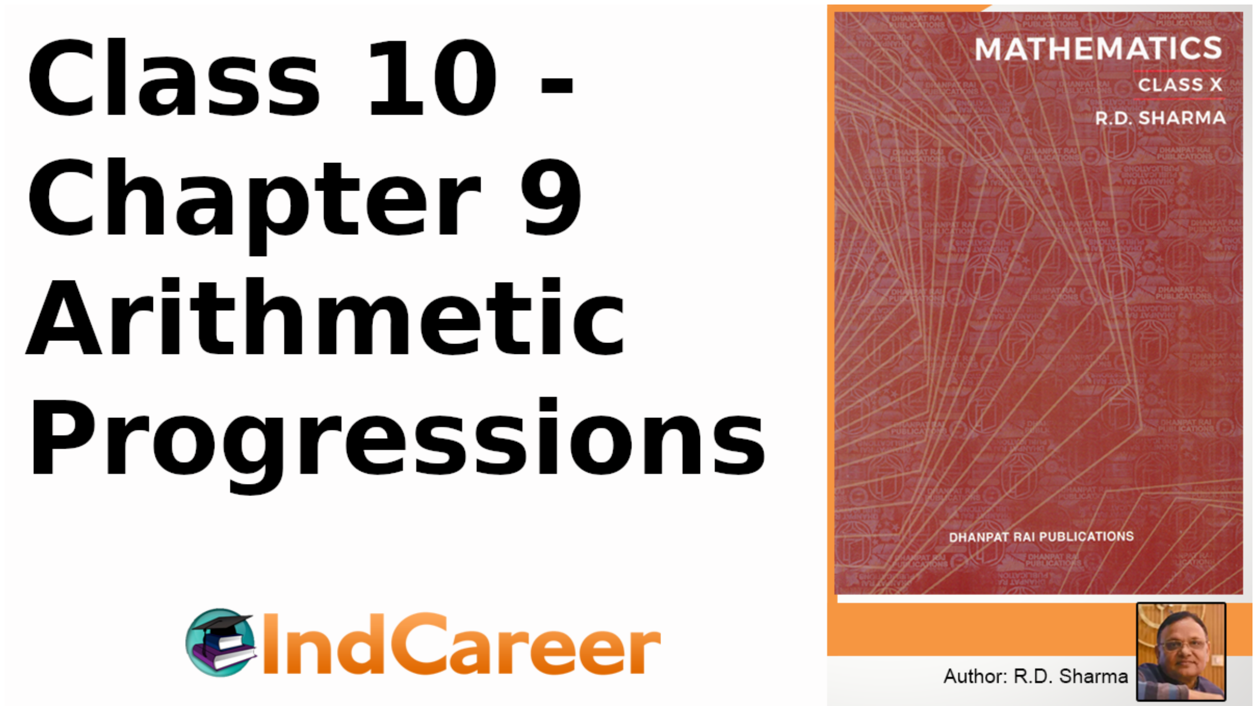

RD Sharma Solutions for Class 10 Maths Chapter 9–Arithmetic Progressions

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Exercise 9.1 Page No: 9.5

1. Write the first terms of each of the following sequences whose n^{th} term are:

(i) $a_n = 3n + 2$

(ii) $a_n = (n – 2)/3$

(iii) $a_n = 3^n$

(iv) $a_n = (3n – 2)/ 5$

(v) $a_n = (-1)^n . 2^n$

(vi) $a_n = n(n – 2)/2$

(vii) $a_n = n^2 – n + 1$

(viii) $a_n = n^2 – n + 1$

(ix) $a_n = (2n – 3)/ 6$

Solutions:

(i) $a_n = 3n + 2$

Given sequence whose $a_n = 3n + 2$

To get the first five terms of given sequence, put n = 1, 2, 3, 4, 5 and we get

$a_1 = (3 \times 1) + 2 = 3 + 2 = 5$

$a_2 = (3 \times 2) + 2 = 6 + 2 = 8$

$a_3 = (3 \times 3) + 2 = 9 + 2 = 11$

$a_4 = (3 \times 4) + 2 = 12 + 2 = 14$

$a_5 = (3 \times 5) + 2 = 15 + 2 = 17$

∴ the required first five terms of the sequence whose n^{th} term, $a_n = 3n + 2$ are 5, 8, 11, 14, 17.

(ii) $a_n = (n – 2)/3$

Given sequence whose

On putting n = 1, 2, 3, 4, 5 then can get the first five terms

$$a_1 = \frac{1-2}{3} = \frac{-1}{3}; a_2 = \frac{2-2}{3} = 0$$

$$a_3 = \frac{3-2}{3} = \frac{1}{3}; a_4 = \frac{4-2}{3} = \frac{2}{3}$$

$$a_n = \frac{n-2}{3} \text{ are } \frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1. \quad a_5 = \frac{5-2}{3} = 1$$

∴ the required first five terms of the sequence whose n^{th} term,

(iii) $a_n = 3^n$

Given sequence whose $a_n = 3^n$

To get the first five terms of given sequence, put n = 1, 2, 3, 4, 5 in the above

$a_1 = 3^1 = 3$;

$a_2 = 3^2 = 9$;

$a_3 = 27$;

$a_4 = 3^4 = 81$;

$a_5 = 3^5 = 243$.

∴ the required first five terms of the sequence whose n^{th} term, $a_n = 3^n$ are 3, 9, 27, 81, 243.

$$a_n = \frac{3n-2}{5}$$

(iv) $a_n = (3n – 2)/ 5$

Given sequence whose

To get the first five terms of the sequence, put n = 1, 2, 3, 4, 5 in the above

And, we get

$$a_1 = \frac{3 \times 1 - 2}{5} = \frac{3-2}{5} = \frac{1}{5}$$

$$a_2 = \frac{3 \times 2 - 2}{5} = \frac{6-2}{5} = \frac{4}{5}$$

$$a_3 = \frac{3 \times 3 - 2}{5} = \frac{9-2}{5} = \frac{7}{5}$$

$$a_4 = \frac{3 \times 4 - 2}{5} = \frac{12-2}{5} = \frac{10}{5}$$

$$a_5 = \frac{3 \times 5 - 2}{5} = \frac{15-2}{5} = \frac{13}{5}$$

∴ the required first five terms of the sequence are 1/5, 4/5, 7/5, 10/5, 13/5

(v) $a_n = (-1)^n 2^n$

Given sequence whose $a_n = (-1)^n 2^n$

To get first five terms of the sequence, put n = 1, 2, 3, 4, 5 in the above.

$a_1 = (-1)^1.2^1 = (-1).2 = -2$

$a_2 = (-1)^2.2^2 = (-1).4 = 4$

$a_3 = (-1)^3.2^3 = (-1).8 = -8$

$a_4 = (-1)^4.2^4 = (-1).16 = 16$

$a_5 = (-1)^5.2^5 = (-1).32 = -32$

∴ the first five terms of the sequence are – 2, 4, – 8, 16, – 32.

(vi) $a_n = n(n – 2)/2$

The given sequence is,

$$a_n = \frac{n(n-2)}{2}$$

To get the first five terms of the sequence, put n = 1, 2, 3, 4, 5.

And, we get

$$a_1 = \frac{1(1-2)}{2} = \frac{1-1}{2} = \frac{-1}{2}$$

$$a_2 = \frac{2(2-2)}{2} = \frac{2.0}{2} = 0$$

$$a_3 = \frac{3(3-2)}{2} = \frac{3.1}{2} = \frac{3}{2}$$

$$a_4 = \frac{4(4-2)}{2} = \frac{4.2}{2} = 4$$

$$a_5 = \frac{5(5-2)}{2} = \frac{5.3}{2} = \frac{15}{2}$$

∴ the required first five terms are -1/2, 0, 3/2, 4, 15/2

(vii) $a_n = n^2 – n + 1$

The given sequence whose, $a_n = n^2 – n + 1$

To get the first five terms of given sequence, put n = 1, 2, 3, 4, 5.

And, we get

$a_1 = 1^2 – 1 + 1 = 1$

$a_2 = 2^2 – 2 + 1 = 3$

$a_3 = 3^2 – 3 + 1 = 7$

$a_4 = 4^2 – 4 + 1 = 13$

$a_5 = 5^2 – 5 + 1 = 21$

∴ the required first five terms of the sequence are 1, 3, 7, 13, 21.

(viii) $a_n = 2n^2 – 3n + 1$

The given sequence whose $a_n = 2n^2 – 3n + 1$

To get the first five terms of the sequence, put n = 1, 2, 3, 4, 5.

And, we get

$a_1 = 2.1^2 – 3.1 + 1 = 2 – 3 + 1 = 0$

$a_2 = 2.2^2 – 3.2 + 1 = 8 – 6 + 1 = 3$

$a_3 = 2.3^2 – 3.3 + 1 = 18 – 9 + 1 = 10$

$a_4 = 2.4^2 – 3.4 + 1 = 32 – 12 + 1 = 21$

$a_5 = 2.5^2 – 3.5 + 1 = 50 – 15 + 1 = 36$

∴ the required first five terms of the sequence are 0, 3, 10, 21, 36.

(ix) $a_n = (2n – 3)/ 6$

Given sequence whose,

$$a_n = \frac{2n - 3}{6}$$

To get the first five terms of the sequence we put n = 1, 2, 3, 4, 5.

And, we get

$$a_1 = \frac{2.1-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$$

$$a_2 = \frac{2.2-3}{6} = \frac{4-3}{6} = \frac{1}{6}$$

$$a_3 = \frac{2.3-3}{6} = \frac{6-3}{6} = \frac{3}{6} = 1/2$$

$$a_4 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$$

$$a_5 = \frac{2.5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$$

∴ the required first five terms of the sequence are -1/6, 1/6, 1/2, 5/6 and 7/6

Exercise 9.2 Page No: 9.8

1. Show that the sequence defined by a_n = 5n – 7 is an A.P., find its common difference.

Solution:

Given, a_n = 5n – 7

Now putting n = 1, 2, 3, 4 we get,

a_1 = 5(1) – 7 = 5 – 7 = -2

a_2 = 5(2) – 7 = 10 – 7 = 3

a_3 = 5(3) – 7 = 15 – 7 = 8

a_4 = 5(4) – 7 = 20 – 7 = 13

We can see that,

$a_2 - a_1$ = 3 – (-2) = 5

$a_3 - a_2$ = 8 – (3) = 5

$a_4 – a_3 = 13 – (8) = 5$

Since the difference between the terms is common, we can conclude that the given sequence defined by $a_n = 5n – 7$ is an A.P with common difference 5.

2. Show that the sequence defined by $a_n = 3n^2 – 5$ is not an A.P.

Solution:

Given, $a_n = 3n^2 – 5$

Now putting n = 1, 2, 3, 4 we get,

$a_1 = 3(1)^2 – 5= 3 – 5 = -2$

$a_2 = 3(2)^2 – 5 = 12 – 5 = 7$

$a_3 = 3(3)^2 – 5 = 27 – 5 = 22$

$a_4 = 3(4)^2 – 5 = 48 – 5 = 43$

We can see that,

$a_2 – a_1 = 7 – (-2) = 9$

$a_3 – a_2 = 22 – 7 = 15$

$a_4 – a_3 = 43 – 22 = 21$

Since the difference between the terms is not common and varying, we can conclude that the given sequence defined by $a_n = 3n^2 – 5$ is not an A.P.

3. The general term of a sequence is given by $a_n = -4n + 15$. Is the sequence an A.P.? If so, find its 15th term and the common difference.

Solution:

Given, $a_n = -4n + 15$

Now putting n = 1, 2, 3, 4 we get,

$a_1 = -4(1) + 15 = -4 + 15 = 11$

$a_2 = -4(2) + 15 = -8 + 15 = 7$

a_3 = -4(3) + 15 = -12 + 15 = 3

a_4 = -4(4) + 15 = -16 + 15 = -1

We can see that,

$a_2 - a_1$ = 7 – (11) = -4

$a_3 - a_2$ = 3 – 7 = -4

$a_4 - a_3$ = -1 – 3 = -4

Since the difference between the terms is common, we can conclude that the given sequence defined by a_n = -4n + 15 is an A.P with common difference of -4.

Hence, the 15th term will be

a_{15} = -4(15) + 15 = -60 + 15 = -45

Exercise 9.3 Page No: 9.11

1. For the following arithmetic progressions write the first term a and the common difference d:

(i) – 5, -1, 3, 7,…

(ii) 1/5, 3/5, 5/5, 7/5,…

(iii) 0.3, 0.55, 0.80, 1.05,…

(iv) -1.1, – 3.1, – 5.1, – 7.1,…

Solution:

We know that if a is the first term and d is the common difference, the arithmetic progression is a, a + d, a + 2d + a + 3d,….

(i) – 5, –1, 3, 7,…

Given arithmetic series is – 5, –1, 3, 7,…

c a, a + d, a + 2d + a + 3d,….

Thus, by comparing these two we get, a = – 5, a + d = 1, a + 2d = 3, a + 3d = 7

First term (a) = – 5

By subtracting second and first term, we get

(a + d) – (a) = d

-1 – (- 5) = d

4 = d

⇒ Common difference (d) = 4.

(ii) 1/5, 3/5, 5/5, 7/5, ………….

Given arithmetic series is 1/5, 3/5, 5/5, 7/5, ……………

It is seen that, it's of the form of 1/5, 2/5, 5/5, 7/5, ……….. a, a + d, a + 2d, a + 3d,

Thus, by comparing these two, we get

a = 1/5, a + d = 3/5, a + 2d = 5/5, a + 3d = 7/5

First term (a) = 1/5

By subtracting first term from second term, we get

d = (a + d)-(a)

d = 3/5 – 1/5

d = 2/5

⇒ common difference (d) = 2/5

(iii) 0.3, 0.55, 0.80, 1.05, …………

Given arithmetic series 0.3, 0.55, 0.80, 1.05, ……….

It is seen that, it's of the form of a, a + d, a + 2d, a + 3d,

Thus, by comparing we get,

a = 0.3, a + d = 0.55, a + 2d = 0.80, a + 3d = 1.05

First term (a) = 0.3.

By subtracting first term from second term. We get

d = (a + d) – (a)

d = 0.55 – 0.3

d = 0.25

⇒ Common difference (d) = 0.25

(iv) –1.1, – 3.1, – 5.1, –7.1, ……..

General series is –1.1, – 3.1, – 5.1, –7.1, ……..

It is seen that, it's of the form of a, a + d, a + 2d, a + 3d, ………..

Thus, by comparing these two, we get

a = –1.1, a + d = –3.1, a + 2d = –5.1, a + 3d = –7.1

First term (a) = –1.1

Common difference (d) = (a + d) – (a)

= -3.1 – (– 1.1)

⇒ Common difference (d) = – 2

2. Write the arithmetic progression when first term a and common difference d are as follows:

(i) a = 4, d = – 3

(ii) a = –1, d = 1/2

(iii) a = –1.5, d = – 0.5

Solution:

We know that, if first term (a) = a and common difference = d, then the arithmetic series is: a, a + d, a + 2d, a + 3d,

(i) a = 4, d = -3

Given, first term (a) = 4

Common difference (d) = -3

Then arithmetic progression is: a, a + d, a + 2d, a + 3d, ……

⇒ 4, 4 – 3, 4 + 2(-3), 4 + 3(-3), ……

⇒ 4, 1, – 2, – 5, – 8 ……..

(ii) a = -1, d = 1/2

Given, first term (a) = -1

Common difference (d) = 1/2

Then arithmetic progression is: a, a + d, a + 2d, a + 3d,

⇒ -1, -1 + 1/2, -1 + 2½, -1 + 3½, …

⇒ -1, -1/2, 0, 1/2

(iii) a = –1.5, d = – 0.5

Given First term (a) = –1.5

Common difference (d) = – 0.5

Then arithmetic progression is; a, a + d, a + 2d, a + 3d, ……

⇒ -1.5, -1.5 + (-0.5), –1.5 + 2(– 0.5), –1.5 + 3(– 0.5)

⇒ – 1.5, – 2, – 2.5, – 3, …….

3. In which of the following situations, the sequence of numbers formed will form an A.P.?

(i) The cost of digging a well for the first metre is Rs 150 and rises by Rs 20 for each succeeding metre.

(ii) The amount of air present in the cylinder when a vacuum pump removes each time 1/4 of their remaining in the cylinder.

(iii) Divya deposited Rs 1000 at compound interest at the rate of 10% per annum. The amount at the end of first year, second year, third year, …, and so on.

Solution:

(i) Given,

Cost of digging a well for the first meter (c_1) = Rs.150.

And, the cost rises by Rs.20 for each succeeding meter

Then,

Cost of digging for the second meter (c_2) = Rs.150 + Rs 20 = Rs 170

Cost of digging for the third meter (c_3) = Rs.170 + Rs 20 = Rs 210

Hence, its clearly seen that the costs of digging a well for different lengths are 150, 170, 190, 210, ….

Evidently, this series is in A·P.

With first term (a) = 150, common difference (d) = 20

(ii) Given,

Let the initial volume of air in a cylinder be V liters each time $3^{th}/4$ of air in a remaining i.e

1 -1/4

First time, the air in cylinder is V.

Second time, the air in cylinder is 3/4 V.

Third time, the air in cylinder is $(3/4)^2$ V.

Thus, series is V, 3/4 V, $(3/4)^2$ V,$(3/4)^3$ V, ….

Hence, the above series is not a A.P.

(iii) Given,

Divya deposited Rs 1000 at compound interest of 10% p.a

So, the amount at the end of first year is = 1000 + 0.1(1000) = Rs 1100

And, the amount at the end of second year is = 1100 + 0.1(1100) = Rs 1210

And, the amount at the end of third year is = 1210 + 0.1(1210) = Rs 1331

Cleary, these amounts 1100, 1210 and 1331 are not in an A.P since the difference between them is not the same.

Exercise 9.4 Page No: 9.24

1. Find:

(i) 10^{th} tent of the AP 1, 4, 7, 10….

(ii) 18^{th} term of the AP √2, 3√2, 5√2, …….

(iii) n^{th} term of the AP 13, 8, 3, -2, ……….

(iv) 10^{th} term of the AP -40, -15, 10, 35, ………….

(v) 8^{th} term of the AP 11, 104, 91, 78, ……………

(vi) 11^{th} tenor of the AP 10.0, 10.5, 11.0, 11.2, …………..

(vii) 9^{th} term of the AP 3/4, 5/4, 7/4 + 9/4, ………..

Solution:

(i) Given A.P. is 1, 4, 7, 10, ……….

First term (a) = 1

Common difference (d) = Second term – First term

= 4 – 1 = 3.

We know that, n^{th} term in an A.P = a + (n – 1)d

Then, 10^{th} term in the A.P is 1 + (10 – 1)3

$= 1 + 9 \times 3$

$= 1 + 27$

$= 28$

∴ 10^{th} term of A. P. is 28

(ii) Given A.P. is √2, 3√2, 5√2, …….

First term (a) = √2

Common difference = Second term – First term

$= 3\sqrt{2} - \sqrt{2}$

$\Rightarrow d = 2\sqrt{2}$

We know that, n^{th} term in an A. P. = a + (n – 1)d

Then, 18^{th} term of A. P. = $\sqrt{2} + (18 - 1)2\sqrt{2}$

$= \sqrt{2} + 17.2\sqrt{2}$

$= \sqrt{2}\,(1+34)$

$= 35\sqrt{2}$

∴ 18^{th} term of A. P. is $35\sqrt{2}$

(iii) Given A. P. is 13, 8, 3, – 2, …………

First term (a) = 13

Common difference (d) = Second term first term

$= 8 - 13 = -5$

We know that, n^{th} term of an A.P. $a_n = a + (n - 1)d$

$= 13 + (n - 1) - 5$

$= 13 - 5n + 5$

∴ n^{th} term of the A.P is $a_n = 18 - 5n$

(iv) Given A. P. is – 40, -15, 10, 35, ……….

First term (a) = -40

Common difference (d) = Second term – fast term

= -15 – (- 40)

= 40 – 15

= 25

We know that, n^{th} term of an A.P. $a_n = a + (n - 1)d$

Then, 10^{th} term of A. P. $a_{10} = -40 + (10 - 1)25$

= – 40 + 9.25

= – 40 + 225

= 185

∴ 10^{th} term of the A. P. is 185

(v) Given sequence is 117, 104, 91, 78, …………

First term (a) = 117

Common difference (d) = Second term – first term

= 104 – 117

= – 13

We know that, n^{th} term = a + (n – 1)d

Then, 8^{th} term = a + (8 – 1)d

= 117 + 7(-13)

= 117 – 91

= 26

∴ 8^{th} term of the A. P. is 26

(vi) Given A. P is 10.0, 10.5, 11.0, 11.5,

First term (a) = 10.0

Common difference (d) = Second term – first term

= 10.5 – 10.0 = 0.5

We know that, n^{th} term $a_n = a + (n – 1)d$

Then, 11^{th} term $a_{11} = 10.0 + (11 – 1)0.5$

= 10.0 + 10 x 0.5

= 10.0 + 5

=15.0

∴ 11^{th} term of the A. P. is 15.0

(vii) Given A. P is 3/4, 5/4, 7/4, 9/4, …………

First term (a) = 3/4

Common difference (d) = Second term – first term

= 5/4 – 3/4

= 2/4

We know that, n^{th} term $a_n = a + (n – 1)d$

Then, 9^{th} term $a_9 = a + (9 – 1)d$

$$= \frac{3}{4} + 8.\frac{2}{4}$$

$$= \frac{3}{4} + \frac{16}{4}$$

$$= \frac{19}{4}$$

∴ 9^{th} term of the A. P. is 19/4.

2.(i) Which term of the AP 3, 8, 13, …. is 248?

(ii) Which term of the AP 84, 80, 76, … is 0?

(iii) Which term of the AP 4. 9, 14, …. is 254?

(iv) Which term of the AP 21. 42, 63, 84, … is 420?

(v) Which term of the AP 121, 117. 113, … is its first negative term?

Solution:

(i) Given A.P. is 3, 8, 13, ………..

First term (a) = 3

Common difference (d) = Second term – first term

= 8 – 3

= 5

We know that, n^{th} term $(a_n) = a + (n – 1)d$

And, given n^{th} term $a_n = 248$

248 = 3+(n – 1)5

248 = -2 + 5n

5n = 250

n =250/5 = 50

∴ 50th term in the A.P is 248.

(ii) Given A. P is 84, 80, 76, …………

First term (a) = 84

Common difference (d) = a_2 – a

= 80 – 84

= – 4

We know that, nth term (a_n) = a +(n – 1)d

And, given nth term is 0

0 = 84 + (n – 1) – 4

84 = +4(n – 1)

n – 1 = 84/4 = 21

n = 21 + 1 = 22

∴ 22nd term in the A.P is 0.

(iii) Given A. P 4, 9, 14, …………

First term (a) = 4

Common difference (d) = a_2 – a_1

= 9 – 4

= 5

We know that, nth term (a_n) = a + (n – 1)d

And, given nth term is 254

4 + (n – 1)5 = 254

$(n - 1) \cdot 5 = 250$

$n - 1 = 250/5 = 50$

$n = 51$

∴ 51^{th} term in the A.P is 254.

(iv) Given A. P 21, 42, 63, 84, ………

$a = 21$, $d = a_2 - a_1$

$= 42 - 21$

$= 21$

We know that, n^{th} term $(a_n) = a + (n - 1)d$

And, given n^{th} term = 420

$21 + (n - 1)21 = 420$

$(n - 1)21 = 399$

$n - 1 = 399/21 = 19$

$n = 20$

∴ 20^{th} term is 420.

(v) Given A.P is 121, 117, 113, ………..

Fiat term (a) = 121

Common difference (d) = 117 – 121

$= -4$

We know that, n^{th} term $a_n = a + (n - 1)d$

And, for some n^{th} term is negative i.e., $a_n < 0$

$121 + (n - 1) - 4 < 0$

$121 + 4 – 4n < 0$

$125 – 4n < 0$

$4n > 125$

$n > 125/4$

$n > 31.25$

The integer which comes after 31.25 is 32.

∴ 32^{nd} term in the A.P will be the first negative term.

3.(i) Is 68 a term of the A.P. 7, 10, 13,… ?

(ii) Is 302 a term of the A.P. 3, 8, 13, …. ?

(iii) Is -150 a term of the A.P. 11, 8, 5, 2, … ?

Solutions:

(i) Given, A.P. 7, 10, 13,…

Here, $a = 7$ and $d = a_2 – a_1 = 10 – 7 = 3$

We know that, n^{th} term $a_n = a + (n – 1)d$

Required to check n^{th} term $a_n = 68$

$a + (n – 1)d = 68$

$7 + (n – 1)3 = 68$

$7 + 3n – 3 = 68$

$3n + 4 = 68$

$3n = 64$

⇒ $n = 64/3$, which is not a whole number.

Therefore, 68 is not a term in the A.P.

(ii) Given, A.P. 3, 8, 13,…

Here, a = 3 and d = $a_2 - a_1$ = 8 – 3 = 5

We know that, n^{th} term $a_n = a + (n – 1)d$

Required to check n^{th} term a_n = 302

a + (n – 1)d = 302

3 + (n – 1)5 = 302

3 + 5n – 5 = 302

5n – 2 = 302

5n = 304

⇒ n = 304/5, which is not a whole number.

Therefore, 302 is not a term in the A.P.

(iii) Given, A.P. 11, 8, 5, 2, …

Here, a = 11 and d = $a_2 - a_1$ = 8 – 11 = -3

We know that, n^{th} term $a_n = a + (n – 1)d$

Required to check n^{th} term a_n = -150

a + (n – 1)d = -150

11 + (n – 1)(-3) = -150

11 – 3n + 3 = -150

3n = 150 + 14

3n = 164

⇒ n = 164/3, which is not a whole number.

Therefore, -150 is not a term in the A.P.

4. How many terms are there in the A.P.?

(i) 7, 10, 13, ….., 43

(ii) -1, -5/6, -2/3, -1/2, … , 10/3

(iii) 7, 13, 19, …, 205

(iv) 18, 15½, 13, …., -47

Solution:

(i) Given, A.P. 7, 10, 13, ….., 43

Here, a = 7 and d = $a_2 - a_1$ = 10 – 7 = 3

We know that, n^{th} term $a_n = a + (n - 1)d$

And, given n^{th} term $a_n = 43$

a + (n – 1)d = 43

7 + (n – 1)(3) = 43

7 + 3n – 3 = 43

3n = 43 – 4

3n = 39

⇒ n = 13

Therefore, there are 13 terms in the given A.P.

(ii) Given, A.P. -1, -5/6, -2/3, -1/2, … , 10/3

Here, a = -1 and d = $a_2 - a_1$ = -5/6 – (-1) = 1/6

We know that, n^{th} term $a_n = a + (n - 1)d$

And, given n^{th} term $a_n = 10/3$

a + (n – 1)d = 10/3

$-1 + (n – 1)(1/6) = 10/3$

$-1 + n/6 – 1/6 = 10/3$

$n/6 = 10/3 + 1 + 1/6$

$n/6 = (20 + 6 + 1)/6$

$n = (20 + 6 + 1)$

$\Rightarrow n = 27$

Therefore, there are 27 terms in the given A.P.

(iii) Given, A.P. 7, 13, 19, …, 205

Here, a = 7 and $d = a_2 – a_1 = 13 – 7 = 6$

We know that, n^{th} term $a_n = a + (n – 1)d$

And, given n^{th} term $a_n = 205$

$a + (n – 1)d = 205$

$7 + (n – 1)(6) = 205$

$7 + 6n – 6 = 205$

$6n = 205 – 1$

$n = 204/6$

$\Rightarrow n = 34$

Therefore, there are 34 terms in the given A.P.

(iv) Given, A.P. 18, 15½, 13, …., -47

Here, a = 7 and $d = a_2 – a_1 = 15½ – 18 = 5/2$

We know that, n^{th} term $a_n = a + (n – 1)d$

And, given n^{th} term $a_n = -47$

a + (n – 1)d = 43

18 + (n – 1)(-5/2) = -47

18 – 5n/2 + 5/2 = -47

36 – 5n + 5 = -94

5n = 94 + 36 + 5

5n = 135

⇒ n = 27

Therefore, there are 27 terms in the given A.P.

5. The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.

Solution:

Given,

a = 5 and d = 3

We know that, n^{th} term a_n = a + (n – 1)d

So, for the given A.P. a_n = 5 + (n – 1)3 = 3n + 2

Also given, last term = 80

⇒ 3n + 2 = 80

3n = 78

n = 78/3 = 26

Therefore, there are 26 terms in the A.P.

6. The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively, find the 40^{th} term.

Solution:

Given,

$a_6 = 19$ and $a_{17} = 41$

We know that, n^{th} term $a_n = a + (n – 1)d$

So,

$a_6 = a + (6-1)d$

⇒ a + 5d = 19 …… (i)

Similarity,

$a_{17} = a + (17 – 1)d$

⇒ a + 16d = 41 …… (ii)

Solving (i) and (ii),

(ii) – (i) ⇒

a + 16d – (a + 5d) = 41 – 19

11d = 22

⇒ d = 2

Using d in (i), we get

a + 5(2) = 19

a = 19 – 10 = 9

Now, the 40^{th} term is given by $a_{40} = 9 + (40 – 1)2 = 9 + 78 = 87$

Therefore the 40^{th} term is 87.

7. If 9^{th} term of an A.P. is zero, prove its 29^{th} term is double the 19^{th} term.

Solution:

Given,

$a_9 = 0$

We know that, n^{th} term $a_n = a + (n – 1)d$

So, $a + (9 – 1)d = 0 \Rightarrow a + 8d = 0$ ……(i)

Now,

29^{th} term is given by $a_{29} = a + (29 – 1)d$

$\Rightarrow a_{29} = a + 28d$

And, $a_{29} = (a + 8d) + 20d$ [using (i)]

$\Rightarrow a_{29} = 20d$ ….. (ii)

Similarly, 19^{th} term is given by $a_{19} = a + (19 – 1)d$

$\Rightarrow a_{19} = a + 18d$

And, $a_{19} = (a + 8d) + 10d$ [using (i)]

$\Rightarrow a_{19} = 10d$ …..(iii)

On comparing (ii) and (iii), it's clearly seen that

$a_{29} = 2(a_{19})$

Therefore, 29^{th} term is double the 19^{th} term.

8. If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero.

Solution:

Given,

10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term.

We know that, n^{th} term $a_n = a + (n – 1)d$

$\Rightarrow 10(a_{10}) = 15(a_{15})$

$10(a + (10 – 1)d) = 15(a + (15 – 1)d)$

$10(a + 9d) = 15(a + 14d)$

$10a + 90d = 15a + 210d$

$5a + 120d = 0$

$5(a + 24d) = 0$

$a + 24d = 0$

$a + (25 - 1)d = 0$

$\Rightarrow a_{25} = 0$

Therefore, the 25th term of the A.P. is zero.

9. The 10th and 18th terms of an A.P. are 41 and 73 respectively. Find 26th term.

Solution:

Given,

$A_{10} = 41$ and $a_{18} = 73$

We know that, nth term $a_n = a + (n - 1)d$

So,

$a_{10} = a + (10 - 1)d$

$\Rightarrow a + 9d = 41$ …… (i)

Similarity,

$a_{18} = a + (18 - 1)d$

$\Rightarrow a + 17d = 73$ …… (ii)

Solving (i) and (ii),

(ii) – (i) $\Rightarrow$

$a + 17d - (a + 9d) = 73 - 41$

$8d = 32$

⇒ $d = 4$

Using d in (i), we get

$a + 9(4) = 41$

$a = 41 – 36 = 5$

Now, the 26th term is given by $a_{26} = 5 + (26 – 1)4 = 5 + 100 = 105$

Therefore the 26th term is 105.

10. In a certain A.P. the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

Solution:

Given,

24th term is twice the 10th term.

We know that, nth term $a_n = a + (n – 1)d$

⇒ $a_{24} = 2(a_{10})$

$a + (24 – 1)d = 2(a + (10 – 1)d)$

$a + 23d = 2(a + 9d)$

$a + 23d = 2a + 18d$

$a = 5d$ …. (1)

Now, the 72nd term can be expressed as

$a_{72} = a + (72 – 1)d$

$= a + 71d$

$= a + 5d + 66d$

$= a + a + 66d$ [using (1)]

$= 2(a + 33d)$

$= 2(a + (34 – 1)d)$

$= 2(a_{34})$

$\Rightarrow a_{72} = 2(a_{34})$

Hence, the 72nd term is twice the 34th term of the given A.P.

11. The 26th, 11th and the last term of an A.P. are 0, 3 and -1/5, respectively. Find the common difference and the number of terms.

Solution:

Given,

$a_{26} = 0$, $a_{11} = 3$ and a_n (last term) = -1/5 of an A.P.

We know that, nth term $a_n = a + (n – 1)d$

Then,

$a_{26} = a + (26 – 1)d$

$\Rightarrow a + 25d = 0$ …..(1)

And,

$a_{11} = a + (11 – 1)d$

$\Rightarrow a + 10d = 3$ …… (2)

Solving (1) and (2),

(1) – (2) $\Rightarrow$

$a + 25d – (a + 10d) = 0 – 3$

$15d = -3$

$\Rightarrow d = -1/5$

Using d in (1), we get

$a + 25(-1/5) = 0$

$a = 5$

Now, given that the last term $a_n = -1/5$

⇒ $5 + (n – 1)(-1/5) = -1/5$

$5 + -n/5 + 1/5 = -1/5$

$25 – n + 1 = -1$

$n = 27$

Therefore, the A.P has 27 terms and its common difference is -1/5.

12. If the n^{th} term of the A.P. 9, 7, 5, …. is same as the n^{th} term of the A.P. 15, 12, 9, … find n.

Solution:

Given,

$A.P_1$ = 9, 7, 5, …. and $A.P_2$ = 15, 12, 9, …

And, we know that, n^{th} term $a_n = a + (n – 1)d$

For $A.P_1$,

a = 9, d = Second term – first term = 9 – 7 = -2

And, its n^{th} term $a_n = 9 + (n – 1)(-2) = 9 – 2n + 2$

$a_n = 11 – 2n$ …..(i)

Similarly, for $A.P_2$

a = 15, d = Second term – first term = 12 – 15 = -3

And, its n^{th} term $a_n = 15 + (n – 1)(-3) = 15 – 3n + 3$

$a_n = 18 – 3n$ …..(ii)

According to the question, its given that

n^{th} term of the $A.P_1$ = n^{th} term of the $A.P_2$

⇒ 11 – 2n = 18 – 3n

n = 7

Therefore, the 7th term of the both the A.Ps are equal.

13. Find the 12th term from the end of the following arithmetic progressions:

(i) 3, 5, 7, 9, …. 201

(ii) 3,8,13, … ,253

(iii) 1, 4, 7, 10, … ,88

Solution:

In order the find the 12th term for the end of an A.P. which has n terms, its done by simply finding the ((n -12) + 1)th of the A.P

And we know, nth term $a_n = a + (n – 1)d$

(i) Given A.P = 3, 5, 7, 9, …. 201

Here, a = 3 and d = (5 – 3) = 2

Now, find the number of terms when the last term is known i.e, 201

$a_n = 3 + (n – 1)2 = 201$

3 + 2n – 2 = 201

2n = 200

n = 100

Hence, the A.P has 100 terms.

So, the 12th term from the end is same as (100 – 12 + 1)th of the A.P which is the 89th term.

⇒ $a_{89} = 3 + (89 – 1)2$

= 3 + 88(2)

= 3 + 176

= 179

Therefore, the 12th term from the end of the A.P is 179.

(ii) Given A.P = 3,8,13, … ,253

Here, a = 3 and d = (8 – 3) = 5

Now, find the number of terms when the last term is known i.e, 253

$a_n = 3 + (n - 1)5 = 253$

3 + 5n – 5 = 253

5n = 253 + 2 = 255

n = 255/5

n = 51

Hence, the A.P has 51 terms.

So, the 12th term from the end is same as (51 – 12 + 1)th of the A.P which is the 40th term.

⇒ $a_{40} = 3 + (40 - 1)5$

= 3 + 39(5)

= 3 + 195

= 198

Therefore, the 12th term from the end of the A.P is 198.

(iii) Given A.P = 1, 4, 7, 10, … ,88

Here, a = 1 and d = (4 – 1) = 3

Now, find the number of terms when the last term is known i.e, 88

$a_n = 1 + (n - 1)3 = 88$

1 + 3n – 3 = 88

3n = 90

n = 30

Hence, the A.P has 30 terms.

So, the 12th term from the end is same as (30 – 12 + 1)th of the A.P which is the 19th term.

⇒ a_{89} = 1 + (19 – 1)3

= 1 + 18(3)

= 1 + 54

= 55

Therefore, the 12th term from the end of the A.P is 55.

14. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term and the common difference.

Solution:

Let's consider the first term and the common difference of the A.P to be a and d respectively.

Then, we know that a_n = a + (n – 1)d

Given conditions,

4th term of an A.P. is three times the first

Expressing this by equation we have,

⇒ a_4 = 3(a)

a + (4 – 1)d = 3a

3d = 2a ⇒ a = 3d/2…….(i)

And,

7th term exceeds twice the third term by 1

⇒ a_7 = 2(a_3) + 1

$a + (7 – 1)d = 2(a + (3–1)d) + 1$

$a + 6d = 2a + 4d + 1$

$a – 2d + 1 = 0$ ….. (ii)

Using (i) in (ii), we have

$3d/2 – 2d + 1 = 0$

$3d – 4d + 2 = 0$

$d = 2$

So, putting d = 2 in (i), we get a

⇒ $a = 3$

Therefore, the first term is 3 and the common difference is 2.

15. Find the second term and the n^{th} term of an A.P. whose 6^{th} term is 12 and the 8^{th} term is 22.

Solution:

Given, in an A.P

$a_6 = 12$ and $a_8 = 22$

We know that $a_n = a + (n – 1)d$

So,

$a_6 = a + (6-1)d = a + 5d = 12$ …. (i)

And,

$a_8 = a + (8-1)d = a + 7d = 22$ ……. (ii)

Solving (i) and (ii), we have

(ii) – (i) ⇒

$a + 7d – (a + 5d) = 22 – 12$

2d = 10

d = 5

Putting d in (i) we get,

a + 5(5) = 12

a = 12 – 25

a = -13

Thus, for the A.P: a = -13 and d = 5

So, the n^{th} term is given by $a_n = a + (n-1)d$

$a_n = -13 + (n-1)5 = -13 + 5n – 5$

⇒ $a_n = 5n – 18$

Hence, the second term is given by $a_2 = 5(2) – 18 = 10 – 18 = -8$

16. How many numbers of two digit are divisible by 3?

Solution:

The first 2 digit number divisible by 3 is 12. And, the last 2 digit number divisible by 3 is 99.

So, this forms an A.P.

12, 15, 18, 21, …. , 99

Where, a = 12 and d = 3

Finding the number of terms in this A.P

⇒ 99 = 12 + (n-1)3

99 = 12 + 3n – 3

90 = 3n

n = 90/3 = 30

Therefore, there are 30 two digit numbers divisible by 3.

17. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.

Solution:

Given, an A.P of 60 terms

And, $a = 7$ and $a_{60} = 125$

We know that $a_n = a + (n – 1)d$

$\Rightarrow a_{60} = 7 + (60 – 1)d = 125$

$7 + 59d = 125$

$59d = 118$

$d = 2$

So, the 32nd term is given by

$a_{32} = 7 + (32 -1)2 = 7 + 62 = 69$

$\Rightarrow a_{32} = 69$

18. The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.

Solution:

Given, in an A.P

The sum of 4th and 8th terms of an A.P. is 24

$\Rightarrow a_4 + a_8 = 24$

And, we know that $a_n = a + (n – 1)d[a + (4-1)d] + [a + (8-1)d] = 24$

$2a + 10d = 24$

$a + 5d = 12$ …. (i)

Also given that,

the sum of the 6th and 10th terms is 34

⇒ $a_6 + a_{10} = 34[a + 5d] + [a + 9d] = 34$

2a + 14d = 34

a + 7d = 17 …… (ii)

Subtracting (i) form (ii), we have

a + 7d – (a + 5d) = 17 – 12

2d = 5

d = 5/2

Using d in (i) we get,

a + 5(5/2) = 12

a = 12 – 25/2

a = -1/2

Therefore, the first term is -1/2 and the common difference is 5/2.

19. The first term of an A.P. is 5 and its 100th term is -292. Find the 50th term of this A.P.

Solution:

Given, an A.P whose

a = 5 and a_{100} = -292

We know that $a_n = a + (n – 1)d$

a100 = 5 + 99d = -292

99d = -297

d = -3

Hence, the 50th term is

$a_{50} = a + 49d = 5 + 49(-3) = 5 – 147 = -142$

20. Find $a_{30} – a_{20}$ for the A.P.

(i) -9, -14, -19, -24 (ii) a, a+d, a+2d, a+3d, ……

Solution:

We know that $a_n = a + (n – 1)d$

So, $a_{30} – a_{20} = (a + 29d) – (a + 19) =10d$

(i) Given A.P. -9, -14, -19, -24

Here, a = -9 and d = -14 – (-9) = = -14 + 9 = -5

So, $a_{30} – a_{20} = 10d$

= 10(-5)

= -50

(ii) Given A.P. a, a+d, a+2d, a+3d, ……

So, $a_{30} – a_{20} = (a + 29d) – (a + 19d)$

=10d

21. Write the expression $a_n – a_k$ for the A.P. a, a+d, a+2d, …..

Hence, find the common difference of the A.P. for which

(i) 11th term is 5 and 13th term is 79.

(ii) $a_{10} – a_5 = 200$

(iii) 20th term is 10 more than the 18th term.

Solution:

Given A.P. a, a+d, a+2d, …..

So, $a_n = a + (n-1)d = a + nd – d$

And, $a_k = a + (k-1)d = a + kd – d$

$a_n – a_k = (a + nd – d) – (a + kd – d)$

$= (n – k)d$

(i) Given 11^{th} term is 5 and 13^{th} term is 79,

Here n = 13 and k = 11,

$a_{13} – a_{11} = (13 – 11)d = 2d$

⇒ 79 – 5 = 2d

d = 74/2 = 37

(ii) Given, $a_{10} – a_5 = 200$

⇒ (10 – 5)d = 200

5d = 200

d = 40

(iii) Given, 20^{th} term is 10 more than the 18^{th} term.

⇒ $a_{20} – a_{18} = 10$

(20 – 18)d = 10

2d = 10

d = 5

22. Find n if the given value of x is the n^{th} term of the given A.P.

(i) 25, 50, 75, 100, ; x = 1000 (ii) -1, -3, -5, -7, …; x = -151

(iii) 5½, 11, 16½, 22, ….; x = 550 (iv) 1, 21/11, 31/11, 41/11, …; x = 171/11

Solution:

(i) Given A.P. 25, 50, 75, 100, …… ,1000

Here, $a = 25$ $d = 50 – 25 = 25$

Last term (n^{th} term) = 1000

We know that $a_n = a + (n – 1)d$

⇒ 1000 = 25 + (n-1)25

1000 = 25 + 25n – 25

n = 1000/25

n = 40

(ii) Given A.P. -1, -3, -5, -7, …., -151

Here, $a = -1$ $d = -3 – (-1) = -2$

Last term (n^{th} term) = -151

We know that $a_n = a + (n – 1)d$

⇒ -151 = -1 + (n-1)(-2)

-151 = -1 – 2n + 2

n = 152/2

n = 76

(iii) Given A.P. 5½, 11, 16½, 22, … , 550

Here, a = 5½ d = 11 – (5½) = 5½ = 11/2

Last term (n^{th} term) = 550

We know that $a_n = a + (n – 1)d$

⇒ 550 = 5½ + (n-1)(11/2)

550 x 2 = 11+ 11n – 11

1100 = 11n

n = 100

(iv) Given A.P. 1, 21/11, 31/11, 41/11, 171/11

Here, a = 1 d = 21/11 – 1 = 10/11

Last term (n^{th} term) = 171/11

We know that a_n = a + (n – 1)d

⇒ 171/11 = 1 + (n-1)10/11

171 = 11 + 10n – 10

n = 170/10

n = 17

23. The eighth term of an A.P is half of its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15^{th} term.

Solution:

Given, an A.P in which,

a_8 = 1/2(a_2)

a_{11} = 1/3(a_4) + 1

We know that a_n = a + (n – 1)d

⇒ a_8 = 1/2(a_2)

a + 7d = 1/2(a + d)

2a + 14d = a + d

a + 13d = 0 …… (i)

And, a_{11} = 1/3(a_4) + 1

a + 10d = 1/3(a + 3d) + 1

3a + 30d = a + 3d + 3

2a + 27d = 3 …… (ii)

Solving (i) and (ii), by (ii) – 2x(i) ⇒

2a + 27d – 2(a + 13d) = 3 – 0

d = 3

Putting d in (i) we get,

a + 13(3) = 0

a = -39

Thus, the 15th term a_{15} = -39 + 14(3) = -39 + 42 = 3

24. Find the arithmetic progression whose third term is 16 and the seventh term exceeds its fifth term by 12.

Solution:

Given, in an A.P

a_3 = 16 and a_7 = a_5 + 12

We know that a_n = a + (n – 1)d

⇒ a + 2d = 16…… (i)

And,

a + 6d = a + 4d + 12

2d = 12

⇒ d = 6

Using d in (i), we have

a + 2(6) = 16

a = 16 – 12 = 4

Hence, the A.P is 4, 10, 16, 22, …….

25. The 7th term of an A.P. is 32 and its 13th term is 62. Find the A.P.

Solution:

Given,

$a_7 = 32$ and $a_{13} = 62$

From $a_n – a_k = (a + nd – d) – (a + kd – d)$

$= (n – k)d$

$a_{13} – a_7 = (13 – 7)d = 62 – 32 = 30$

$6d = 30$

$d = 5$

Now,

$a_7 = a + (7 – 1)5 = 32$

$a + 30 = 32$

$a = 2$

Hence, the A.P is 2, 7, 12, 17, ……

26. Which term of the A.P. 3, 10, 17, …. will be 84 more than its 13th term ?

Solution:

Given, A.P. 3, 10, 17, ….

Here, $a = 3$ and $d = 10 – 3 = 7$

According the question,

$a_n = a_{13} + 84$

Using $a_n = a + (n – 1)d$,

3 + (n – 1)7 = 3 + (13 – 1)7 + 84

3 + 7n – 7 = 3 + 84 + 84

7n = 168 + 7

n = 175/7

n = 25

Therefore, it the 25^{th} term which is 84 more than its 13^{th} term.

27. Two arithmetic progressions have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000^{th} terms?

Solution:

Let the two A.Ps be $A.P_1$ and $A.P_2$

For $A.P_1$ the first term = a and the common difference = d

And for $A.P_2$ the first term = b and the common difference = d

So, from the question we have

$a_{100} – b_{100} = 100$

(a + 99d) – (b + 99d) = 100

a – b = 100

Now, the difference between their 1000^{th} terms is,

(a + 999d) – (b + 999d) = a – b = 100

Therefore, the difference between their 1000^{th} terms is also 100.

Exercise 9.5 Page No: 9.30

1. Find the value of x for which (8x + 4), (6x – 2) and (2x + 7) are in A.P.

Solution:

Given,

(8x + 4), (6x – 2) and (2x + 7) are in A.P.

So, the common difference between the consecutive terms should be the same.

$(6x - 2) - (8x + 4) = (2x + 7) - (6x - 2)$

$\Rightarrow 6x - 2 - 8x - 4 = 2x + 7 - 6x + 2$

$\Rightarrow -2x - 6 = -4x + 9$

$\Rightarrow -2x + 4x = 9 + 6$

$\Rightarrow 2x = 15$

Therefore, $x = 15/2$

2. If x + 1, 3x and 4x + 2 are in A.P., find the value of x.

Solution:

Given,

x + 1, 3x and 4x + 2 are in A.P.

So, the common difference between the consecutive terms should be the same.

$3x - x - 1 = 4x + 2 - 3x$

$\Rightarrow 2x - 1 = x + 2$

$\Rightarrow 2x - x = 2 + 1$

$\Rightarrow x = 3$

Therefore, x = 3

3. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P.

Solution:

If $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ have to be in A.P. then, It should satisfy the condition,

2b = a + c [for a, b, c are in A.P]

Thus,

$2(a^2 + b^2) = (a - b)^2 + (a + b)^2$

$2(a^2 + b^2) = a^2 + b^2 - 2ab + a^2 + b^2 + 2ab$

$2(a^2 + b^2) = 2a^2 + 2b^2 = 2(a^2 + b^2)$

LHS = RHS

Hence proved.

4. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

Solution:

Let's consider the three terms of the A.P. to be a – d, a, a + d

so, the sum of three terms = 21

$\Rightarrow a - d + a + a + d = 21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$

And, product of the first and 3rd = 2nd term + 6

$\Rightarrow (a - d)(a + d) = a + 6$

$a^2 - d^2 = a + 6$

$\Rightarrow (7)^2 - d^2 = 7 + 6$

$\Rightarrow 49 - d^2 = 13$

$\Rightarrow d^2 = 49 - 13 = 36$

$\Rightarrow d^2 = (6)^2$

⇒ d = 6

Hence, the terms are 7 – 6, 7, 7 + 6 ⇒ 1, 7, 13

5. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Solution:

Let the three numbers of the A.P. be a – d, a, a + d

From the question,

Sum of these numbers = 27

a – d + a + a + d = 27

⇒ 3a = 27

a = 27/3 = 9

Now, product of these numbers = 648

(a – d)(a)(a + d) = 648

$a(a^2 – d^2) = 648$

$a^2 – 648/a = d^2$

$9^2 – (648/9) = d^2$

$9^3 – 648 = 9d^2$

$729 – 648 = 9d^2$

$81 = 9d^2$

$d^2 = 9$

d = 3 or -3

Hence, the terms are 9-3, 9 and 9+3 ⇒ 6, 9, 12 or 12, 9, 6 (for d = -3)

6. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.

Solution:

Let's consider the four terms of the A.P. to be (a – 3d), (a – d), (a + d) and (a + 3d).

From the question,

Sum of these terms = 50

⇒ (a – 3d) + (a – d) + (a + d) + (a + 3d) = 50

⇒ a – 3d + a – d + a + d + a – 3d= 50

⇒ 4a = 50

⇒ a = 50/4 = 25/2

And, also given that the greatest number = 4 x least number

⇒ a + 3d = 4 (a – 3d)

⇒ a + 3d = 4a – 12d

⇒ 4a – a = 3d + 12d

⇒3a = 15d

⇒a = 5d

Using the value of a in the above equation, we have

⇒25/2 = 5d

⇒ d = 5/2

So, the terms will be:

(a – 3d) = (25/2 – 3(5/2)), (a – d) = (25/2 – 5/2), (25/2 + 5/2) and (25/2 + 3(5/2)).

⇒ 5, 10, 15, 20

Exercise 9.6 Page No: 9.50

1. Find the sum of the following arithmetic progressions:

(i) 50, 46, 42, … to 10 terms

(ii) 1, 3, 5, 7, … to 12 terms

(iii) 3, 9/2, 6, 15/2, … to 25 terms

(iv) 41, 36, 31, … to 12 terms

(v) a + b, a – b, a – 3b, … to 22 terms

(vi) $(x - y)^2$, $(x^2 + y^2)$, $(x + y)^2$, to 22 tams

$$\text{(vii)}\ \frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \text{to n terms}$$

(viii) – 26, – 24, – 22, …. to 36 terms

Solution:

In an A.P if the first term = a, common difference = d, and if there are n terms.

Then, sum of n terms is given by:

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

(i) Given A.P.is 50, 46, 42 to 10 term.

First term (a) = 50

Common difference (d) = 46 – 50 = – 4

n^{th} term (n) = 10

$$\text{Then } S_{10} = \frac{10}{2}\{2.50 + (10 - 1) - 4\}$$

= 5{100 – 9.4}

= 5{100 – 36}

= 5 × 64

∴ S_{10} = 320

(ii) Given A.P is, 1, 3, 5, 7, …..to 12 terms.

First term (a) = 1

Common difference (d) = 3 – 1 = 2

n^{th} term (n) = 12

$$\text{Sum of n}^{\text{th}} \text{ terms } S_{12} = \frac{12}{2} \times \{2.1 + (12-1).2\}$$

= 6 × {2 + 22} = 6.24

∴ S_{12} = 144

(iii) Given A.P. is 3, 9/2, 6, 15/2, … to 25 terms

First term (a) = 3

Common difference (d) = 9/2 – 3 = 3/2

Sum of n terms S_n, given n = 25

$$S_{25} = \frac{n}{2}(2a + (n-1)d)$$

$$S_{25} = \frac{25}{2}(2.3 + 25 - 1) \times \frac{3}{2}$$

$$= \frac{25}{2}\left(6 + 24.\frac{3}{2}\right)$$

$$= \frac{25}{2}\ (6 + 36)$$

$$= \frac{25}{2}\ (42)$$

$\therefore S_{25} = 525$

(iv) Given expression is 41, 36, 31, ….. to 12 terms.

First term (a) = 41

Common difference (d) = 36 – 41 = -5

Sum of n terms S_n, given n = 12

$$S_{12} = \frac{n}{2}(2a + (n-1)d)$$

$$S_{12} = \frac{12}{6}(2.41 + 12 - 1) \times -5$$

$$= 6\big(82 + 11 \times (-5)\big)$$

$$= 6 \times 27$$

$$= 162$$

$\therefore\ S_{12} = 162.$

(v) a + b, a – b, a – 3b, ….. to 22 terms

First term (a) = a + b

Common difference (d) = a – b – a – b = -2b

Sum of n terms S_n = n/2{2a(n – 1). d}

Here n = 22

S_{22} = 22/2{2.(a + b) + (22 – 1). -2b}

= 11{2(a + b) – 22b)

= 11{2a – 20b}

= 22a – 440b

∴S_{22} = 22a – 440b

(vi) $(x – y)^2$,$(x^2 + y^2)$, $(x + y)^2$,… to n terms

First term (a) = $(x – y)^2$

Common difference (d) = $x^2 + y^2 – (x – y)^2$

= $x^2 + y^2 – (x^2 + y^2 – 2xy)$

= $x^2 + y^2 – x^2 + y^2 + 2xy$

= 2xy

Sum of n^{th} terms S_n = n/2{2a + (n – 1). d}

= n/2{2$(x – y)^2$ + (n – 1). 2xy}

= n{$(x – y)^2$ + (n – 1)xy}

∴ S_n = n{$(x — y)^2$ + (n — 1). xy)

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots\ldots$ to n terms

First term (a) $= \frac{x-y}{x+y}$

Common difference (d) $= \frac{3x-2y}{x+y} - \frac{x-y}{x+y}$

$= \frac{3x-2y-x-y}{x+y}$

$= \frac{2x-y}{x+y}$

Sum of n terms $S_n = \frac{n}{2}(2a + (n-1).d)$

$= \frac{n}{2}\left[2.\frac{x-y}{x+y} = +(n-1).2 - \frac{2x-y}{x+y}\right\}$

$= \frac{n}{2(x+y)}\{2(x-y) + (n-1)(2x-y)\}$

$= \frac{n}{2(x+y)}, \{2x - 2y + 2nx - ny - 2x + y\}$

$= \frac{n}{2(x+y)}\{n(2x - y) - y\}$

$\therefore S_n = \frac{n}{2(x+y)}\{n(2x - y) - y\}$

(viii) Given expression -26, – 24. -22, to 36 terms

First term (a) = -26

Common difference (d) = -24 – (-26)

= -24 + 26 = 2

Sum of n terms, S_n = n/2{2a + (n – 1)d) for n = 36

S_n = 36/2{2(-26) + (36 – 1)2}

= 18[-52 + 70]

= 18×18

= 324

∴ S_n = 324

2. Find the sum to n terms of the A.P. 5, 2, –1, – 4, –7, …

Solution:

Given AP is 5, 2, -1, -4, -7, …..

Here, a = 5, d = 2 – 5 = -3

We know that,

S_n = n/2{2a + (n – 1)d}

= n/2{2.5 + (n – 1) – 3}

= n/2{10 – 3(n – 1)}

= n/2{13 – 3n)

∴ S_n = n/2(13 – 3n)

3. Find the sum of n terms of an A.P. whose the terms is given by a_n = 5 – 6n.

Solution:

Given nth term of the A.P as a_n = 5 – 6n

Put n = 1, we get

a_1 = 5 – 6.1 = -1

So, first term (a) = -1

Last term (a_n) = 5 – 6n = 1

Then, S_n = n/2(-1 + 5 – 6n)

= n/2(4 – 6n) = n(2 – 3n)

4. Find the sum of last ten terms of the A.P. : 8, 10, 12, 14, .. , 126

Solution:

Given A.P. 8, 10, 12, 14, .. , 126

Here, a = 8 , d = 10 – 8 = 2

We know that, a_n = a + (n – 1)d

So, to find the number of terms

126 = 8 + (n – 1)2

126 = 8 + 2n – 2

2n = 120

n = 60

Next, let's find the 51^{st} term

a_{51} = 8 + 50(2) = 108

So, the sum of last ten terms is the sum of a_{51} + a_{52} + a_{53} + ……. + a_{60}

Here, n = 10, a = 108 and l = 126

S = 10/2 [108 + 126]

= 5(234)

= 1170

Hence, the sum of last ten terms of the A.P is 1170.

5. Find the sum of first 15 terms of each of the following sequences having n^{th} term as:

(i) $a_n = 3 + 4n$

(ii) $b_n = 5 + 2n$

(iii) $x_n = 6 – n$

(iv) $y_n = 9 – 5n$

Solution:

(i) Given an A.P. whose n^{th} term is given by $a_n = 3 + 4n$

To find the sum of the n terms of the given A.P., using the formula,

$S_n = n(a + l)/ 2$

Where, a = the first term l = the last term.

Putting n = 1 in the given a_n, we get

a = 3 + 4(1) = 3 + 4 = 7

For the last term (l), here n = 15

$a_{15} = 3 + 4(15) = 63$

So, $S_n = 15(7 + 63)/2$

= 15 x 35

= 525

Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = 525$

(ii) Given an A.P. whose n^{th} term is given by $b_n = 5 + 2n$

To find the sum of the n terms of the given A.P., using the formula,

$S_n = n(a + l)/ 2$

Where, a = the first term l = the last term.

Putting n = 1 in the given b_n, we get

a = 5 + 2(1) = 5 + 2 = 7

For the last term (l), here n = 15

a_{15} = 5 + 2(15) = 35

So, S_n = 15(7 + 35)/2

= 15 x 21

= 315

Therefore, the sum of the 15 terms of the given A.P. is S_{15} = 315

(iii) Given an A.P. whose n^{th} term is given by x_n = 6 – n

To find the sum of the n terms of the given A.P., using the formula

S_n = n(a + l)/ 2

Where, a = the first term l = the last term.

Putting n = 1 in the given x_n, we get

a = 6 – 1 = 5

For the last term (l), here n = 15

a_{15} = 6 – 15 = -9

So, S_n = 15(5 – 9)/2

= 15 x (-2)

= -30

Therefore, the sum of the 15 terms of the given A.P. is S_{15} = -30

(iv) Given an A.P. whose n^{th} term is given by y_n = 9 – 5n

To find the sum of the n terms of the given A.P., using the formula,

$S_n = n(a + l)/ 2$

Where, a = the first term l = the last term.

Putting n = 1 in the given y_n, we get

a = 9 – 5(1) = 9 – 5 = 4

For the last term (l), here n = 15

a_{15} = 9 – 5(15) = -66

So, S_n = 15(4 – 66)/2

= 15 x (-31)

= -465

Therefore, the sum of the 15 terms of the given A.P. is S_{15} = -465

6. Find the sum of first 20 terms the sequence whose n^{th} term is a_n = An + B.

Solution:

Given an A.P. whose nth term is given by, a_n = An + B

We need to find the sum of first 20 terms.

To find the sum of the n terms of the given A.P., we use the formula,

$S_n = n(a + l)/ 2$

Where, a = the first term l = the last term,

Putting n = 1 in the given a_n, we get

a = A(1) + B = A + B

For the last term (l), here n = 20

A_{20} = A(20) + B = 20A + B

S_{20} = 20/2((A + B) + 20A + B)

= 10[21A + 2B]

= 210A + 20B

Therefore, the sum of the first 20 terms of the given A.P. is 210 A + 20B

7. Find the sum of first 25 terms of an A.P whose n^{th} term is given by $a_n = 2 – 3n$.

Solution:

Given an A.P. whose n^{th} term is given by $a_n = 2 – 3n$

To find the sum of the n terms of the given A.P., we use the formula,

$S_n = n(a + l)/ 2$

Where, a = the first term l = the last term.

Putting n = 1 in the given a_n, we get

a = 2 – 3(1) = -1

For the last term (l), here n = 25

$a_{25} = 2 – 3(25) = -73$

So, $S_n = 25(-1 – 73)/2$

= 25 x (-37)

= -925

Therefore, the sum of the 25 terms of the given A.P. is $S_{25} = -925$

8. Find the sum of first 25 terms of an A.P whose n^{th} term is given by $a_n = 7 – 3n$.

Solution:

Given an A.P. whose n^{th} term is given by $a_n = 7 – 3n$

To find the sum of the n terms of the given A.P., we use the formula,

$S_n = n(a + l)/ 2$

Where, a = the first term l = the last term.

Putting n = 1 in the given a_n, we get

a = 7 – 3(1) = 7 – 3 = 4

For the last term (l), here n = 25

a_{15} = 7 – 3(25) = -68

So, S_n = 25(4 – 68)/2

= 25 x (-32)

= -800

Therefore, the sum of the 15 terms of the given A.P. is S_{25} = -800

9. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, . . ., is 116. Find the last term.

Solution:

Given the sum of the certain number of terms of an A.P. = 116

We know that, S_n = n/2[2a + (n − 1)d]

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms So for the given A.P.(25, 22, 19,…)

Here we have, the first term (a) = 25

The sum of n terms S_n = 116

Common difference of the A.P. (d) = a_2 – a_1 = 22 – 25 = -3

Now, substituting values in S_n

⟹ 116 = n/2[2(25) + (n − 1)(−3)]

⟹ (n/2)[50 + (−3n + 3)] = 116

⟹ $(n/2)[53 - 3n] = 116$

⟹ $53n - 3n^2 = 116 \times 2$

Thus, we get the following quadratic equation,

$3n^2 - 53n + 232 = 0$

By factorization method of solving, we have

⟹ $3n^2 - 24n - 29n + 232 = 0$

⟹ $3n(n - 8) - 29(n - 8) = 0$

⟹ $(3n - 29)(n - 8) = 0$

So, $3n - 29 = 0$

⟹ $n = 29/3$

Also, $n - 8 = 0$

⟹ $n = 8$

Since, n cannot be a fraction, so the number of terms is taken as 8.

So, the term is:

$a_8 = a_1 + 7d = 25 + 7(-3) = 25 - 21 = 4$

Hence, the last term of the given A.P. such that the sum of the terms is 116 is 4.

10. (i) How many terms of the sequence 18, 16, 14…. should be taken so that their sum is zero.

(ii) How many terms are there in the A.P. whose first and fifth terms are -14 and 2 respectively and the sum of the terms is 40?

(iii) How many terms of the A.P. 9, 17, 25, . . . must be taken so that their sum is 636?

(iv) How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?

(v) How many terms of the A.P. is 27, 24, 21. . . should be taken that their sum is zero?

Solution:

(i) Given AP. is 18, 16, 14, …

We know that,

S_n = n/2[2a + (n − 1)d]

Here,

The first term (a) = 18

The sum of n terms (S_n) = 0 (given)

Common difference of the A.P.

(d) = a_2 – a_1 = 16 – 18 = – 2

So, on substituting the values in S_n

⇒ 0 = n/2[2(18) + (n − 1)(−2)]

⇒ 0 = n/2[36 + (−2n + 2)]

⇒ 0 = n/2[38 − 2n] Further, n/2

⇒ n = 0 Or, 38 – 2n = 0

⇒ 2n = 38

⇒ n = 19

Since, the number of terms cannot be zer0, hence the number of terms (n) should be 19.

(ii) Given, the first term (a) = -14, Filth term (a_5) = 2, Sum of terms (S_n) = 40 of the A.P.

If the common difference is taken as d.

Then, a_5 = a + 4d

⇒ 2 = -14 + 4d

⇒ 2 + 14 = 4d

$\Rightarrow 4d = 16$

$\Rightarrow d = 4$

Next, we know that $S_n = n/2[2a + (n - 1)d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

Now, on substituting the values in S_n

$\Rightarrow 40 = n/2[2(-14) + (n - 1)(4)]$

$\Rightarrow 40 = n/2[-28 + (4n - 4)]$

$\Rightarrow 40 = n/2[-32 + 4n]$

$\Rightarrow 40(2) = -32n + 4n^2$

So, we get the following quadratic equation,

$4n^2 - 32n - 80 = 0$

$\Rightarrow n^2 - 8n - 20 = 0$

On solving by factorization method, we get

$n^2 - 10n + 2n - 20 = 0$

$\Rightarrow n(n - 10) + 2(n - 10) = 0$

$\Rightarrow (n + 2)(n - 10) = 0$

Either, $n + 2 = 0$

$\Rightarrow n = -2$

Or, $n - 10 = 0$

$\Rightarrow n = 10$

Since the number of terms cannot be negative.

Therefore, the number of terms (n) is 10.

(iii) Given AP is 9, 17, 25,…

We know that,

$S_n = n/2[2a + (n - 1)d]$

Here we have,

The first term (a) = 9 and the sum of n terms (S_n) = 636

Common difference of the A.P. (d) = $a_2 - a_1 = 17 - 9 = 8$

Substituting the values in S_n, we get

$\Rightarrow 636 = n/2[2(9) + (n - 1)(8)]$

$\Rightarrow 636 = n/2[18 + (8n - 8)]$

$\Rightarrow 636(2) = (n)[10 + 8n]$

$\Rightarrow 1271 = 10n + 8n^2$

Now, we get the following quadratic equation,

$\Rightarrow 8n^2 + 10n - 1272 = 0$

$\Rightarrow 4n^2 + 5n - 636 = 0$

On solving by factorisation method, we have

$\Rightarrow 4n^2 - 48n + 53n - 636 = 0$

$\Rightarrow 4n(n - 12) + 53(n - 12) = 0$

$\Rightarrow (4n + 53)(n - 12) = 0$

Either $4n + 53 = 0 \Rightarrow n = -53/4$

Or, $n - 12 = 0 \Rightarrow n = 12$

Since, the number of terms cannot be a fraction.

Therefore, the number of terms (n) is 12.

(iv) Given A.P. is 63, 60, 57,…

We know that,

$S_n = n/2[2a + (n - 1)d]$

Here we have,

the first term (a) = 63

The sum of n terms (S_n) = 693

Common difference of the A.P. (d) = $a_2 - a_1 = 60 - 63 = -3$

On substituting the values in S_n we get

$\Rightarrow 693 = n/2[2(63) + (n - 1)(-3)]$

$\Rightarrow 693 = n/2[126+(-3n + 3)]$

$\Rightarrow 693 = n/2[129 - 3n]$

$\Rightarrow 693(2) = 129n - 3n^2$

Now, we get the following quadratic equation.

$\Rightarrow 3n^2 - 129n + 1386 = 0$

$\Rightarrow n^2 - 43n + 462$

Solving by factorisation method, we have

$\Rightarrow n^2 - 22n - 21n + 462 = 0$

$\Rightarrow n(n - 22) -21(n - 22) = 0$

$\Rightarrow (n - 22) (n - 21) = 0$

Either, $n - 22 = 0 \Rightarrow n = 22$

Or, $n - 21 = 0 \Rightarrow n = 21$

Now, the 22^{nd} term will be $a_{22} = a_1 + 21d = 63 + 21(-3) = 63 - 63 = 0$

So, the sum of 22 as well as 21 terms is 693.

Therefore, the number of terms (n) is 21 or 22.

(v) Given A.P. is 27, 24, 21. . .

We know that,

$S_n = n/2[2a + (n - 1)d]$

Here we have, the first term (a) = 27

The sum of n terms (S_n) = 0

Common difference of the A.P. (d) = $a_2 - a_1 = 24 - 27 = -3$

On substituting the values in S_n, we get

$\Rightarrow 0 = n/2[2(27) + (n - 1)(-3)]$

$\Rightarrow 0 = (n)[54 + (n - 1)(-3)]$

$\Rightarrow 0 = (n)[54 - 3n + 3]$

$\Rightarrow 0 = n[57 - 3n]$ Further we have, $n = 0$ Or, $57 - 3n = 0$

$\Rightarrow 3n = 57$

$\Rightarrow n = 19$

The number of terms cannot be zero,

Hence, the numbers of terms (n) is 19.

11. Find the sum of the first

(i) 11 terms of the A.P. : 2, 6, 10, 14, . . .

(ii) 13 terms of the A.P. : -6, 0, 6, 12, . . .

(iii) 51 terms of the A.P. : whose second term is 2 and fourth term is 8.

Solution:

We know that the sum of terms for different arithmetic progressions is given by

$S_n = n/2[2a + (n - 1)d]$

Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

(i) Given A.P 2, 6, 10, 14,… to 11 terms.

Common difference (d) = $a_2 - a_1 = 10 - 6 = 4$

Number of terms (n) = 11

First term for the given A.P. (a) = 2

So,

$S_{11} = 11/2[2(2) + (11 - 1)4]$

$= 11/2[2(2) + (10)4]$

$= 11/2[4 + 40]$

$= 11 \times 22$

$= 242$

Hence, the sum of first 11 terms for the given A.P. is 242

(ii) Given A.P. – 6, 0, 6, 12, … to 13 terms.

Common difference (d) = $a_2 - a_1 = 6 - 0 = 6$

Number of terms (n) = 13

First term (a) = -6

So,

$S_{13} = 13/2[2(-6) + (13 - 1)6]$

= 13/2[(−12) + (12)6]

= 13/2[60] = 390

Hence, the sum of first 13 terms for the given A.P. is 390

(iii) 51 terms of an AP whose a_2 = 2 and a_4 = 8

We know that, a_2 = a + d

2 = a + d …(2)

Also, a_4 = a + 3d

8 = a + 3d … (2)

Subtracting (1) from (2), we have

2d = 6

d = 3

Substituting d = 3 in (1), we get

2 = a + 3

⇒ a = -1

Given that the number of terms (n) = 51

First term (a) = -1

So,

S_n = 51/2[2(−1) + (51 − 1)(3)]

= 51/2[−2 + 150]

= 51/2[148]

= 3774

Hence, the sum of first 51 terms for the A.P. is 3774.

12. Find the sum of

(i) the first 15 multiples of 8

(ii) the first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.

(iii) all 3 – digit natural numbers which are divisible by 13.

(iv) all 3 – digit natural numbers which are multiples of 11.

Solution:

We know that the sum of terms for an A.P is given by

$S_n = n/2[2a + (n - 1)d]$

Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

(i) Given, first 15 multiples of 8.

These multiples form an A.P: 8, 16, 24, …… , 120

Here, a = 8 , d = 61 – 8 = 8 and the number of terms(n) = 15

Now, finding the sum of 15 terms, we have

\

$$S_n = \frac{15}{2}[2(8) + (15 - 1)8]$$

$$= \frac{15}{2}[16 + (14)8]$$

$$= \frac{15}{2}[16 + 12]$$

$$= \frac{15}{2}[128]$$

$$= 960$$

Hence, the sum of the first 15 multiples of 8 is 960

(ii)(a) First 40 positive integers divisible by 3.

Hence, the first multiple is 3 and the 40^{th} multiple is 120.

And, these terms will form an A.P. with the common difference of 3.

Here, First term (a) = 3

Number of terms (n) = 40

Common difference (d) = 3

So, the sum of 40 terms

S_{40} = 40/2[2(3) + (40 − 1)3]

= 20[6 + (39)3]

= 20(6 + 117)

= 20(123) = 2460

Thus, the sum of first 40 multiples of 3 is 2460.

(b) First 40 positive integers divisible by 5

Hence, the first multiple is 5 and the 40th multiple is 200.

And, these terms will form an A.P. with the common difference of 5.

Here, First term (a) = 5

Number of terms (n) = 40

Common difference (d) = 5

So, the sum of 40 terms

S_{40} = 40/2[2(5) + (40 − 1)5]

= 20[10 + (39)5]

= 20 (10 + 195)

= 20 (205) = 4100

Hence, the sum of first 40 multiples of 5 is 4100.

(c) First 40 positive integers divisible by 6

Hence, the first multiple is 6 and the 40th multiple is 240.

And, these terms will form an A.P. with the common difference of 6.

Here, First term (a) = 6

Number of terms (n) = 40

Common difference (d) = 6

So, the sum of 40 terms

S_{40} = 40/2[2(6) + (40 − 1)6]

= 20[12 + (39)6]

=20(12 + 234)

= 20(246) = 4920

Hence, the sum of first 40 multiples of 6 is 4920.

(iii) All 3 digit natural number which are divisible by 13.

So, we know that the first 3 digit multiple of 13 is 104 and the last 3 digit multiple of 13 is 988.

And, these terms form an A.P. with the common difference of 13.

Here, first term (a) = 104 and the last term (l) = 988

Common difference (d) = 13

Finding the number of terms in the A.P. by, $a_n = a + (n - 1)d$

We have,

988 = 104 + (n – 1)13

⟹ 988 = 104 + 13n -13

⟹ 988 = 91 + 13n

⟹ 13n = 897

⟹ n = 69

Now, using the formula for the sum of n terms, we get

S_{69} = 69/2[2(104) + (69 − 1)13]

= 69/2[208 + 884]

= 69/2[1092]

= 69(546)

= 37674

Hence, the sum of all 3 digit multiples of 13 is 37674.

(iv) All 3 digit natural number which are multiples of 11.

So, we know that the first 3 digit multiple of 11 is 110 and the last 3 digit multiple of 13 is 990.

And, these terms form an A.P. with the common difference of 11.

Here, first term (a) = 110 and the last term (l) = 990

Common difference (d) = 11

Finding the number of terms in the A.P. by, $a_n = a + (n - 1)d$

We get,

990 = 110 + (n – 1)11

⟹ 990 = 110 + 11n -11

⟹ 990 = 99 + 11n

⟹ 11n = 891

⟹ n = 81

Now, using the formula for the sum of n terms, we get

S_{81} = 81/2[2(110) + (81 − 1)11]

= 81/2[220 + 880]

= 81/2[1100]

= 81(550)

= 44550

Hence, the sum of all 3 digit multiples of 11 is 44550.

13. Find the sum:

(i) 2 + 4 + 6 + . . . + 200

(ii) 3 + 11 + 19 + . . . + 803

(iii) (-5) + (-8) + (-11) + . . . + (- 230)

(iv) 1 + 3 + 5 + 7 + . . . + 199

$$(v)\ 7 + 10\frac{1}{2} + 14 + \cdots + 84$$

(vi) 34 + 32 + 30 + . . . + 10

(vii) 25 + 28 + 31 + . . . + 100

Solution:

We know that the sum of terms for an A.P is given by

S_n = n/2[2a + (n − 1)d]

Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

Or S_n = n/2[a + l]

Where; a = first term for the given A.P. ;l = last term for the given A.P

(i) Given series. 2 + 4 + 6 + . . . + 200 which is an A.P

Where, a = 2 ,d = 4 – 2 = 2 and last term (a_n = l) = 200

We know that, a_n = a + (n – 1)d

So,

200 = 2 + (n – 1)2

200 = 2 + 2n – 2

n = 200/2 = 100

Now, for the sum of these 100 terms

S_{100} = 100/2 [2 + 200]

= 50(202)

= 10100

Hence, the sum of terms of the given series is 10100.

(ii) Given series. 3 + 11 + 19 + . . . + 803 which is an A.P

Where, a = 3 ,d = 11 – 3 = 8 and last term (a_n = l) = 803

We know that, $a_n = a + (n – 1)d$

So,

803 = 3 + (n – 1)8

803 = 3 + 8n – 8

n = 808/8 = 101

Now, for the sum of these 101 terms

S_{101} = 101/2 [3 + 803]

= 101(806)/2

= 101 x 403

= 40703

Hence, the sum of terms of the given series is 40703.

(iii) Given series (-5) + (-8) + (-11) + . . . + (- 230) which is an A.P

Where, a = -5 ,d = -8 – (-5) = -3 and last term (a_n = l) = -230

We know that, $a_n = a + (n – 1)d$

So,

-230 = -5 + (n – 1)(-3)

-230 = -5 – 3n + 3

3n = -2 + 230

n = 228/3 = 76

Now, for the sum of these 76 terms

S_{76} = 76/2 [-5 + (-230)]

= 38 x (-235)

= -8930

Hence, the sum of terms of the given series is -8930.

(iv) Given series. 1 + 3 + 5 + 7 + . . . + 199 which is an A.P

Where, a = 1 ,d = 3 – 1 = 2 and last term (a_n = l) = 199

We know that, a_n = a + (n – 1)d

So,

199 = 1 + (n – 1)2

199 = 1 + 2n – 2

n = 200/2 = 100

Now, for the sum of these 100 terms

S_{100} = 100/2 [1 + 199]

= 50(200)

= 10000

Hence, the sum of terms of the given series is 10000.

$$7 + 10\frac{1}{2} + 14 + \cdots + 84$$

(v) Given series which is an A.P

Where, a = 7, d = 10 ½ – 7 = (21 – 14)/2 = 7/2 and last term (a_n = l) = 84

We know that, a_n = a + (n – 1)d

So,

$84 = 7 + (n – 1)(7/2)$

$168 = 14 + 7n – 7$

$n = (168 – 7)/7 = 161/7 = 23$

Now, for the sum of these 23 terms

$S_{23} = 23/2 [7 + 84]$

$= 23(91)/2$

$= 2093/2$

Hence, the sum of terms of the given series is 2093/2.

(vi) Given series, 34 + 32 + 30 + . . . + 10 which is an A.P

Where, a = 34 ,d = 32 – 34 = -2 and last term ($a_n = l$) = 10

We know that, $a_n = a + (n – 1)d$

So,

$10 = 34 + (n – 1)(-2)$

$10 = 34 – 2n + 2$

$n = (36 – 10)/2 = 13$

Now, for the sum of these 13 terms

$S_{13} = 13/2 [34 + 10]$

$= 13(44)/2$

$= 13 \times 22$

$= 286$

Hence, the sum of terms of the given series is 286.

(vii) Given series, 25 + 28 + 31 + . . . + 100 which is an A.P

Where, $a = 25$, $d = 28 – 25 = 3$ and last term ($a_n = l$) = 100

We know that, $a_n = a + (n – 1)d$

So,

$100 = 25 + (n – 1)(3)$

$100 = 25 + 3n – 3$

$n = (100 – 22)/3 = 26$

Now, for the sum of these 26 terms

$S_{100} = 26/2\ [25 + 100]$

$= 13(125)$

$= 1625$

Hence, the sum of terms of the given series is 1625.

14. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

Given, the first term of the A.P (a) = 17

The last term of the A.P (l) = 350

The common difference (d) of the A.P. = 9

Let the number of terms be n. And, we know that; $l = a + (n – 1)d$

So, $350 = 17 + (n- 1)\ 9$

$\Rightarrow 350 = 17 + 9n – 9$

$\Rightarrow 350 = 8 + 9n$

$\Rightarrow 350 – 8 = 9n$

Thus we get, $n = 38$

Now, finding the sum of terms

S_n = n/2[a + l]

= 38/2(17 + 350)

= 19 × 367

= 6973

Hence, the number of terms is of the A.P is 38 and their sum is 6973.

15. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.

Solution:

Let's consider the first term as a and the common difference as d.

Given,

a_3 = 7 …. (1) and,

$a_7 = 3a_3 + 2$ …. (2)

So, using (1) in (2), we get,

a_7 = 3(7) + 2 = 21 + 2 = 23 …. (3)

Also, we know that

a_n = a +(n – 1)d

So, the 3th term (for n = 3),

a_3 = a + (3 – 1)d

⇒ 7 = a + 2d (Using 1)

⇒ a = 7 – 2d …. (4)

Similarly, for the 7th term (n = 7),

a_7 = a + (7 – 1) d 24 = a + 6d = 23 (Using 3)

a = 23 – 6d …. (5)

Subtracting (4) from (5), we get,

a – a = (23 – 6d) – (7 – 2d)

⟹ 0 = 23 – 6d – 7 + 2d

⟹ 0 = 16 – 4d

⟹ 4d = 16

⟹ d = 4

Now, to find a, we substitute the value of d in (4), a =7 – 2(4)

⟹ a = 7 – 8

a = -1

Hence, for the A.P. a = -1 and d = 4

For finding the sum, we know that

S_n = n/2[2a + (n − 1)d] and n = 20 (given)

S_{20} = 20/2[2(−1) + (20 − 1)(4)]

= (10)[-2 + (19)(4)]

= (10)[-2 + 76]

= (10)[74]

= 740

Hence, the sum of first 20 terms for the given A.P. is 740

16. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Solution:

Given,

The first term of the A.P (a) = 2

The last term of the A.P (l) = 50

Sum of all the terms S_n = 442

So, let the common difference of the A.P. be taken as d.

The sum of all the terms is given as,

442 = (n/2)(2 + 50)

⟹ 442 = (n/2)(52)

⟹ 26n = 442

⟹ n = 17

Now, the last term is expressed as

50 = 2 + (17 – 1)d

⟹ 50 = 2 + 16d

⟹ 16d = 48

⟹ d = 3

Thus, the common difference of the A.P. is d = 3.

17. If 12^{th} term of an A.P. is -13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Solution:

Let us take the first term as a and the common difference as d.

Given,

a_{12} = -13 S_4 = 24

Also, we know that a_n = a + (n – 1)d

So, for the 12th term

a_{12} = a + (12 – 1)d = -13

⇒ a + 11d = -13

a = -13 – 11d …. (1)

And, we that for sum of terms

S_n = n/2[2a + (n − 1)d]

Here, n = 4

S_4 = 4/2[2(a) + (4 − 1)d]

⇒ 24 = (2)[2a + (3)(d)]

⇒ 24 = 4a + 6d

⇒ 4a = 24 – 6d

$$\Rightarrow a = 6 - \frac{6}{4}d \; \ldots.(2)$$

Subtracting (1) from (2), we have

$$\Rightarrow a - a = (6 - \frac{6}{4}d) - (-13 - 11d)$$

$$\Rightarrow 0 = 6 - \frac{6}{4}d \; + \; 13d \; + \; 11d$$

$$\Rightarrow 0 = 19 + \frac{44d \; - 6d}{4s}$$

Further simplifying for d, we get,

$$\Rightarrow 0 = 19 + \frac{38}{4}d$$

$$\Rightarrow -19 = \frac{19}{2}d$$

$\Rightarrow$ -19 × 2 = 19d

$\Rightarrow$ d = – 2

On substituting the value of d in (1), we find a

a = -13 – 11(-2)

a = -13 + 22

a = 9

Next, the sum of 10 term is given by

S_{10} = 10/2[2(9) + (10 − 1)(−2)]

= (5)[19 + (9)(-2)]

= (5)(18 – 18) = 0

Thus, the sum of first 10 terms for the given A.P. is S_{10} = 0.

18.

Q.18. Find the sum of n terms of the series

$$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+.....$$

Solution:

Given:

First term, $a = 4 - \frac{1}{n}$

So, common difference is

$$d=\left(4-\frac{2}{n}\right)-\left(4-\frac{1}{n}\right)$$

$$=\frac{4n-2-4n+1}{n}=\frac{-1}{n}$$

By using the formula,

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

We get,

$$=\frac{n}{2}\left[2\left(4-\frac{1}{n}\right)+(n-1)\left(-\frac{1}{n}\right)\right]$$

$$=\frac{n}{2}\left[8-\frac{2}{n}+\left(-1+\frac{1}{n}\right)\right]$$

$$=\frac{n}{2}\left[\frac{8n-2-n+1}{n}\right]=\frac{1}{2}(7n-1)$$

Hence, the sum of n terms of the series is $\frac{1}{2}(7n-1)$.

19. In an A.P., if the first term is 22, the common difference is – 4 and the sum to n terms is 64, find n.

Solution:

Given that,

a = 22, d = – 4 and S_n = 64

Let us consider the number of terms as n.

For sum of terms in an A.P, we know that

$S_n = n/2[2a + (n - 1)d]$

Where; a = first term for the given A.P. d = common difference of the given A.P. n = number of terms

So,

$\Rightarrow S_n = n/2[2(22) + (n - 1)(-4)]$

$\Rightarrow 64 = n/2[2(22) + (n - 1)(-4)]$

$\Rightarrow 64(2) = n(48 - 4n)$

$\Rightarrow 128 = 48n - 4n^2$

After rearranging the terms, we have a quadratic equation

$4n^2 - 48n + 128 = 0$,

$n^2 - 12n + 32 = 0$ [dividing by 4 on both sides]

$n^2 - 12n + 32 = 0$

Solving by factorisation method,

$n^2 - 8n - 4n + 32 = 0$

$n (n - 8) - 4 (n - 8) = 0$

$(n - 8) (n - 4) = 0$

So, we get $n - 8 = 0 \Rightarrow n = 8$

Or, $n - 4 = 0 \Rightarrow n = 4$

Hence, the number of terms can be either n = 4 or 8.

20. In an A.P., if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?

Solution:

Let's take the first term as a and the common difference to be d

Given that,

$a_5 = 30$ and $a_{12} = 65$

And, we know that $a_n = a + (n – 1)d$

So,

$a_5 = a + (5 – 1)d$

$30 = a + 4d$

$a = 30 – 4d$ …. (i)

Similarly, $a_{12} = a + (12 – 1) d$

$65 = a + 11d$

$a = 65 – 11d$ …. (ii)

Subtracting (i) from (ii), we have

$a – a = (65 – 11d) – (30 – 4d)$

$0 = 65 – 11d – 30 + 4d$

$0 = 35 – 7d$

$7d = 35$

$d = 5$

Putting d in (i), we get

$a = 30 – 4(5)$

$a = 30 – 20$

$a = 10$

Thus for the A.P; d = 5 and a = 10

Next, to find the sum of first 20 terms of this A.P., we use the following formula for the sum of n terms of an A.P.,

$S_n = n/2[2a + (n - 1)d]$

Where;

a = first term of the given A.P.

d = common difference of the given A.P.

n = number of terms

Here n = 20, so we have

$S_{20} = 20/2[2(10) + (20 - 1)(5)]$

= (10)[20 + (19)(5)]

= (10)[20 + 95]

= (10)[115]

= 1150

Hence, the sum of first 20 terms for the given A.P. is 1150

21. Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Solution:

Let's take the first term as a and the common difference as d.

Given that,

$a_2 = 14$ and $a_3 = 18$

And, we know that $a_n = a + (n – 1)d$

So,

$a_2 = a + (2 – 1)d$

⟹ $14 = a + d$

⟹ $a = 14 – d$ …. (i)

Similarly,

$a_3 = a + (3 – 1)d$

⟹ $18 = a + 2d$

⟹ $a = 18 – 2d$ …. (ii)

Subtracting (i) from (ii), we have

$a – a = (18 – 2d) – (14 – d)$

$0 = 18 – 2d – 14 + d$

$0 = 4 – d$

$d = 4$

Putting d in (i), to find a

$a = 14 – 4$

$a = 10$

Thus, for the A.P. d = 4 and a = 10

Now, to find sum of terms

$S_n = n/2(2a + (n − 1)d)$

Where,

a = the first term of the A.P.

d = common difference of the A.P.

n = number of terms So, using the formula for

n = 51,

⟹ S_{51} = 51/2[2(10) + (51 – 1)(4)]

= 51/2[20 + (40)4]

= 51/2[220]

= 51(110)

= 5610

Hence, the sum of the first 51 terms of the given A.P. is 5610

22. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of n terms.

Solution:

Given,

Sum of 7 terms of an A.P. is 49

⟹ S_7 = 49

And, sum of 17 terms of an A.P. is 289

⟹ S_{17} = 289

Let the first term of the A.P be a and common difference as d.

And, we know that the sum of n terms of an A.P is

S_n = n/2[2a + (n − 1)d]

So,

S_7 = 49 = 7/2[2a + (7 – 1)d]

= 7/2 [2a + 6d]

= 7[a + 3d]

⟹ 7a + 21d = 49

a + 3d = 7 ….. (i)

Similarly,

S_{17} = 17/2[2a + (17 – 1)d]

= 17/2 [2a + 16d]

= 17[a + 8d]

⇒ 17[a + 8d] = 289

a + 8d = 17 ….. (ii)

Now, subtracting (i) from (ii), we have

a + 8d – (a + 3d) = 17 – 7

5d = 10

d = 2

Putting d in (i), we find a

a + 3(2) = 7

a = 7 – 6 = 1

So, for the A.P: a = 1 and d = 2

For the sum of n terms is given by,

S_n = n/2[2(1) + (n − 1)(2)]

= n/2[2 + 2n – 2]

= n/2[2n]

= n^2

Therefore, the sum of n terms of the A.P is given by n^2.

23. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution:

Sum of first n terms of an A.P is given by S_n = n/2(2a + (n − 1)d)

Given,

First term (a) = 5, last term (a_n) = 45 and sum of n terms (S_n) = 400

Now, we know that

a_n = a + (n – 1)d

⟹ 45 = 5 + (n – 1)d

⟹ 40 = nd – d

⟹ nd – d = 40 …. (1)

Also,

S_n = n/2(2(a) + (n − 1)d)

400 = n/2(2(5) + (n − 1)d)

800 = n (10 + nd – d)

800 = n (10 + 40) [using (1)]

⟹ n = 16

Putting n in (1), we find d

nd – d = 40

16d – d = 40

15d = 40

d = 8/3

Therefore, the common difference of the given A.P. is 8/3.

24. In an A.P. the first term is 8, n^{th} term is 33 and the sum of first n term is 123. Find n and the d, the common difference.

Solution:

Given,

The first term of the A.P (a) = 8

The nth term of the A.P (l) = 33

And, the sum of all the terms S_n = 123

Let the common difference of the A.P. be d.

So, find the number of terms by

123 = (n/2)(8 + 33)

123 = (n/2)(41)

n = (123 x 2)/ 41

n = 246/41

n = 6

Next, to find the common difference of the A.P. we know that

l = a + (n – 1)d

33 = 8 + (6 – 1)d

33 = 8 + 5d

5d = 25

d = 5

Thus, the number of terms is n = 6 and the common difference of the A.P. is d = 5.

25. In an A.P. the first term is 22, n^{th} term is -11 and the sum of first n term is 66. Find n and the d, the common difference.

Solution:

Given,

The first term of the A.P (a) = 22

The nth term of the A.P (l) = -11

And, sum of all the terms S_n = 66

Let the common difference of the A.P. be d.

So, finding the number of terms by

66 = (n/2)[22 + (−11)]

66 = (n/2)[22 − 11]

(66)(2) = n(11)

6 × 2 = n

n = 12

Now, for finding d

We know that, l = a + (n – 1)d

– 11 = 22 + (12 – 1)d

-11 = 22 + 11d

11d = – 33

d = – 3

Hence, the number of terms is n = 12 and the common difference d = -3

26. The first and the last terms of an A.P. are 7 and 49 respectively. If sum of all its terms is 420, find the common difference.

Solution:

Given,

First term (a) = 7, last term (a_n) = 49 and sum of n terms (S_n) = 420

Now, we know that

$a_n = a + (n - 1)d$

⟹ 49 = 7 + (n – 1)d

⟹ 43 = nd – d

⟹ nd – d = 42 ….. (1)

Next,

$S_n = n/2(2(7) + (n - 1)d)$

⟹ 840 = n[14 + nd – d]

⟹ 840 = n[14 + 42] [using (1)]

⟹ 840 = 54n

⟹ n = 15 …. (2)

So, by substituting (2) in (1), we have

nd – d = 42

⟹ 15d – d = 42

⟹ 14d = 42

⟹ d = 3

Therefore, the common difference of the given A.P. is 3.

27. The first and the last terms of an A.P are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Solution:

Given,

First term (a) = 5 and the last term (l) = 45

Also, $S_n = 400$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 45 = 5 + (n - 1)d$

$\Rightarrow 40 = nd - d$

$\Rightarrow nd - d = 40$ ….. (1)

Next,

$S_n = n/2(2(5) + (n - 1)d)$

$\Rightarrow 400 = n[10 + nd - d]$

$\Rightarrow 800 = n[10 + 40]$ [using (1)]

$\Rightarrow 800 = 50n$

$\Rightarrow n = 16$ …. (2)

So, by substituting (2) in (1), we have

$nd - d = 40$

$\Rightarrow 16d - d = 40$

$\Rightarrow 15d = 40$

$\Rightarrow d = 8/3$

Therefore, the common difference of the given A.P. is 8/3.

28. The sum of first q terms of an A.P. is 162. The ratio of its 6^{th} term to its 13^{th} term is 1: 2. Find the first and 15^{th} term of the A.P.

Solution:

Let a be the first term and d be the common difference.

And we know that, sum of first n terms is:

$S_n = n/2(2a + (n - 1)d)$

Also, nth term is given by $a_n = a + (n - 1)d$

From the question, we have

$S_q = 162$ and $a_6 : a_{13} = 1 : 2$

So,

$2a_6 = a_{13}$

$\Rightarrow 2 [a + (6 – 1d)] = a + (13 – 1)d$

$\Rightarrow 2a + 10d = a + 12d$

$\Rightarrow a = 2d$ …. (1)

And, $S_9 = 162$

$\Rightarrow S_9 = 9/2(2a + (9 - 1)d)$

$\Rightarrow 162 = 9/2(2a + 8d)$

$\Rightarrow 162 \times 2 = 9[4d + 8d]$ [from (1)]

$\Rightarrow 324 = 9 \times 12d$

$\Rightarrow d = 3$

$\Rightarrow a = 2(3)$ [from (1)]

$\Rightarrow a = 6$

Hence, the first term of the A.P. is 6

For the 15^{th} term, $a_{15} = a + 14d = 6 + 14 \times 3 = 6 + 42$

Therefore, $a_{15} = 48$

29. If the 10^{th} term of an A.P. is 21 and the sum of its first 10 terms is 120, find its n^{th} term.

Solution:

Let's consider a to be the first term and d be the common difference.

And we know that, sum of first n terms is:

$S_n = n/2(2a + (n - 1)d)$ and n^{th} term is given by: $a_n = a + (n - 1)d$

Now, from the question we have

$S_{10} = 120$

$\Rightarrow 120 = 10/2(2a + (10 - 1)d)$

$\Rightarrow 120 = 5(2a + 9d)$

$\Rightarrow 24 = 2a + 9d$ …. (1)

Also given that, $a_{10} = 21$

$\Rightarrow 21 = a + (10 - 1)d$

$\Rightarrow 21 = a + 9d$ …. (2)

Subtracting (2) from (1), we get

$24 - 21 = 2a + 9d - a - 9d$

$\Rightarrow a = 3$

Now, on putting a = 3 in equation (2), we get

$3 + 9d = 21$

$9d = 18$

$d = 2$

Thus, we have the first term(a) = 3 and the common difference(d) = 2

Therefore, the n^{th} term is given by

$a_n = a + (n - 1)d = 3 + (n - 1)2$

$= 3 + 2n - 2$

$= 2n + 1$

Hence, the n^{th} term of the A.P is $(a_n) = 2n + 1$.

30. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28th term of this A.P.

Solution:

Let's take a to be the first term and d to be the common difference.

And we know that, sum of first n terms

$S_n = n/2(2a + (n - 1)d)$

Given that sum of the first 7 terms of an A.P. is 63.

$S_7 = 63$

And sum of next 7 terms is 161.

So, the sum of first 14 terms = Sum of first 7 terms + sum of next 7 terms

$S_{14} = 63 + 161 = 224$

Now, having

$S_7 = 7/2(2a + (7 - 1)d)$

$\Rightarrow 63(2) = 7(2a + 6d)$

$\Rightarrow 9 \times 2 = 2a + 6d$

$\Rightarrow 2a + 6d = 18 \ldots. (1)$

And,

$S_{14} = 14/2(2a + (14 - 1)d)$

$\Rightarrow 224 = 7(2a + 13d)$

$\Rightarrow 32 = 2a + 13d \ldots. (2)$

Now, subtracting (1) from (2), we get

$\Rightarrow 13d - 6d = 32 - 18$

$\Rightarrow 7d = 14$

$\Rightarrow$ d = 2

Using d in (1), we have

2a + 6(2) = 18

2a = 18 – 12

a = 3

Thus, from n^{th} term

$\Rightarrow a_{28}$ = a + (28 – 1)d

= 3 + 27 (2)

= 3 + 54 = 57

Therefore, the 28^{th} term is 57.

31. The sum of first seven terms of an A.P. is 182. If its 4^{th} and 17^{th} terms are in ratio 1: 5, find the A.P.

Solution:

Given that,

S_{17} = 182

And, we know that the sum of first n term is:

S_n = n/2(2a + (n − 1)d)

So,

S_7 = 7/2(2a + (7 − 1)d)

182 × 2 = 7(2a + 6d)

364 = 14a + 42d

26 = a + 3d

a = 26 – 3d … (1)

Also, it's given that 4^{th} term and 17^{th} term are in a ratio of 1: 5. So, we have

$\Rightarrow 5(a_4) = 1(a_{17})$

$\Rightarrow 5\ (a + 3d) = 1\ (a + 16d)$

$\Rightarrow 5a + 15d = a + 16d$

$\Rightarrow 4a = d$ …. (2)

Now, substituting (2) in (1), we get

$\Rightarrow 4\ (\ 26 – 3d\ ) = d$

$\Rightarrow 104 – 12d = d$

$\Rightarrow 104 = 13d$

$\Rightarrow d = 8$

Putting d in (2), we get

$\Rightarrow 4a = d$

$\Rightarrow 4a = 8$

$\Rightarrow a = 2$

Therefore, the first term is 2 and the common difference is 8. So, the A.P. is 2, 10, 18, 26, ...

32. The n^{th} term of an A.P is given by (-4n + 15). Find the sum of first 20 terms of this A.P.

Solution:

Given,

The n^{th} term of the A.P = (-4n + 15)

So, by putting n = 1 and n = 20 we can find the first ans 20^{th} term of the A.P

a = (-4(1) + 15) = 11

And,

$a_{20} = (-4(20) + 15) = -65$

Now, for find the sum of 20 terms of this A.P we have the first and last term.

So, using the formula

$S_n = n/2(a + l)$

$S_{20} = 20/2(11 + (-65))$

$= 10(-54)$

$= -540$

Therefore, the sum of first 20 terms of this A.P. is -540.

33. In an A.P. the sum of first ten terms is -150 and the sum of its next 10 term is -550. Find the A.P.

Solution:

Let's take a to be the first term and d to be the common difference.

And we know that, sum of first n terms

$S_n = n/2(2a + (n - 1)d)$

Given that sum of the first 10 terms of an A.P. is -150.

$S_{10} = -150$

And the sum of next 10 terms is -550.

So, the sum of first 20 terms = Sum of first 10 terms + sum of next 10 terms

$S_{20} = -150 + -550 = -700$

Now, having

$S_{10} = 10/2(2a + (10 - 1)d)$

$\Rightarrow -150 = 5(2a + 9d)$

$\Rightarrow -30 = 2a + 9d$

⟹ $2a + 9d = -30$ . . . . (1)

And,

$S_{20} = 20/2(2a + (20 - 1)d)$

⟹ $-700 = 10(2a + 19d)$

⟹ $-70 = 2a + 19d$ …. (2)

Now, subtracting (1) from (2), we get

⟹ $19d – 9d = -70 – (-30)$

⟹ $10d = -40$

⟹ $d = -4$

Using d in (1), we have

$2a + 9(-4) = -30$

$2a = -30 + 36$

$a = 6/2 = 3$

Hence, we have a = 3 and d = -4

So, the A.P is 3, -1, -5, -9, -13,…..

34. Sum of the first 14 terms of an A.P. is 1505 and its first term is 10. Find its 25th term.

Solution

Given,

First term of the A.P is 1505 and

$S_{14} = 1505$

We know that, the sum of first n terms is

$S_n = n/2(2a + (n - 1)d)$

So,

S_{14} = 14/2(2(10) + (14 − 1)d) = 1505

7(20 + 13d) = 1505

20 + 13d = 215

13d = 215 – 20

d = 195/13

d =15

Thus, the 25th term is given by

a_{25} = 10 + (25 -1)15

= 10 + (24)15

= 10 + 360

= 370

Therefore, the 25th term of the A.P is 370

35. In an A.P. , the first term is 2, the last term is 29 and the sum of the terms is 155. Find the common difference of the A.P.

Solution:

Given,

The first term of the A.P. (a) = 2

The last term of the A.P. (l) = 29

And, sum of all the terms (S_n) = 155

Let the common difference of the A.P. be d.

So, find the number of terms by sum of terms formula

S_n = n/2 (a + l)

155 = n/2(2 + 29)

155(2) = n(31)

31n = 310

n = 10

Using n for the last term, we have

l = a + (n – 1)d

29 = 2 + (10 – 1)d

29 = 2 + (9)d

29 – 2 = 9d

9d = 27

d = 3

Hence, the common difference of the A.P. is d = 3

36. The first and the last term of an A.P are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

Given,

In an A.P first term (a) = 17 and the last term (l) = 350

And, the common difference (d) = 9

We know that,

a_n = a + (n – 1)d

so,

a_n = l = 17 + (n – 1)9 = 350

17 + 9n – 9 = 350

$9n = 350 - 8$

$n = 342/9$

$n = 38$

So, the sum of all the term of the A.P is given by

$S_n = n/2\ (a + l)$

$= 38/2(17 + 350)$

$= 19(367)$

$= 6973$

Therefore, the sum of terms of the A.P is 6973.

37. Find the number of terms of the A.P. –12, –9, –6, . . . , 21. If 1 is added to each term of this A.P., then find the sum of all terms of the A.P. thus obtained.

Solution:

Given,

First term, $a = -12$

Common difference, $d = a_2 - a_1 = -9 - (-12)$

$d = -9 + 12 = 3$

And, we know that n^{th} term $= a_n = a + (n - 1)d$

$\Rightarrow 21 = -12 + (n - 1)3$

$\Rightarrow 21 = -12 + 3n - 3$

$\Rightarrow 21 = 3n - 15$

$\Rightarrow 36 = 3n$

$\Rightarrow\ n = 12$

Thus, the number of terms is 12.

Now, if 1 is added to each of the 12 terms, the sum will increase by 12.

Hence, the sum of all the terms of the A.P. so obtained is

$\Longrightarrow S_{12} + 12 = 12/2[a + l] + 12$

$= 6[-12 + 21] + 12$

$= 6 \times 9 + 12$

$= 66$

Therefore, the sum after adding 1 to each of the terms in the A.P is 66.

Chapterwise RD Sharma Solutions for Class 10 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.