=1$$

Question 4.

Solution:

$$(1+\cot^2\theta)\sin^2\theta$$
$$=\operatorname{cosec}^2\theta \times \frac{1}{\operatorname{cosec}^2\theta}$$
$$=1$$

Question 5.

Solution:

$$\sin^2\theta+\frac{1}{1+\tan^2\theta}$$
$$=\sin^2\theta+\frac{1}{\sec^2\theta}$$
$$=\sin^2\theta+\frac{1}{1/\cos^2\theta}$$
$$=\sin^2\theta+\cos^2\theta$$
$$=1$$

Question 6.

Solution:

$\cot^2\theta + \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$

$= \frac{-\sin^2\theta}{\sin^2\theta}$ $\quad ....(\sin^2\theta + \cos^2\theta = 1)$

$= -1$

Question 7.

Solution:

$\sin\theta\cos(90^\circ - \theta) + \cos\theta\sin(90^\circ - \theta)$

$= \sin\theta \times \sin\theta + \cos\theta \times \cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

Question 8.

Solution:

$\mathrm{cosec}^2(90^\circ - \theta) - \tan^2\theta$

$= \sec^2\theta - \tan^2\theta$

$= 1$

Question 9.

Solution:

$\sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$

$= \sec^2\theta(1 - \sin^2\theta)$

$= \sec^2\theta \times \cos^2\theta$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta$

$= 1$

Question 10.

Solution:

$$\text{cosec}^2\theta(1+\cos\theta)(1-\cos\theta)$$
$$=\text{cosec}^2\theta(1-\cos^2\theta)$$
$$=\text{cosec}^2\theta \times \sin^2\theta$$
$$=\frac{1}{\sin^2\theta}\times\sin^2\theta$$
$$=1$$

Question 11.

Solution:

$$\sin^2\theta\cos^2\theta(1+\tan^2\theta)(1+\cot^2\theta)$$
$$=\sin^2\theta \times \cos^2\theta \times \sec^2\theta \times \text{cosec}^2\theta$$
$$=\sin^2\theta \times \cos^2\theta \times \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}$$
$$=1$$

Question 12.

Solution:

$$(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$$
$$=\sec^2\theta(1-\sin^2\theta)$$
$$=\sec^2\theta \times \cos^2\theta$$
$$=\frac{1}{\cos^2\theta}\times\cos^2\theta$$
$$=1$$

Question 13.

Solution:

$3\cot^2\theta - 3\operatorname{cosec}^2\theta$
$= 3(\cot^2\theta - \operatorname{cosec}^2\theta)$
$= 3 \times (-1)$
$= -3$

Question 14.

Solution:

$$4\tan^2\theta - \frac{4}{\cos^2\theta}$$
$$= 4\frac{\sin^2\theta}{\cos^2\theta} - \frac{4}{\cos^2\theta}$$
$$= \frac{4\sin^2\theta - 4}{\cos^2\theta}$$
$$= \frac{4(\sin^2\theta - 1)}{\cos^2\theta}$$
$$= \frac{4 \times (-\cos^2\theta)}{\cos^2\theta}$$
$$= -4$$

Question 15.

Solution:

$$\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \operatorname{cosec}^2\theta}$$
$$= \frac{-1}{-1} \quad \ldots(1 + \tan^2\theta = \sec^2\theta \text{ and } 1 + \cot^2\theta = \operatorname{cosec}^2\theta)$$
$$= 1$$

Question 16.

Solution:

$\sin\theta = \frac{1}{2} \Rightarrow \sin^2\theta = \frac{1}{4}$

$\therefore 3\cot^2\theta + 3 = \frac{3\cos^2\theta}{\sin^2\theta} + 3$

$= \frac{3\cos^2\theta + 3\sin^2\theta}{\sin^2\theta}$

$= \frac{3(\cos^2\theta + \sin^2\theta)}{\sin^2\theta}$

$= \frac{3 \times 1}{1/4}$

$= 3 \times 4$

$= 12$

Question 17.

Solution:

$\cos\theta = \frac{2}{3} \Rightarrow \cos^2\theta = \frac{4}{9}$

$\therefore 4 + 4\tan^2\theta = 4(1 + \tan^2\theta)$

$= 4\left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right)$

$= 4\left(\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}\right)$

$= 4 \times \frac{1}{\cos^2\theta}$

$= 4 \times \frac{1}{4/9}$

$= 9$

Question 18.

Solution:

$\cos\theta = \frac{7}{25} \Rightarrow \cos^2\theta = \frac{49}{625}$

$\Rightarrow \sin^2\theta = 1 - \sin^2\theta = 1 - \frac{49}{625} = \frac{576}{625}$

$\Rightarrow \sin\theta = \frac{24}{25}$

$\tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\frac{24}{25} \times \frac{7}{25}}$

$= \frac{625}{168}$

Question 19.

Solution:

$\cos\ \theta = \frac{2}{3}$

$\Rightarrow \sec\theta = \frac{1}{\cos\theta} = \frac{3}{2}$

$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\frac{3}{2} - 1}{\frac{3}{2} + 1} = \frac{\frac{3-2}{2}}{\frac{3+2}{2}} = \frac{1}{5}$

Question 20.

Solution:

$5\tan\theta = 4$

$\Rightarrow \tan\theta = \frac{4}{5}$

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} = \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}} = \frac{\frac{5-4}{5}}{\frac{5+4}{5}} = \frac{1}{9}$$

Question 21.

Solution:

$3\cot\theta = 4$

$\Rightarrow \cot\theta = \frac{4}{3}$

$$\frac{2\cos\theta + \sin\theta}{4\cos\theta - \sin\theta} = \frac{\frac{2\cos\theta}{\sin\theta} + \frac{\sin\theta}{\sin\theta}}{\frac{4\cos\theta}{\sin\theta} - \frac{\sin\theta}{\sin\theta}} = \frac{2\cot\theta + 1}{4\cot\theta - 1} = \frac{2 \times \frac{4}{3} + 1}{4 \times \frac{4}{3} - 1} = \frac{\frac{8}{3} + 1}{\frac{16}{3} - 1} = \frac{\frac{8+3}{3}}{\frac{16-3}{3}} = \frac{11}{13}$$

Question 22.

Solution:

$\cot\theta = \frac{1}{\sqrt{3}} \Rightarrow \cot^2\theta = \frac{1}{3}$

Now, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\Rightarrow \operatorname{cosec}^2\theta = 1 + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$

$\Rightarrow \sin^2\theta = \frac{1}{\operatorname{cosec}^2\theta} = \frac{3}{4}$

$$\frac{1 - \cos^2\theta}{2 - \sin^2\theta} = \frac{\sin^2\theta}{2 - \sin^2\theta} = \frac{\frac{3}{4}}{2 - \frac{3}{4}} = \frac{\frac{3}{4}}{\frac{8-3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$$

Question 23.

Solution:

$$\tan\theta = \frac{1}{\sqrt{5}} \Rightarrow \tan^2\theta = \frac{1}{5}$$

$$\Rightarrow \sec^2\theta = 1 + \tan^2\theta = 1 + \frac{1}{5} = \frac{5+1}{5} = \frac{6}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1/\sqrt{5}} = \sqrt{5}$$

$$\text{Now, } \operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \left(\sqrt{5}\right)^2 = 1 + 5 = 6$$

$$\therefore \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{6 - \frac{6}{5}}{6 + \frac{6}{5}} = \frac{\frac{30-6}{5}}{\frac{30+6}{5}} = \frac{24}{36} = \frac{2}{3}$$

Question 24.

Solution:

$$\text{Given, } \cot A = \frac{4}{3}$$

$$A + B = 90^\circ$$

$$\Rightarrow A = 90^\circ - B$$

$$\Rightarrow \cot A = \cot(90^\circ - B)$$

$$\Rightarrow \cot A = \tan B$$

$$\Rightarrow \cot A = \tan B = \frac{4}{3}$$

Question 25.

Solution:

$$\text{Given, } \cos B = \frac{3}{5}$$

$$A + B = 90^\circ$$

$$\Rightarrow B = 90^\circ - A$$

$$\Rightarrow \cos B = \cos(90^\circ - A)$$

$$\Rightarrow \cos B = \sin A$$

$$\Rightarrow \cos B = \sin A = \frac{3}{5}$$

Question 26.

Solution:

$$\sqrt{3}\sin\theta = \cos\theta$$
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan\theta = \tan 30°$$
$$\Rightarrow \theta = 30°$$

Question 27.

Solution:

$$\tan 10° \tan 20° \tan 70° \tan 80°$$
$$= \tan 10° \times \tan 20° \times \tan(90° - 20°) \times \tan(90° - 10°)$$
$$= \tan 10° \times \tan 20° \times \cot 20° \times \cot 10°$$
$$= (\tan 10° \cot 10°)(\tan 20° \cot 20°)$$
$$= 1 \times 1$$
$$= 1$$

Question 28.

Solution:

$$\tan 1° \tan 2° \ldots\ldots\ldots \tan 89°$$
$$= [\tan 1° \tan 89°][\tan 2° \tan 88°] \ldots\ldots\ldots [\tan 44° \tan 66°] \tan 45°$$
$$= [\tan 1° \tan(90° - 1°)][\tan 2° \tan(90° - 2°)] \ldots. [\tan 44° \tan(90° - 44°)](1)$$
$$= [\tan 1° \cot 1°][\tan 2° \cot 2°] \ldots. [\tan 44° \cot 44°]$$
$$= 1 \times 1 \times \ldots\ldots\ldots \times 1$$
$$= 1$$

Question 29.

Solution:

Since $\cos 90^\circ = 0$,
$\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 180^\circ = 0$

Question 30.

Solution:

Given, $\tan A = \frac{5}{12}$

$\therefore (\sin A + \cos A) \sec A$

$= (\sin A + \cos A) \times \frac{1}{\cos A}$

$= \frac{\sin A}{\cos A} + \frac{\cos A}{\cos A}$

$= \tan A + 1$

$= \frac{5}{12} + 1$

$= \frac{5 + 12}{12}$

$= \frac{17}{12}$

Question 31.

Solution:

$\sin\theta = \cos(\theta - 45^\circ)$
$\Rightarrow \cos(90^\circ - \theta) = \cos(\theta - 45^\circ)$
$\Rightarrow 90^\circ - \theta = \theta - 45^\circ$
$\Rightarrow 2\theta = 135^\circ$
$\Rightarrow \theta = 67.5^\circ$

Question 32.

Solution:

$$\frac{\sin 50^\circ}{\cos 40^\circ}+\frac{\text{cosec } 40^\circ}{\sec 50^\circ}-4\cos 50^\circ \text{ cosec } 40^\circ$$
$$=\frac{\sin(90^\circ-40^\circ)}{\cos 40^\circ}+\frac{\text{cosec}(90^\circ-50^\circ)}{\sec 50^\circ}-4\cos 50^\circ \text{ cosec}(90^\circ-50^\circ)$$
$$=\frac{\cos 40^\circ}{\cos 40^\circ}+\frac{\sec 50^\circ}{\sec 50^\circ}-4\cos 50^\circ \sec 50^\circ$$
$$=1+1-4\cos 50^\circ \times \frac{1}{\cos 50^\circ}$$
$$=2-4\times 1$$
$$=2-4$$
$$=-2$$

Question 33.

Solution:

$$\sin 48^\circ \sec 42^\circ + \cos 48^\circ \text{ cosec } 42^\circ$$
$$=\sin 48^\circ \sec(90^\circ-48^\circ)+\cos 48^\circ \text{ cosec}(90^\circ-48^\circ)$$
$$=\sin 48^\circ \text{ cosec } 48^\circ+\cos 48^\circ \sec 48^\circ$$
$$=\sin 48^\circ\times\frac{1}{\sin 48^\circ}+\cos 48^\circ\times\frac{1}{\cos 48^\circ}$$
$$=1+1$$
$$=2$$

Question 34.

Solution:

$x = a\sin\theta$ and $y = b\cos\theta$

Now,

$$b^2x^2+a^2y^2$$
$$=b^2(a\sin\theta)^2+a^2(b\cos\theta)^2$$
$$=a^2b^2\sin^2\theta+a^2b^2\cos^2\theta$$
$$=a^2b^2(\sin^2\theta+\cos^2\theta)$$
$$=a^2b^2\times 1$$
$$=a^2b^2$$

Question 35.

Solution:

Given, $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$

We know that,

$1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow 1 + \left(\frac{5}{x}\right)^2 = (5x)^2$

$\Rightarrow 1 + \frac{25}{x^2} = 25x^2$

$\Rightarrow 25x^2 - \frac{25}{x^2} = 1$

$\Rightarrow 25\left(x^2 - \frac{1}{x^2}\right) = 1$

$\Rightarrow 5\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{5}$

Question 36.

Solution:

Given, $\operatorname{cosec}\theta = 2x$ and $\cot\theta = \frac{2}{x}$

We know that

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (2x)^2 - \left(\frac{2}{x}\right)^2 = 1$

$\Rightarrow 4x^2 - \frac{4}{x^2} = 1$

$\Rightarrow 4\left(x^2 - \frac{1}{x^2}\right) = 1$

$\Rightarrow 2 \times 2\left(x^2 - \frac{1}{x^2}\right) = 1$

$\Rightarrow 2\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{2}$

Question 37.

Solution:

$\sec\theta + \tan\theta = x \quad ....(i)$

Now, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow x(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{x} \quad ....(ii)$

Adding (i) and (ii), we get

$2\sec\theta = x + \frac{1}{x}$

$\Rightarrow \sec\theta = \frac{1}{2}\left(x + \frac{1}{x}\right) = \frac{1}{2}\left(\frac{x^2+1}{x}\right) = \frac{x^2+1}{2x}$

Question 38.

Solution:

$$\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$$

$$= \frac{\cos 38^\circ \operatorname{cosec}(90^\circ - 38^\circ)}{\tan 18^\circ \tan 35^\circ \left(\sqrt{3}\right) \tan(90^\circ - 18^\circ) \tan(90^\circ - 35^\circ)}$$

$$= \frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \tan 35^\circ \left(\sqrt{3}\right) \cot 18^\circ \cot 35^\circ}$$

$$= \frac{\cos 38^\circ \times \frac{1}{\cos 38^\circ}}{(\tan 18^\circ \cot 18^\circ)(\tan 35^\circ \cot 35^\circ)\left(\sqrt{3}\right)}$$

$$= \frac{1}{\sqrt{3}}$$

Question 39.

Solution:

$\sin\theta = x$

$\Rightarrow \sin^2\theta = x^2$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta = 1 - x^2$

$\Rightarrow \cos\theta = \sqrt{1 - x^2}$

Now, $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sqrt{1 - x^2}}{x}$

Question 40.

Solution:

$\sec\theta = x$

$\Rightarrow \dfrac{1}{\cos\theta} = x$

$\Rightarrow \cos\theta = \dfrac{1}{x}$

$\Rightarrow \cos^2\theta = \dfrac{1}{x^2}$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta = 1 - \dfrac{1}{x^2} = \dfrac{x^2 - 1}{x}$

$\Rightarrow \sin\theta = \dfrac{\sqrt{x^2 - 1}}{x}$

Now, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{\sqrt{x^2 - 1}}{x}}{\frac{1}{x}} = \sqrt{x^2 - 1}$

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He was born on January 2, 1946 in a village of Delhi. He graduated from Kirori Mal College, University of Delhi. After completing his M.Sc. in Mathematics in 1969, he joined N.A.S. College, Meerut, as a lecturer. In 1976, he was awarded a fellowship for 3 years and joined the University of Delhi for his Ph.D. Thereafter, he was promoted as a reader in N.A.S. College, Meerut. In 1999, he joined M.M.H. College, Ghaziabad, as a reader and took voluntary retirement in 2003. He has authored more than 75 titles ranging from Nursery to M. Sc. He has also written books for competitive examinations right from the clerical grade to the I.A.S. level.

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