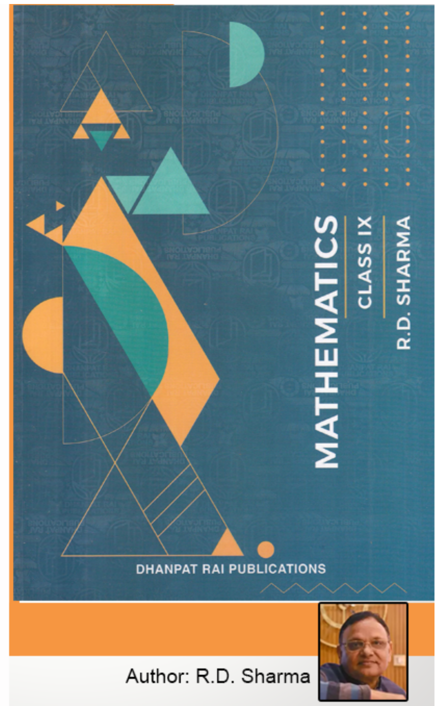
Class 9 - Chapter 6 Factorization Of Polynomials



RD Sharma Solutions for Class 9 Maths Chapter 6–Factorization Of Polynomials

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Exercise 6.1 Page No: 6.2

Question 1: Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 – 4x + 15$

(ii) $y^2 + 2\sqrt{3}$

(iii) $3\sqrt{x} + \sqrt{2}x$

(iv) $x – 4/x$

(v) $x^{12} + y^3 + t^{50}$

Solution:

(i) $3x^2 – 4x + 15$

It is a polynomial of x.

(ii) $y^2 + 2\sqrt{3}$

It is a polynomial of y.

(iii) $3\sqrt{x} + \sqrt{2}x$

It is not a polynomial since the exponent of $3\sqrt{x}$ is a rational term.

(iv) $x – 4/x$

It is not a polynomial since the exponent of $– 4/x$ is not a positive term.

(v) $x^{12} + y^3 + t^{50}$

It is a three variable polynomial, x, y and t.

Question 2: Write the coefficient of x^2 in each of the following:

(i) $17 – 2x + 7x^2$

(ii) $9 – 12x + x^3$

(iii) ∏/6 $x^2 – 3x + 4$

(iv) √3x – 7

Solution:

(i) $17 - 2x + 7x^2$

Coefficient of $x^2 = 7$

(ii) $9 - 12x + x^3$

Coefficient of $x^2 = 0$

(iii) ∏/6 $x^2 - 3x + 4$

Coefficient of x^2 = ∏/6

(iv) √3x – 7

Coefficient of $x^2 = 0$

Question 3: Write the degrees of each of the following polynomials:

(i) $7x^3 + 4x^2 - 3x + 12$

(ii) $12 - x + 2x^3$

(iii) 5y – √2

(iv) 7

(v) 0

Solution:

As we know, degree is the highest power in the polynomial

(i) Degree of the polynomial $7x^3 + 4x^2 - 3x + 12$ is 3

(ii) Degree of the polynomial $12 - x + 2x^3$ is 3

(iii) Degree of the polynomial 5y – √2 is 1

(iv) Degree of the polynomial 7 is 0

(v) Degree of the polynomial 0 is undefined.

Question 4: Classify the following polynomials as linear, quadratic, cubic and biquadratic polynomials:

(i) $x + x^2 + 4$

(ii) $3x – 2$

(iii) $2x + x^2$

(iv) 3y

(v) $t^2 + 1$

(vi) $7t^4 + 4t^3 + 3t – 2$

Solution:

(i) $x + x^2 + 4$: It is a quadratic polynomial as its degree is 2.

(ii) $3x – 2$: It is a linear polynomial as its degree is 1.

(iii) $2x + x^2$: It is a quadratic polynomial as its degree is 2.

(iv) 3y: It is a linear polynomial as its degree is 1.

(v) $t^2+ 1$: It is a quadratic polynomial as its degree is 2.

(vi) $7t^4 + 4t^3 + 3t – 2$: It is a biquadratic polynomial as its degree is 4.

Exercise 6.2 Page No: 6.8

Question 1: If $f(x) = 2x^3 – 13x^2 + 17x + 12$, find

(i) f (2)

(ii) f (-3)

(iii) f(0)

Solution:

$f(x) = 2x^3 – 13x^2 + 17x + 12$

(i) $f(2) = 2(2)^3 – 13(2)^2 + 17(2) + 12$

= 2 x 8 – 13 x 4 + 17 x 2 + 12

= 16 – 52 + 34 + 12

= 62 – 52

= 10

(ii) $f(-3) = 2(-3)^3 – 13(-3)^2 + 17 x (-3) + 12$

= 2 x (-27) – 13 x 9 + 17 x (-3) + 12

= -54 – 117 -51 + 12

= -222 + 12

= -210

(iii) $f(0) = 2 x (0)^3 – 13(0)^2 + 17 x 0 + 12$

= 0-0 + 0+ 12

= 12

Question 2: Verify whether the indicated numbers are zeros of the polynomials corresponding to them in the following cases:

(i) f(x) = 3x + 1, x = −1/3

(ii) $f(x) = x^2 – 1$**, x = 1,−1**

(iii) $g(x) = 3x^2 – 2$ **, x = 2/√3 , −2/√3**

(iv) $p(x) = x^3 – 6x^2 + 11x – 6$ **, x = 1, 2, 3**

(v) f(x) = 5x – π, x = 4/5

(vi) $f(x) = x^2$ **, x = 0**

(vii) f(x) = lx + m, x = −m/l

(viii) f(x) = 2x + 1, x = 1/2

Solution:

(i) f(x) = 3x + 1, x = −1/3

f(x) = 3x + 1

Substitute x = −1/3 in f(x)

f(−1/3) = 3(−1/3) + 1

= -1 + 1

= 0

Since, the result is 0, so x = −1/3 is the root of 3x + 1

(ii) f(x) = x^2 – 1, x = 1,−1

f(x) = x^2 – 1

Given that x = (1 , -1)

Substitute x = 1 in f(x)

f(1) = 1^2 – 1

= 1 – 1

= 0

Now, substitute x = (-1) in f(x)

f(-1) = $(-1)^2$ – 1

= 1 – 1

= 0

Since , the results when x = 1 and x = -1 are 0, so (1 , -1) are the roots of the polynomial f(x) = x^2 – 1

(iii) g(x) = $3x^2$ – 2 , x = 2/√3 , −2/√3

$g(x) = 3x^2 – 2$

Substitute $x = 2/\sqrt{3}$ in g(x)

$g(2/\sqrt{3}) = 3(2/\sqrt{3})^2 – 2$

$= 3(4/3) – 2$

$= 4 – 2$

$= 2 \neq 0$

Now, Substitute $x = −2/\sqrt{3}$ in g(x)

$g(2/\sqrt{3}) = 3(-2/\sqrt{3})^2 – 2$

$= 3(4/3) – 2$

$= 4 – 2$

$= 2 \neq 0$

Since, the results when $x = 2/\sqrt{3}$ and $x = −2/\sqrt{3}$) are not 0. Therefore ($2/\sqrt{3}$, $−2/\sqrt{3}$) are not zeros of $3x^2–2$.

(iv) $p(x) = x^3 – 6x^2 + 11x – 6$, x = 1, 2, 3

$p(1) = 1^3 – 6(1)^2 + 11x\ 1 – 6 = 1 – 6 + 11 – 6 = 0$

$p(2) = 2^3 – 6(2)^2 + 11×2 – 6 = 8 – 24 + 22 – 6 = 0$

$p(3) = 3^3 – 6(3)^2 + 11×3 – 6 = 27 – 54 + 33 – 6 = 0$

Therefore, x = 1, 2, 3 are zeros of p(x).

(v) $f(x) = 5x – \pi$, x = 4/5

$f(4/5) = 5 \times 4/5 – \pi = 4 – \pi \neq 0$

Therefore, x = 4/5 is not a zeros of f(x).

(vi) $f(x) = x^2$, x = 0

$f(0) = 0^2 = 0$

Therefore, x = 0 is a zero of f(x).

(vii) f(x) = lx + m, x = −m/l

f(−m/l) = l x −m/l + m = -m + m = 0

Therefore, x = −m/l is a zero of f(x).

(viii) f(x) = 2x + 1, x = ½

f(1/2) = 2x 1/2 + 1 = 1 + 1 = 2 ≠ 0

Therefore, x = ½ is not a zero of f(x).

Exercise 6.3 Page No: 6.14

In each of the following, using the remainder theorem, find the remainder when f(x) is divided by g(x) and verify the by actual division : (1 – 8)

Question 1: f(x) = $x^3 + 4x^2 - 3x + 10$, g(x) = x + 4

Solution:

f(x) = $x^3 + 4x^2 - 3x + 10$, g(x) = x + 4

Put g(x) =0

⇒ x + 4 = 0 or x = -4

Remainder = f(-4)

Now,

f(-4) = $(-4)^3 + 4(-4)^2 - 3(-4) + 10 = -64 + 64 + 12 + 10 = 22$

Actual Division:

$$
\begin{array}{r|llll}
 & x^2 & -3 & & \\
\hline
x+4 & x^3 & +4x^2 & -3x & +10 \\
 & - & & & \\
 & x^3 & +4x^2 & & \\
\hline
 & & 0 & -3x & +10 \\
 & & - & -3x & -12 \\
\hline
 & & & & 22
\end{array}
$$

Question 2: f(x) = $4x^4 - 3x^3 - 2x^2 + x - 7$, g(x) = x – 1

Solution:

f(x) = $4x^4 - 3x^3 - 2x^2 + x - 7$

Put g(x) =0

⇒ x – 1 = 0 or x = 1

Remainder = f(1)

Now,

f(1) = $4(1)^4 - 3(1)^3 - 2(1)^2 + (1) - 7 = 4 - 3 - 2 + 1 - 7 = -7$

Actual Division:

$$
\begin{array}{r|rrrrr}
 & 4x^3 & +x^2 & -x & & \\
\hline
x-1 & 4x^4 & -3x^3 & -2x^2 & +x & -7 \\
 & - & & & & \\
 & 4x^4 & -4x^3 & & & \\
\hline
 & & x^3 & -2x^2 & +x & -7 \\
 & & - & & & \\
 & & x^3 & -x^2 & & \\
\hline
 & & & -x^2 & +x & -7 \\
 & & & - & & \\
 & & & -x^2 & +x & \\
\hline
 & & & & 0 & -7
\end{array}
$$

Question 3: f(x) = $2x^4 - 6X^3 + 2x^2 - x + 2$, g(x) = x + 2

Solution:

f(x) = $2x^4 - 6X^3 + 2x^2 - x + 2$, g(x) = x + 2

Put g(x) = 0

⇒ x + 2 = 0 or x = -2

Remainder = f(-2)

Now,

f(-2) = $2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2 = 32 + 48 + 8 + 2 + 2 = 92$

Actual Division:

$$
\begin{array}{r|l}
 & 2x^3 \quad -10x^2 \quad +22x \quad -45 \\
x+2 & 2x^4 \quad -6x^3 \quad +2x^2 \quad -x \quad +2 \\
 & -\ (2x^4 \quad +4x^3) \\
\hline
 & -10x^3 \quad +2x^2 \quad -x \quad +2 \\
 & -\ (-10x^3 \quad -20x^2) \\
\hline
 & 22x^2 \quad -x \quad +2 \\
 & -\ (22x^2 \quad +44x) \\
\hline
 & -45x \quad +2 \\
 & -\ (-45x \quad -90) \\
\hline
 & 92
\end{array}
$$

Question 4: f(x) = $4x^3$ – $12x^2$ + 14x – 3, g(x) = 2x – 1

Solution:

f(x) = $4x^3$ – $12x^2$ + 14x – 3, g(x) = 2x – 1

Put g(x) =0

⇒ 2x -1 =0 or x = 1/2

Remainder = f(1/2)

Now,

f(1/2) = $4(1/2)^3$ – $12(1/2)^2$ + 14(1/2) – 3 = ½ – 3 + 7 – 3 = 3/2

Actual Division:

$$
\begin{array}{r|llll}
 & 2x^2 & -5x & +\frac{9}{2} & \\
\hline
2x-1 & 4x^3 & -12x^2 & +14x & -3 \\
 & - & & & \\
 & 4x^3 & -2x^2 & & \\
\hline
 & & -10x^2 & +14x & -3 \\
 & & - & & \\
 & & -10x^2 & +5x & \\
\hline
 & & & 9x & -3 \\
 & & & - & \\
 & & & 9x & -\frac{9}{2} \\
\hline
 & & & & \frac{3}{2}
\end{array}
$$

Question 5: f(x) = $x^3 - 6x^2 + 2x - 4$, g(x) = 1 – 2x

Solution:

f(x) = $x^3 - 6x^2 + 2x - 4$, g(x) = 1 – 2x

Put g(x) = 0

⇒ 1 – 2x = 0 or x = 1/2

Remainder = f(1/2)

Now,

f(1/2) = $(1/2)^3 - 6(1/2)^2 + 2(1/2) - 4$ = 1 + 1/8 – 4 – 3/2 = -35/8

Actual Division:

$$
\begin{array}{r|llll}
 & -\frac{x^2}{2} & +\frac{11x}{4} & +\frac{3}{8} & \\
\hline
-2x+1 & x^3 & -6x^2 & +2x & -4 \\
 & - & & & \\
 & x^3 & -\frac{x^2}{2} & & \\
\hline
 & & -\frac{11x^2}{2} & +2x & -4 \\
 & & - & & \\
 & & -\frac{11x^2}{2} & +\frac{11x}{4} & \\
\hline
 & & & -\frac{3x}{4} & -4 \\
 & & & - & \\
 & & & -\frac{3x}{4} & +\frac{3}{8} \\
\hline
 & & & & -\frac{35}{8}
\end{array}
$$

Question 6: f(x) = $x^4 - 3x^2 + 4$, g(x) = x – 2

Solution:

f(x) = $x^4 - 3x^2 + 4$, g(x) = x – 2

Put g(x) = 0

⇒ x – 2 = 0 or x = 2

Remainder = f(2)

Now,

f(2) = $(2)^4 - 3(2)^2 + 4 = 16 - 12 + 4 = 8$

Actual Division:

$$
\begin{array}{r|rrrrr}
 & x^3 & +2x^2 & +x & +2 & \\
\hline
x-2 & x^4 & +0x^3 & -3x^2 & +0x & +4 \\
- & x^4 & -2x^3 & & & \\
\hline
 & & 2x^3 & -3x^2 & +0x & +4 \\
 & - & 2x^3 & -4x^2 & & \\
\hline
 & & & x^2 & +0x & +4 \\
 & & - & x^2 & -2x & \\
\hline
 & & & & 2x & +4 \\
 & & & - & 2x & -4 \\
\hline
 & & & & & 8
\end{array}
$$

Question 7: $f(x) = 9x^3 – 3x^2 + x – 5$, $g(x) = x – 2/3$

Solution:

$f(x) = 9x^3 – 3x^2 + x – 5$, $g(x) = x – 2/3$

Put g(x) = 0

⇒ x – 2/3 = 0 or x = 2/3

Remainder = f(2/3)

Now,

$f(2/3) = 9(2/3)^3 – 3(2/3)^2 + (2/3) – 5 = 8/3 – 4/3 + 2/3 – 5/1 = -3$

Actual Division:

$$\begin{array}{rllll} & 9x^2 & +3x & +3 & \\ \cline{2-5} x-\frac{2}{3} \,\big) & 9x^3 & -3x^2 & +x & -5 \\ - & 9x^3 & -6x^2 & & \\ \hline & & 3x^2 & +x & -5 \\ & - & 3x^2 & -2x & \\ \hline & & & 3x & -5 \\ & & - & 3x & -2 \\ \hline & & & & -3 \end{array}$$

Exercise 6.4 Page No: 6.24

In each of the following, use factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) or, not: (1-7)

Question 1: $f(x) = x^3 – 6x^2 + 11x – 6$; $g(x) = x – 3$

Solution:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x -3 = 0

or x = 3

Remainder = f(3)

Now,

$f(3) = (3)^3 – 6(3)^2 +11 \times 3 – 6$

= 27 – 54 + 33 – 6

= 60 – 60

= 0

Therefore, g(x) is a factor of f(x)

Question 2: $f(x) = 3X^4 + 17x^3 + 9x^2 – 7x – 10; g(x) = x + 5$

Solution:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x + 5 = 0, then x = -5

Remainder = f(-5)

Now,

$f(3) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 – 7(-5) – 10$

= 3 x 625 + 17 x (-125) + 9 x (25) – 7 x (-5) – 10

= 1875 -2125 + 225 + 35 – 10

= 0

Therefore, g(x) is a factor of f(x).

Question 3: $f(x) = x^5 + 3x^4 – x^3 – 3x^2 + 5x + 15$, g(x) = x + 3

Solution:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x + 3 = 0, then x = -3

Remainder = f(-3)

Now,

$f(-3) = (-3)^5 + 3(-3)^4 – (-3)^3 – 3(-3)^2 + 5(-3) + 15$

= -243 + 3 x 81 -(-27)-3 x 9 + 5(-3) + 15

= -243 +243 + 27-27- 15 + 15

= 0

Therefore, g(x) is a factor of f(x).

Question 4: f(x) = $x^3 – 6x^2 – 19x + 84$, g(x) = x – 7

Solution:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x – 7 = 0, then x = 7

Remainder = f(7)

Now,

f(7) = $(7)^3 – 6(7)^2 – 19 \times 7 + 84$

= 343 – 294 – 133 + 84

= 343 + 84 – 294 – 133

= 0

Therefore, g(x) is a factor of f(x).

Question 5: f(x) = $3x^3 + x^2 – 20x + 12$, g(x) = 3x – 2

Solution:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = 3x – 2 = 0, then x = 2/3

Remainder = f(2/3)

Now,

f(2/3) = $3(2/3)^3 + (2/3)^2 – 20(2/3) + 12$

= 3 x 8/27 + 4/9 – 40/3 + 12

= 8/9 + 4/9 – 40/3 + 12

= 0/9

= 0

Therefore, g(x) is a factor of f(x).

Question 6: f(x) = $2x^3 - 9x^2 + x + 12$, g(x) = 3 – 2x

Solution:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = 3 – 2x = 0, then x = 3/2

Remainder = f(3/2)

Now,

f(3/2) = $2(3/2)^3 - 9(3/2)^2 + (3/2) + 12$

= 2 x 27/8 – 9 x 9/4 + 3/2 + 12

= 27/4 – 81/4 + 3/2 + 12

= 0/4

= 0

Therefore, g(x) is a factor of f(x).

Question 7: f(x) = $x^3 - 6x^2 + 11x - 6$, g(x) = $x^2 - 3x + 2$

Solution:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = 0

or $x^2 - 3x + 2 = 0$

$x^2 - x - 2x + 2 = 0$

x(x – 1) – 2(x – 1) = 0

(x – 1) (x – 2) = 0

Therefore x = 1 or x = 2

Now,

f(1) = $(1)^3$ – $6(1)^2$ + 11(1) – 6 = 1-6+11-6= 12- 12 = 0

f(2) = $(2)^3$ – $6(2)^2$ + 11(2) – 6 = 8 – 24 + 22 – 6 = 30 – 30 = 0

⇒ f(1) = 0 and f(2) = 0

Which implies g(x) is factor of f(x).

Question 8: Show that (x – 2), (x + 3) and (x – 4) are factors of $x^3 – 3x^2 – 10x + 24$.

Solution:

Let f(x) = $x^3 – 3x^2 – 10x + 24$

If x – 2 = 0, then x = 2,

If x + 3 = 0 then x = -3,

and If x – 4 = 0 then x = 4

Now,

f(2) = $(2)^3$ – $3(2)^2$ – 10 x 2 + 24 = 8 – 12 – 20 + 24 = 32 – 32 = 0

f(-3) = $(-3)^3$ – $3(-3)^2$ – 10 (-3) + 24 = -27 -27 + 30 + 24 = -54 + 54 = 0

f(4) = $(4)^3$ – $3(4)^2$ – 10 x 4 + 24 = 64-48 -40 + 24 = 88 – 88 = 0

f(2) = 0

f(-3) = 0

f(4) = 0

Hence (x – 2), (x + 3) and (x – 4) are the factors of f(x)

Question 9: Show that (x + 4), (x – 3) and (x – 7) are factors of $x^3 - 6x^2 - 19x + 84$.

Solution:

Let $f(x) = x^3 - 6x^2 - 19x + 84$

If x + 4 = 0, then x = -4

If x – 3 = 0, then x = 3

and if x – 7 = 0, then x = 7

Now,

$f(-4) = (-4)^3 - 6(-4)^2 - 19(-4) + 84 = -64 - 96 + 76 + 84 = 160 - 160 = 0$

f(-4) = 0

$f(3) = (3)^3 - 6(3)^2 - 19 \times 3 + 84 = 27 - 54 - 57 + 84 = 111 - 111 = 0$

f(3) = 0

$f(7) = (7)^3 - 6(7)^2 - 19 \times 7 + 84 = 343 - 294 - 133 + 84 = 427 - 427 = 0$

f(7) = 0

Hence (x + 4), (x – 3), (x – 7) are the factors of f(x).

Exercise 6.5 Page No: 6.32

Using factor theorem, factorize each of the following polynomials:

Question 1: $x^3 + 6x^2 + 11x + 6$

Solution:

Let $f(x) = x^3 + 6x^2 + 11x + 6$

Step 1: Find the factors of constant term

Here constant term = 6

Factors of 6 are ±1, ±2, ±3, ±6

Step 2: Find the factors of f(x)

Let x + 1 = 0

⇒ x = -1

Put the value of x in f(x)

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$

= -1 + 6 -11 + 6

= 12 – 12

= 0

So, (x + 1) is the factor of f(x)

Let x + 2 = 0

⇒ x = -2

Put the value of x in f(x)

$f(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6 = -8 + 24 - 22 + 6 = 0$

So, (x + 2) is the factor of f(x)

Let x + 3 = 0

⇒ x = -3

Put the value of x in f(x)

$f(-3) = (-3)^3 + 6(-3)^2 + 11(-3) + 6 = -27 + 54 - 33 + 6 = 0$

So, (x + 3) is the factor of f(x)

Hence, f(x) = (x + 1)(x + 2)(x + 3)

Question 2: $x^3 + 2x^2 – x – 2$

Solution:

Let $f(x) = x^3 + 2x^2 – x – 2$

Constant term = -2

Factors of -2 are ±1, ±2

Let x – 1 = 0

⇒ x = 1

Put the value of x in f(x)

$f(1) = (1)^3 + 2(1)^2 – 1 – 2 = 1 + 2 – 1 – 2 = 0$

So, (x – 1) is factor of f(x)

Let x + 1 = 0

⇒ x = -1

Put the value of x in f(x)

$f(-1) = (-1)^3 + 2(-1)^2 – 1 – 2 = -1 + 2 + 1 – 2 = 0$

(x + 1) is a factor of f(x)

Let x + 2 = 0

⇒ x = -2

Put the value of x in f(x)

$f(-2) = (-2)^3 + 2(-2)^2 – (-2) – 2 = -8 + 8 + 2 – 2 = 0$

(x + 2) is a factor of f(x)

Let x – 2 = 0

⇒ x = 2

Put the value of x in f(x)

$f(2) = (2)^3 + 2(2)^2 – 2 – 2 = 8 + 8 – 2 – 2 = 12 \neq 0$

(x – 2) is not a factor of f(x)

Hence f(x) = (x + 1)(x- 1)(x+2)

Question 3: $x^3 – 6x^2 + 3x + 10$

Solution:

Let $f(x) = x^3 – 6x^2 + 3x + 10$

Constant term = 10

Factors of 10 are ±1, ±2, ±5, ±10

Let x + 1 = 0 or x = -1

$f(-1) = (-1)^3 – 6(-1)^2 + 3(-1) + 10 = 10 – 10 = 0$

f(-1) = 0

Let x + 2 = 0 or x = -2

$f(-2) = (-2)^3 – 6(-2)^2 + 3(-2) + 10 = -8 – 24 – 6 + 10 = -28$

$f(-2) \neq 0$

Let x – 2 = 0 or x = 2

$f(2) = (2)^3 – 6(2)^2 + 3(2) + 10 = 8 – 24 + 6 + 10 = 0$

f(2) = 0

Let x – 5 = 0 or x = 5

$f(5) = (5)^3 – 6(5)^2 + 3(5) + 10 = 125 – 150 + 15 + 10 = 0$

f(5) = 0

Therefore, (x + 1), (x – 2) and (x-5) are factors of f(x)

Hence f(x) = (x + 1) (x – 2) (x-5)

Question 4: $x^4 - 7x^3 + 9x^2 + 7x - 10$

Solution:

Let $f(x) = x^4 - 7x^3 + 9x^2 + 7x - 10$

Constant term = -10

Factors of -10 are ±1, ±2, ±5, ±10

Let x – 1 = 0 or x = 1

$f(1) = (1)^4 - 7(1)^3 + 9(1)^2 + 7(1) - 10 = 1 - 7 + 9 + 7 - 10 = 0$

f(1) = 0

Let x + 1 = 0 or x = -1

$f(-1) = (-1)^4 - 7(-1)^3 + 9(-1)^2 + 7(-1) - 10 = 1 + 7 + 9 - 7 - 10 = 0$

f(-1) = 0

Let x – 2 = 0 or x = 2

$f(2) = (2)^4 - 7(2)^3 + 9(2)^2 + 7(2) - 10 = 16 - 56 + 36 + 14 - 10 = 0$

f(2) = 0

Let x – 5 = 0 or x = 5

$f(5) = (5)^4 - 7(5)^3 + 9(5)^2 + 7(5) - 10 = 625 - 875 + 225 + 35 - 10 = 0$

f(5) = 0

Therefore, (x – 1), (x + 1), (x – 2) and (x-5) are factors of f(x)

Hence f(x) = (x – 1) (x + 1) (x – 2) (x-5)

Question 5: $x^4 - 2x^3 - 7x^2 + 8x + 12$

Solution:

$f(x) = x^4 – 2x^3 – 7x^2 + 8x + 12$

Constant term = 12

Factors of 12 are ±1, ±2, ±3, ±4, ±6, ±12

Let x – 1 = 0 or x = 1

$f(1) = (1)^4 – 2(1)^3 – 7(1)^2 + 8(1) + 12 = 1 – 2 – 7 + 8 + 12 = 12$

$f(1) \neq 0$

Let x + 1 = 0 or x = -1

$f(-1) = (-1)^4 – 2(-1)^3 – 7(-1)^2 + 8(-1) + 12 = 1 + 2 – 7 – 8 + 12 = 0$

f(-1) = 0

Let x +2 = 0 or x = -2

$f(-2) = (-2)^4 – 2(-2)^3 – 7(-2)^2 + 8(-2) + 12 = 16 + 16 – 28 – 16 + 12 = 0$

f(-2) = 0

Let x – 2 = 0 or x = 2

$f(2) = (2)^4 – 2(2)^3 – 7(2)^2 + 8(2) + 12 = 16 – 16 – 28 + 16 + 12 = 0$

f(2) = 0

Let x – 3 = 0 or x = 3

$f(3) = (3)^4 – 2(3)^3 – 7(3)^2 + 8(3) + 12 = 0$

f(3) = 0

Therefore, (x + 1), (x + 2), (x – 2) and (x-3) are factors of f(x)

Hence f(x) = (x + 1)(x + 2) (x – 2) (x-3)

Question 6: $x^4 + 10x^3 + 35x^2 + 50x + 24$

Solution:

Let $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$

Constant term = 24

Factors of 24 are ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

Let x + 1 = 0 or x = -1

$f(-1) = (-1)^4 + 10(-1)^3 + 35(-1)^2 + 50(-1) + 24 = 1 – 10 + 35 – 50 + 24 = 0$

f(1) = 0

(x + 1) is a factor of f(x)

Likewise, (x + 2),(x + 3),(x + 4) are also the factors of f(x)

Hence f(x) = (x + 1) (x + 2)(x + 3)(x + 4)

Question 7: $2x^4 – 7x^3 – 13x^2 + 63x – 45$

Solution:

Let $f(x) = 2x^4 – 7x^3 – 13x^2 + 63x – 45$

Constant term = -45

Factors of -45 are ±1, ±3, ±5, ±9, ±15, ±45

Here coefficient of x^4 is 2. So possible rational roots of f(x) are

±1, ±3, ±5, ±9, ±15, ±45, ±1/2,±3/2,±5/2,±9/2,±15/2,±45/2

Let x – 1 = 0 or x = 1

$f(1) = 2(1)^4 – 7(1)^3 – 13(1)^2 + 63(1) – 45 = 2 – 7 – 13 + 63 – 45 = 0$

f(1) = 0

f(x) can be written as,

$f(x) = (x-1) (2x^3 – 5x^2 -18x +45)$

or f(x) =(x-1)g(x) …(1)

Let x – 3 = 0 or x = 3

$f(3) = 2(3)^4 – 7(3)^3 – 13(3)^2 + 63(3) – 45 = = 162 – 189 – 117 + 189 – 45= 0$

f(3) = 0

Now, we are available with 2 factors of f(x), (x – 1) and (x – 3)

Here $g(x) = 2x^2 (x-3) + x(x-3) -15(x-3)$

Taking (x-3) as common

$= (x-3)(2x^2 + x – 15)$

$= (x-3)(2x^2+6x – 5x -15)$

= (x-3)(2x-5)(x+3)

= (x-3)(x+3)(2x-5) ….(2)

From (1) and (2)

f(x) =(x-1) (x-3)(x+3)(2x-5)

Exercise VSAQs Page No: 6.33

Question 1: Define zero or root of a polynomial

Solution:

zero or root, is a solution to the polynomial equation, f(y) = 0.

It is that value of y that makes the polynomial equal to zero.

Question 2: If x = 1/2 is a zero of the polynomial $f(x) = 8x^3 + ax^2 – 4x + 2$, find the value of a.

Solution:

If x = 1/2 is a zero of the polynomial f(x), then f(1/2) = 0

$8(1/2)^3 + a(1/2)^2 – 4(1/2) + 2 = 0$

8 x 1/8 + a/4 – 2 + 2 = 0

1 + a/4 = 0

a = -4

Question 3: Write the remainder when the polynomial f(x) = $x^3 + x^2 - 3x + 2$ is divided by x + 1.

Solution:

Using factor theorem,

Put x + 1 = 0 or x = -1

f(-1) is the remainder.

Now,

f(-1) = $(-1)^3 + (-1)^2 - 3(-1) + 2$

= -1 + 1 + 3 + 2

= 5

Therefore 5 is the remainder.

Question 4: Find the remainder when $x^3 + 4x^2 + 4x - 3$ if divided by x

Solution:

Using factor theorem,

Put x = 0

f(0) is the remainder.

Now,

f(0) = $0^3 + 4(0)^2 + 4\times0 - 3 = -3$

Therefore -3 is the remainder.

Question 5: If x+1 is a factor of $x^3 + a$, then write the value of a.

Solution:

Let $f(x) = x^3 + a$

If x+1 is a factor of $x^3 + a$ then f(-1) = 0

$(-1)^3 + a = 0$

-1 + a = 0

or a = 1

Question 6: If $f(x) = x^4 – 2x^3 + 3x^2 – ax – b$ when divided by x – 1, the remainder is 6, then find the value of a+b.

Solution:

From the statement, we have f(1) = 6

$(1)^4 – 2(1)^3 + 3(1)^2 – a(1) – b = 6$

1 – 2 + 3 – a – b = 6

2 – a – b = 6

a + b = -4

Chapterwise RD Sharma Solutions for Class 9 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.