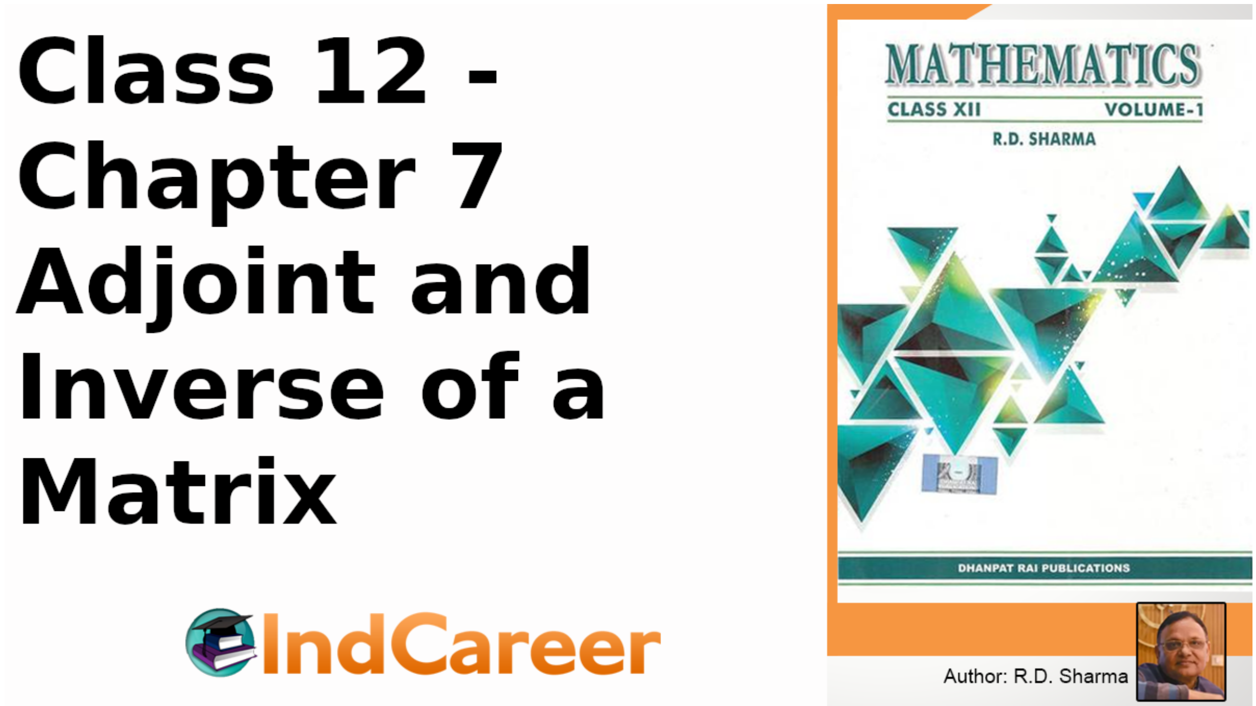


# RD Sharma Solutions for Class 12 Maths Chapter 7–Adjoint and Inverse of a Matrix

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## RD Sharma Solutions for Class 12 Maths Chapter 7–Adjoint and Inverse of a Matrix

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**Exercise 7.1 Page No: 7.22**

**1. Find the adjoint of each of the following matrices:**

$$(i)\begin{bmatrix}-3 & 5\\ 2 & 4\end{bmatrix}$$

$$(ii)\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$(iii)\begin{bmatrix}cos\alpha & sin\alpha\\ sin\alpha & cos\alpha\end{bmatrix}$$

$$(iv)\begin{bmatrix}1 & tan\frac{\alpha}{2}\\ -tan\frac{\alpha}{2} & 1\end{bmatrix}$$

**Verify that (adj A) A = |A| I = A (adj A) for the above matrices.**

**Solution:**

(i) Let

A =

$$\begin{bmatrix}-3 & 5\\ 2 & 4\end{bmatrix}$$

Cofactors of A are

$C_{11} = 4$

$C_{12} = -2$

$C_{21} = -5$

$C_{22} = -3$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}^T$

= $\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}$

Now, (adj A) A = $\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -12-10 & 20-20 \\ 6-6 & -10-12 \end{bmatrix}$

(adj A)A = $\begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

And, |A| I = $\begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (-22)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Also, A (adj A) = $\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -12-10 & 20-20 \\ 6-6 & -10-12 \end{bmatrix}$

A (adj A) = $\begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Hence, (adj A) A = |A|I = A (adj A)

(ii) Let

A =

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Therefore cofactors of A are

$C_{11}$ = d

$C_{12}$ = – c

$C_{21} = -b$

$C_{22} = a$

We know that, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

Therefore by substituting these values we get,

(adj A) = $\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T$

= $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now, (adj A) A = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ad - bc & bd - bd \\ -ac + ac & -bc + ad \end{bmatrix}$

(adj A)A = $\begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}$

And, |A|.I = $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (ad - bc) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}$

Also,

A (adj A) = $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}$

Hence, (adj A) A = |A| I = A (adj A)

(iii) Let

A =

$\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Therefore cofactors of A are

$C_{11} = \cos\alpha$

$C_{12} = -\sin\alpha$

$C_{21} = -\sin\alpha$

$C_{22} = \cos\alpha$

We know that, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}^T$

= $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Now, (adj A) A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} -\sin^2\alpha + \cos^2\alpha & \cos\alpha.\sin\alpha - \sin\alpha.\cos\alpha \\ -\cos\alpha\sin\alpha + \sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

(adj A) A = $\begin{bmatrix} \cos2\alpha & 0 \\ 0 & \cos2\alpha \end{bmatrix}$

And, |A|I = $\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $(\cos^2\alpha - \sin^2\alpha) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 0 \\ 0 & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$

= $\begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix}$

Also, A (adj A)

= $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 0 \\ 0 & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$

= $\begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix}$

Hence, (adj A) A = |A|I = A (adj A)

(iv) Let

A =

$$\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Therefore cofactors of A are

$C_{11} = 1$

$C_{12} = \tan \alpha/2$

$C_{21} = -\tan \alpha/2$

$C_{22} = 1$

We know that, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}^T$

= $\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

Now, (adj A) A = $\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & \tan\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} - \tan\frac{\alpha}{2} & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

(adj A)A = $\begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & 0 \\ 0 & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

And, |A|.I = $\begin{vmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \left(1 + \tan^2\frac{\alpha}{2}\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & 0 \\ 0 & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

Also, A (adj A) = $\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & \tan\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} - \tan\frac{\alpha}{2} & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & 0 \\ 0 & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

Hence, (adj A) A = |A|I = A (adj A)

**2. Compute the adjoint of each of the following matrices.**

$(i) \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} (ii) \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} (iv) \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

**Solution:**

(i) Let

A =

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Therefore cofactors of A are

$C_{11} = -3$

$C_{21} = 2$

$C_{31} = 2$

$C_{12} = 2$

$C_{22} = -3$

$C_{23} = 2$

$C_{13} = 2$

$C_{23} = 2$

$C_{33} = -3$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

Now, (adj A) A = $\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

= $\begin{bmatrix} -3+4+4 & -6+2+4 & -6+4+2 \\ 2-3+4 & 4-3+4 & 4-6+2 \\ 2+4-6 & 4+2-6 & 4+4-3 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Also, |A|I = $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (-3+4+4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Then, A (adj A) = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

$= \begin{bmatrix} -3+4+4 & -6+2+4 & -6+4+2 \\ 2-3+4 & 4-3+4 & 4-6+2 \\ 2+4-6 & 4+2-6 & 4+4-3 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Since, (adj A) A = |A|I = A (adj A)

(ii) Let

A =

$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Cofactors of A

$C_{11} = 2$

$C_{21} = 3$

$C_{31} = -13$

$C_{12} = -3$

$C_{22} = 6$

$C_{32} = 9$

$C_{13} = 5$

$C_{23} = -3$

$C_{33} = -1$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}^T$

adj A = $\begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$

Now, (adj A) A = $\begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

= $\begin{bmatrix} 2 + 6 + 13 & 4 + 9 - 13 & 10 + 3 - 13 \\ -3 + 12 - 9 & -6 + 18 + 9 & -15 + 6 + 9 \\ 5 - 6 + 1 & 10 - 9 - 1 & 25 - 3 - 1 \end{bmatrix}$

= $\begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$

Also, |A|I = $\begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= [1(3-1) - 2(2+1) + 5(2+3)] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= 21 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$

Then, A (adj A) = $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2-6+25 & 3+12-15 & -13+18-5 \\ 4-9+5 & 6+18-3 & -26+27-1 \\ -2-3+5 & -3+6-3 & 13+9-1 \end{bmatrix}$

$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$

Hence, (adj A) A = |A|I = A (adj A)

(iii) Let

A =

$\begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$

Therefore cofactors of A

$C_{11} = -22$

$C_{21} = 11$

$C_{31} = -11$

$C_{12} = 4$

$C_{22} = -2$

$C_{32} = 2$

$C_{13} = 16$

$C_{23} = -8$

$C_{33} = 8$

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

Now by substituting the values in above matrix we get,

= $\begin{bmatrix} -22 & 4 & 16 \\ 11 & -2 & -8 \\ -11 & 2 & 8 \end{bmatrix}^T$

adj A = $\begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}$

Now, (adj A) A = $\begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$

= $\begin{bmatrix} -44 + 44 + 0 & 22 + 22 - 44 & -66 + 55 + 11 \\ 8 - 8 + 0 & -4 - 4 + 8 & 12 - 10 - 2 \\ 32 - 32 + 0 & -16 - 16 + 32 & 48 - 40 - -8 \end{bmatrix}$

= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Now, |A|I = $\begin{vmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= [2(– 2 – 20) + 1(– 4 – 0) + 3(16 – 0)] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= (– 44 – 4 + 48) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Then, A (adj A) = $\begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}\begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}$

= $\begin{bmatrix} -44-4+48 & 22+2-24 & -22-2+24 \\ -88+8+80 & 44-4-40 & -44+4+40 \\ 0+16-16 & 0-8+8 & 0+8-8 \end{bmatrix}$

= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Hence, (adj A) A = |A|I = A (adj A)

(iv) Let

A =

$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

Therefore cofactors of A

$C_{11}$ = 3

$C_{21}$ = – 1

$C_{31}$ = 1

$C_{12}$ = – 15

$C_{22}$ = 7

$C_{32}$ = – 5

$C_{13}$ = 4

$C_{23}$ = – 2

$C_{33}$ = 2

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 3 & -15 & 4 \\ -3 & 7 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$

adj A = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$

Now, (adj A) A = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 6-5+1 & 0-1+1 & -3+0+3 \\ -30+35-5 & 0+7-5 & 15-0-15 \\ 8-10+2 & 0-2+2 & -4-0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Also, $|A|I = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= [2(3-0) + 0(15-0) - 1(5-1)] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= (6-4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Then, A (adj A) $= \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 6+0-4 & -2+0+2 & 2-0-2 \\ 15-15+0 & -5+7+0 & 5-5+0 \\ 3-15+12 & -1+7-6 & 1-5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Hence, (adj A) A = |A|I = A (adj A)

**3.** $For\ the\ matrix\ A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix},\ show\ that\ A(adj\,A) = 0$

**Solution:**

Given

A =

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$$

Therefore cofactors of A

$C_{11} = 30$

$C_{21} = 12$

$C_{31} = -3$

$C_{12} = -20$

$C_{22} = -8$

$C_{32} = 2$

$C_{13} = -50$

$C_{23} = -20$

$C_{33} = 5$

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

By substituting these values in above matrix we get,

$= \begin{bmatrix} 30 & -20 & -50 \\ 12 & -8 & -20 \\ -3 & 2 & 5 \end{bmatrix}^T$

So, adj (A) = $\begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix}$

Now, A (adj A) = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix} \begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix}$

$= \begin{bmatrix} 30 + 20 - 50 & 12 + 8 - 20 & -3 - 2 + 5 \\ 60 - 60 + 0 & 24 - 24 + 0 & -6 + 6 + 0 \\ 540 - 40 - 500 & 216 - 16 - 200 & -54 + 4 + 50 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Hence, A (adj A) = 0

$$4. \ If \ A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}, \ show \ that \ adj\, A = A$$

**Solution:**

Given

A =

$$\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

Cofactors of A

$C_{11} = -4$

$C_{21} = -3$

$C_{31} = -3$

$C_{12} = 1$

$C_{22} = 0$

$C_{32} = 1$

$C_{13} = 4$

$C_{23} = 4$

$C_{33} = 3$

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

Hence, adj A = A

$$\mathbf{5.}\ \boldsymbol{If\ A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix},\ show\ that\ adj\, A = 3A^T.}$$

**Solution:**

Given

A =

$$\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

Cofactors of A are

$C_{11}$ = – 3

$C_{21}$ = 6

$C_{31}$ = 6

$C_{12}$ = – 6

$C_{22}$ = 3

$C_{32} = -6$

$C_{13} = -6$

$C_{23} = -6$

$C_{33} = 3$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

Now, $3A^T = 3\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

Hence, adj A = $3.A^T$

$$6.\ Find\ A(adjA)\ for\ the\ matrix\ A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$$

**Solution:**

Given

A =

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$$

Cofactors of A are

$C_{11}$ = 9

$C_{21}$ = 19

$C_{31}$ = – 4

$C_{12}$ = 4

$C_{22}$ = 14

$C_{32}$ = 1

$C_{13}$ = 8

$C_{23}$ = 3

$C_{33}$ = 2

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$

Now, A adj A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}\begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$

= $\begin{bmatrix} 9-8+24 & 19-28+9 & -4-2+6 \\ 0+8-8 & 0+28-3 & 0+2-2 \\ -36+20+16 & -76+70+6 & 16+5+4 \end{bmatrix}$

= $\begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

Hence, A adj A = 25 $I_3$

**7. Find the inverse of each of the following matrices:**

$(i) \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ $(ii) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $(iii) \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$

$(iv) \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$

**Solution:**

(i) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.

Now, |A| = cos θ (cos θ) + sin θ (sin θ)

= 1

Hence, $A^{-1}$ exists.

Cofactors of A are

$C_{11} = \cos\theta$

$C_{12} = \sin\theta$

$C_{21} = -\sin\theta$

$C_{22} = \cos\theta$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^T$

= $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

$A^{-1} = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(ii) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.

Now, |A| = – 1 ≠ 0

Hence, $A^{-1}$ exists.

Cofactors of A are

$C_{11} = 0$

$C_{12} = -1$

$C_{21} = -1$

$C_{22} = 0$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}^T$

= $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(iii) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.

Now, $|A| = \frac{a + abc}{a} - bc = \frac{a + abc - abc}{a} = 1 \neq 0$

Hence, $A^{-1}$ exists.

Cofactors of A are

$C_{11} = \frac{1 + bc}{a}$

$C_{12} = -c$

$C_{21} = -b$

$C_{22} = a$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} \frac{1 + bc}{a} & -c \\ -b & a \end{bmatrix}^T$

$= \begin{bmatrix} \frac{1 + bc}{a} & -b \\ -c & a \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

$A^{-1} = \frac{1}{1}\begin{bmatrix} \frac{1 + bc}{a} & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1 + bc}{a} & -b \\ -c & a \end{bmatrix}$

**8. Find the inverse of each of the following matrices.**

$(i)\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} (ii)\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix} (iii)\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} (iv)\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$(v)\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} (vi)\begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix} (vii)\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & sin\alpha \\ 0 & sin\alpha & -cos\alpha \end{bmatrix}$

**Solution:**

**(i) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.**

**|A| =**

**= 1(6 – 1) – 2(4 – 3) + 3(2 – 9)**

**= 5 – 2 – 21**

**= – 18≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11} = 5$**

**$C_{21} = -1$**

**$C_{31} = -7$**

**$C_{12} = -1$**

**$C_{22} = -7$**

**$C_{32} = 5$**

**$C_{13} = -7$**

**$C_{23} = 5$**

**$C_{33} = -1$**

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

So, $A^{-1} = \frac{1}{(-18)} \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} \frac{-5}{18} & \frac{1}{18} & \frac{7}{18} \\ \frac{1}{18} & \frac{7}{18} & \frac{-5}{18} \\ \frac{7}{18} & \frac{-5}{18} & \frac{1}{18} \end{bmatrix}$

**(ii) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.**

**|A| =**

$$1\begin{vmatrix}-1 & -1\\ 3 & -1\end{vmatrix} - 2\begin{vmatrix}1 & -1\\ 2 & -1\end{vmatrix} + 5\begin{vmatrix}1 & -1\\ 2 & 3\end{vmatrix}$$

**= 1 (1 + 3) – 2 (– 1 + 2) + 5 (3 + 2)**

**= 4 – 2 + 25**

**= 27≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11}$ = 4**

**$C_{21}$ = 17**

**$C_{31}$ = 3**

**$C_{12}$ = – 1**

**$C_{22}$ = – 11**

**$C_{32}$ = 6**

**$C_{13}$ = 5**

**$C_{23}$ = 1**

**$C_{33}$ = – 3**

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$= \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

So, $A^{-1} = \frac{1}{(27)}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{3}{27} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{6}{27} \\ \frac{5}{27} & \frac{1}{27} & \frac{-3}{27} \end{bmatrix} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{1}{9} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9} \\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9} \end{bmatrix}$

**(iii) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.**

**|A| =**

$$2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}$$

**= 2(4 – 1) + 1(– 2 + 1) + 1(1 – 2)**

**= 6 – 2**

**= – 4≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11} = 3$**

**$C_{21} = 1$**

**$C_{31} = – 1$**

**$C_{12} = + 1$**

**$C_{22} = 3$**

**$C_{32} = 1$**

**$C_{13} = – 1$**

**$C_{23} = 1$**

**$C_{33} = 3$**

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

So, $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & \frac{-1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{-1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

**(iv) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.**

**|A| =**

$2\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0\begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$

**= 2(3 – 0) – 0 – 1(5)**

**= 6 – 5**

**= 1≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11} = 3$**

**$C_{21} = -1$**

**$C_{31} = 1$**

**$C_{12} = -15$**

**$C_{22} = 6$**

**$C_{32} = -5$**

**$C_{13} = 5$**

**$C_{23} = -2$**

**$C_{33} = 2$**

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

So, $A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

**(v) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.**

**|A| =**

$$0\begin{vmatrix} -3 & 0 \\ -3 & 4 \end{vmatrix} - 1\begin{vmatrix} 4 & 4 \\ 3 & 4 \end{vmatrix} - 1\begin{vmatrix} 4 & -3 \\ 3 & -3 \end{vmatrix}$$

**= 0 – 1 (16 – 12) – 1 (– 12 + 9)**

**= – 4 + 3**

**= – 1≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11}$ = 0**

**$C_{21}$ = – 1**

**$C_{31}$ = 1**

**$C_{12}$ = – 4**

**$C_{22}$ = 3**

**$C_{32}$ = – 4**

**$C_{13}$ = – 3**

**$C_{23}$ = 3**

**$C_{33}$ = – 4**

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 0 & -4 & -3 \\ -1 & 3 & 3 \\ 1 & -4 & -4 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

So, $A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

**(vi) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.**

**|A| =**

$$0\begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - 0\begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1\begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix}$$

**= 0 – 0 – 1(– 12 + 8)**

**= 4≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11}$ = – 8**

**$C_{21}$ = 4**

**$C_{31}$ = 4**

**$C_{12}$ = 11**

**$C_{22}$ = – 2**

**$C_{32}$ = – 3**

**$C_{13}$ = – 4**

**$C_{23}$ = 0**

**$C_{33}$ = 0**

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

So, $A^{-1} = \frac{1}{4}\begin{bmatrix} 8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 2 & 1 & 1 \\ \frac{11}{4} & \frac{-1}{2} & \frac{-3}{4} \\ -1 & 0 & 0 \end{bmatrix}$

**(vii) The criteria of existence of inverse matrix is the determinant of a given matrix should not equal to zero.**

**|A| =**

$$1\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix}$$

**– 0 + 0**

**= – (cos² α – sin² α)**

**= – 1≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11}$ = – 1**

**$C_{21}$ = 0**

**$C_{31}$ = 0**

**$C_{12}$ = 0**

**$C_{22}$ = – cos α**

**$C_{32}$ = – sin α**

**$C_{13}$ = 0**

**$C_{23}$ = – sin α**

**$C_{33}$ = cos α**

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A

So, $A^{-1} = \frac{1}{-1}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

**9. Find the inverse of each of the following matrices and verify that $A^{-1}A = I_3$.**

$(i)\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} (ii)\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

**Solution:**

**(i) We have**

**|A| =**

$1\begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix}$

**= 1(16 – 9) – 3(4 – 3) + 3(3 – 4)**

**= 7 – 3 – 3**

**= 1≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11} = 7$**

**$C_{21} = -3$**

**$C_{31} = -3$**

**$C_{12} = -1$**

**$C_{22} = 1$**

**$C_{32} = 0$**

**$C_{13} = -1$**

**$C_{23} = 0$**

**$C_{33} = 1$**

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A = $\frac{1}{1}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Also, $A^{-1}A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

= $\begin{bmatrix} 7-3-3 & 21-12-9 & 21-9-12 \\ -1+1+0 & -3+4+0 & -3+3+0 \\ -1+0+1 & -3+0+3 & -3+0+4 \end{bmatrix}$

= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence, $A^{-1}A = I_3$

**(ii) We have**

**|A| =**

Now, $A^{-1} = \frac{1}{|A|}$ adj A = $\frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

Also, $A^{-1}.A = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2+3-3 & 3+4-7 & 1+1-2 \\ -6+3+3 & -9+4+7 & -3+1+2 \\ 18-15-3 & 27-20-7 & 9-5-2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence, $A^{-1}.A = I_3$

**= 2(8 – 7) – 3(6 – 3) + 1(21 – 12)**

**= 2 – 9 + 9**

**= 2≠ 0**

**Hence, $A^{-1}$ exists**

**Cofactors of A are**

**$C_{11}$ = 1**

**$C_{21}$ = 1**

**$C_{31}$ = – 1**

**$C_{12}$ = – 3**

**$C_{22}$ = 1**

**$C_{32}$ = 1**

**$C_{13}$ = 9**

**$C_{23}$ = – 5**

**$C_{33} = -1$**

We know that adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj A = $\frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

Also, $A^{-1}.A = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2+3-3 & 3+4-7 & 1+1-2 \\ -6+3+3 & -9+4+7 & -3+1+2 \\ 18-15-3 & 27-20-7 & 9-5-2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence, $A^{-1}.A = I_3$

**10. For the following pair of matrices verify that $(AB)^{-1} = B^{-1}A^{-1}$.**

$$(i) A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \ and \ B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} (ii) A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \ and \ B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

**Solution:**

**(i) Given**

A = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$,

$|A| = 1 \neq 0$

Then, adj A = $\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

B = $\begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$,

$|B| = -10 \neq 0$

Then, adj B = $\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}$

$B^{-1} = \dfrac{\text{adj } B}{|B|} = -\frac{1}{10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}$

Also, A.B = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 12 + 6 & 18 + 4 \\ 28 + 15 & 42 + 10 \end{bmatrix}$

AB = $\begin{bmatrix} 18 & 22 \\ 43 & 52 \end{bmatrix}$

$|AB| = 936 - 946 = -10 \neq 0$

Adj (AB) = $\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix}$

$(AB)^{-1} = \dfrac{\text{adj } AB}{|AB|} = \frac{1}{-10}\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$

Now $B^{-1}A^{-1} = \frac{1}{-10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} 10 + 42 & -4 - 18 \\ -15 - 28 & 6 + 12 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$

**Hence, $(AB)^{-1} = B^{-1}A^{-1}$**

**(ii) Given**

$|A| = 1 \neq 0$

Adj A = $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

B = $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

$|B| = 1 \neq 0$

adj B = $\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

$B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{1}\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

Also, AB = $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

= $\begin{bmatrix} 11 & 14 \\ 29 & 37 \end{bmatrix}$

$|AB| = 407 - 406 = 1 \neq 0$

And, adj (AB) = $\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

$(AB)^{-1} = \frac{\text{adj } AB}{|AB|} = \frac{1}{1}\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

= $\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

Now, $B^{-1}A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

= $\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

**Hence, $(AB)^{-1} = B^{-1}A^{-1}$**

$$11.\ Let\ A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\ and\ B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}.\ Find (AB)^{-1}$$

**Solution:**

**Given**

A = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$

|A| = 15 – 14 = 1 ≠ 0

Therefore adj A = $\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \frac{\text{adj}}{|A|} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

B = $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

|B| = 54 – 56 = – 2 ≠ 0

adj B = $\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$

$B^{-1} = \frac{\text{adjB}}{|B|} = \frac{1}{-2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$

Now, $(AB)^{-1} = B^{-1}A^{-1}$

$= \frac{1}{-2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 45 + 49 & -18 - 21 \\ -40 - 42 & 16 + 18 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$

$(AB)^{-1} = \begin{bmatrix} -47 & \frac{39}{2} \\ 41 & -17 \end{bmatrix}$

**12.** $\boldsymbol{Given\ A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix},\ compute\ A^{-1}\ and\ show\ that\ 2A^{-1} = 9I - A.}$

**Solution:**

**Given**

$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$|A| = 14 - 12 = 2 \neq 0$

adj $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

To Show: $2A^{-1} = 9I - A$

We have

L.H.S $= 2A^{-1} = 2.\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

R.H.S $= 9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

Hence, $2A^{-1} = 9I - A$

**13.** $If\ A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, *then show that* $A - 3I = 2(I + 3A^{-1})$.

**Solution:**

**Given**

A = $\begin{bmatrix}4 & 5\\2 & 1\end{bmatrix}$

$|A| = 4 - 10 = -6 \neq 0$

adj A = $\begin{bmatrix}1 & -5\\-2 & 4\end{bmatrix}$

$A^{-1} = \frac{1}{-6}\begin{bmatrix}1 & -5\\-2 & 4\end{bmatrix}$

To Show: $A - 3I = 2(I + 3A^{-1})$

We have

LHS = $A - 3I$

$= \begin{bmatrix}4 & 5\\2 & 1\end{bmatrix} - 3\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & 5\\2 & -2\end{bmatrix}$

R.H.S = $2(I + 3A^{-1}) = 2I + 6A^{-1}$

$= 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + 6\frac{1}{6}\begin{bmatrix}-1 & 5\\2 & -4\end{bmatrix}$

$= \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix} + \begin{bmatrix}-1 & 5\\2 & -4\end{bmatrix}$

$= \begin{bmatrix}1 & 5\\2 & -2\end{bmatrix}$

Hence, $A - 3I = 2(I + 3A^{-1})$

14. *Find the inverse of the matrix* $A = \begin{bmatrix}a & b\\c & \frac{1+bc}{a}\end{bmatrix}$, *and show that* $aA^{-1} = (a^2 + bc + 1)I - aA$.

**Solution:**

A = $\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$

Now, |A| = $\frac{a+abc}{a} - bc = \frac{a+abc-abc}{a} = 1 \neq 0$

Hence, $A^{-1}$ exists.

Cofactors of A are

$C_{11} = \frac{1+bc}{a}$

$C_{12} = -c$

$C_{21} = -b$

$C_{22} = a$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

Adj A = $\begin{bmatrix} \frac{1+bc}{a} & -c \\ -b & a \end{bmatrix}^T$

$= \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.adj\ A$

$A^{-1} = \frac{1}{1}\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

To show $a A^{-1} = (a^2 + bc + 1) I - aA$.

LHS = $a A^{-1}$

$= a\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

$= \begin{bmatrix} 1 + bc & -ab \\ -ac & a^2 \end{bmatrix}$

RHS = $(a^2 + bc + 1) I - a A$

$= \begin{bmatrix} a2 + bc + 1 & 0 \\ 0 & a2 + bc + 1 \end{bmatrix} - \begin{bmatrix} a^2 & ab \\ ac & 1 + bc \end{bmatrix} = \begin{bmatrix} 1 + bc & -ab \\ -ac & a^2 \end{bmatrix}$

Hence, LHS = RHS

$$15.\ Given\ A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix},\ B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}.Compute(AB)^{-1}$$

**Solution:**

**Given**

**A =**

$$\begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

**and $B^{-1}$ =**

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

**Here, $(AB)^{-1} = B^{-1} A^{-1}$**

**$|A| = -5 + 4 = -1$**

**Cofactors of A are**

**$C_{11} = -1$**

**$C_{21} = 8$**

**$C_{31} = -12$**

**$C_{12} = 0$**

**$C_{22} = 1$**

**$C_{32} = -2$**

**$C_{13} = 1$**

**$C_{23} = -10$**

**$C_{33} = 15$**

Adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} -1 & 0 & 1 \\ 8 & 1 & -10 \\ -12 & -2 & 15 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A = \frac{1}{-1}\begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$

$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$

= $\begin{bmatrix} 1+0-3 & -8-3+30 & 12+6-45 \\ 1+0-3 & -8-4+30 & 12+8-45 \\ 1+0-4 & -8-3+40 & 12+6-60 \end{bmatrix}$

Hence, $(AB)^{-1} = \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$

$16.\ Let\ F(\alpha) = \begin{bmatrix} cos\alpha & -sin\alpha & 0 \\ sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\ and\ G(\beta) = \begin{bmatrix} cos\beta & 0 & sin\beta \\ 0 & 1 & 0 \\ -sin\beta & 0 & cos\beta \end{bmatrix}.Show\ that$

**(i) $[F(\alpha)]^{-1} = F(-\alpha)$**

**(ii) $[G(\beta)]^{-1} = G(-\beta)$**

**(iii) $[F(\alpha)G(\beta)]^{-1} = G(-\beta)F(-\alpha)$**

**Solution:**

**(i) Given**

**F (α) =**

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**$|F(\alpha)| = \cos^2\alpha + \sin^2\alpha = 1 \neq 0$**

**Cofactors of A are**

**$C_{11} = \cos\alpha$**

**$C_{21} = \sin\alpha$**

**$C_{31} = 0$**

**$C_{12} = -\sin\alpha$**

**$C_{22} = \cos\alpha$**

**$C_{32} = 0$**

**$C_{13} = 0$**

**$C_{23} = 0$**

**$C_{33} = 1$**

Adj F (α) = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$

So, adj F (α) = $\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $[F(\alpha)]^{-1} = \frac{1}{|F(\alpha)|}$ adj F (α) $= \frac{1}{1}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ...... (i)

And, F (– α) = $\begin{bmatrix} \cos(-\alpha) & \sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ...... (ii)

Hence, $[F(\alpha)]^{-1} = F(-\alpha)$

**(ii) We have**

**$|G(\beta)| = \cos^2\beta + \sin^2\beta = 1$**

**Cofactors of A are**

**$C_{11} = \cos\beta$**

**$C_{21} = 0$**

**$C_{31} = -\sin\beta$**

**$C_{12} = 0$**

**$C_{22}$ = 1**

**$C_{32}$ = 0**

**$C_{13}$ = sin β**

**$C_{23}$ = 0**

**$C_{33}$ = cos β**

Adj G (β) = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}^T$

So, adj G (β) = $\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$

Now, $[G(\beta)]^{-1} = \frac{1}{1}\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$ …… (i)

And, G (– β) = $\begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ \sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix}$

= $\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$ …… (ii)

Hence, $[G(\beta)]^{-1}$ = G – β)

**(iii) Now we have to show that$[F(\alpha)\, G(\beta)]^{-1} = G(-\beta)\, F(-\alpha)$**

**We have already know that$[G(\beta)]^{-1} = G(-\beta)$$[F(\alpha)]^{-1} = F(-\alpha)$**

**And LHS = [F (α) G (β)] $^{-1}$**

**= [G (β)] $^{-1}$ [F (α)] $^{-1}$**

**= G (– β) F (– α)**

**Hence = RHS**

$17.\ If\ A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\ verify\ that\ A^2 - 4A + I = 0,\ where\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\ and\ O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.Hence\ find\ A^{-1}.$

**Solution:**

**Consider,**

$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$

$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

$4A = 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7-8+1 & 12-2+0 \\ 4-4+0 & 7-8+1 \end{bmatrix}$

Hence, $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Now, $A^2 - 4A + I = 0$

$A.A - 4A = -I$

Multiply by $A^{-1}$ both sides we get

$A.A(A^{-1}) - 4A A^{-1} = -IA^{-1}$

$AI - 4I = -A^{-1}$

$A^{-1} = 4I - AI = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

18. *Show that* $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ *satisfies the equation* $A^2 + 4A - 42I = 0$. *Hence find* $A^{-1}$.

**Solution:**

**Given**

$A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 64 + 10 & -40 + 20 \\ -16 + 8 & 10 + 16 \end{bmatrix}$

$= \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$

$4A = 4\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix}$

$42I = 42\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix}$

Now,

$A^2 + 4A - 42I = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix}$

$= \begin{bmatrix} 74 - 74 & -20 + 20 \\ -8 + 8 & 42 - 42 \end{bmatrix}$

Hence, $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Now, $A^2 + 4A - 42I = 0$

$= A^{-1} A. A + 4 A^{-1}.A - 42 A^{-1}I = 0$

$= IA + 4I - 42A^{-1} = 0$

$= 42A^{-1} = A + 4I$

$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ $= A^{-1} = \frac{1}{42}[A + 4I]$

$= \frac{1}{42}\left[\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right]$

$A^{-1} = \frac{1}{42}\left[\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}\right]$

19. $If\ A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ $show\ that\ A^2 - 5A + 7I = 0. Hence\ find\ A^{-1}.$

**Solution:**

**Given**

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

Now, $A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

So, $A^2 - 5A + 7I = 0$

Multiply by $A^{-1}$ both sides

$A.A A^{-1} - 5A. A^{-1} + 7I. A^{-1} = 0$

$A - 5I + 7 A^{-1} = 0$

$A^{-1} = \frac{1}{7}[5I - A]$

$A^{-1} = \frac{1}{7}.\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{7}.\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

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**Exercise 7.2 Page No: 7.34**

**Find the inverse of the following matrices by using elementary row transformations:**

1. $\begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix}$

**Solution:**

For row transformation we have

A = IA

$\Rightarrow \begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_1 \to \frac{1}{7} r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{1}{7} \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_2 \to r_2 - 4r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{1}{7} \\ 0 & \frac{-25}{7} \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ -\frac{4}{7} & 1 \end{bmatrix} A$

Applying $r_2 \to -\frac{7}{25} r_2$

$\Rightarrow \begin{bmatrix} 1 & \frac{1}{7} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix} A$

Applying $r_1 \to r_1 - \frac{1}{7} r_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{21}{175} & \frac{1}{25} \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix} A$

So, as we know that

I = $A^{-1}A$

Therefore

$\Rightarrow A^{-1} = \begin{bmatrix} \frac{21}{175} & \frac{1}{25} \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix}$

2. $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

**Solution:**

For row transformation we have,

A = IA

$\Rightarrow \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_1 \to \frac{1}{5} r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{2}{5} \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_2 \to r_2 - 2r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ -\frac{2}{5} & 1 \end{bmatrix} A$

Applying $r_2 \to 5r_2$

$\Rightarrow \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ -2 & 5 \end{bmatrix} A$

Applying $r_1 \to r_1 - \frac{2}{5} r_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} A$

So, as we know that

I = $A^{-1}$A

Therefore

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

**3.** $\begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix}$

**Solution:**

For row transformation we have

A = IA

$\Rightarrow \begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_2 \rightarrow r_2 + 3r_1$

$\Rightarrow \begin{bmatrix} 1 & 6 \\ 0 & 23 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} A$

Applying $r_2 \rightarrow \frac{1}{23} r_2$

$\Rightarrow \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{23} & \frac{1}{23} \end{bmatrix} A$

Applying $r_1 \rightarrow r_1 - 6r_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{23} & \frac{-6}{23} \\ \frac{3}{23} & \frac{1}{23} \end{bmatrix} A$

So, as we know that

I = $A^{-1}$A

Therefore

$\Rightarrow A^{-1} = \frac{1}{23} \begin{bmatrix} 5 & -6 \\ 3 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

**Solution:**

For elementary row operation we have

A = IA

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_1 \to \frac{1}{2} r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_2 \to r_2 - r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A$

Applying $r_2 \to 2r_2$

$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & 2 \end{bmatrix} A$

Applying $r_1 \to r_1 - \frac{5}{2} r_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$

So, as we know that

I = $A^{-1}$A

Therefore

$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

**Solution:**

For elementary row operation we have

A = IA

$\Rightarrow \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_1 \to \frac{1}{3} r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{10}{3} \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_2 \to r_2 - 2r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{10}{3} \\ 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -\frac{2}{3} & 1 \end{bmatrix} A$

Applying $r_2 \to 3r_2$

$\Rightarrow \begin{bmatrix} 1 & \frac{10}{3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -2 & 3 \end{bmatrix} A$

Applying $r_1 \to r_1 - \frac{10}{3} r_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$

So, as we know that

I = $A^{-1}$A

Therefore

$\Rightarrow A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

6. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

**Solution:**

For elementary row operation we have,

A = IA

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \leftrightarrow r_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_3 \rightarrow r_3 - 3r_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 - 2r_2$ and $r_3 \to r_3 + 5r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying $r_3 \to \frac{1}{2} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

Applying $r_1 \to r_1 + r_3$ and $r_2 \to r_2 - 2r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$$

So, as we know that

$I = A^{-1}A$

Therefore

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

**7.** $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

**Solution:**

For elementary row operation we have,

A = IA

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to \frac{1}{2} r_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to r_2 - 5r_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_3 \to r_3 - r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{1}{2} & -1 & 1 \end{bmatrix} A$$

Applying $r_3 \to 2r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $r_1 \to r_1 + \frac{1}{2}r_3$ and $r_2 \to r_2 - \frac{5}{2}r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

So, as we know that

$I = A^{-1}A$

Therefore

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

8. $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

**Solution:**

For row transformation we have

A = IA

$$\Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to \frac{1}{2} r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to r_2 - 2r_1 \; and \; r_3 \to r_3 - 3r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 - \frac{3}{2}r_2$ and $r_3 \to r_3 - \frac{5}{2}r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & -\frac{3}{2} & 0 \\ -1 & 1 & 0 \\ 1 & -\frac{5}{2} & 1 \end{bmatrix} A$$

Applying $r_3 \to 2r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{3}{2} & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

Applying $r_1 \to r_1 - \frac{1}{2}r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

So, as we know that

$I = A^{-1}A$

Therefore

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$

# Chapterwise RD Sharma Solutions for Class 12 Maths :


- Chapter 1–Relation
- Chapter 2–Functions
- Chapter 3–Binary Operations
- Chapter 4–Inverse Trigonometric Functions
- Chapter 5–Algebra of Matrices
- Chapter 6–Determinants
- Chapter 7–Adjoint and Inverse of a Matrix
- Chapter 8–Solution of Simultaneous Linear Equations
- Chapter 9–Continuity
- Chapter 10–Differentiability
- Chapter 11–Differentiation
- Chapter 12–Higher Order Derivatives
- Chapter 13–Derivatives as a Rate Measurer
- Chapter 14–Differentials, Errors and Approximations
- Chapter 15–Mean Value Theorems
- Chapter 16–Tangents and Normals
- Chapter 17–Increasing and Decreasing Functions
- Chapter 18–Maxima and Minima
- Chapter 19–Indefinite Integrals

# About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.