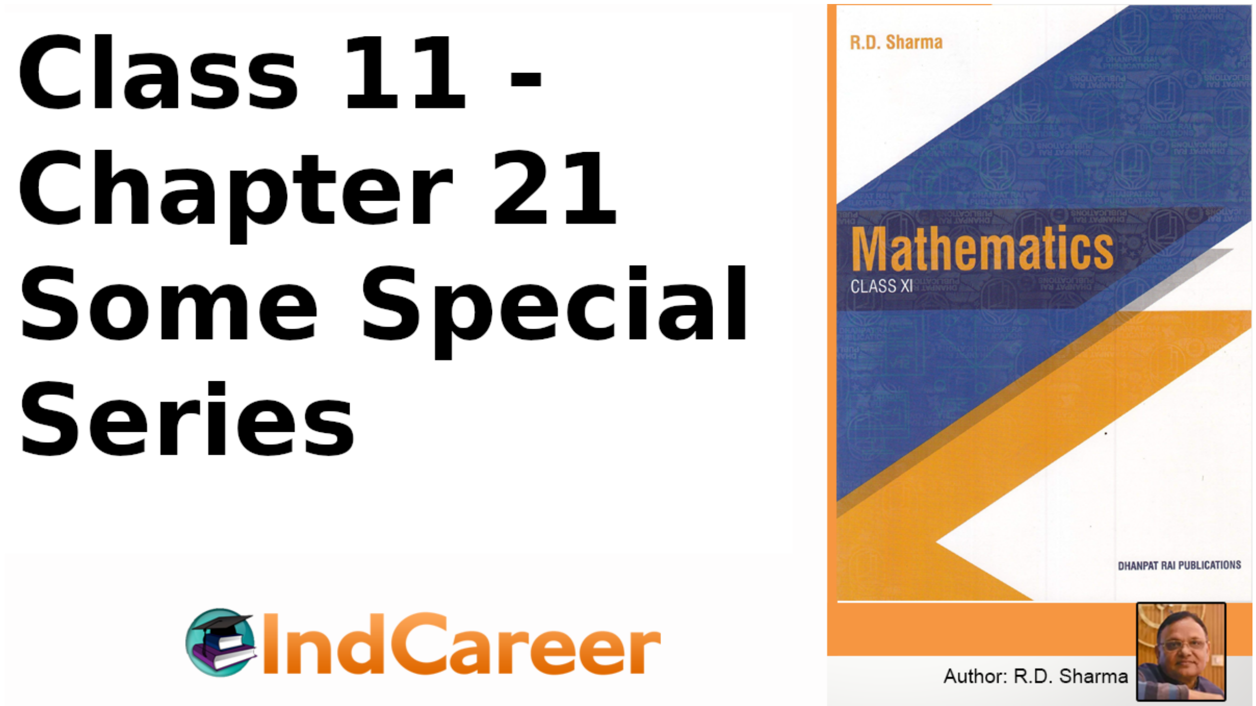

RD Sharma Solutions for Class 11 Maths Chapter 21–Some Special Series

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EXERCISE 21.1 PAGE NO: 21.10

Find the sum of the following series to n terms:

1. $1^3 + 3^3 + 5^3 + 7^3 + \ldots\ldots$

Solution:

Let T_n be the nth term of the given series.

We have:

$T_n = [1 + (n - 1)2]^3$

$= (2n - 1)^3$

$= (2n)^3 - 3(2n)^2 . 1 + 3.1^2.2n - 1^3$ [Since, $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b$]

$= 8n^3 - 12n^2 + 6n - 1$

Now, let S_n be the sum of n terms of the given series.

We have:

$$\begin{aligned} S_n &= \sum_{k=1}^{n} T_k \\ &= \sum_{k=1}^{n} [2k-1]^3 \\ &= \sum_{k=1}^{n} [8k^3 - 1 - 6k(2k-1)] \\ &= \sum_{k=1}^{n} [8k^3 - 1 - 12k^2 + 6k] \\ &= \sum_{k=1}^{n} [8k^3 - 1 - 12k^2 + 6k] \\ &= 8\sum_{k=1}^{n} k^3 - \sum_{k=1}^{n} 1 - 12\sum_{k=1}^{n} k^2 + 6\sum_{k=1}^{n} k \\ &= \frac{8n^2(n+1)^2}{4} - n - \frac{12n(n+1)(2n+1)}{6} + \frac{6\,n(n+1)}{2} \end{aligned}$$

Upon simplification we get,

$= 2n^2(n + 1)^2 - n - 2n(n + 1)(2n + 1) + 3n(n + 1)$

= n (n + 1) [2n (n + 1) – 2 (2n + 1) + 3] – n

= n (n + 1) [$2n^2$ – 2n + 1] – n

= n [$2n^3$ – $2n^2$ + n + $2n^2$ – 2n + 1 – 1]

= n [$2n^3$ – n]

= n^2 [$2n^2$ – 1]

∴ The sum of the series is n^2 [$2n^2$ – 1]

2. $2^3 + 4^3 + 6^3 + 8^3 + \ldots\ldots\ldots$

Solution:

Let T_n be the nth term of the given series.

We have:

$T_n = (2n)^3$

$= 8n^3$

Now, let S_n be the sum of n terms of the given series.

We have:

$$S_n = \sum_{k=1}^{n} 8k^3$$
$$= 8\sum_{k=1}^{n} k^3$$
$$= 8\left[\frac{n(n+1)}{2}\right]^2$$
$$= 8 \times \frac{n^2(n+1)^2}{4}$$
$$= 2n^2 (n+1)^2$$
$$= 2\{n (n+1)\}^2$$

∴ The sum of the series is $2\{n (n + 1)\}^2$

3. 1.2.5 + 2.3.6 + 3.4.7 + ……..

Solution:

Let T_n be the nth term of the given series.

We have:

$T_n = n (n + 1) (n + 4)$

$= n (n^2 + 5n + 4)$

$= n^3 + 5n^2 + 4n$

Now, let S_n be the sum of n terms of the given series.

We have:

$$S_n = \sum_{k=1}^{n} T_k$$
$$= \sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} 5k^2 + \sum_{k=1}^{n} 4k$$
$$= \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

Upon simplification we get,

$$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}$$
$$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1)$$
$$= \frac{n(n+1)}{2}\left(\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 4\right)$$
$$= \frac{n(n+1)}{2}\left(\frac{n^2+n}{2} + \frac{10n+5}{3} + 4\right)$$
$$= \frac{n(n+1)}{2}\left(\frac{3n^2+3n+20n+10+24}{6}\right)$$
$$= \frac{n}{12}(n+1)(3n^2+23n+34)$$

$$= \frac{n(n+1)}{2}\left(\frac{3n^2+3n+20n+10+24}{6}\right)$$
$$= \frac{n}{12}(n+1)(3n^2+23n+34)$$

∴ The sum of the series is
$$= \frac{n}{12}(n+1)(3n^2+23n+34)$$

4. 1.2.4 + 2.3.7 + 3.4.10 + … to n terms.

Solution:

Let T_n be the nth term of the given series.

We have:

T_n = n (n + 1) (3n + 1)

= n ($3n^2$ + 4n + 1)

= $3n^3 + 4n^2 + n$

Now, let S_n be the sum of n terms of the given series.

We have:

$$S_n = \sum_{k=1}^{n} T_k$$
$$= \sum_{k=1}^{n} 3k^3 + \sum_{k=1}^{n} 4k^2 + \sum_{k=1}^{n} k$$
$$= 3\sum_{k=1}^{n} k^3 + 4\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$$

Upon simplification we get,

$$= \frac{3n^2(n+1)^2}{4} + \frac{4n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$
$$= \frac{3n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}$$
$$= \frac{n(n+1)}{2}\left(\frac{3n(n+1)}{2} + \frac{4(2n+1)}{3} + 1\right)$$
$$= \frac{n(n+1)}{2}\left(\frac{3n^2+3n}{2} + \frac{8n+4}{3} + 1\right)$$
$$= \frac{n(n+1)}{2}\left(\frac{9n^2+9n+16n+8+6}{6}\right)$$
$$= \frac{n}{12}(n+1)(9n^2+25n+14)$$

∴ The sum of the series is

$$= \frac{n}{12}(n+1)(9n^2+25n+14)$$

5. 1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) + … to n terms

Solution:

Let T_n be the nth term of the given series.

We have:

T_n = n(n+1)/2

= (n^2 + n)/2

Now, let S_n be the sum of n terms of the given series.

We have:

$$
\begin{aligned}
S_n &= \sum_{k=1}^{n} T_k \\
&= \sum_{k=1}^{n} \left(\frac{k^2+k}{2}\right) \\
&= \frac{1}{2} \sum_{k=1}^{n} \left(k^2 + k\right) \\
&= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right] \\
&= \frac{n(n+1)}{4}\left(\frac{2n+1}{3} + 1\right) \\
&= \frac{n(n+1)}{4}\left(\frac{2n+4}{3}\right) \\
&= \frac{n(n+1)(2n+4)}{12} \\
&= \frac{n(n+1)(n+2)}{6}
\end{aligned}
$$

∴ The sum of the series is [n(n+1)(n+2)]/6

EXERCISE 21.2 PAGE NO: 21.18

Sum the following series to n terms:

1. 3 + 5 + 9 + 15 + 23 + ………….

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.

We have,

$S_n = 3 + 5 + 9 + 15 + 23 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ … (1)

Equation (1) can be rewritten as:

$S_n = 3 + 5 + 9 + 15 + 23 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ ……..(2)

By subtracting (2) from (1) we get

$S_n = 3 + 5 + 9 + 15 + 23 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$

$S_n = 3 + 5 + 9 + 15 + 23 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$

$0 = 3 + [2 + 4 + 6 + 8 + \ldots + (T_n - T_{n-1})] - T_n$

The difference between the successive terms are 5-3 = 2, 9-5 = 4, 15-9 = 6,

So these differences are in A.P

Now,

$$3 + \left[\frac{(n-1)}{2}\left\{4 + (n-2)2\right\}\right] - T_n = 0$$
$$3 + \left[\frac{(n-1)}{2}(2n)\right] = T_n$$
$$3 + n(n-1) = T_n$$

Now,

$$\begin{aligned} S_n &= \sum_{k=1}^{n} T_k \\ &= \sum_{k=1}^{n}\{3 + k(k-1)\} \\ &= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 3 - \sum_{k=1}^{n} k \\ &= \frac{n(n+1)(2n+1)}{6} + 3n - \frac{n(n+1)}{2} \\ &= \frac{n}{3}\left[\frac{(n+1)(2n+1)}{2} + 9 - \frac{3}{2}(n+1)\right] \\ &= \frac{n[n^2+8]}{3} \\ &= \frac{n}{3}(n^2+8) \end{aligned}$$

∴ The sum of the series is n/3 (n^2 + 8)

2. 2 + 5 + 10 + 17 + 26 + ………..

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.

We have,

S_n = 2 + 5 + 10 + 17 + 26 + …………. + T_{n-1} + T_n … (1)

Equation (1) can be rewritten as:

S_n = 2 + 5 + 10 + 17 + 26 + …………. + T_{n-1} + T_n ……..(2)

By subtracting (2) from (1) we get

S_n = 2 + 5 + 10 + 17 + 26 + …………. + T_{n-1} + T_n

S_n = 2 + 5 + 10 + 17 + 26 + …………. + T_{n-1} + T_n

0 = 2 + [3 + 5 + 7 + 9 + … + (T_n – T_{n-1})] – T_n

The difference between the successive terms are 3, 5, 7, 9

So these differences are in A.P

Now,

$$2+\left[\frac{(n-1)}{2}\{6+(n-2)2\}\right]-T_n=0$$

$$2+\left[n^2-1\right]=T_n$$

$$\left[n^2+1\right]=T_n$$

Now,

$$S_n=\sum_{k=1}^{n}T_k$$

$$=\sum_{k=1}^{n}\left(k^2+1\right)$$

$$=\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}1$$

$$=\frac{n(n+1)(2n+1)}{6}+n$$

$$=\frac{n(n+1)(2n+1)+6n}{6}$$

$$=\frac{n\left(2n^2+3n+7\right)}{6}$$

$$=\frac{n}{6}(2n^2+3n+7)$$

∴ The sum of the series is n/6 ($2n^2$ + 3n + 7)

3. 1 + 3 + 7 + 13 + 21 + …

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.

We have,

S_n = 1 + 3 + 7 + 13 + 21 + …………. + T_{n-1} + T_n … (1)

Equation (1) can be rewritten as:

S_n = 1 + 3 + 7 + 13 + 21 + …………. + T_{n-1} + T_n ……..(2)

By subtracting (2) from (1) we get

S_n = 1 + 3 + 7 + 13 + 21 + …………. + T_{n-1} + T_n

S_n = 1 + 3 + 7 + 13 + 21 + …………. + T_{n-1} + T_n

0 = 1 + [2 + 4 + 6 + 8 + … + (T_n – T_{n-1})] – T_n

The difference between the successive terms are 2, 4, 6, 8

So these differences are in A.P

Now,

$$1+\left[\frac{(n-1)}{2}\{4+(n-2)2\}\right]-T_n=0$$
$$1+\left[n^2-n\right]=T_n$$
$$\left[n^2-n+1\right]=T_n$$

Now,

$$S_n=\sum_{k=1}^{n}T_k$$
$$=\sum_{k=1}^{n}\left(k^2-k+1\right)$$
$$=\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}1-\sum_{k=1}^{n}k$$
$$=\frac{n(n+1)(2n+1)}{6}+n-\frac{n(n+1)}{2}$$
$$=\frac{n(n+1)}{2}\left(\frac{2n-2}{3}\right)+n$$
$$=n\left(\frac{n^2-1+3}{3}\right)$$
$$=\frac{n}{3}(n^2+2)$$

∴ The sum of the series is n/3 (n^2 + 2)

4. 3 + 7 + 14 + 24 + 37 + …

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.

We have,

S_n = 3 + 7 + 14 + 24 + 37 + …………. + T_{n-1} + T_n … (1)

Equation (1) can be rewritten as:

S_n = 3 + 7 + 14 + 24 + 37 + …………. + T_{n-1} + T_n ……..(2)

By subtracting (2) from (1) we get

$S_n = 3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$

$S_n = 3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$

$0 = 3 + [4 + 7 + 10 + 13 + \ldots + (T_n - T_{n-1})] - T_n$

The difference between the successive terms are 4, 7, 10, 13

So these differences are in A.P

Now,

$$3+\left[\frac{(n-1)}{2}\left\{8+(n-2)3\right\}\right]-T_n=0$$
$$3+\left[\frac{(n-1)}{2}(3n+2)\right]-T_n=0$$
$$\left[\frac{3n^2-n+4}{2}\right]=T_n$$
$$\left[\frac{3}{2}n^2-\frac{n}{2}+2\right]=T_n$$

Now,

$$\begin{aligned}S_n &= \sum_{k=1}^{n} T_k\\ &= \sum_{k=1}^{n}\left(\frac{3}{2}k^2-\frac{k}{2}+2\right)\\ &= \frac{3}{2}\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}2-\frac{1}{2}\sum_{k=1}^{n}k\\ &= \frac{n(n+1)(2n+1)}{4}+2n-\frac{n(n+1)}{4}\\ &= \frac{n(n+1)(2n)+8n}{4}\\ &= \frac{(n+1)(2n^2)+8n}{4}\\ &= \frac{n}{2}\left[n(n+1)+4\right]\\ &= \frac{n}{2}\left[n^2+n+4\right]\end{aligned}$$

∴ The sum of the series is $n/2\ [n^2 + n + 4]$

5. 1 + 3 + 6 + 10 + 15 + …

Solution:

Let T_n be the nth term and S_n be the sum to n terms of the given series.

We have,

$S_n = 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ … (1)

Equation (1) can be rewritten as:

$S_n = 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$ ……..(2)

By subtracting (2) from (1) we get

$S_n = 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$

$S_n = 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots\ldots + T_{n-1} + T_n$

$0 = 1 + [2 + 3 + 4 + 5 + \ldots + (T_n – T_{n-1})] – T_n$

The difference between the successive terms are 2, 3, 4, 5

So these differences are in A.P

Now,

$$1 + \left[\frac{(n-1)}{2}(4 + (n-2)1)\right] - T_n = 0$$

$$1 + \left[\frac{(n-1)}{2}(n+2)\right] - T_n = 0$$

$$\left[\frac{n^2+n}{2}\right] = T_n$$

Now,

$$S_n = \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n}\left(\frac{k^2+k}{2}\right)$$

$$= \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{2}\sum_{k=1}^{n} k$$

$$= \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$= \frac{n(n+1)}{4}\left(\frac{2n+1}{3} + 1\right)$$

$$= \frac{n(n+1)}{4}\left(\frac{2n+4}{3}\right)$$

$$= \frac{n(n+1)}{2}\left(\frac{n+2}{3}\right)$$

$$= \frac{n(n+1)(n+2)}{6}$$

$$= \frac{n}{6}(n+1)(n+2)$$

∴ The sum of the series is n/6 (n+1) (n+2)

Chapterwise RD Sharma Solutions for Class 11 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.