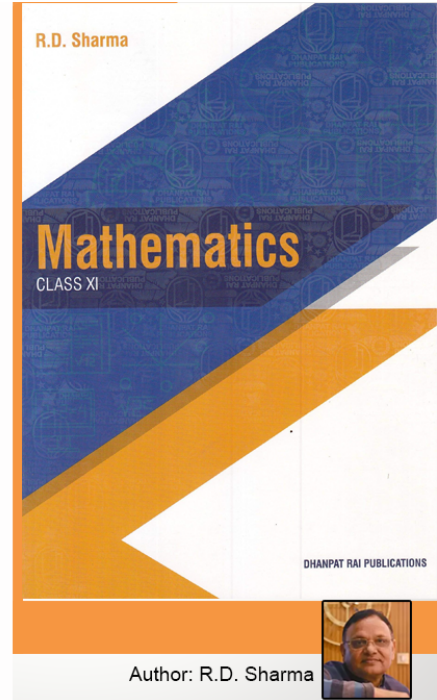


Class 11 - Chapter 21 Some Special Series



RD Sharma Solutions for Class 11 Maths Chapter 21–Some Special Series

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RD Sharma Solutions for Class 11 Maths Chapter 21–Some Special Series

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EXERCISE 21.1 PAGE NO: 21.10

Find the sum of the following series to n terms:

1. $1^3 + 3^3 + 5^3 + 7^3 + \dots$

Solution:

Let T_n be the nth term of the given series.

We have:

$$\begin{aligned} T_n &= [1 + (n - 1)2]^3 \\ &= (2n - 1)^3 \\ &= (2n)^3 - 3(2n)^2 \cdot 1 + 3 \cdot 1^2 \cdot 2n - 1^3 \text{ [Since, } (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \text{]} \\ &= 8n^3 - 12n^2 + 6n - 1 \end{aligned}$$

Now, let S_n be the sum of n terms of the given series.

We have:

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n [2k - 1]^3 \\ &= \sum_{k=1}^n [8k^3 - 1 - 6k(2k - 1)] \\ &= \sum_{k=1}^n [8k^3 - 1 - 12k^2 + 6k] \\ &= \sum_{k=1}^n [8k^3 - 1 - 12k^2 + 6k] \\ &= 8 \sum_{k=1}^n k^3 - \sum_{k=1}^n 1 - 12 \sum_{k=1}^n k^2 + 6 \sum_{k=1}^n k \\ &= \frac{8n^2(n+1)^2}{4} - n - \frac{12n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2} \end{aligned}$$

Upon simplification we get,

$$= 2n^2(n+1)^2 - n - 2n(n+1)(2n+1) + 3n(n+1)$$

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$$= n(n+1)[2n(n+1) - 2(2n+1) + 3] - n$$

$$= n(n+1)[2n^2 - 2n + 1] - n$$

$$= n[2n^3 - 2n^2 + n + 2n^2 - 2n + 1 - 1]$$

$$= n[2n^3 - n]$$

$$= n^2[2n^2 - 1]$$

∴ The sum of the series is $n^2[2n^2 - 1]$

2. $2^3 + 4^3 + 6^3 + 8^3 + \dots$

Solution:

Let T_n be the n th term of the given series.

We have:

$$T_n = (2n)^3$$

$$= 8n^3$$

Now, let S_n be the sum of n terms of the given series.

We have:

$$\begin{aligned} S_n &= \sum_{k=1}^n 8k^3 \\ &= 8 \sum_{k=1}^n k^3 \\ &= 8 \left[\frac{n(n+1)}{2} \right]^2 \\ &= 8 \times \frac{n^2(n+1)^2}{4} \\ &= 2n^2(n+1)^2 \\ &= 2\{n(n+1)\}^2 \end{aligned}$$

∴ The sum of the series is $2\{n(n+1)\}^2$

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3. $1.2.5 + 2.3.6 + 3.4.7 + \dots$

Solution:

Let T_n be the n th term of the given series.

We have:

$$T_n = n(n+1)(n+4)$$

$$= n(n^2 + 5n + 4)$$

$$= n^3 + 5n^2 + 4n$$

Now, let S_n be the sum of n terms of the given series.

We have:

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n k^3 + \sum_{k=1}^n 5k^2 + \sum_{k=1}^n 4k \\ &= \sum_{k=1}^n k^3 + 5 \sum_{k=1}^n k^2 + 4 \sum_{k=1}^n k \end{aligned}$$

Upon simplification we get,

$$\begin{aligned} &= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} \\ &= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1) \\ &= \frac{n(n+1)}{2} \left(\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 4 \right) \\ &= \frac{n(n+1)}{2} \left(\frac{n^2+n}{2} + \frac{10n+5}{3} + 4 \right) \\ &= \frac{n(n+1)}{2} \left(\frac{3n^2+3n+20n+10+24}{6} \right) \\ &= \frac{n}{12} (n+1)(3n^2 + 23n + 34) \end{aligned} \qquad \begin{aligned} &= \frac{n(n+1)}{2} \left(\frac{3n^2+3n+20n+10+24}{6} \right) \\ &= \frac{n}{12} (n+1)(3n^2 + 23n + 34) \end{aligned}$$

\therefore The sum of the series is
 $= \frac{n}{12} (n+1)(3n^2 + 23n + 34)$

4. $1.2.4 + 2.3.7 + 3.4.10 + \dots$ to n terms.

Solution:

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Let T_n be the n th term of the given series.

We have:

$$T_n = n(n+1)(3n+1)$$

$$= n(3n^2 + 4n + 1)$$

$$= 3n^3 + 4n^2 + n$$

Now, let S_n be the sum of n terms of the given series.

We have:

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n 3k^3 + \sum_{k=1}^n 4k^2 + \sum_{k=1}^n k \\ &= 3 \sum_{k=1}^n k^3 + 4 \sum_{k=1}^n k^2 + \sum_{k=1}^n k \end{aligned}$$

Upon simplification we get,

$$\begin{aligned} &= \frac{3n^2(n+1)^2}{4} + \frac{4n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \\ &= \frac{3n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2} \\ &= \frac{n(n+1)}{2} \left(\frac{3n(n+1)}{2} + \frac{4(2n+1)}{3} + 1 \right) \\ &= \frac{n(n+1)}{2} \left(\frac{3n^2+3n}{2} + \frac{8n+4}{3} + 1 \right) \\ &= \frac{n(n+1)}{2} \left(\frac{9n^2+9n+16n+8+6}{6} \right) \\ &= \frac{n}{12} (n+1)(9n^2 + 25n + 14) \end{aligned}$$

\therefore The sum of the series is

$$= \frac{n}{12} (n+1)(9n^2 + 25n + 14)$$

5. $1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) + \dots$ to n terms

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Solution:

Let T_n be the n th term of the given series.

We have:

$$T_n = n(n+1)/2$$

$$= (n^2 + n)/2$$

Now, let S_n be the sum of n terms of the given series.

We have:

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n \left(\frac{k^2+k}{2} \right) \\ &= \frac{1}{2} \sum_{k=1}^n (k^2 + k) \\ &= \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right] \\ &= \frac{n(n+1)}{4} \left(\frac{2n+1}{3} + 1 \right) \\ &= \frac{n(n+1)}{4} \left(\frac{2n+4}{3} \right) \\ &= \frac{n(n+1)(2n+4)}{12} \\ &= \frac{n(n+1)(n+2)}{6} \end{aligned}$$

\therefore The sum of the series is $[n(n+1)(n+2)]/6$

EXERCISE 21.2 PAGE NO: 21.18

Sum the following series to n terms:

1. $3 + 5 + 9 + 15 + 23 + \dots$

Solution:

Let T_n be the n th term and S_n be the sum to n terms of the given series.

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We have,

$$S_n = 3 + 5 + 9 + 15 + 23 + \dots + T_{n-1} + T_n \dots (1)$$

Equation (1) can be rewritten as:

$$S_n = 3 + 5 + 9 + 15 + 23 + \dots + T_{n-1} + T_n \dots (2)$$

By subtracting (2) from (1) we get

$$S_n = 3 + 5 + 9 + 15 + 23 + \dots + T_{n-1} + T_n$$

$$S_n = 3 + 5 + 9 + 15 + 23 + \dots + T_{n-1} + T_n$$

$$0 = 3 + [2 + 4 + 6 + 8 + \dots + (T_n - T_{n-1})] - T_n$$

The difference between the successive terms are $5-3 = 2$, $9-5 = 4$, $15-9 = 6$,

So these differences are in A.P

Now,

$$3 + \left[\frac{(n-1)}{2} \{4 + (n-2)2\} \right] - T_n = 0$$

$$3 + \left[\frac{(n-1)}{2} (2n) \right] = T_n$$

$$3 + n(n-1) = T_n$$

Now,

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n \{3 + k(k-1)\} \\ &= \sum_{k=1}^n k^2 + \sum_{k=1}^n 3 - \sum_{k=1}^n k \\ &= \frac{n(n+1)(2n+1)}{6} + 3n - \frac{n(n+1)}{2} \\ &= \frac{n}{3} \left[\frac{(n+1)(2n+1)}{2} + 9 - \frac{3}{2}(n+1) \right] \\ &= \frac{n[n^2+8]}{3} \\ &= \frac{n}{3}(n^2+8) \end{aligned}$$

∴ The sum of the series is $n/3 (n^2 + 8)$

2. $2 + 5 + 10 + 17 + 26 + \dots$

Solution:

Let T_n be the n th term and S_n be the sum to n terms of the given series.

We have,

$$S_n = 2 + 5 + 10 + 17 + 26 + \dots + T_{n-1} + T_n \dots (1)$$

Equation (1) can be rewritten as:

$$S_n = 2 + 5 + 10 + 17 + 26 + \dots + T_{n-1} + T_n \dots (2)$$

By subtracting (2) from (1) we get

$$S_n = 2 + 5 + 10 + 17 + 26 + \dots + T_{n-1} + T_n$$

$$S_n = 2 + 5 + 10 + 17 + 26 + \dots + T_{n-1} + T_n$$

$$0 = 2 + [3 + 5 + 7 + 9 + \dots + (T_n - T_{n-1})] - T_n$$

The difference between the successive terms are 3, 5, 7, 9

So these differences are in A.P

Now,

$$2 + \left[\frac{(n-1)}{2} \{6 + (n-2)2\} \right] - T_n = 0$$

$$2 + [n^2 - 1] = T_n$$

$$[n^2 + 1] = T_n$$

Now,

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n (k^2 + 1) \\ &= \sum_{k=1}^n k^2 + \sum_{k=1}^n 1 \\ &= \frac{n(n+1)(2n+1)}{6} + n \\ &= \frac{n(n+1)(2n+1)+6n}{6} \\ &= \frac{n(2n^2+3n+7)}{6} \\ &= \frac{n}{6}(2n^2 + 3n + 7) \end{aligned}$$

∴ The sum of the series is $\frac{n}{6}(2n^2 + 3n + 7)$

3. $1 + 3 + 7 + 13 + 21 + \dots$

Solution:

Let T_n be the n th term and S_n be the sum to n terms of the given series.

We have,

$$S_n = 1 + 3 + 7 + 13 + 21 + \dots + T_{n-1} + T_n \dots (1)$$

Equation (1) can be rewritten as:

$$S_n = 1 + 3 + 7 + 13 + 21 + \dots + T_{n-1} + T_n \dots (2)$$

By subtracting (2) from (1) we get

$$S_n = 1 + 3 + 7 + 13 + 21 + \dots + T_{n-1} + T_n$$

$$S_n = 1 + 3 + 7 + 13 + 21 + \dots + T_{n-1} + T_n$$

$$0 = 1 + [2 + 4 + 6 + 8 + \dots + (T_n - T_{n-1})] - T_n$$

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The difference between the successive terms are 2, 4, 6, 8

So these differences are in A.P

Now,

$$1 + \left[\frac{(n-1)}{2} \{4 + (n-2)2\} \right] - T_n = 0$$

$$1 + [n^2 - n] = T_n$$

$$[n^2 - n + 1] = T_n$$

Now,

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n (k^2 - k + 1) \\ &= \sum_{k=1}^n k^2 + \sum_{k=1}^n 1 - \sum_{k=1}^n k \\ &= \frac{n(n+1)(2n+1)}{6} + n - \frac{n(n+1)}{2} \\ &= \frac{n(n+1)}{2} \left(\frac{2n-2}{3} \right) + n \\ &= n \left(\frac{n^2-1+3}{3} \right) \\ &= \frac{n}{3} (n^2 + 2) \end{aligned}$$

∴ The sum of the series is $\frac{n}{3} (n^2 + 2)$

4. 3 + 7 + 14 + 24 + 37 + ...

Solution:

Let T_n be the n th term and S_n be the sum to n terms of the given series.

We have,

$$S_n = 3 + 7 + 14 + 24 + 37 + \dots + T_{n-1} + T_n \dots (1)$$

Equation (1) can be rewritten as:

$$S_n = 3 + 7 + 14 + 24 + 37 + \dots + T_{n-1} + T_n \dots (2)$$

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By subtracting (2) from (1) we get

$$S_n = 3 + 7 + 14 + 24 + 37 + \dots + T_{n-1} + T_n$$

$$S_n = 3 + 7 + 14 + 24 + 37 + \dots + T_{n-1} + T_n$$

$$0 = 3 + [4 + 7 + 10 + 13 + \dots + (T_n - T_{n-1})] - T_n$$

The difference between the successive terms are 4, 7, 10, 13

So these differences are in A.P

Now,

$$3 + \left[\frac{(n-1)}{2} \{8 + (n-2)3\} \right] - T_n = 0$$

$$3 + \left[\frac{(n-1)}{2} (3n + 2) \right] - T_n = 0$$

$$\left[\frac{3n^2 - n + 4}{2} \right] = T_n$$

$$\left[\frac{3}{2}n^2 - \frac{n}{2} + 2 \right] = T_n$$

Now,

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n \left(\frac{3}{2}k^2 - \frac{k}{2} + 2 \right) \\ &= \frac{3}{2} \sum_{k=1}^n k^2 + \sum_{k=1}^n 2 - \frac{1}{2} \sum_{k=1}^n k \\ &= \frac{n(n+1)(2n+1)}{4} + 2n - \frac{n(n+1)}{4} \\ &= \frac{n(n+1)(2n)+8n}{4} \\ &= \frac{(n+1)(2n^2)+8n}{4} \\ &= \frac{n}{2} [n(n+1) + 4] \\ &= \frac{n}{2} [n^2 + n + 4] \end{aligned}$$

∴ The sum of the series is $\frac{n}{2} [n^2 + n + 4]$

5. $1 + 3 + 6 + 10 + 15 + \dots$

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Solution:

Let T_n be the n th term and S_n be the sum to n terms of the given series.

We have,

$$S_n = 1 + 3 + 6 + 10 + 15 + \dots + T_{n-1} + T_n \dots (1)$$

Equation (1) can be rewritten as:

$$S_n = 1 + 3 + 6 + 10 + 15 + \dots + T_{n-1} + T_n \dots (2)$$

By subtracting (2) from (1) we get

$$S_n = 1 + 3 + 6 + 10 + 15 + \dots + T_{n-1} + T_n$$

$$S_n = 1 + 3 + 6 + 10 + 15 + \dots + T_{n-1} + T_n$$

$$0 = 1 + [2 + 3 + 4 + 5 + \dots + (T_n - T_{n-1})] - T_n$$

The difference between the successive terms are 2, 3, 4, 5

So these differences are in A.P

Now,

$$1 + \left[\frac{(n-1)}{2} (4 + (n-2)1) \right] - T_n = 0$$

$$1 + \left[\frac{(n-1)}{2} (n+2) \right] - T_n = 0$$

$$\left[\frac{n^2+n}{2} \right] = T_n$$

Now,

$$S_n = \sum_{k=1}^n T_k$$

$$= \sum_{k=1}^n \left(\frac{k^2+k}{2} \right)$$

$$= \frac{1}{2} \sum_{k=1}^n k^2 + \frac{1}{2} \sum_{k=1}^n k$$

$$= \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$= \frac{n(n+1)}{4} \left(\frac{2n+1}{3} + 1 \right)$$

$$= \frac{n(n+1)}{4} \left(\frac{2n+4}{3} \right)$$

$$= \frac{n(n+1)}{2} \left(\frac{n+2}{3} \right)$$

$$= \frac{n(n+1)(n+2)}{6}$$

$$= \frac{n}{6} (n+1)(n+2)$$

∴ The sum of the series is $\frac{n}{6} (n+1)(n+2)$



Chapterwise RD Sharma Solutions for Class 11 Maths :

- Chapter 1–Sets
- Chapter 2–Relations
- Chapter 3–Functions
- Chapter 4–Measurement of Angles
- Chapter 5–Trigonometric Functions
- Chapter 6–Graphs of Trigonometric Functions
- Chapter 7–Values of Trigonometric Functions at Sum or Difference of Angles
- Chapter 8–Transformation Formulae
- Chapter 9–Values of Trigonometric Functions at Multiples and Submultiples of an Angle
- Chapter 10–Sine and Cosine Formulae and their Applications
- Chapter 11–Trigonometric Equations
- Chapter 12–Mathematical Induction
- Chapter 13–Complex Numbers
- Chapter 14–Quadratic Equations
- Chapter 15–Linear Inequations
- Chapter 16–Permutations
- Chapter 17–Combinations
- Chapter 18–Binomial Theorem
- Chapter 19–Arithmetic Progressions
- Chapter 20–Geometric Progressions

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- Chapter 21–Some Special Series
- Chapter 22–Brief review of Cartesian System of Rectangular Coordinates
- Chapter 23–The Straight Lines
- Chapter 24–The Circle
- Chapter 25–Parabola
- Chapter 26–Ellipse
- Chapter 27–Hyperbola
- Chapter 28–Introduction to Three Dimensional Coordinate Geometry
- Chapter 29–Limits
- Chapter 30–Derivatives
- Chapter 31–Mathematical Reasoning
- Chapter 32–Statistics
- Chapter 33–Probability

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About RD Sharma

RD Sharma isn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many CBSE students lose their dread of maths. Sunday Times profiles the tutor turned internet star

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.

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