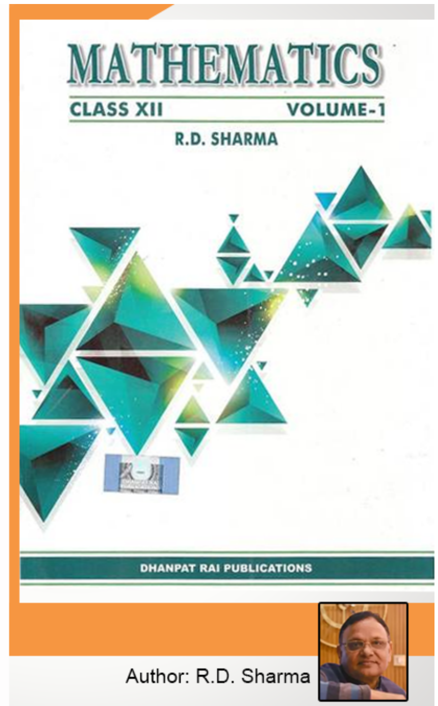
Class 12 - Chapter 8 Solution of Simultaneous Linear Equations



RD Sharma Solutions for Class 12 Maths Chapter 8–Solution of Simultaneous Linear Equations

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Exercise 8.1 Page No: 8.14

1. Solve the following system of equations by matrix method:

(i) $5x + 7y + 2 = 0$

$4x + 6y + 3 = 0$

(ii) $5x + 2y = 3$

$3x + 2y = 5$

(iii) $3x + 4y - 5 = 0$

$x - y + 3 = 0$

(iv) $3x + y = 19$

$3x - y = 23$

(v) $3x + 7y = 4$

$x + 2y = -1$

(vi) $3x + y = 7$

$5x + 3y = 12$

Solution:

(i) Given $5x + 7y + 2 = 0$ and $4x + 6y + 3 = 0$

The above system of equations can be written as

$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$ B = $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = 30 - 28 = 2$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 6 = 6$

$C_{12} = (-1)^{1+2}\, 4 = -4$

$C_{21} = (-1)^{2+1}\, 7 = -7$

$C_{22} = (-1)^{2+2}\, 5 = 5$

Also, adj A = $\begin{bmatrix} 6 & -4 \\ -7 & 5 \end{bmatrix}^T$

= $\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -12 + 21 \\ 8 - 15 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ \frac{-7}{2} \end{bmatrix}$$

Hence, x = 9/2 and y = -7/2

(ii) Given 5x + 2y = 3

3x + 2y = 5

The above system of equations can be written as

$$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \text{ Or } AX = B$$

Where A = $\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = 10 - 6 = 4$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 2 = 2$

$C_{12} = (-1)^{1+2}\, 3 = -3$

$C_{21} = (-1)^{2+1}\, 2 = -2$

$C_{22} = (-1)^{2+2}\, 2 = 5$

Also, adj A = $\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}^T$

$= \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6-10 \\ -9+25 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Hence, x = -1 and y = 4

(iii) Given 3x + 4y – 5 = 0

x – y + 3 = 0

The above system of equations can be written as

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix} \text{ Or } AX = B$$

Where $A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = -3 - 4 = -7$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1} - 1 = -1$

$C_{12} = (-1)^{1+2}\, 1 = -1$

$C_{21} = (-1)^{2+1}\, 4 = -4$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} -1 & -1 \\ -4 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{7}\begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -5 + 12 \\ -5 - 9 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Hence, X = 1 Y = – 2

(iv) Given 3x + y = 19

3x – y = 23

The above system of equations can be written as

$\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}$ B = $\begin{bmatrix} 19 \\ 23 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = -3 - 3 = -6$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1} - 1 = -1$

$C_{12} = (-1)^{1+2}\ 3 = -3$

$C_{21} = (-1)^{2+1}\ 1 = -1$

$C_{22} = (-1)^{2+2}\ 3 = 3$

Also, adj A = $\begin{bmatrix} -1 & -3 \\ -1 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{-6} \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -19 - 23 \\ -57 + 69 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -42 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

Hence, $x = 7$ and $y = -2$

(v) Given 3x + 7y = 4

x + 2y = -1

The above system of equations can be written as

$\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = 6 - 7 = -1$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 2 = 2$

$C_{12} = (-1)^{1+2}\, 1 = -1$

$C_{21} = (-1)^{2+1}\, 7 = -7$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

Now, X = A $^{-1}$B

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 8 + 7 \\ -4 - 3 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 15 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -15 \\ 7 \end{bmatrix}$$

Hence, X = – 15 Y = 7

(vi) Given 3x + y = 7

5x + 3y = 12

The above system of equations can be written as

$\begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 7 \\ 12 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = 9 - 5 = 4$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 3 = 3$

$C_{12} = (-1)^{1+2}\, 5 = -5$

$C_{21} = (-1)^{2+1}\, 1 = -1$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}^T$

= $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 21 - 12 \\ -35 + 36 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ \frac{1}{4} \end{bmatrix}$$

Hence, $X = \frac{9}{4}$ $Y = \frac{1}{4}$

2. Solve the following system of equations by matrix method:

(i) x + y –z = 3

2x + 3y + z = 10

3x – y – 7z = 1

(ii) x + y + z = 3

$2x - y + z = -1$

$2x + y - 3z = -9$

(iii) $6x - 12y + 25z = 4$

$4x + 15y - 20z = 3$

$2x + 18y + 15z = 10$

(iv) $3x + 4y + 7z = 14$

$2x - y + 3z = 4$

$x + 2y - 3z = 0$

(v) $(2/x) - (3/y) + (3/z) = 10$

$(1/x) + (1/y) + (1/z) = 10$

$(3/x) - (1/y) + (2/z) = 13$

(vi) $5x + 3y + z = 16$

$2x + y + 3z = 19$

$x + 2y + 4z = 25$

(vii) $3x + 4y + 2z = 8$

$2y - 3z = 3$

$x - 2y + 6z = -2$

(viii) $2x + y + z = 2$

$x + 3y - z = 5$

$3x + y - 2z = 6$

(ix) $2x + 6y = 2$

$3x - z = -8$

$2x - y + z = -3$

(x) $2y - z = 1$

$x - y + z = 2$

$2x - y = 0$

(xi) $8x + 4y + 3z = 18$

$2x + y + z = 5$

$x + 2y + z = 5$

(xii) $x + y + z = 6$

$x + 2z = 7$

$3x + y + z = 12$

(xiii) $(2/x) + (3/y) + (10/z) = 4$,

$(4/x) - (6/y) + (5/z) = 1$,

$(6/x) + (9/y) - (20/z) = 2$, $x, y, z \neq 0$

(xiv) $x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Solution:

(i) Given $x + y - z = 3$

$2x + 3y + z = 10$

$3x - y - 7z = 1$

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$ Or A X = B

$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

Now, $|A| = 1\begin{vmatrix} 3 & 1 \\ -1 & -7 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix}$

= (– 20) – 1(– 17) – 1(11)

= – 20 + 17 + 11 = 8

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1} - 21 + 1 = -20$

$C_{21} = (-1)^{2+1} - 7 - 1 = 8$

$C_{31} = (-1)^{3+1}\ 1 + 3 = 4$

$C_{12} = (-1)^{1+2} - 14 - 3 = 17$

$C_{22} = (-1)^{2+1} - 7 + 3 = -4$

$C_{32} = (-1)^{3+1}\ 1 + 2 = -3$

$C_{13} = (-1)^{1+2} - 2 - 9 = -11$

$C_{23} = (-1)^{2+1} - 1 - 3 = 4$

$C_{33} = (-1)^{3+1}\ 3 - 2 = 1$

Adj A = $\begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^T$

= $\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$

Now, X = $A^{-1}B$ = $\frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

X = $\frac{1}{8}\begin{bmatrix} -60 + 80 + 4 \\ 51 - 40 - 3 \\ -33 + 40 + 1 \end{bmatrix}$

X = $\frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

Hence, X = 3, Y = 1 and Z = 1

(ii) Given x + y + z = 3

2x – y + z = -1

2x + y – 3z = -9

The given system can be written in matrix form as:

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$ Or A X = B

A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$

Now, |A| = $1\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}$

= (3 – 1) – 1(– 6 – 2) + 1(2 + 2)

= 2 + 8 + 4

= 14

So, the above system has a unique solution, given by

X = $A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 3 - 1 = 2$

$C_{21} = (-1)^{2+1} - 3 - 1 = 4$

$C_{31} = (-1)^{3+1}\ 1 + 1 = 2$

$C_{12} = (-1)^{1+2} - 6 - 2 = 8$

$C_{22} = (-1)^{2+1} - 3 - 2 = -5$

$C_{32} = (-1)^{3+1}\ 1 - 2 = 1$

$C_{13} = (-1)^{1+2}\ 2 + 2 = 4$

$C_{23} = (-1)^{2+1}\ 1 - 2 = 1$

$C_{33} = (-1)^{3+1} - 1 - 2 = -3$

Adj A = $\begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}^T$

= $\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$

Now, X = A $^{-1}$B = $\frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$

X = $\frac{1}{14}\begin{bmatrix} -16 \\ 20 \\ 38 \end{bmatrix}$

X = $\frac{1}{7}\begin{bmatrix} \frac{-8}{7} \\ \frac{10}{7} \\ \frac{19}{7} \end{bmatrix}$

Hence, X = $\frac{-8}{7}$,Y = $\frac{10}{7}$ and Z = $\frac{19}{7}$

X = A $^{-1}$B

Cofactors of A are

C_{11} = (– 1)$^{1+1}$ (225 + 360) = 585

C_{21} = (– 1)$^{2+1}$ (– 180 – 450) = 630

C_{31} = (– 1)$^{3+1}$ (240 – 375) = – 135

C_{12} = (– 1)$^{1+2}$ (60 + 40) = – 100

C_{22} = $(-1)^{2+1}$ (90 – 50) = 40

C_{32} = $(-1)^{3+1}$ (– 120 – 100) = 220

C_{13} = $(-1)^{1+2}$ (72 – 30) = 42

C_{23} = $(-1)^{2+1}$(108 + 24) = – 132

C_{33} = $(-1)^{3+1}$ (90 + 48) = 138

$$\text{Adj } A = \begin{bmatrix} 585 & -100 & 42 \\ 630 & 40 & -132 \\ -135 & 220 & 138 \end{bmatrix}^T$$

$$= \begin{bmatrix} 585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{5760}\begin{bmatrix} 585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

$$X = \frac{1}{5760}\begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$X = \frac{1}{7}\begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Hence, $X = \frac{1}{2}, Y = \frac{1}{3}$ and $Z = \frac{1}{5}$

(iv) Given 3x + 4y + 7z = 14

2x – y + 3z = 4

x + 2y – 3z = 0

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$ Or A X = B

$A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$

Now, $|A| = 3\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} + 7\begin{vmatrix} 2 & 3 \\ 2 & -3 \end{vmatrix}$

= 3(3 – 6) – 4(– 6 – 3) + 7(4 + 1)

= – 9 + 36 + 35

= 62

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 3 - 6 = -3$

$C_{21} = (-1)^{2+1} - 12 - 14 = 26$

$C_{31} = (-1)^{3+1} 12 + 7 = 19$

$C_{12} = (-1)^{1+2} - 6 - 3 = 9$

$C_{22} = (-1)^{2+1} - 3 - 7 = -10$

$C_{32} = (-1)^{3+1}\ 9 - 14 = 5$

$C_{13} = (-1)^{1+2}\ 4 + 1 = 5$

$C_{23} = (-1)^{2+1}\ 6 - 4 = -2$

$C_{33} = (-1)^{3+1} - 3 - 8 = -11$

$$\mathrm{Adj}\ A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -5 & -2 \\ 19 & 5 & -11 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\,\mathrm{adj}\ A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$X = \frac{1}{62}\begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix}$$

$$X = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, X = 1, Y = 1 and Z = 1

(v) Given (2/x) – (3/y) + (3/z) = 10

(1/x) + (1/y) + (1/z) = 10

(3/x) – (1/y) + (2/z) = 13

= 5(4 – 6) – 3(8 – 3) + 1(4 – 2)

= – 10 – 15 + 3

= – 22

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}(4 - 6) = -2$

$C_{21} = (-1)^{2+1}(12 - 2) = -10$

$C_{31} = (-1)^{3+1}(9 - 1) = 8$

$C_{12} = (-1)^{1+2} (8 - 3) = -5$

$C_{22} = (-1)^{2+1}\ 20 - 1 = 19$

$C_{32} = (-1)^{3+1}\ 15 - 2 = -13$

$C_{13} = (-1)^{1+2} (4 - 2) = 2$

$C_{23} = (-1)^{2+1}\ 10 - 3 = -7$

$C_{33} = (-1)^{3+1}\ 5 - 6 = -1$

Adj A = $\begin{bmatrix} -2 & -5 & -3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T$

= $\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & 13 \\ 3 & -7 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

Now, $X = A^{-1}B = \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}\begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$

$X = \frac{1}{-22}\begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix}$

$X = \frac{1}{-22}\begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

Hence, X = 1, Y = 2 and Z = 5

(vi) Given 5x + 3y + z = 16

2x + y + 3z = 19

x + 2y + 4z = 25

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$ Or A X = B

$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$

Now, $|A| = 3\begin{vmatrix} 2 & -3 \\ -2 & 6 \end{vmatrix} - 4\begin{vmatrix} 0 & -3 \\ 1 & 6 \end{vmatrix} + 2\begin{vmatrix} 0 & 2 \\ 1 & -2 \end{vmatrix}$

= 3(12 – 6) – 4(0 + 3) + 2(0 – 2)

= 18 – 12 – 4

= 2

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}(12 - 6) = 6$

$C_{21} = (-1)^{2+1}(24 + 4) = -28$

$C_{31} = (-1)^{3+1}(-12 - 4) = -16$

$C_{12} = (-1)^{1+2}(0 + 3) = -3$

$C_{22} = (-1)^{2+1}\ 18 - 2 = 16$

$C_{32} = (-1)^{3+1} - 9 - 0 = 9$

$C_{13} = (-1)^{1+2}(0 - 2) = -2$

$C_{23} = (-1)^{2+1}(-6 - 4) = 10$

$C_{33} = (-1)^{3+1}\ 6 - 0 = 6$

Adj A = $\begin{bmatrix} 6 & -3 & 2 \\ -28 & 16 & 10 \\ -16 & -9 & 6 \end{bmatrix}^T$

= $\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$

$X = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

Hence, X = – 2, Y = 3 and Z = 1

(vii) Given 3x + 4y + 2z = 8

2y – 3z = 3

x – 2y + 6z = -2

The given system can be written in matrix form as:

$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ Or A X = B

$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

Now, $|A| = 2\begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$

= 2(– 6 + 1) – 1(– 2 + 3) + 1(1 – 9)

= – 10 – 1 – 8

= – 19

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1} - 6 + 1 = -5$

$C_{21} = (-1)^{2+1}(24 + 4) = -28$

$C_{31} = (-1)^{3+1} - 1 - 3 = -4$

$C_{12} = (-1)^{1+2} - 2 + 3 = -1$

$C_{22} = (-1)^{2+1} - 4 - 3 = -7$

$C_{32} = (-1)^{3+1} - 2 - 1 = 3$

$C_{13} = (-1)^{1+2} 1 - 9 = -8$

$C_{23} = (-1)^{2+1} 2 - 3 = -1$

$C_{33} = (-1)^{3+1}\ 6 - 1 = 5$

(viii) Given 2x + y + z = 2

x + 3y – z = 5

3x + y – 2z = 6

= 2(– 6 + 1) – 1(– 2 + 3) + 1(1 – 9)

= – 10 – 1 – 8

= – 19

So, the above system has a unique solution, given by

X = $A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1} - 6 + 1 = -5$

$C_{21} = (-1)^{2+1}(24 + 4) = -28$

$C_{31} = (-1)^{3+1} - 1 - 3 = -4$

$C_{12} = (-1)^{1+2} - 2 + 3 = -1$

$C_{22} = (-1)^{2+1} - 4 - 3 = -7$

$C_{32} = (-1)^{3+1} - 2 - 1 = 3$

$C_{13} = (-1)^{1+2} 1 - 9 = -8$

$C_{23} = (-1)^{2+1} 2 - 3 = -1$

$C_{33} = (-1)^{3+1} 6 - 1 = 5$

$$\text{Adj } A = \begin{bmatrix} -5 & -1 & -8 \\ 3 & -7 & 1 \\ -4 & 3 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

Now, $X = A^{-1}B = \frac{1}{-19}\begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

$X = \frac{1}{-19}\begin{bmatrix} -10 + 15 - 24 \\ -2 - 35 + 18 \\ -16 + 5 + 30 \end{bmatrix}$

$X = \frac{1}{-19}\begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

Hence, X = 1, Y = 1 and Z = – 1

(ix) Given 2x + 6y = 2

3x – z = -8

2x – y + z = -3

The given system can be written in matrix form as:

$\begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$ Or A X = B

$A = \begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$

Now, $|A| = 2\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} - 6\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} + 0$

= 2(0 – 1) – 6(3 + 2)

= – 2 – 30

= – 32

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 0 - 1 = -1$

$C_{21} = (-1)^{2+1} 6 + 0 = -6$

$C_{31} = (-1)^{3+1} - 6 = -6$

$C_{12} = (-1)^{1+2}\ 3 + 2 = 5$

$C_{22} = (-1)^{2+1}\ 2 - 0 = 2$

$C_{32} = (-1)^{3+1} - 2 - 0 = 2$

$C_{13} = (-1)^{1+2} - 3 - 0 = -3$

$C_{23} = (-1)^{2+1} - 2 - 12 = 14$

$C_{33} = (-1)^{3+1}\ 0 - 18 = -18$

Adj A = $\begin{bmatrix} -1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18 \end{bmatrix}^T$

= $\begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{-32}\begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}\begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$

$X = \frac{1}{62}\begin{bmatrix} -2 + 48 + 18 \\ -10 - 16 - 6 \\ -6 - 112 + 54 \end{bmatrix}$

$X = \frac{1}{62}\begin{bmatrix} 64 \\ -32 \\ -64 \end{bmatrix}$

$X = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

Hence, X = – 2, Y = 1 and Z = 2

(x) Given 2y – z = 1

x – y + z = 2

2x – y = 0

The given system can be written in matrix form as:

$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

AX = B

Now, $|A| = 0 - 2\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$

= 0 + 4 – 1

= 3

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 1 - 0 = 1$

$C_{21} = (-1)^{2+1} 1 - 2 = 1$

$C_{31} = (-1)^{3+1} 0 + 1 = 1$

$C_{12} = (-1)^{1+2} - 2 - 0 = 2$

$C_{22} = (-1)^{2+1} - 1 - 0 = -1$

$C_{32} = (-1)^{3+1}\ 0 - 2 = 2$

$C_{13} = (-1)^{1+2}\ 4 - 0 = 4$

$C_{23} = (-1)^{2+1}\ 2 - 0 = -2$

$C_{33} = (-1)^{3+1} - 1 + 2 = 1$

Adj A = $\begin{bmatrix}1 & 2 & 4\\1 & -1 & -2\\1 & 2 & 1\end{bmatrix}^T$

= $\begin{bmatrix}1 & 1 & 1\\2 & -1 & 2\\4 & -2 & 1\end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

Now, $X = A^{-1}B = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\2 & -1 & 2\\4 & -2 & 1\end{bmatrix}\begin{bmatrix}2\\0\\1\end{bmatrix}$

(xi) Given 8x + 4y + 3z = 18

2x + y + z = 5

x + 2y + z = 5

The given system can be written in matrix form as:

$$\begin{bmatrix}8 & 4 & 3\\2 & 1 & 1\\1 & 2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}18\\5\\5\end{bmatrix}$$

AX = B

Now, $|A| = 8\begin{vmatrix}1 & 1\\2 & 1\end{vmatrix} - 4\begin{vmatrix}2 & 1\\1 & 1\end{vmatrix} + 3\begin{vmatrix}2 & 1\\1 & 2\end{vmatrix}$

= 8(– 1) – 4(1) + 3(3)

= – 8 – 4 + 9

= – 3

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 1 - 2 = -1$

$C_{21} = (-1)^{2+1}\ 4 - 6 = 2$

$C_{31} = (-1)^{3+1}\ 4 - 3 = 1$

$C_{12} = (-1)^{1+2}\ 2 - 1 = -1$

$C_{22} = (-1)^{2+1}\ 8 - 3 = 5$

$C_{32} = (-1)^{3+1}\ 8 - 6 = -2$

$C_{13} = (-1)^{1+2}\ 4 - 1 = 3$

$C_{23} = (-1)^{2+1}\ 16 - 4 = -12$

$C_{33} = (-1)^{3+1}\ 8 - 8 = 0$

Adj A = $\begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T$

= $\begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{-3}\begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}\begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix} -18 + 10 + 5 \\ -18 + 25 - 10 \\ 54 - 60 + 0 \end{bmatrix}$

$X = \frac{1}{-3}\begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

Hence, X = 1, Y = 1 and Z = 2

(xii) Given x + y + z = 6

x + 2z = 7

3x + y + z = 12

The given system can be written in matrix form as:

$$\begin{bmatrix}1 & 1 & 1\\1 & 0 & 2\\3 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\17\\12\end{bmatrix}$$

A X = B

Now, $|A| = 1\begin{vmatrix}0 & 2\\1 & 1\end{vmatrix} - 1\begin{vmatrix}1 & 2\\3 & 1\end{vmatrix} + 1\begin{vmatrix}1 & 0\\3 & 1\end{vmatrix}$

= 1(– 2) – 1(1 – 6) + 1(1)

= – 2 + 5 + 1

= 4

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 0 - 2 = -2$

$C_{21} = (-1)^{2+1}\ 1 - 1 = 0$

$C_{31} = (-1)^{3+1}\ 2 - 0 = 2$

$C_{12} = (-1)^{1+2}\ 1 - 6 = 5$

$C_{22} = (-1)^{2+1}\ 1 - 3 = -2$

$C_{32} = (-1)^{3+1}\ 2 - 1 = -1$

$C_{13} = (-1)^{1+2}\ 1 - 0 = 1$

$C_{23} = (-1)^{2+1}\ 1 - 3 = 2$

$C_{33} = (-1)^{3+1}\ 0 - 1 = -1$

$$X = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Hence, X = 3, Y = 1 and Z = 2

(xiii) Given (2/x) + (3/y) + (10/z) = 4,

(4/x) – (6/y) + (5/z) = 1,

(6/x) + (9/y) – (20/z) = 2, x, y, z ≠ 0

The given system can be written in matrix form as:

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

AX = B

Now,

|A| = 2(75) – 3(– 110) + 10(72)

= 150 + 330 + 720

= 1200

So, the above system has a unique solution, given by

X = A $^{-1}$B

Cofactors of A are

$C_{11} = (-1)^{1+1}$ 120 – 45 = 75

$C_{21} = (-1)^{2+1}$ – 60 – 90 = 150

$C_{31} = (-1)^{3+1}$ 15 + 60 = 75

$C_{12} = (-1)^{1+2}$ – 80 – 30 = 110

$C_{22} = (-1)^{2+1} - 40 - 60 = -100$

$C_{32} = (-1)^{3+1}\ 10 - 40 = 30$

$C_{13} = (-1)^{1+2}\ 36 + 36 = 72$

$C_{23} = (-1)^{2+1}\ 18 - 18 = 0$

$C_{33} = (-1)^{3+1} - 12 - 12 = -24$

$$\text{Adj } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$X = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Hence, X = 2, Y = 3 and Z = 5

(xiv) Given x – y + 2z = 7

3x + 4y – 5z = -5

2x – y + 3z = 12

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

A X = B

Now,

|A| = 1(12 – 5) + 1(9 + 10) + 2(– 3 – 8)

= 7 + 19 – 22

= 4

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}$ 12 – 5 = 7

$C_{21} = (-1)^{2+1}$ – 3 + 2 = 1

$C_{31} = (-1)^{3+1}$ 5 – 8 = – 3

$C_{12} = (-1)^{1+2}$ 9 + 10 = – 19

$C_{22} = (-1)^{2+1}$ 3 – 4 = – 1

$C_{32} = (-1)^{3+1}$ – 5 – 6 = 11

$C_{13} = (-1)^{1+2}$ – 3 – 8 = – 11

$C_{23} = (-1)^{2+1}$ – 1 + 2 = – 1

$C_{33} = (-1)^{3+1}$ 4 + 3 = 7

Adj A = $\begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & -7 \end{bmatrix}^T$

= $\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

Hence, X = 2, Y = 1 and Z = 3

3. Show that each one of the following systems of linear equations is consistent and also find their solutions:

(i) 6x + 4y = 2

9x + 6y = 3

(ii) 2x + 3y = 5

6x + 9y = 15

(iii) 5x + 3y + 7z = 4

3x + 26y + 2z = 9

7x + 2y + 10z = 5

(v) x + y + z = 6

x + 2y + 3z = 14

x + 4y + 7z = 30

(vi) 2x + 2y – 2z = 1

4x + 4y – z = 2

6x + 6y + 2z = 3

Solution:

(i) Given 6x + 4y = 2

9x + 6y = 3

The above system of equations can be written as

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ Or } AX = B$$

Where $A = \begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 36 – 36 = 0

So, A is singular, Now X will be consistence if (Adj A) x B = 0

$C_{11} = (-1)^{1+1}\, 6 = 6$

$C_{12} = (-1)^{1+2}\, 9 = -9$

$C_{21} = (-1)^{2+1}\, 4 = -4$

$C_{22} = (-1)^{2+2}\, 6 = 6$

Also, adj A = $\begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}^T$

= $\begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

= $\begin{bmatrix} 12 - 12 \\ -18 + 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Thus, AX = B will be infinite solution,

Let y = k

Hence, 6x = 2 – 4k or 9x = 3 – 6k

X = $\frac{1-2k}{3}$

Hence, X = $\frac{1-2k}{3}$, Y = k

(ii) Given 2x + 3y = 5

6x + 9y = 15

The above system of equations can be written as

$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ 15 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 18 – 18 = 0

So, A is singular,

Now X will be consistence if (Adj A) x B = 0

$C_{11} = (-1)^{1+1}\, 9 = 9$

$C_{12} = (-1)^{1+2}\, 6 = -6$

$C_{21} = (-1)^{2+1}\, 3 = -3$

$C_{22} = (-1)^{2+2}\, 2 = 2$

Also, adj A = $\begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T$

= $\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$

(Adj A).B = $\left[\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\right]\begin{bmatrix} 5 \\ 15 \end{bmatrix}$

= $\begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Thus, AX = B will be infinite solution,

Let y = k

Hence,

2x = 5 – 3k or X = $\frac{5-3k}{2}$

x = 15 – 9k or X = $\frac{5-3k}{2}$

Hence, X = $\frac{5-3k}{2}$, Y = k

(iii) Given 5x + 3y + 7z = 4

3x + 26y + 2z = 9

7x + 2y + 10z = 5

|A| = 5(260 – 4) – 3(30 – 14) + 7(6 – 182)

= 5(256) – 3(16) + 7(176)

|A| = 0

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

(Adj A) x B≠0 or (Adj A) x B = 0

Cofactors of A are

$C_{11} = (-1)^{1+1}$ 260 – 4 = 256

$C_{21} = (-1)^{2+1}$ 30 – 14 = – 16

$C_{31} = (-1)^{3+1}$ 6 – 182 = – 176

$C_{12} = (-1)^{1+2}$ 30 – 14 = – 16

$C_{22} = (-1)^{2+1}$ 50 – 49 = 1

$C_{32} = (-1)^{3+1}$ 10 – 21 = 11

$C_{13} = (-1)^{1+2}$ 6 – 182 = – 176

$C_{23} = (-1)^{2+1}$ 10 – 21 = 11

$C_{33} = (-1)^{3+1}$ 130 – 9 = 121

$$\text{Adj } A = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}^T$$

$$= \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, AX = B has infinite many solution

Let z = k

Then, 5x + 3y = 4 – 7k

3x + 26y = 9 – 2k

This can be written as

$$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4-7k \\ 9-2k \end{bmatrix}$$

$|A| = 121$

Adj A = $\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$

Now, $X = A^{-1}B = \frac{1}{|A|}\text{Adj } A \times B$

$= \frac{1}{121}\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 4-7k \\ 9-2k \end{bmatrix}$

$= \frac{1}{121}\begin{bmatrix} 77-176k \\ 11k + 33 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{7-16k}{11} \\ \frac{k + 3}{11} \end{bmatrix}$$

There values of x, y and z satisfy the third equation

(v) Given x + y + z = 6

x + 2y + 3z = 14

x + 4y + 7z = 30

This can be written as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

|A| = 1(2) – 1(4) + 1(2)

= 2 – 4 + 2

|A| = 0

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

(Adj A) x B≠0 or (Adj A) x B = 0

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 14 - 12 = 2$

$C_{21} = (-1)^{2+1}\ 7 - 4 = -3$

$C_{31} = (-1)^{3+1}\ 3 - 2 = 1$

$C_{12} = (-1)^{1+2}\ 7 - 3 = -4$

$C_{22} = (-1)^{2+1}\ 7 - 1 = 6$

$C_{32} = (-1)^{3+1}\ 3 - 1 = 2$

$C_{13} = (-1)^{1+2}\ 4 - 2 = 2$

$C_{23} = (-1)^{2+1}\ 4 - 1 = -3$

$C_{33} = (-1)^{3+1}\ 2 - 1 = 1$

$$\text{Adj } A = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 6 & -3 \\ 1 & -2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -3 & 1 \\ -4 & 1 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 1 & -2 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, AX = B has infinite many solution

Let z = k

Then, x + y = 6 – k

x + 2y = 14 – 3k

This can be written as:

$$\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6-k\\14-3k\end{bmatrix}$$

$|A| = 1$

Adj A = $\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}$

Now, $X = A^{-1}B = \frac{1}{|A|}\text{Adj } A \times B$

$= \frac{1}{1}\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}6-k\\14-3k\end{bmatrix}$

$= \frac{1}{1}\begin{bmatrix}12-2k-14+3k\\-6+k+14-3k\end{bmatrix}$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-2+k\\8-2k\end{bmatrix}$$

There values of x, y and z satisfy the third equation

Hence, x = k – 2, y = 8 – 2k, z = k

(vi) Given x + y + z = 6

x + 2y + 3z = 14

x + 4y + 7z = 30

This can be written as

$$\begin{bmatrix}2 & 2 & -2\\4 & 4 & -1\\6 & 6 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$|A| = 2(14) – 2(14) – 2(0)$

$|A| = 0$

So, A is singular. Thus, the given system is either inconsistent or it is consistent

with infinitely many solution according to as:

$(\text{Adj } A) \times B \neq 0$ or $(\text{Adj } A) \times B = 0$

Cofactors of A are:

$C_{11} = (– 1)^{1 + 1} 8 + 6 = 14$

$C_{21} = (– 1)^{2 + 1} 4 + 12 = – 16$

$C_{31} = (– 1)^{3 + 1} – 2 + 8 = 6$

$C_{12} = (– 1)^{1 + 2} 8 + 6 = – 14$

$C_{22} = (– 1)^{2 + 1} 4 + 12 = 16$

$C_{32} = (– 1)^{3 + 1} – 2 + 8 = – 6$

$C_{13} = (– 1)^{1 + 2} 24 – 24 = 0$

$C_{23} = (– 1)^{2 + 1} 12 – 12 = 0$

$C_{33} = (– 1)^{3 + 1} 8 – 8 = 0$

Adj A = $\begin{bmatrix} 14 & -14 & 6 \\ -16 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}^T$

= $\begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

Adj A x B = $\begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, AX = B has infinite many solution

Let z = k

Then, 2x + 2y = 1 + 2k

4x + 4y = 2 + k

This can be written as:

$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 + 2k \\ 2 + k \end{bmatrix}$

Hence, |A| = 0 z = 0

Hence, the given equation doesn't satisfy.

4. Show that each one of the following systems of linear equations is consistent:

(i) 2x + 5y = 7

6x + 15y = 13

(ii) 2x + 3y = 5

6x + 9y = 10

(iii) 4x – 2y = 3

6x – 3y = 5

(iv) 4x – 5y – 2z = 2

5x – 4y + 2z = -2

2x + 2y + 8z = -1

(v) 3x – y – 2z = 2

2y – z = -1

3x – 5y = 3

(vi) x + y – 2z = 5

x – 2y + z = -2

-2x + y + z = 4

Solution:

(i) Given 2x + 5y = 7

6x + 15y = 13

The above system of equations can be written as

$\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}$ B = $\begin{bmatrix} 7 \\ 13 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 30 – 30 = 0

So, A is singular,

Now X will be consistence if (Adj A) x B = 0

$C_{11} = (-1)^{1+1}\ 15 = 15$

$C_{12} = (-1)^{1+2}\ 6 = -6$

$C_{21} = (-1)^{2+1}\ 5 = -5$

$C_{22} = (-1)^{2+2}\, 2 = 2$

Also, adj A = $\begin{bmatrix} 15 & -6 \\ -5 & 2 \end{bmatrix}^T$

$= \begin{bmatrix} 15 & -5 \\ -5 & 2 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 15 & -5 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 13 \end{bmatrix}$

$= \begin{bmatrix} 105 - 65 \\ -35 + 26 \end{bmatrix} = \begin{bmatrix} 40 \\ -16 \end{bmatrix}$

$\neq 0$

Hence, the given system is inconsistent.

(ii) Given 2x + 3y = 5

6x + 9y = 10

The above system of equations can be written as

$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ 10 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 18 – 18 = 0

So, A is singular,

Now X will be consistence if (Adj A) x B = 0

$C_{11} = (-1)^{1+1}\, 9 = 9$

$C_{12} = (-1)^{1+2}\, 6 = -6$

$C_{21} = (-1)^{2+1}\, 3 = -3$

$C_{22} = (-1)^{2+2}\, 2 = 2$

Also, adj A = $\begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T$

= $\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

= $\begin{bmatrix} 45 - 30 \\ -30 + 20 \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \end{bmatrix} \neq 0$

Hence, the given system is inconsistent.

(iii) Given 4x – 2y = 3

6x – 3y = 5

The above system of equations can be written as

$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = – 12 + 12 = 0

So, A is singular,

Now X will be consistence if (Adj A) x B = 0

$C_{11} = (-1)^{1+1} - 3 = -3$

$C_{12} = (-1)^{1+2}\, 6 = -6$

$C_{21} = (-1)^{2+1} - 2 = 2$

$C_{22} = (-1)^{2+2}\, 4 = 4$

Also, adj A = $\begin{bmatrix} -3 & -2 \\ -6 & 4 \end{bmatrix}^T$

= $\begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

= $\begin{bmatrix} -9 + 10 \\ -18 + 20 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Hence, the given system is inconsistent.

(iv) Given 4x – 5y – 2z = 2

5x – 4y + 2z = -2

2x + 2y + 8z = -1

$$\begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

|A| = 4(– 36) + 5(36) – 2(18)

|A| = 0

Cofactors of A are:

$C_{11} = (-1)^{1+1} - 32 - 4 = -36$

$C_{21} = (-1)^{2+1} - 40 + 4 = -36$

$C_{31} = (-1)^{3+1} - 10 - 8 = -18$

$C_{12} = (-1)^{1+2}\ 40 - 4 = -36$

$C_{22} = (-1)^{2+1}\ 32 + 4 = 36$

$C_{32} = (-1)^{3+1}\ 8 + 10 = -18$

$C_{13} = (-1)^{1+2}\ 10 + 8 = 18$

$C_{23} = (-1)^{2+1}\ 8 + 10 = -18$

$C_{33} = (-1)^{3+1} - 16 + 25 = 9$

$$\text{Adj } A = \begin{bmatrix} -36 & -34 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}^T$$

$$= \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -72 - 72 + 18 \\ -72 - 72 + 18 \\ 36 + 36 - 9 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the above system is inconsistent.

(v) Given $3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$|A| = 3(-5) + 1(3) - 2(-6)$

$|A| = 0$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 0 - 5 = -5$

$C_{21} = (-1)^{2+1}\ 0 - 10 = 10$

$C_{31} = (-1)^{3+1}\ 1 + 4 = 5$

$C_{12} = (-1)^{1+2}\ 0 + 3 = -3$

$C_{22} = (-1)^{2+1}\ 0 + 6 = 6$

$C_{32} = (-1)^{3+1} - 3 - 0 = 3$

$C_{13} = (-1)^{1+2}\ 0 - 6 = -6$

$C_{23} = (-1)^{2+1} - 15 + 3 = 12$

$C_{33} = (-1)^{3+1}\ 6 - 0 = 6$

$$\text{Adj } A = \begin{bmatrix} -5 & 3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10 - 10 + 15 \\ 6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the above system is inconsistent.

(vi) Given $x + y - 2z = 5$

$x - 2y + z = -2$

-2x + y + z = 4

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$|A| = 1(-3) - 1(3) - 2(-3) = -3 - 3 + 6$

$|A| = 0$

Cofactors of A are:

$C_{11} = (-1)^{1+1} - 2 - 1 = -3$

$C_{21} = (-1)^{2+1}\ 1 + 2 = -3$

$C_{31} = (-1)^{3+1}\ 1 - 4 = -3$

$C_{12} = (-1)^{1+2}\ 1 + 2 = -3$

$C_{22} = (-1)^{2+1}\ 1 - 4 = -3$

$C_{32} = (-1)^{3+1}\ 1 + 2 = -3$

$C_{13} = (-1)^{1+2}\ 1 - 4 = -3$

$C_{23} = (-1)^{2+1}\ 1 + 2 = -3$

$C_{33} = (-1)^{3+1} - 2 - 1 = -3$

Adj A = $\begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}^T$

= $\begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$

Adj A x B = $\begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$

= $\begin{bmatrix} -15 + 6 - 12 \\ -15 + 6 - 12 \\ -15 + 6 - 12 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Hence, the above system is inconsistent.

$$5.\ If A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} and\ B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & 1 & -5 \end{bmatrix} are\ two\ square\ matrices.$$
$$Find\ AB\ and\ hence\ solve\ the\ system\ of\ linear\ equations:$$

x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

Solution:

A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

AB = $\begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$

AB = $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Now, we can see that it is AB = 6I. Where I is the unit Matrix

Or, $A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix}$$

$= \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$

$X = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

Hence, x = 2, y = – 1 and z = 4

$$6. If\ A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix},\ find\ A^{-1}\ and\ hence\ solve\ the\ system\ of\ linear\ equations:$$

2x – 3y + 5z = 11, 3x + 2y – 4z = -5, x + y – 2z = -3.

Solution:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$|A| = 2(0) + 3(-2) + 5(1)$

$= -1$

Now, the cofactors of A

$C_{11} = (-1)^{1+1} - 4 + 4 = 0$

$C_{21} = (-1)^{2+1}\ 6 - 5 = -1$

$C_{31} = (-1)^{3+1}\ 12 - 10 = 2$

$C_{12} = (-1)^{1+2} - 6 + 4 = 2$

$C_{22} = (-1)^{2+1} - 4 - 5 = -9$

$C_{32} = (-1)^{3+1} - 8 - 15 = 23$

$C_{13} = (-1)^{1+2}\ 3 - 2 = 1$

$C_{23} = (-1)^{2+1}\ 2 + 3 = -5$

$C_{33} = (-1)^{3+1}\ 4 + 9 = 13$

Adj A = $\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Now the given equation can be written as:

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

A X = B

Or, $X = A^{-1}B$

$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22+45+69 \\ -11-25+39 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Hence, x = 1, y = 2 and z = 3

$7. Find\ A^{-2},\ if\ A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}.\ Hence\ solve\ the\ following\ system\ of\ linear\ equations:$

x + 2y + 5z = 10, x – y – z = -2, 2x + 3y – z = -11.

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

$|A| = 1(1 + 3) + 2(– 1 + 2) + 5(3 + 2)$

$= 4 + 2 + 25$

$= 27$

Now, the cofactors of A

$C_{11} = (– 1)^{1 + 1}\ 1 + 3 = 4$

$C_{21} = (– 1)^{2 + 1} – 2 – 15 = 17$

$C_{31} = (– 1)^{3 + 1} – 2 + 5 = 3$

$C_{12} = (– 1)^{1 + 2} – 1 + 2 = – 1$

$C_{22} = (– 1)^{2 + 1} – 1 – 10 = – 11$

$C_{32} = (– 1)^{3 + 1} – 1 – 5 = 6$

$C_{13} = (– 1)^{1 + 2}\ 3 + 2 = 5$

$C_{23} = (– 1)^{2 + 1}\ 3 – 4 = 1$

$C_{33} = (– 1)^{3 + 1} – 1 – 2 = – 3$

Adj A = $\begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T = \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}\begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 40 - 34 - 33 \\ -10 + 22 - 66 \\ 50 - 2 + 33 \end{bmatrix}$$

$X = \frac{1}{27}\begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix}$

$X = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$

Hence, $x = -1$, $y = -2$ and $z = 3$

$8.\,(i)\,If\,A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix},\ find\,A^{-1}.\,Using\,A^{-1},\ solve\,the system\,of\,linear\,equations:$

$x-2y=10, 2x+y+3z=8, -2y+z=7$

Solution:

Given

A = $\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$

|A| = 1(1 + 6) + 2(2 – 0) + 0

= 7 + 4

= 11

Now, the cofactors of A

$C_{11} = (-1)^{1+1}\ 1 + 6 = 7$

$C_{21} = (-1)^{2+1} - 2 - 0 = 2$

$C_{31} = (-1)^{3+1} - 6 - 0 = -6$

$C_{12} = (-1)^{1+2}\ 2 - 0 = -2$

$C_{22} = (-1)^{2+1}\ 1 - 0 = 1$

$C_{32} = (-1)^{3+1}\ 3 - 0 = -3$

$C_{13} = (-1)^{1+2} - 4 - 0 = -4$

$C_{23} = (-1)^{2+1} - 2 - 0 = 2$

$C_{33} = (-1)^{3+1}\ 1 + 4 = 5$

Adj A = $\begin{bmatrix} 7 & 2 & -4 \\ -2 & 1 & -3 \\ -6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{11}\begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}$

Now the given equation can be written as:

$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{11}\begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$

$X = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$

$X = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$

Hence, x = 4, y = – 3 and z = 1

$(ii)\ A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix},\ find\ A^{-1}\ and\ hence\ solve\ the system\ of\ linear\ equations:$

$3x - 4y + 2z = -1, 2x + 3y + 5z = 7, x + z = 2$

Solution:

Given

$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

|A| = 3(3 – 0) + 4(2 – 5) + 2(0 – 3)

= 9 – 12 – 6

= – 9

Now, the cofactors of A

$C_{11} = (-1)^{1+1}\ 3 - 0 = 3$

$C_{21} = (-1)^{2+1} - 4 - 0 = 4$

$C_{31} = (-1)^{3+1} - 20 - 6 = -26$

$C_{12} = (-1)^{1+2}\ 2 - 5 = 3$

$C_{22} = (-1)^{2+1}\ 3 - 2 = 1$

$C_{32} = (-1)^{3+1}\ 15 - 4 = -11$

$C_{13} = (-1)^{1+2}\ 0 - 3 = -3$

$C_{23} = (-1)^{2+1}\ 0 + 4 = -4$

$C_{33} = (-1)^{3+1}\ 9 + 8 = 17$

Adj A = $\begin{bmatrix} 3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -4 & 27 \end{bmatrix}^T = \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{-9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$

Now the given equation can be written as:

$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{-9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & 11 \\ -3 & -4 & 17 \end{bmatrix}\begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-9}\begin{bmatrix} -3 + 28 - 52 \\ 21 + 7 + 22 \\ 3 - 28 + 34 \end{bmatrix}$

$X = \frac{1}{-9}\begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix}$

$X = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

Hence, x = 3, y = 2 and z = – 1

$$(iii)\ A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix},\ and\ B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}\ find\ AB.\ Hence\ solve\ thesystem\ of\ linear\ equations:$$

$$x-2y=10, 2x+y+3z=8\ and\ -2y+z=7$$

Solution:

A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

AB = $\begin{bmatrix} 7+4-0 & 2-2+0 & -6+6+0 \\ 14-2-12 & 4+1+6 & -12-3+15 \\ 0-4+4 & 0-2+2 & 0+6+5 \end{bmatrix}$

AB = $\begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Now, we can see that it is AB = 11I. Where I is the unit Matrix

Or, A^{-1} = $\frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

A X = B

Or, X = A^{-1}B

= $\frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70+16-42 \\ -20+8-21 \\ -40+16+35 \end{bmatrix}$$

$= \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$

$X = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$

Hence, x = 4, y = – 3 and z = 1

$(iv)\ If\ A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix},\ find\ A^{-1}. Using\ A^{-1}\ solve\ the\ system\ of\ linearequations:$

$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$

Solution:

Given

$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

|A| = 1(– 1 – 1) – 2(– 2 – 0) + 0

= – 2 + 4

= 2

Now, the cofactors of A

$C_{11} = (-1)^{1+1} - 1 - 1 = -2$

$C_{21} = (-1)^{2+1}\ 2 - 0 = 2$

$C_{31} = (-1)^{3+1} - 2 - 0 = -2$

$C_{12} = (-1)^{1+2}\ 2 - 0 = -2$

$C_{22} = (-1)^{2+1}\ 1 - 0 = 1$

$C_{32} = (-1)^{3+1} - 1 - 0 = 1$

$C_{13} = (-1)^{1+2} - 2 - 0 = -2$

$C_{23} = (-1)^{2+1} - 1 - 0 = 1$

$C_{33} = (-1)^{3+1} - 1 + 4 = 3$

$$\text{Adj } A = \begin{bmatrix} -2 & -2 & -2 \\ 2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} -2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|}$ adj A

$A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{2}\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 10-16+0 \\ 20-8-7 \\ 0-16+7 \end{bmatrix}$$

$X = \frac{1}{2}\begin{bmatrix} -6 \\ 5 \\ -9 \end{bmatrix}$

Hence, x = – 3, y = 2.5 and z = – 4.5

$(v)\ Given\ A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$
$find\ BA\ and\ Use\ this\ to\ solve\ the\ system\ of\ linear\ equations\ y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17$

Solution:

Given

B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$

BA = $\begin{bmatrix} 2+4-0 & 2-2+0 & -4+4+0 \\ -4-12+16 & 4+6-4 & -8-12+20 \\ 0-4+8 & 0-2+2 & 0-4+10 \end{bmatrix}$

BA = $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Now, we can see that it is BA = 6I. Where I is the unit Matrix

Or, B^{-1} = $\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 17 \end{bmatrix}$$

A X = B

Or, X = B^{-1}A

= $\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \\ 17 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 14+6-68 \\ -28+6-68 \\ 28-3+85 \end{bmatrix}$$

$= \frac{1}{6}\begin{bmatrix} -48 \\ -90 \\ 110 \end{bmatrix}$

$X = \begin{bmatrix} -8 \\ -15 \\ \frac{110}{6} \end{bmatrix}$

Hence, $x = -8$, $y = -15$ and $z = \frac{110}{6}$

9. The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 1. By adding second and third number to five times the first number, we get 6. Find the three numbers by using matrices.

Solution:

Let the numbers are x, y, z

$x + y + z = 2$

…… (i)

Also, $2y + (x + z) + 1$

$x + 2y + z = 1$ …… (ii)

Again,

$x + z + 5(x) = 6$

$5x + y + z = 6$ …… (iii)

$A X = B$

$|A| = 1(1) – 1(– 4) + 1(– 9)$

$= 1 + 4 – 9$

$= – 4$

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(2-1) = 1$

$C_{12} = (-1)^{1+2}(1-5) = 4$

$C_{13} = (-1)^{1+3}(1-10) = -9$

$C_{21} = (-1)^{2+1}(1-1) = 0$

$C_{22} = (-1)^{2+2}(1-5) = -4$

$C_{23} = (-1)^{2+3}(1-5) = 4$

$C_{31} = (-1)^{3+1}(1-2) = -1$

$C_{32} = (-1)^{3+2}(1-1) = 0$

$C_{33} = (-1)^{3+3}(2-1) = 1$

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\, B$$

$$\text{Adj } A = \begin{bmatrix} 1 & 4 & -9 \\ 0 & -4 & 4 \\ -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$$X = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$X = \frac{1}{-4}\begin{bmatrix} 2-6 \\ 8-4 \\ -18+4+6 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

$$\text{Hence, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

10. An amount of ₹10,000 is put into three investments at the rate of 10, 12 and 15% per annum. The combined incomes are ₹1310 and the combined income of first and second

investment is ₹ 190 short of the income from the third. Find the investment in each using matrix method.

Solution:

Let the numbers are x, y, and z

x + y + z = 10,000 …… (i)

Also,

0.1x + 0.12y + 0.15z = 1310 …… (ii)

Again,

0.1x + 0.12y – 0.15z = – 190 …… (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.12 & 0.15 \\ 0.1 & 0.12 & -0.15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

A X = B

|A| = 1(– 0.036) – 1(– 0.03) + 1(0)

= – 0.006

Hence, the unique solution given by x = $A^{-1}B$

C_{11} = – 0.036

C_{12} = 0.27

C_{13} = 0

C_{21} = 0.27

C_{22} = – 0.25

C_{23} = – 0.02

C_{31} = 0.03

C_{32} = – 0.05

C_{33} = 0.02

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\, B$$

$$\text{Adj } A = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.27 & -0.25 & -0.05 \\ 0.03 & -0.02 & 0.02 \end{bmatrix}^T = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}$$

$$X = \frac{1}{-0.006}\begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}\begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

$$X = \frac{1}{-0.006}\begin{bmatrix} -12 \\ -18 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$$

Hence, x = Rs 2000, y = Rs 3000 and z = Rs 5000

Exercise 8.2 Page No: 8.20

Solve the following systems of homogeneous linear equations by matrix method:

1. 2x – y + z = 0

3x + 2y – z = 0

x + 4y + 3z = 0

Solution:

Given

2x – y + z = 0

3x + 2y – z = 0

X + 4y + 3z = 0

The system can be written as

A X = 0

Now, |A| = 2(6 + 4) + 1(9 + 1) + 1(12 – 2)

|A| = 2(10) + 10 + 10

|A| = 40 ≠ 0

Since, |A|≠ 0, hence x = y = z = 0 is the only solution of this homogeneous equation.

2. 2x – y + 2z = 0

5x + 3y – z = 0

X + 5y – 5z = 0

Solution:

Given 2x – y + 2z = 0

5x + 3y – z = 0

X + 5y – 5z = 0

The system can be written as

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 2(– 15 + 5) + 1(– 25 + 1) + 2(25 – 3)

|A| = – 20 – 24 + 44

|A| = 0

Hence, the system has infinite solutions

Let z = k

2x – y = – 2k

5x + 3y = k

$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

A X = B

$|A| = 6 + 5 = 11 \neq 0$ So, A^{-1} exist

Now adj A = $\begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$

$X = A^{-1} B = \frac{1}{|A|} (\text{adj } A)B = \frac{1}{11} \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} -2k \\ k \end{bmatrix}$

$X = \begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$

Hence, $x = \frac{-5k}{11}$, $y = \frac{12k}{11}$ and z = k

3. 3x – y + 2z = 0

4x + 3y + 3z = 0

5x + 7y + 4z = 0

Given 3x – y + 2z = 0

4x + 3y + 3z = 0

5x + 7y + 4z = 0

The system can be written as

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 3(12 – 21) + 1(16 – 15) + 2(28 – 15)

|A| = – 27 + 1 + 26

|A| = 0

Hence, the system has infinite solutions

Let z = k

3x – y = – 2k

4x + 3y = – 3k

$$\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$$

A X = B

$|A| = 9 + 4 = 13 \neq 0$ So, A^{-1} exist

Now adj A = $\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$

X = A^{-1} B = $\frac{1}{|A|}$ $(\text{adj } A)B = \frac{1}{13}\begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -2k \\ -3k \end{bmatrix}$

X = $\begin{bmatrix} \frac{-9k}{13} \\ \frac{-k}{13} \end{bmatrix}$

Hence, x = $\frac{-9k}{13}$, y = $\frac{-k}{13}$ and z = k

4. x + y – 6z = 0

x – y + 2z = 0

– 3x + y + 2z = 0

Solution:

Given x + y – 6z = 0

x – y + 2z = 0

– 3x + y + 2z = 0

The system can be written as

$$\begin{bmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 1(– 2 – 2) – 1(2 + 6) – 6(1 – 3)

|A| = – 4 – 8 + 12

|A| = 0

Hence, the system has infinite solutions

Let z = k

x + y = 6k

x – y = – 2k

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

A X = B

|A| = – 1 – 1 = – 2 ≠0 So, A^{-1} exist

Now adj A = $\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

X = A^{-1} B = $\frac{1}{|A|}(\text{adj } A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 6k \\ -2k \end{bmatrix}$

X = $\frac{1}{-2}\begin{bmatrix} -6k + 2k \\ -6k - 2k \end{bmatrix}$

X = $\begin{bmatrix} -4k \\ -8k \end{bmatrix}$

Hence, x = 2k, y = 4k and z = k

Chapterwise RD Sharma Solutions for Class 12 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.