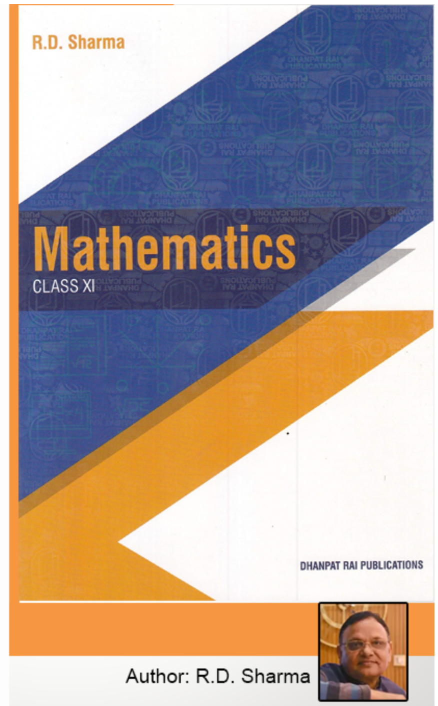
Class 11 - Chapter 22
Brief review of Cartesian System of Rectangular Coordinates



RD Sharma Solutions for Class 11 Maths Chapter 22–Brief review of Cartesian System of Rectangular Coordinates

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RD Sharma Solutions for Class 11 Maths Chapter 22–Brief review of Cartesian System of Rectangular Coordinates

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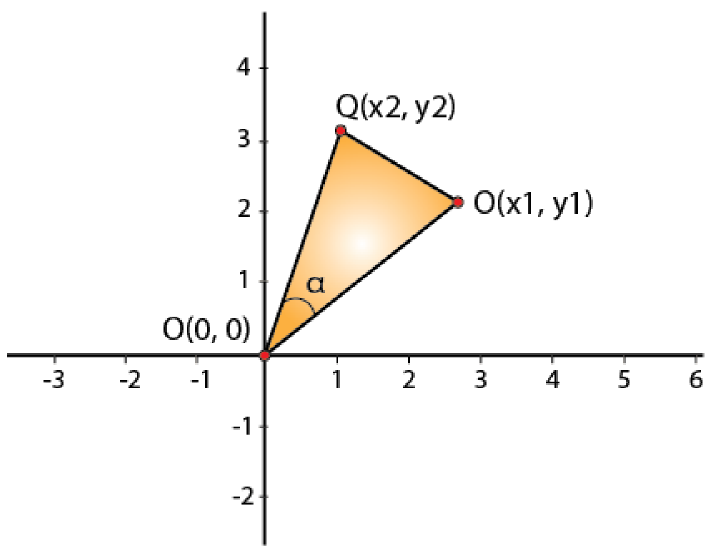
EXERCISE 22.1 PAGE NO: 22.12

1. If the line segment joining the points P(x_1, y_1) and Q(x_2, y_2) subtends an angle α at the origin O, prove that : OP. OQ cos α = x_1 x_2 + y_1 y_2.

Solution:

Given,

Two points P and Q subtends an angle α at the origin as shown in figure:



From figure we can see that points O, P and Q forms a triangle.

Clearly in ΔOPQ we have:

$\cos\alpha = \frac{OP^2+OQ^2-PQ^2}{2OP.OQ}$ {from cosine formula}

2 OP.OQ cos α = $OP^2 + OQ^2 - PQ^2$ … equation (1)

We know that the, coordinates of O are (0, 0) ⇒ $x_2 = 0$ and $y_2 = 0$

Coordinates of P are (x_1, y_1) ⇒ $x_1 = x_1$ and $y_1 = y_1$

By using distance formula we have:

$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2}$

$= \sqrt{x_1^2 + y_1^2}$

Similarly, $OQ = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$

$= \sqrt{x_2^2 + y_2^2}$

And, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

∴ $OP^2 + OQ^2 - PQ^2 = x_1^2 + y_1^2 + x_2^2 + y_2^2 - \{(x_2 - x_1)^2 + (y_2 - y_1)^2\}$

By using $(a-b)^2 = a^2 + b^2 - 2ab$

∴ $OP^2 + OQ^2 - PQ^2 = 2x_1 x_2 + 2y_1 y_2$ …. Equation (2)

So now from equation (1) and (2) we have:

$2OP.\,OQ\cos\alpha = 2x_1x_2 + 2y_1y_2$

$OP.\,OQ\cos\alpha = x_1x_2 + y_1y_2$

Hence Proved.

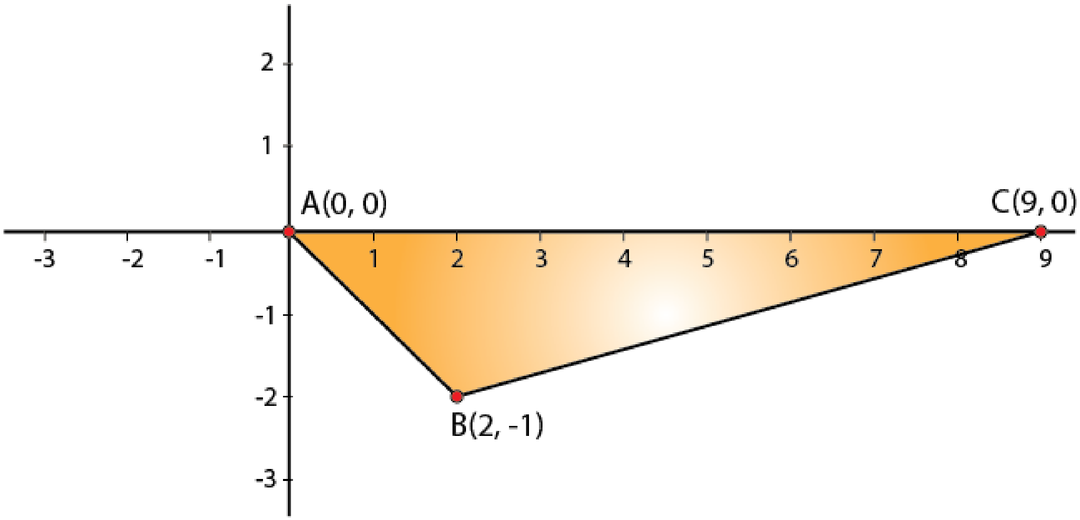
2. The vertices of a triangle ABC are A(0, 0), B (2, -1) and C (9, 0). Find cos B.

Solution:

Given:

The coordinates of triangle.

From the figure,



By using cosine formula,

In ΔABC, we have:

$$\cos B = \frac{AB^2 + BC^2 - AC^2}{2AB.BC}$$

Now by using distance formula we have:

$$AB = \sqrt{(2-0)^2 + (-1-0)^2} = \sqrt{5}$$

$$BC = \sqrt{(9-2)^2 + (0-(-1))^2} = \sqrt{7^2 + 1^2} = \sqrt{50}$$

$$\text{And, } AC = \sqrt{(9-0)^2 + (0-0)^2} = 9$$

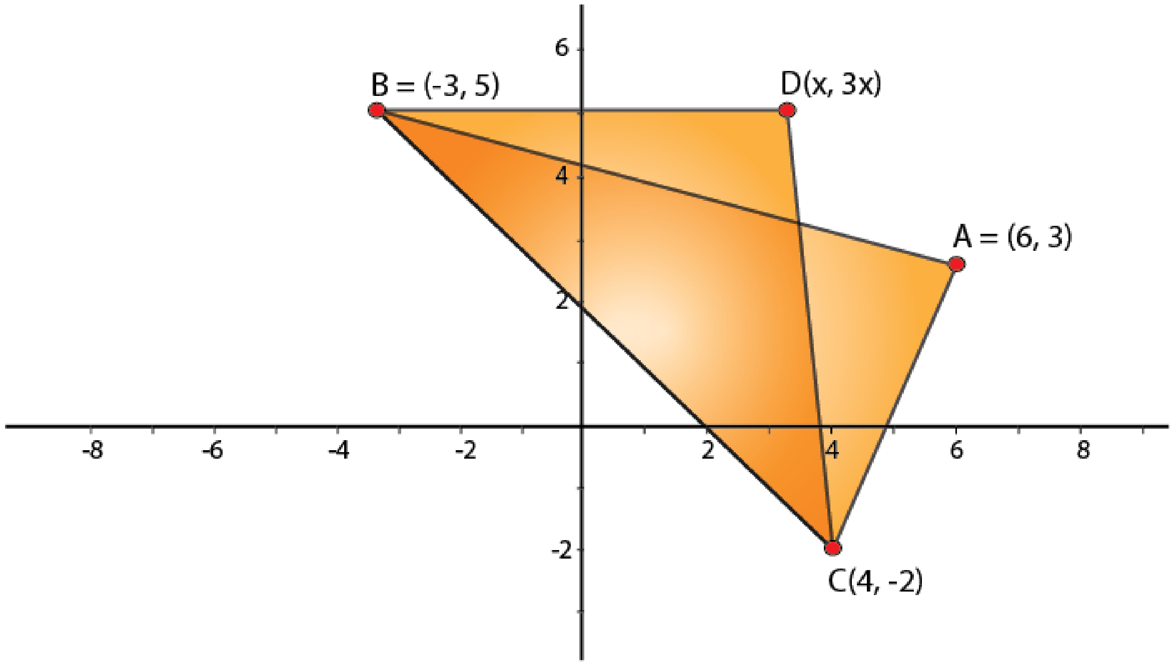
Now substitute the obtained values in the cosine formula, we get

$$\therefore \cos B = \frac{(\sqrt{5})^2 + (\sqrt{50})^2 - 9^2}{2\sqrt{5}\sqrt{50}} = \frac{55-81}{2\sqrt{5}\sqrt{2\times 25}} = \frac{-26}{10\sqrt{10}} = \frac{-13}{5\sqrt{10}}$$

3. Four points A (6, 3), B (-3, 5), C (4, -2) and D (x, 3x) are given in such a way that , find x.

Solution:

Given:
The coordinates of triangle are shown in the below figure.
Also, $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$



Now let us consider Area of a ΔPQR

Where, $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ be the 3 vertices of ΔPQR.

So, Area of $(\Delta PQR) = ½ [x_1(y_2 – y_3) + x_2(y_3 – y_1) + x_3(y_1 – y_2)]$

$$\text{Area of } (\Delta DBC) = \frac{1}{2}[x(5 - (-2)) + (-3)(-2 - 3x) + 4(3x - 5)]$$

$$= \frac{1}{2}[7x + 6 + 9x + 12x - 20] = 14x - 7$$

$$\text{Similarly, area of } (\Delta ABC) = \frac{1}{2}[6(5 - (-2)) + (-3)(-2 - 3) + 4(3 - 5)]$$

$$= \frac{1}{2}[42 + 15 - 8] = \frac{49}{2} = 24.5$$

$$\therefore \frac{\Delta DBC}{\Delta ABC} = \frac{1}{2} = \frac{14x-7}{24.5}$$

$24.5 = 28x – 14$

$28x = 38.5$

$x = 38.5/28$

$= 1.375$

24.5 = 28x – 14

28x = 38.5

x = 38.5/28

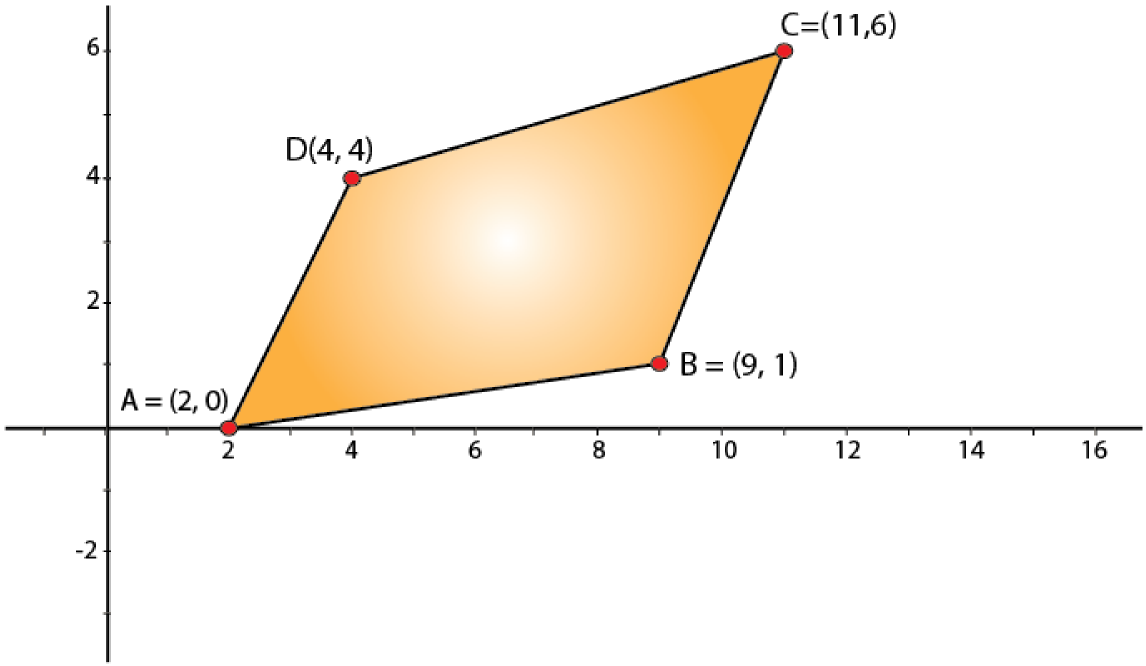
= 1.375

4. The points A (2, 0), B (9, 1), C (11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Solution:

Given:

The coordinates of 4 points that form a quadrilateral is shown in the below figure



Now by using distance formula, we have:

$$AB = \sqrt{(9-2)^2 + (1-0)^2} = \sqrt{7^2 + 1} = \sqrt{50}$$
$$BC = \sqrt{(11-9)^2 + (6-1)^2} = \sqrt{2^2 + 5^2} = \sqrt{29}$$

It is clear that, AB ≠ BC [quad ABCD does not have all 4 sides equal.]

∴ ABCD is not a Rhombus

EXERCISE 22.2 PAGE NO: 22.18

1. Find the locus of a point equidistant from the point (2, 4) and the y-axis.

Solution:

Let P (h, k) be any point on the locus and let A (2, 4) and B (0, k).

Then, PA = PB

$PA^2 = PB^2$

By using distance formula:

Distance of (h, k) from (2, 4) = $\sqrt{(h-2)^2 + (k-4)^2}$

Distance of (h, k) from (0, k) = $\sqrt{(h-0)^2 + (k-k)^2}$

So both the distances are same.

$\therefore \sqrt{(h-2)^2 + (k-4)^2} = \sqrt{(h-0)^2 + (k-k)^2}$

By squaring on both the sides we get,

$(h-2)^2 + (k-4)^2 = (h-0)^2 + (k-k)^2$

$h^2 + 4 - 4h + k^2 - 8k + 16 = h^2 + 0$

$k^2 - 4h - 8k + 20 = 0$

Replace (h, k) with (x, y)

∴ The locus of point equidistant from (2, 4) and y-axis is

$y^2 - 4x - 8y + 20 = 0$

2. Find the equation of the locus of a point which moves such that the ratio of its distance from (2, 0) and (1, 3) is 5: 4.

Solution:

Let P (h, k) be any point on the locus and let A (2, 0) and B (1, 3).

So then, PA/ BP = 5/4

$PA^2 = BP^2 = 25/16$

Distance of (h, k) from (2, 0) = $\sqrt{(h-2)^2+(k-0)^2}$

Distance of (h, k) from (1, 3) = $\sqrt{(h-1)^2+(k-3)^2}$

So,

$$\frac{\sqrt{(h-2)^2+(k-0)^2}}{\sqrt{(h-1)^2+(k-3)^2}}=\frac{5}{4}$$

By squaring on both the sides we get,

$$16\{(h-2)^2+k^2\}=25\{(h-1)^2+(k-3)^2\}$$

$$16\{h^2+4-4h+k^2\}=25\{h^2-2h+1+k^2-6k+9\}$$

$$9h^2+9k^2+14h-150k+186=0$$

Replace (h, k) with (x, y)

∴ The locus of a point which moves such that the ratio of its distance from (2, 0) and (1, 3) is 5: 4 which is

$$9x^2+9y^2+14x-150y+186=0$$

$$9h^2+9k^2+14h-150k+186=0$$

Replace (h, k) with (x, y)

∴ The locus of a point which moves such that the ratio of its distance from (2, 0) and (1, 3) is 5: 4 which is

$$9x^2+9y^2+14x-150y+186=0$$

3. A point moves as so that the difference of its distances from (ae, 0) and (-ae, 0) is 2a, prove that the equation to its locus is

, where $b^2 = a^2(e^2 - 1)$.

Solution:

Let P (h, k) be any point on the locus and let A (ae, 0) and B (-ae, 0).

Where, PA – PB = 2a

Distance of (h, k) from (ae, 0) = $\sqrt{(h-ae)^2+(k-0)^2}$

Distance of (h, k) from (-ae, 0) = $\sqrt{(h-(-ae))^2+(k-0)^2}$

So,

$$\sqrt{(h-ae)^2+(k-0)^2} - \sqrt{(h-(-ae))^2+(k-0)^2} = 2a$$

$$\sqrt{(h-ae)^2+(k-0)^2} = 2a + \sqrt{(h+ae)^2+(k-0)^2}$$

By squaring on both the sides we get:

$$(h-ae)^2+(k-0)^2 = \left\{2a + \sqrt{(h+ae)^2+(k-0)^2}\right\}^2$$

$$\Rightarrow h^2+a^2e^2-2aeh+k^2 = 4a^2+\{(h+ae)^2+k^2\}+4a\sqrt{(h+ae)^2+(k-0)^2}$$

$$\Rightarrow h^2+a^2e^2-2aeh+k^2$$
$$= 4a^2+h^2+2aeh+a^2e^2+k^2+4a\sqrt{(h+ae)^2+(k-0)^2}$$

$$-4aeh-4a^2 = 4a\sqrt{(h+ae)^2+(k-0)^2}$$

$$-4a(eh+a) = 4a\sqrt{(h+ae)^2+(k-0)^2}$$

Now again let us square on both the sides we get,

$(eh + a)^2 = (h + ae)^2 + (k – 0)^2$

$e^2h^2 + a^2 + 2aeh = h^2 + a^2e^2 + 2aeh + k^2$

$h^2 (e^2 – 1) – k^2 = a^2 (e^2 – 1)$

$$\frac{h^2}{a^2} - \frac{k^2}{a^2(e^2-1)} = 1$$
$$\frac{h^2}{a^2} - \frac{k^2}{b^2} = 1 \ [where, b^2 = a^2(e^2-1)]$$

Now let us replace (h, k) with (x, y)

The locus of a point such that the difference of its distances from (ae, 0) and (-ae, 0) is 2a.

Where $b^2 = a^2 (e^2 – 1)$

Hence proved.

4. Find the locus of a point such that the sum of its distances from (0, 2) and (0, -2) is 6.

Solution:

Let P (h, k) be any point on the locus and let A (0, 2) and B (0, -2).

Where, PA – PB = 6

Distance of (h, k) from (0, 2) = $\sqrt{(h-0)^2 + (k-2)^2}$

Distance of (h, k) from (0, -2) = $\sqrt{(h-0)^2 + (k-(-2))^2}$

So,

$$\sqrt{(h)^2 + (k-2)^2} + \sqrt{(h)^2 + (k+2)^2} = 6$$

$$\sqrt{(h)^2 + (k-2)^2} = 6 - \sqrt{(h)^2 + (k+2)^2}$$

By squaring on both the sides we get,

$$h^2 + (k-2)^2 = \left\{6 - \sqrt{h^2 + (k+2)^2}\right\}^2$$

$$\Rightarrow h^2 + 4 - 4k + k^2 = 36 + \{h^2 + k^2 + 4k + 4\} - 12\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow -8k - 36 = -12\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow -4(2k+9) = -12\sqrt{h^2 + (k+2)^2}$$

Now, again let us square on both the sides we get,

$$(2k+9)^2 = \left\{3\sqrt{h^2 + (k+2)^2}\right\}^2$$

$$4k^2 + 81 + 36k = 9(h^2 + k^2 + 4k + 4)$$

$$9h^2 + 5k^2 = 45$$

By replacing (h, k) with (x, y)

∴ The locus of a point is

$$9x^2 + 5y^2 = 45$$

5. Find the locus of a point which is equidistant from (1, 3) and x-axis.

Solution:

Let P (h, k) be any point on the locus and let A (1, 3) and B (h, 0).

Where, PA = PB

Distance of (h, k) from (1, 3) = $\sqrt{(h-1)^2+(k-3)^2}$

Distance of (h, k) from (h, 0) = $\sqrt{(h-h)^2+(k-0)^2}$

It is given that both distance are same.

So,

$\sqrt{(h-1)^2+(k-3)^2} = \sqrt{(h-h)^2+(k-0)^2}$

Now, let us square on both the sides we get,

$(h-1)^2+(k-3)^2 = (h-h)^2+(k-0)^2$

$h^2+1-2h+k^2-6k+9 = k^2+0$

$h^2-2h-6k+10 = 0$

By replacing (h, k) with (x, y),

∴ The locus of point equidistant from (1, 3) and x-axis is

$x^2-2x-6y+10=0$

6. Find the locus of a point which moves such that its distance from the origin is three times is distance from x-axis.

Solution:

Let P (h, k) be any point on the locus and let A (0, 0) and B (h, 0).

Where, PA = 3PB

Distance of (h, k) from (0, 0) = $\sqrt{(h-0)^2+(k-0)^2}$

Distance of (h, k) from (h, 0) = $\sqrt{(h-h)^2+(k-0)^2}$

So, where PA = 3PB

$\therefore \sqrt{(h-0)^2+(k-0)^2} = 3\sqrt{(h-h)^2+(k-0)^2}$

Now by squaring on both the sides we get,

$h^2 + k^2 = 9k^2$

$h^2 = 8k^2$

By replacing (h, k) with (x, y)

∴ The locus of point is $x^2 = 8y^2$

EXERCISE 22.3 PAGE NO: 22.21

1. What does the equation $(x – a)^2 + (y – b)^2 = r^2$ become when the axes are transferred to parallel axes through the point (a-c, b)?

Solution:

Given:

The equation, $(x – a)^2 + (y – b)^2 = r^2$

The given equation $(x – a)^2 + (y – b)^2 = r^2$ can be transformed into the new equation by changing x by x – a + c and y by y – b, i.e. substitution of x by x + a and y by y + b.

$((x + a – c) – a)^2 + ((y – b) – b)^2 = r^2$

$(x – c)^2 + y^2 = r^2$

$x^2 + c^2 – 2cx + y^2 = r^2$

$x^2 + y^2 - 2cx = r^2 – c^2$

Hence, the transformed equation is $x^2 + y^2 - 2cx = r^2 – c^2$

2. What does the equation $(a – b)(x^2 + y^2) – 2abx = 0$ become if the origin is shifted to the point (ab / (a-b), 0) without rotation?

Solution:

Given:

The equation $(a – b)(x^2 + y^2) – 2abx = 0$

The given equation $(a – b)(x^2 + y^2) – 2abx = 0$ can be transformed into new equation by changing x by [X + ab / (a-b)] and y by Y

$$(a-b)\left[\left(X+\frac{ab}{a-b}\right)^2+Y^2\right]-2ab\times\left(X+\frac{ab}{a-b}\right)=0$$

Upon expansion we get,

$$(a-b)\left(X^2+\frac{a^2b^2}{(a-b)^2}+\frac{2abX}{a-b}+Y^2\right)-2abX-\frac{2a^2b^2}{a-b}=0$$

Now let us simplify,

$$(a-b)\left(X^2+Y^2\right)+\frac{a^2b^2}{a-b}+2abX-2abX-\frac{2a^2b^2}{a-b}=0$$

$$(a-b)\left(X^2+Y^2\right)-\frac{a^2b^2}{a-b}=0$$

By taking LCM we get,

$$(a-b)^2\left(X^2+Y^2\right)=a^2b^2$$

Hence, the transformed equation is $(a - b)^2 (X^2 + Y^2) = a^2 b^2$

3. Find what the following equations become when the origin is shifted to the point (1, 1)?

(i) $x^2 + xy - 3x - y + 2 = 0$

(ii) $x^2 - y^2 - 2x + 2y = 0$

(iii) $xy - x - y + 1 = 0$

(iv) $xy - y^2 - x + y = 0$

Solution:

(i) $x^2 + xy - 3x - y + 2 = 0$

Firstly let us substitute the value of x by x + 1 and y by y + 1

Then,

$(x + 1)^2 + (x + 1) (y + 1) - 3(x + 1) - (y + 1) + 2 = 0$

$x^2 + 1 + 2x + xy + x + y + 1 - 3x - 3 - y - 1 + 2 = 0$

Upon simplification we get,

$x^2 + xy = 0$

∴ The transformed equation is $x^2 + xy = 0$.

(ii) $x^2 – y^2 – 2x + 2y = 0$

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$(x + 1)^2 – (y + 1)^2 – 2(x + 1) + 2(y + 1) = 0$

$x^2 + 1 + 2x – y^2 – 1 – 2y – 2x – 2 + 2y + 2 = 0$

Upon simplification we get,

$x^2 – y^2 = 0$

∴ The transformed equation is $x^2 – y^2 = 0$.

(iii) xy – x – y + 1 = 0

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$(x + 1) (y + 1) – (x + 1) – (y + 1) + 1 = 0$

$xy + x + y + 1 – x – 1 – y – 1 + 1 = 0$

Upon simplification we get,

$xy = 0$

∴ The transformed equation is xy = 0.

(iv) $xy – y^2 – x + y = 0$

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$(x + 1) (y + 1) – (y + 1)^2 – (x + 1) + (y + 1) = 0$

$xy + x + y + 1 – y^2 – 1 – 2y – x – 1 + y + 1 = 0$

Upon simplification we get,

$xy – y^2 = 0$

∴ The transformed equation is $xy – y^2 = 0$.

4. At what point the origin be shifted so that the equation $x^2 + xy – 3x + 2 = 0$ does not contain any first-degree term and constant term?

Solution:

Given:

The equation $x^2 + xy – 3x + 2 = 0$

We know that the origin has been shifted from (0, 0) to (p, q)

So any arbitrary point (x, y) will also be converted as (x + p, y + q).

The new equation is:

$(x + p)^2 + (x + p)(y + q) – 3(x + p) + 2 = 0$

Upon simplification,

$x^2 + p^2 + 2px + xy + py + qx + pq – 3x – 3p + 2 = 0$

$x^2 + xy + x(2p + q – 3) + y(q – 1) + p^2 + pq – 3p – q + 2 = 0$

For no first degree term, we have $2p + q – 3 = 0$ and $p – 1 = 0$, and

For no constant term we have $p^2 + pq – 3p – q + 2 = 0$.

By solving these simultaneous equations we have p = 1 and q = 1 from first equation.

The values p = 1 and q = 1 satisfies $p^2 + pq – 3p – q + 2 = 0$.

Hence, the point to which origin must be shifted is (p, q) = (1, 1).

5. Verify that the area of the triangle with vertices (2, 3), (5, 7) and (-3 -1) remains invariant under the translation of axes when the origin is shifted to the point (-1, 3).

Solution:

Given:

The points (2, 3), (5, 7), and (-3, -1).

The area of triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$= ½ [x_1(y_2 – y_3) + x_2(y_3 -y_1) + x_3(y_1 – y_2)]$

The area of given triangle = ½ [2(7+1) + 5(-1-3) – 3(3-7)]

= ½ [16 – 20 + 12]

= ½ [8]

= 4

Origin shifted to point (-1, 3), the new coordinates of the triangle are (3, 0), (6, 4), and (-2, -4) obtained from subtracting a point (-1, 3).

The new area of triangle = ½ [3(4-(-4)) + 6(-4-0) – 2(0-4)]

= ½ [24-24+8]

= ½ [8]

= 4

Since the area of the triangle before and after the translation after shifting of origin remains same, i.e. 4.

∴ We can say that the area of a triangle is invariant to shifting of origin.

Chapterwise RD Sharma Solutions for Class 11 Maths :

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- Chapter 2–Relations
- Chapter 3–Functions
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- Chapter 5–Trigonometric Functions
- Chapter 6–Graphs of Trigonometric Functions
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- Chapter 10–Sine and Cosine Formulae and their Applications
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About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.