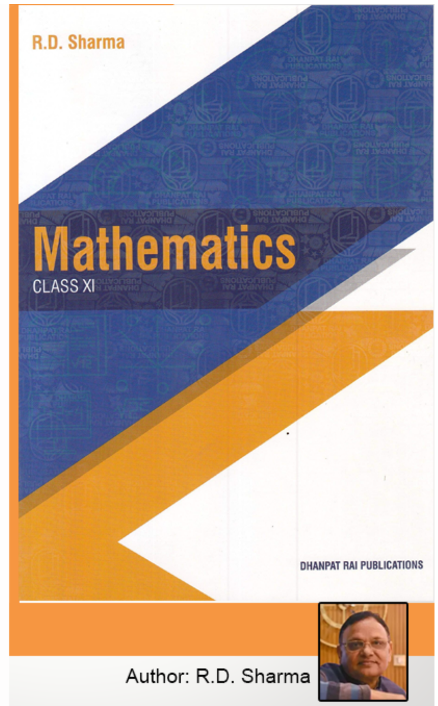
Class 11 - Chapter 5 Trigonometric Functions



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EXERCISE 5.1 PAGE NO: 5.18

Prove the following identities:

1. $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

Solution:

Let us consider LHS: $\sec^4 x - \sec^2 x$

$(\sec^2 x)^2 - \sec^2 x$

By using the formula, $\sec^2 \theta = 1 + \tan^2 \theta$.

$(1 + \tan^2 x)^2 - (1 + \tan^2 x)$

$1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$

$\tan^4 x + \tan^2 x$

= RHS

∴ LHS = RHS

Hence proved.

2. $\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$

Solution:

Let us consider LHS: $\sin^6 x + \cos^6 x$

$(\sin^2 x)^3 + (\cos^2 x)^3$

By using the formula, $a^3 + b^3 = (a + b) (a^2 + b^2 - ab)$

$(\sin^2 x + \cos^2 x) [(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cos^2 x]$

By using the formula, $\sin^2 x + \cos^2 x = 1$ and $a^2 + b^2 = (a + b)^2 - 2ab$

$1 \times [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x$

$1^2 - 3\sin^2 x \cos^2 x$

$1 - 3\sin^2 x \cos^2 x$

= RHS

∴ LHS = RHS

Hence proved.

3. (cosec x – sin x) (sec x – cos x) (tan x + cot x) = 1

Solution:

Let us consider LHS: (cosec x – sin x) (sec x – cos x) (tan x + cot x)

By using the formulas

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$\left(\frac{1}{\sin x} - \sin x\right)\left(\frac{1}{\cos x} - \cos x\right)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$\frac{1-\sin^2 x}{\sin x} \times \frac{1-\cos^2 x}{\cos x} \times \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

By using the formula,

$\text{Sin}^2 x + \cos^2 x = 1;$

$$\frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} \times \frac{1}{\sin x \cos x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

4. cosec x (sec x – 1) – cot x (1 – cos x) = tan x – sin x

Solution:

Let us consider LHS: cosec x (sec x – 1) – cot x (1 – cos x)

By using the formulas

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$\frac{1}{\sin x}\left(\frac{1}{\cos x}-1\right)-\frac{\cos x}{\sin x}(1-\cos x)$$
$$\frac{1}{\sin x}\left(\frac{1-\cos x}{\cos x}\right)-\frac{\cos x}{\sin x}(1-\cos x)$$
$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{1}{\cos x}-\cos x\right)$$
$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{1-\cos^2 x}{\cos x}\right)$$

By using the formula, $1 – \cos^2 x = \sin^2 x$;

$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{\sin^2 x}{\cos x}\right)$$

$$(1-\cos x)\left(\frac{\sin x}{\cos x}\right)$$

$$\frac{\sin x}{\cos x}-\sin x$$

$$\tan x-\sin x$$

= RHS

∴ LHS = RHS

Hence Proved.

5.

$$\frac{1-\sin x\cos x}{\cos x\left(\sec x-\operatorname{cosec} x\right)}\cdot\frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}=\sin x$$

Solution:

Let us consider the LHS:

$$\frac{1-\sin x\cos x}{\cos x\left(\sec x-\operatorname{cosec} x\right)}\cdot\frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}$$

By using the formula,

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

Now,

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{1}{\cos x}-\frac{1}{\sin x}\right)}\times\frac{(\sin x)^2-(\cos x)^2}{(\sin x)^3+(\cos x)^3}$$

By using the formula, $a^3 + b^3 = (a + b)\,(a^2 + b^2 - ab)$

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{\sin x-\cos x}{\cos x\sin x}\right)}\times\frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x+\cos x)\,[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

$$\frac{\sin x\,(1-\sin x\cos x)}{\sin x-\cos x}\times\frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x+\cos x)\,[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

$$\frac{\sin x\,(1-\sin x\cos x)}{1}\times\frac{1}{[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x\,(1-\sin x\cos x)\times\frac{1}{(1-\sin x\cos x)}$$

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{1}{\cos x}-\frac{1}{\sin x}\right)}\times\frac{(\sin x)^2-(\cos x)^2}{(\sin x)^3+(\cos x)^3}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{\sin x-\cos x}{\cos x\sin x}\right)}\times\frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x + \cos x)\left[(\sin x)^2 + (\cos x)^2 - \sin x\cos x\right]}$$

$$\frac{\sin x(1-\sin x\cos x)}{\sin x-\cos x}\times\frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x + \cos x)\left[(\sin x)^2 + (\cos x)^2 - \sin x\cos x\right]}$$

$$\frac{\sin x(1-\sin x\cos x)}{1}\times\frac{1}{\left[(\sin x)^2 + (\cos x)^2 - \sin x\cos x\right]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x(1-\sin x\cos x)\times\frac{1}{(1-\sin x\cos x)}$$

sin x

= RHS

∴ LHS = RHS

Hence Proved.

6.

$$\frac{\tan x}{1-\cot x}+\frac{\cot x}{1-tan x}=(\sec x\ cosec\ x+1)$$

Solution:

Let us consider the LHS:

$$\frac{\tan x}{1-\cot x}+\frac{\cot x}{1-tan x}$$

By using the formula,

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

By using the formula, $a^3 – b^3 = (a – b) (a^2 + b^2 + ab)$

By using the formula,

cosec θ = 1/sin θ,

sec θ = 1/cos θ;

cosec x × sec x + 1

sec x cosec x + 1

=RHS

∴ LHS = RHS

Hence Proved.

7.

Solution:

Let us consider LHS:

By using the formula $a^3 \pm b^3 = (a \pm b) (a^2 + b^2 \mp ab)$

$$\frac{(\sin x+\cos x)\,[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}{\sin x+\cos x} + \frac{(\sin x - \cos x)\,[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x - \cos x}$$

We know, $\sin^2 x + \cos^2 x = 1$.

1 – sinx cosx + 1 + sinx cosx

2

= RHS

∴ LHS = RHS

Hence Proved.

8. (sec x sec y + tan x tan y)^2 – (sec x tan y + tan x sec y)^2 = 1

Solution:

Let us consider LHS:

(sec x sec y + tan x tan y)^2 – (sec x tan y + tan x sec y)^2

Expanding the above equation we get,[(sec x sec y)^2 + (tan x tan y)^2 + 2 (sec x sec y) (tan x tan y)] – [(sec x tan y)^2 + (tan x sec y)^2 + 2 (sec x tan y) (tan x sec y)] [sec^2 x sec^2 y + tan^2 x tan^2 y + 2 (sec x sec y) (tan x tan y)] – [sec^2 x tan^2 y + tan^2 x sec^2 y + 2 (sec^2 x tan^2 y) (tan x sec y)]

sec^2 x sec^2 y – sec^2 x tan^2 y + tan^2 x tan^2 y – tan^2 x sec^2 y

sec^2 x (sec^2 y – tan^2 y) + tan^2 x (tan^2 y – sec^2 y)

sec^2 x (sec^2 y – tan^2 y) – tan^2 x (sec^2 y – tan^2 y)

We know, sec^2 x – tan^2 x = 1.

sec^2 x × 1 – tan^2 x × 1

sec^2 x – tan^2 x

1 = RHS

∴ LHS = RHS

Hence proved.

9.

$$\frac{\cos x}{1-\sin x} = \frac{1+\cos x+\sin x}{1+\cos x-\sin x}$$

Solution:

Let us Consider RHS:

$$\frac{1+\cos x+\sin x}{1+\cos x-\sin x}$$

$$\frac{(1+\cos x)+(\sin x)}{(1+\cos x)-(\sin x)}$$

$$\frac{(1+\cos x)+(\sin x)}{(1+\cos x)-(\sin x)}\times\frac{(1+\cos x)+(\sin x)}{(1+\cos x)+(\sin x)}$$

$$\frac{[(1+\cos x)+(\sin x)]^2}{(1+\cos x)^2-(\sin x)^2}$$

$$\frac{[(1+\cos x)+(\sin x)]^2}{(1+\cos x)^2-(\sin x)^2}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{1+\cos^2 x+2\cos x+\sin^2 x+2\sin x+2\sin x\cos x}{1+\cos^2 x+2\cos x-\sin^2 x}$$

We know, $\sin^2 x+\cos^2 x=1$.

$$\frac{1+1+2\cos x+2\sin x+2\sin x\cos x}{(1-\sin^2 x)+\cos^2 x+2\cos x}$$

We know, $1-\cos^2 x=\sin^2 x$.

$$\frac{2+2\cos x+2\sin x+2\sin x\cos x}{\cos^2 x+\cos^2 x+2\cos x}$$

$$\frac{2+2\cos x+2\sin x+2\sin x\cos x}{2\cos^2 x+2\cos x}$$

$$\frac{2+2\cos x+2\sin x+2\sin x\cos x}{\cos^2 x+\cos^2 x+2\cos x}$$

$$\frac{1+\cos x+\sin x+\sin x\cos x}{\cos x(\cos x+1)}$$

$$\frac{1(1+\cos x)+\sin x(\cos x+1)}{\cos x(\cos x+1)}$$

$$\frac{(1+\sin x)(\cos x+1)}{\cos x(\cos x+1)}$$

$$\frac{1+\sin x}{\cos x}\times\frac{\cos x}{\cos x}$$

$$\frac{(1+\sin x)\cos x}{\cos^2 x}$$

We know, $1 - \sin^2 x = \cos^2 x$.

$$\frac{(1+\sin x)\cos x}{1-\sin^2 x}$$

$$\frac{(1+\sin x)\cos x}{(1-\sin x)(1+\sin x)}$$

$$\frac{\cos x}{1-\sin x}$$

= LHS

∴ LHS = RHS

Hence Proved.

10.

$$\frac{\tan^3 x}{1+\tan^2 x}+\frac{\cot^3 x}{1+\cot^2 x}=\frac{1-2\sin^2 x\cos^2 x}{\sin x\cos x}$$

Solution:

Let us consider LHS:

$$\frac{\tan^3 x}{1+\tan^2 x}+\frac{\cot^3 x}{1+\cot^2 x}$$

By using the formulas,

$1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \operatorname{cosec}^2 x$

$$\frac{\tan^3 x}{\sec^2 x}+\frac{\cot^3 x}{\operatorname{cosec}^2 x}$$

$$\frac{\frac{\sin^3 x}{\cos^3 x}}{\frac{1}{\cos^2 x}}+\frac{\frac{\cos^3 x}{\sin^3 x}}{\frac{1}{\sin^2 x}}$$

$$\frac{\sin^3 x}{\cos x}+\frac{\cos^3 x}{\sin x}$$

= RHS

∴ LHS = RHS

Hence Proved.

11.

$$1-\frac{\sin^2 x}{1+\cot x}-\frac{\cos^2 x}{1+\tan x}=\sin x\cos x$$

Solution:

Let us consider LHS:

$$1-\frac{\sin^2 x}{1+\cot x}-\frac{\cos^2 x}{1+\tan x}$$

By using the formula,

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$1-\frac{\sin^2 x}{1+\frac{\cos x}{\sin x}}-\frac{\cos^2 x}{1+\frac{\sin x}{\cos x}}$$

$$1-\frac{\sin^3 x}{\sin x+\cos x}-\frac{\cos^3 x}{\sin x+\cos x}$$

$$\frac{\sin x+\cos x-(\sin^3 x+\cos^3 x)}{\sin x+\cos x}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{\sin x+\cos x-((\sin x+\cos x)(\sin x)^2\ +\ (\cos x)^2-\sin x\ \cos x))}{\sin x+\cos x}$$

$$\frac{(\sin x+\cos x)(1-\sin^2 x-\cos^2 x+\sin x\cos x)}{\sin x+\cos x}$$

$$1-(\sin^2 x+\cos^2 x)+\sin x\cos x$$

We know, $\sin^2 x + \cos^2 x = 1$.

$1 - 1 + \sin x \cos x$

Sin x cos x

= RHS

∴ LHS = RHS

Hence proved.

12.

Solution:

Let us consider LHS:

$$\left(\frac{1}{\sec^2 x-\cos^2 x}+\frac{1}{cosec^2 x-\sin^2 x}\right)\sin^2 x\cos^2 x$$

By using the formula,

cosec θ = 1/sin θ,

sec θ = 1/cos θ;

$$\left(\frac{\cos^2 x-\cos^2 x\sin^4 x+\sin^2 x-\sin^2 x\cos^4 x}{(1+\sin^2 x)(1-\sin^2 x)(1+\cos^2 x)(1-\cos^2 x)}\right)\sin^2 x\cos^2 x$$

We know, $\sin^2 x+\cos^2 x=1$.

$$\left(\frac{1-\cos^2 x\sin^4 x-\sin^2 x\cos^4 x}{(1+\sin^2 x)\cos^2 x(1+\cos^2 x)\sin^2 x}\right)\sin^2 x\cos^2 x$$

$$\left(\frac{1-\cos^2 x\sin^2 x(\sin^2 x+\cos^2 x)}{(1+\sin^2 x)(1+\cos^2 x)}\right)$$

$$\frac{1-\sin^2 x\cos^2 x}{2+\sin^2 x\cos^2 x}$$

= RHS

∴ LHS = RHS

Hence proved.

13. (1 + tan α tan β)^2 + (tan α – tan β)^2 = sec^2 α sec^2 β

Solution:

Let us consider LHS: (1 + tan α tan β)^2 + (tan α – tan β)^2

1+ tan^2 α tan^2 β + 2 tan α tan β + tan^2 α + tan^2 β – 2 tan α tan β

$1 + \tan^2 \alpha \tan^2 \beta + \tan^2 \alpha + \tan^2 \beta$

$\tan^2 \alpha (\tan^2 \beta + 1) + 1 (1 + \tan^2 \beta)$

$(1 + \tan^2 \beta) (1 + \tan^2 \alpha)$

We know, $1 + \tan^2 \theta = \sec^2 \theta$

So,

$\sec^2 \alpha \sec^2 \beta$

= RHS

∴ LHS = RHS

Hence proved.

EXERCISE 5.2 PAGE NO: 5.25

1. Find the values of the other five trigonometric functions in each of the following:

(i) cot x = 12/5, x in quadrant III

(ii) cos x = -1/2, x in quadrant II

(iii) tan x = 3/4, x in quadrant III

(iv) sin x = 3/5, x in quadrant I

Solution:

(i) cot x = 12/5, x in quadrant III

In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.

By using the formulas,

tan x = 1/cot x

= 1/(12/5)

= 5/12

cosec x = $-\sqrt{(1 + \cot^2 x)}$

= $-\sqrt{(1 + (12/5)^2)}$

= $-\sqrt{(25+144)/25}$

= $-\sqrt{(169/25)}$

= -13/5

sin x = 1/cosec x

= 1/(-13/5)

= -5/13

cos x = $-\sqrt{(1 - \sin^2 x)}$

= $-\sqrt{(1 - (-5/13)^2)}$

= $-\sqrt{(169-25)/169}$

= $-\sqrt{(144/169)}$

= -12/13

sec x = 1/cos x

= 1/(-12/13)

= -13/12

∴ sin x = -5/13, cos x = -12/13, tan x = 5/12, cosec x = -13/5, sec x = -13/12

(ii) cos x = -1/2, x in quadrant II

In second quadrant, sin x and cosec x are positive. tan x, cot x, cos x, sec x are negative.

By using the formulas,

sin x = $\sqrt{(1 - \cos^2 x)}$

= $\sqrt{(1 - (-1/2)^2)}$

$= \sqrt{(4-1)/4}$

$= \sqrt{(3/4)}$

$= \sqrt{3}/2$

tan x = sin x/cos x

$= (\sqrt{3}/2)/(-1/2)$

$= -\sqrt{3}$

cot x = 1/tan x

$= 1/-\sqrt{3}$

$= -1/\sqrt{3}$

cosec x = 1/sin x

$= 1/(\sqrt{3}/2)$

$= 2/\sqrt{3}$

sec x = 1/cos x

= 1/(-1/2)

= -2

∴ sin x = $\sqrt{3}/2$, tan x = $-\sqrt{3}$, cosec x = $2/\sqrt{3}$, cot x = $-1/\sqrt{3}$ sec x = -2

(iii) tan x = 3/4, x in quadrant III

In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.

By using the formulas,

sin x = $\sqrt{(1 - \cos^2 x)}$

$= -\sqrt{(1-(-4/5)^2)}$

$= -\sqrt{(25-16)/25}$

$= -\sqrt{(9/25)}$

$= -3/5$

cos x = 1/sec x

= 1/(-5/4)

= -4/5

cot x = 1/tan x

= 1/(3/4)

= 4/3

cosec x = 1/sin x

= 1/(-3/5)

= -5/3

sec x = $-\sqrt{(1 + \tan^2 x)}$

$= -\sqrt{(1+(3/4)^2)}$

$= -\sqrt{(16+9)/16}$

$= -\sqrt{(25/16)}$

= -5/4

∴ sin x = -3/5, cos x = -4/5, cosec x = -5/3, sec x = -5/4, cot x = 4/3

(iv) sin x = 3/5, x in quadrant I

In first quadrant, all trigonometric ratios are positive.

So, by using the formulas,

tan x = sin x/cos x

= (3/5)/(4/5)

= 3/4

cosec x = 1/sin x

= 1/(3/5)

= 5/3

cos x = $\sqrt{(1-\sin^2 x)}$

= $\sqrt{(1 - (-3/5)^2)}$

= $\sqrt{(25-9)/25}$

= $\sqrt{(16/25)}$

= 4/5

sec x = 1/cos x

= 1/(4/5)

= 5/4

cot x = 1/tan x

= 1/(3/4)

= 4/3

∴ cos x = 4/5, tan x = 3/4, cosec x = 5/3, sec x = 5/4, cot x = 4/3

2. If sin x = 12/13 and lies in the second quadrant, find the value of sec x + tan x.

Solution:

Given:

Sin x = 12/13 and x lies in the second quadrant.

We know, in second quadrant, sin x and cosec x are positive and all other ratios are negative.

By using the formulas,

Cos x = $\sqrt{(1-\sin^2 x)}$

= – $\sqrt{(1-(12/13)^2)}$

= – $\sqrt{(1- (144/169))}$

= – $\sqrt{(169-144)/169}$

= -$\sqrt{(25/169)}$

= – 5/13

We know,

tan x = sin x/cos x

sec x = 1/cos x

Now,

tan x = (12/13)/(-5/13)

= -12/5

sec x = 1/(-5/13)

= -13/5

Sec x + tan x = -13/5 + (-12/5)

= (-13-12)/5

= -25/5

= -5

∴ Sec x + tan x = -5

3. If sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2 find the value of 8 tan x -√5 sec y.

Solution:

Given:

sin x = 3/5, tan y = 1/2 and $\pi/2 < x < \pi < y < 3\pi/2$

We know that, x is in second quadrant and y is in third quadrant.

In second quadrant, cos x and tan x are negative.

In third quadrant, sec y is negative.

By using the formula,

$\cos x = -\sqrt{(1-\sin^2 x)}$

tan x = sin x/cos x

Now,

$\cos x = -\sqrt{(1-\sin^2 x)}$

$= -\sqrt{(1 - (3/5)^2)}$

$= -\sqrt{(1 - 9/25)}$

$= -\sqrt{((25-9)/25)}$

$= -\sqrt{(16/25)}$

$= -4/5$

tan x = sin x/cos x

= (3/5)/(-4/5)

= 3/5 × -5/4

= -3/4

We know that $\sec y = -\sqrt{(1+\tan^2 y)}$

$= -\sqrt{(1 + (1/2)^2)}$

$= -\sqrt{(1 + 1/4)}$

$= -\sqrt{((4+1)/4)}$

$= -\sqrt{(5/4)}$

$= -\sqrt{5}/2$

Now, $8 \tan x - \sqrt{5} \sec y = 8(-3/4) - \sqrt{5}(-\sqrt{5}/2)$

$= -6 + 5/2$

$= (-12+5)/2$

$= -7/2$

$\therefore 8 \tan x - \sqrt{5} \sec y = -7/2$

4. If sin x + cos x = 0 and x lies in the fourth quadrant, find sin x and cos x.

Solution:

Given:

Sin x + cos x = 0 and x lies in fourth quadrant.

Sin x = -cos x

Sin x/cos x = -1

So, tan x = -1 (since, tan x = sin x/cos x)

We know that, in fourth quadrant, cos x and sec x are positive and all other ratios are negative.

By using the formulas,

$\text{Sec } x = \sqrt{(1 + \tan^2 x)}$

$\text{Cos } x = 1/\sec x$

$\text{Sin } x = -\sqrt{(1 - \cos^2 x)}$

Now,

$\text{Sec } x = \sqrt{(1 + \tan^2 x)}$

$= \sqrt{(1 + (-1)^2)}$

$= \sqrt{2}$

Cos x = 1/sec x

$= 1/\sqrt{2}$

Sin x = $-\sqrt{(1 - \cos^2 x)}$

$= -\sqrt{(1 - (1/\sqrt{2})^2)}$

$= -\sqrt{(1 - (1/2))}$

$= -\sqrt{((2-1)/2)}$

$= -\sqrt{(1/2)}$

$= -1/\sqrt{2}$

∴ sin x = $-1/\sqrt{2}$ and cos x = $1/\sqrt{2}$

5. If cos x = -3/5 and π<x<3π/2 find the values of other five trigonometric functions and hence evaluate

Solution:

Given:

cos x= -3/5 and $\pi < x < 3\pi/2$

We know that in the third quadrant, tan x and cot x are positive and all other rations are negative.

By using the formulas,

Sin x = $-\sqrt{(1-\cos^2 x)}$

Tan x = sin x/cos x

Cot x = 1/tan x

Sec x = 1/cos x

Cosec x = 1/sin x

Now,

Sin x = – $\sqrt{(1-\cos^2 x)}$

= – $\sqrt{(1-(-3/5)^2)}$

= – $\sqrt{(1-9/25)}$

= – $\sqrt{((25-9)/25)}$

= – $\sqrt{(16/25)}$

= – 4/5

Tan x = sin x/cos x

= (-4/5)/(-3/5)

= -4/5 × -5/3

= 4/3

Cot x = 1/tan x

= 1/(4/3)

= 3/4

Sec x = 1/cos x

= 1/(-3/5)

= -5/3

Cosec x = 1/sin x

= 1/(-4/5)

= -5/4

∴

= [(-5/4) + (3/4)] / [(-5/3) – (4/3)]

= [(-5+3)/4] / [(-5-4)/3]

= [-2/4] / [-9/3]

= [-1/2] / [-3]

= 1/6

EXERCISE 5.3 PAGE NO: 5.39

1. Find the values of the following trigonometric ratios:

(i) sin 5π/3

(ii) sin 17π

(iii) tan 11π/6

(iv) cos (-25π/4)

(v) tan 7π/4

(vi) sin 17π/6

(vii) cos 19π/6

(viii) sin (-11π/6)

(ix) cosec (-20π/3)

(x) tan (-13π/4)

(xi) cos 19π/4

(xii) sin 41π/4

(xiii) cos 39π/4

(xiv) sin 151π/6

Solution:

(i) sin 5π/3

$5\pi/3 = (5\pi/3 \times 180)^o$

$= 300^o$

$= (90 \times 3 + 30)^o$

Since, 300^o lies in IV quadrant in which sine function is negative.

$\sin 5\pi/3 = \sin (300)^o$

$= \sin (90 \times 3 + 30)^o$

$= – \cos 30^o$

$= – \sqrt{3}/2$

(ii) sin 17π

$\text{Sin } 17\pi = \sin 3060^o$

$= \sin (90 \times 34 + 0)^o$

Since, 3060^o lies in the negative direction of x-axis i.e., on boundary line of II and III quadrants.

$\text{Sin } 17\pi = \sin (90 \times 34 + 0)^o$

$= – \sin 0^o$

$= 0$

(iii) tan 11π/6

$\tan 11\pi/6 = (11/6 \times 180)^o$

$= 330^o$

Since, 330^o lies in the IV quadrant in which tangent function is negative.

$\tan 11\pi/6 = \tan (300)^o$

$= \tan (90 \times 3 + 60)^o$

$= – \cot 60^o$

= – 1/√3

(iv) cos (-25π/4)

cos (-25π/4) = cos (-1125)°

= cos (1125)°

Since, 1125° lies in the I quadrant in which cosine function is positive.

cos (1125)° = cos (90×12 + 45)°

= cos 45°

= 1/√2

(v) tan 7π/4

tan 7π/4 = tan 315°

= tan (90×3 + 45)°

Since, 315° lies in the IV quadrant in which tangent function is negative.

tan 315° = tan (90×3 + 45)°

= – cot 45°

= -1

(vi) sin 17π/6

sin 17π/6 = sin 510°

= sin (90×5 + 60)°

Since, 510° lies in the II quadrant in which sine function is positive.

sin 510° = sin (90×5 + 60)°

= cos 60°

= 1/2

(vii) cos 19π/6

cos 19π/6 = cos 570°

= cos (90×6 + 30)°

Since, 570° lies in III quadrant in which cosine function is negative.

cos 570° = cos (90×6 + 30)°

= – cos 30°

= – √3/2

(viii) sin (-11π/6)

sin (-11π/6) = sin (-330°)

= – sin (90×3 + 60)°

Since, 330° lies in the IV quadrant in which the sine function is negative.

sin (-330°) = – sin (90×3 + 60)°

= – (-cos 60°)

= – (-1/2)

= 1/2

(ix) cosec (-20π/3)

cosec (-20π/3) = cosec (-1200)°

= – cosec (1200)°

= – cosec (90×13 + 30)°

Since, 1200° lies in the II quadrant in which cosec function is positive.

cosec (-1200)° = – cosec (90×13 + 30)°

= – sec 30°

= -2/√3

(x) tan (-13π/4)

tan (-13π/4) = tan (-585)°

= – tan (90×6 + 45)°

Since, 585° lies in the III quadrant in which the tangent function is positive.

tan (-585)° = – tan (90×6 + 45)°

= – tan 45°

= -1

(xi) cos 19π/4

cos 19π/4 = cos 855°

= cos (90×9 + 45)°

Since, 855° lies in the II quadrant in which the cosine function is negative.

cos 855° = cos (90×9 + 45)°

= – sin 45°

= – 1/√2

(xii) sin 41π/4

sin 41π/4 = sin 1845°

= sin (90×20 + 45)°

Since, 1845° lies in the I quadrant in which the sine function is positive.

sin 1845° = sin (90×20 + 45)°

= sin 45°

= 1/√2

(xiii) cos 39π/4

cos 39π/4 = cos 1755°

= cos (90×19 + 45)°

Since, 1755° lies in the IV quadrant in which the cosine function is positive.

cos 1755° = cos (90×19 + 45)°

= sin 45°

= 1/√2

(xiv) sin 151π/6

sin 151π/6 = sin 4530°

= sin (90×50 + 30)°

Since, 4530° lies in the III quadrant in which the sine function is negative.

sin 4530° = sin (90×50 + 30)°

= – sin 30°

= -1/2

2. prove that:

(i) tan 225° cot 405° + tan 765° cot 675° = 0

(ii) sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6 = 1/2

(iii) cos 24° + cos 55° + cos 125° + cos 204° + cos 300° = 1/2

(iv) tan (-125°) cot (-405°) – tan (-765°) cot (675°) = 0

(v) cos 570° sin 510° + sin (-330°) cos (-390°) = 0

(vi) tan 11π/3 – 2 sin 4π/6 – 3/4 cosec^2 π/4 + 4 cos^2 17π/6 = (3 – 4√3)/2

(vii) 3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4 = 1

Solution:

(i) tan 225° cot 405° + tan 765° cot 675° = 0

Let us consider LHS:

tan 225° cot 405° + tan 765° cot 675°

tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)

We know that when n is odd, cot → tan.

tan 45° cot 45° + tan 45° [-tan 45°]

tan 45° cot 45° – tan 45° tan 45°

1 × 1 – 1 × 1

1 – 1

0 = RHS

∴ LHS = RHS

Hence proved.

(ii) sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6 = 1/2

Let us consider LHS:

sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6

sin 480° cos 690° + cos 780° sin 1050°

sin (90° × 5 + 30°) cos (90° × 7 + 60°) + cos (90° × 8 + 60°) sin (90° × 11 + 60°)

We know that when n is odd, sin → cos and cos → sin.

cos 30° sin 60° + cos 60° [-cos 60°]

√3/2 × √3/2 – 1/2 × 1/2

3/4 – 1/4

2/4

1/2

= RHS

∴ LHS = RHS

Hence proved.

(iii) cos 24° + cos 55° + cos 125° + cos 204° + cos 300° = 1/2

Let us consider LHS:

cos 24° + cos 55° + cos 125° + cos 204° + cos 300°

cos 24° + cos (90° × 1 – 35°) + cos (90° × 1 + 35°) + cos (90° × 2 + 24°) + cos (90° × 3 + 30°)

We know that when n is odd, cos → sin.

cos 24° + sin 35° – sin 35° – cos 24° + sin 30°

0 + 0 + 1/2

1/2

= RHS

∴ LHS = RHS

Hence proved.

(iv) tan (-125°) cot (-405°) – tan (-765°) cot (675°) = 0

Let us consider LHS:

tan (-125°) cot (-405°) – tan (-765°) cot (675°)

We know that tan (-x) = -tan (x) and cot (-x) = -cot (x).[-tan (225°)] [-cot (405°)] – [-tan (765°)] cot (675°)

tan (225°) cot (405°) + tan (765°) cot (675°)

tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)

$\tan 45° \cot 45° + \tan 45° [-\tan 45°]$

$1 \times 1 + 1 \times (-1)$

$1 – 1$

0

= RHS

∴ LHS = RHS

Hence proved.

(v) $\cos 570° \sin 510° + \sin (-330°) \cos (-390°) = 0$

Let us consider LHS:

$\cos 570° \sin 510° + \sin (-330°) \cos (-390°)$

We know that $\sin (-x) = -\sin (x)$ and $\cos (-x) = +\cos (x)$.

$\cos 570° \sin 510° + [-\sin (330°)] \cos (390°)$

$\cos 570° \sin 510° – \sin (330°) \cos (390°)$

$\cos (90° \times 6 + 30°) \sin (90° \times 5 + 60°) – \sin (90° \times 3 + 60°) \cos (90° \times 4 + 30°)$

We know that cos is negative at $90° + \theta$ i.e. in Q_2 and when n is odd, sin → cos and cos → sin.

$-\cos 30° \cos 60° – [-\cos 60°] \cos 30°$

$-\cos 30° \cos 60° + \cos 60° \cos 30°$

0

= RHS

∴ LHS = RHS

Hence proved.

(vi) $\tan 11\pi/3 – 2 \sin 4\pi/6 – 3/4 \operatorname{cosec}^2 \pi/4 + 4 \cos^2 17\pi/6 = (3 – 4\sqrt{3})/2$

Let us consider LHS:

tan 11π/3 – 2 sin 4π/6 – 3/4 $cosec^2$ π/4 + 4 cos^2 17π/6

tan (11 × 180°)/3 – 2 sin (4 × 180°)/6 – 3/4 $cosec^2$ 180°/4 + 4 cos^2 (17 × 180°)/6

tan 660° – 2 sin 120° – 3/4 $(cosec\ 45°)^2$ + 4 $(cos\ 510°)^2$

tan (90° × 7 + 30°) – 2 sin (90° × 1 + 30°) – 3/4 $[cosec\ 45°]^2$ + 4 $[cos\ (90° × 5 + 60°)]^2$

We know that tan and cos is negative at 90° + θ i.e. in Q_2 and when n is odd, tan → cot, sin → cos and cos → sin.[-cot 30°] – 2 cos 30° – 3/4 $[cosec\ 45°]^2$ + $[-sin\ 60°]^2$

– cot 30° – 2 cos 30° – 3/4 $[cosec\ 45°]^2$ + $[sin\ 60°]^2$

-√3 – 2√3/2 – 3/4 $(\sqrt{2})^2$ + 4 $(\sqrt{3}/2)^2$

-√3 – √3 – 6/4 + 12/4

(3 – 4√3)/2

= RHS

∴ LHS = RHS

Hence proved.

(vii) 3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4 = 1

Let us consider LHS:

3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4

3 sin 180°/6 sec 180°/3 – 4 sin 5(180°)/6 cot 180°/4

3 sin 30° sec 60° – 4 sin 150° cot 45°

3 sin 30° sec 60° – 4 sin (90° × 1 + 60°) cot 45°

We know that when n is odd, sin → cos.

3 sin 30° sec 60° – 4 cos 60° cot 45°

3 (1/2) (2) – 4 (1/2) (1)

3 – 2

1

= RHS

∴ LHS = RHS

Hence proved.

3. Prove that:

(i)

(ii)

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

(iii)

$$\frac{sin(\pi + x)\; cos(\frac{\pi}{2} + x)\; tan(\frac{3\pi}{2} - x)\; cot(2\pi - x)}{sin(2\pi - x)\; cos(2\pi + x)\; cosec(-x)\; sin(\frac{3\pi}{2} - x)} = 1$$

(iv)

$$\left\{1 + cot\,x - sec(\frac{\pi}{2} + x)\right\}\left\{1 + cot\,x + sec(\frac{\pi}{2} + x)\right\} = 2\,cot\,x$$

(v)

$$\frac{tan(\frac{\pi}{2} - x)\; sec(\pi - x)\; sin(-x)}{sin(\pi + x)\; cot(2\pi - x)\; cosec(\frac{\pi}{2} - x)} = 1$$

Solution:

(i)

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)} = 1$$

Let us consider LHS:

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)}$$

$$\frac{cos\, x\, cosec\, x[-cot\, x]}{[-cosec\, x]\, cos\, x\, cot\, x}$$

$$\frac{-cos\, x cosec\, x\, cot\, x}{-cosec\, x\, cos\, x\, cot\, x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(ii)

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

Let us consider LHS:

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)}$$

$$\frac{cosec\,(90^o + x) + cot(90^o \times 5 + x)}{cosec(90^o - x) + tan(90^o \times 2 - x)} + \frac{tan(90^o \times 2 + x) + sec(90^o \times 2 - x)}{tan(90^o \times 4 + x) - sec(-x)}$$

We know that when n is odd, cosec → sec and also sec (-x) = sec x.

$$\frac{sec\, x + cot(90^o \times 5 + x)}{cosec(90^o - x) + tan(90^o \times 2 - x)} + \frac{tan(90^o \times 2 + x) + sec(90^o \times 2 - x)}{tan(90^o \times 4 + x) - sec(-x)}$$

$$\frac{sec\, x - tan\, x}{sec\, x - tan\, x} + \frac{tan\, x - sec\, x}{tan\, x - sec\, x}$$

1 + 1

2 = RHS

∴ LHS = RHS

Hence proved.

(iii)

$$\frac{sin(\pi+x)\ cos(\frac{\pi}{2}+x)\ tan(\frac{3\pi}{2}-x)\ cot(2\pi-x)}{sin(2\pi-x)\ cos(2\pi+x)\ cosec(-x)\ sin(\frac{3\pi}{2}-x)} = 1$$

Let us consider LHS:

$$\frac{sin(\pi+x)\ cos(\frac{\pi}{2}+x)\ tan(\frac{3\pi}{2}-x)\ cot(2\pi-x)}{sin(2\pi-x)\ cos(2\pi+x)\ cosec(-x)\ sin(\frac{3\pi}{2}-x)}$$

$$\frac{sin(180^o-x)\ cos(90^o+x)\ tan(270^o-x)\ cot(360^o-x)}{sin(360^o-x)\ cos(360^o+x)\ \ cosec(-x)\ sin(270^o-x)}$$

We know that cosec (-x) = - cosec x.

$$\frac{sin(90^o\times 2-x)\ cos(90^o\times 1+x)\ tan(90^o\times 3-x)\ cot(90^o\times 4-x)}{sin(90^o\times 4-x)\ cos(90^o\times 4+x)\ [-cosec(x)]\ sin(90^o\times 3-x)}$$

We know that when n is odd, cos → sin, tan → cot and sin → cos.

$$\frac{(-sin\,x)(-sin\,x)\,cot\,x\,(-cot\,x)}{(-sin\,x)\ cos\,x\ (-cosec\,x)(-cos\,x)}$$

$$\frac{sin^2x\,cot^2x}{sin\,x\,cosec\,x\,cos\,x\,cos\,x}$$

$$\frac{sin^2x\ \times \frac{cos^2x}{sin^2x}}{sin\,x\ \times \frac{1}{sin\,x}\times cos^2x}$$

$$\frac{cos^2x}{cos^2x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(iv)

$$\left\{1 + cot\,x - sec(\frac{\pi}{2} + x)\right\}\left\{1 + cot\,x + sec(\frac{\pi}{2} + x)\right\} = 2\,cot\,x$$

Let us consider LHS:

$$\left\{1 + cot\,x - sec(\frac{\pi}{2} + x)\right\}\left\{1 + cot\,x + sec(\frac{\pi}{2} + x)\right\}$$

{1 + cot x – (-cosec x)} {1 + cot x + (-cosec x)}

{1 + cot x + cosec x} {1 + cot x – cosec x}

{(1 + cot x) + (cosec x)} {(1 + cot x) – (cosec x)}

By using the formula, (a + b) (a – b) = $a^2 - b^2$

$(1 + \cot x)^2 - (\text{cosec } x)^2$

1 + $\cot^2 x$ + 2 cot x – $\text{cosec}^2 x$

We know that 1 + $\cot^2 x$ = $\text{cosec}^2 x$

$\text{cosec}^2 x$ + 2 cot x – $\text{cosec}^2 x$

2 cot x = RHS

∴ LHS = RHS

Hence proved.

(v)

$$\frac{tan(\frac{\pi}{2}-x)\ sec(\pi-x)\ sin(-x)}{sin(\pi+x)\ cot(2\pi-x)\ cosec(\frac{\pi}{2}-x)}=1$$

Let us consider LHS:

$$\frac{tan(\frac{\pi}{2}-x)\ sec(\pi-x)\ sin(-x)}{sin(\pi+x)\ cot(2\pi-x)\ cosec(\frac{\pi}{2}-x)}$$

$$\frac{tan(90^o-x)\ sec(180^o-x)\ sin(-x)}{sin(180^o+x)\ cot(360^o-x)cosec(90^o-x)}$$

We know that sin (-x) = -sin x.

$$\frac{tan(90^o\times 1-x)\ sec(90^o\times 2-x)\ [-sin(x)]}{sin(90^o\times 2+x)\ cot(90^o\times 4-x)cosec(90^o\times 1-x)}$$

We know that when n is odd, tan $\rightarrow$ cot and cosec $\rightarrow$ sec.

$$\frac{(cot\,x)(-sec\,x)(-sin\,x)}{(-sin\,x)(-cot\,x)(sec\,x)}$$

$$\frac{cot\,x\ sec\,x\ sin\,x}{sin\,x\ cot\,x\ sec\,x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

4. Prove that: $\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9 = 2$

Solution:

Let us consider LHS:

$\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9$

$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 7\pi/18 + \sin^2 8\pi/18$

$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 (\pi/2 – 2\pi/18) + \sin^2 (\pi/2 – \pi/18)$

We know that when n is odd, sin → cos.

$\sin^2 \pi/18 + \sin^2 2\pi/18 + \cos^2 2\pi/18 + \cos^2 2\pi/18$

when rearranged,

$\sin^2 \pi/18 + \cos^2 2\pi/18 + \sin^2 \pi/18 + \cos^2 2\pi/18$

We know that $\sin^2 + \cos^2 x = 1$.

So,

1 + 1

2 = RHS

∴ LHS = RHS

Hence proved.

Chapterwise RD Sharma Solutions for Class 11 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.