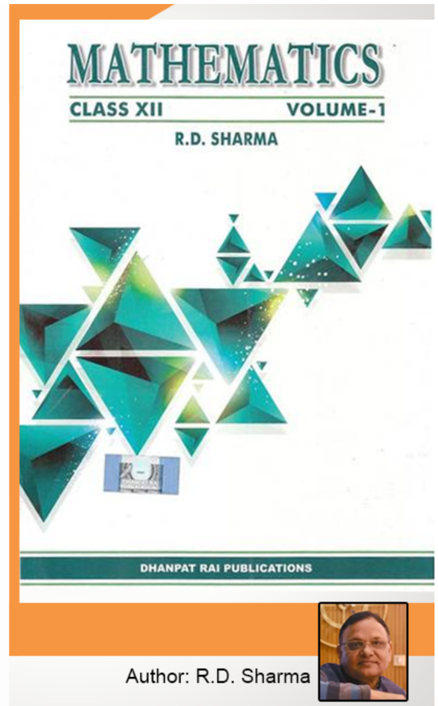
Class 12 - Chapter 4 Inverse Trigonometric Functions



RD Sharma Solutions for Class 12 Maths Chapter 4–Inverse Trigonometric Functions

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Exercise 4.1 Page No: 4.6

1. Find the principal value of the following:

$$(i)\ sin^{-1}(-\sqrt{\frac{3}{2}})\ (ii)\ sin^{-1}(\cos\frac{2\pi}{3})\ (iii)\ \sin^{-1}(\frac{\sqrt{3}-1}{2\sqrt{2}})\ (iv)\ \sin^{-1}(\frac{\sqrt{3}+1}{2\sqrt{2}})$$

$$(v)\ \sin^{-1}(\cos\frac{3\pi}{4})\ (vi)\ \sin^{-1}(\tan\frac{5\pi}{4})$$

Solution:

$$(i) Let \sin^{-1}(\frac{-\sqrt{3}}{2}) = y$$

$$Then\ siny = (\frac{-\sqrt{3}}{2})$$

$$= -\sin(\frac{\pi}{3})$$

$$= sin(-\frac{\pi}{3})$$

$$We\ know\ that\ the\ principal\ value\ of\ \sin^{-1} is \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$And - sin\frac{\pi}{3} = sin(\frac{-\pi}{3})$$

$$Therefore\ principal\ value\ of \sin^{-1}(\frac{-\sqrt{3}}{2}) = \frac{-\pi}{3}$$

$$(ii) Let \sin^{-1}(\cos\frac{2\pi}{3}) = y$$

$$Then\ siny = \cos(\frac{2\pi}{3})$$

$$= -\sin(\frac{\pi}{2} + \frac{\pi}{6})$$

$$= -sin(\frac{\pi}{6})$$

$$We\ know\ that\ the\ principal\ value\ of\ \sin^{-1} is \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$And\ - sin(\frac{\pi}{6}) = cos(\frac{2\pi}{3})$$

$$Therefore\ principal\ value\ of\ \sin^{-1}(\cos\frac{2\pi}{3})\ is \frac{-\pi}{6}$$

(iii) Given functions can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)$$

Taking 1/√2 as common from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking √3/2 as common, and 1/√2 from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{2}} \times \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the values,

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

Taking LCM and cross multiplying we get,

$$= \frac{\pi}{12}$$

(iv) The given question can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

Taking 1/√2 as common from the above equation we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} + \frac{1}{2}\times\frac{1}{\sqrt{2}}\right)$$

Taking √3/2 as common, and 1/√2 from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\times\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{1}{\sqrt{2}}\times\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the corresponding values we get

$$= \frac{\pi}{3} + \frac{\pi}{4}$$

$$= \frac{7\pi}{12}$$

(v) Let

$\sin^{-1}\left(\cos\frac{3\pi}{4}\right) = y$

Then above equation can be written as

$\sin y = \cos\frac{3\pi}{4} = -\sin\left(\pi - \frac{3\pi}{4}\right) = -\sin(\frac{\pi}{4})$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore above equation becomes,

$-\sin\left(\frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

Therefore the principal value of $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$ is $-\frac{\pi}{4}$

(vi) Let

$y = \sin^{-1}\left(\tan\frac{5\pi}{4}\right)$

Therefore above equation can be written as

$\text{Sin } y = \left(\tan\frac{5\pi}{4}\right) = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1 = \sin\left(\frac{\pi}{2}\right)$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\left(\frac{\pi}{2}\right) = \tan\frac{5\pi}{4}$

Therefore the principal value of $\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$ is $\frac{\pi}{2}$.

2.

(i)

(ii)

Solution:

(i) The given question can be written as,

$$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}} = \sin^{-1}\frac{1}{2} - \sin^{-1}\left(2 \times \frac{1}{\sqrt{2}}\sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\frac{1}{2} - \sin^{-1}(1)$$

By substituting the corresponding values, we get

$$= \frac{\pi}{6} - \frac{\pi}{2}$$

$$= -\frac{\pi}{3}$$

(ii) Given question can be written as

We know that $\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \pi/3$

$= \sin^{-1}\left\{\cos\left(\frac{\pi}{3}\right)\right\}$

Now substituting the values we get,

$= \sin^{-1}\left\{\frac{1}{2}\right\}$

$= \frac{\pi}{6}$

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Exercise 4.2 Page No: 4.10

1. Find the domain of definition of $f(x) = \cos^{-1}(x^2 – 4)$

Solution:

Given $f(x) = \cos^{-1}(x^2 – 4)$

We know that domain of $\cos^{-1}(x^2 – 4)$ lies in the interval [-1, 1]

Therefore, we can write as

$-1 \le x^2 – 4 \le 1$

$4 – 1 \le x^2 \le 1 + 4$

$3 \le x^2 \le 5$

$\pm\sqrt{3} \le x \le \pm\sqrt{5}$

$– \sqrt{5} \le x \le – \sqrt{3}$ and $\sqrt{3} \le x \le \sqrt{5}$

Therefore domain of $\cos^{-1}(x^2 – 4)$ is $[-\sqrt{5}, –\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

2. Find the domain of $f(x) = \cos^{-1} 2x + \sin^{-1} x$.

Solution:

Given that f(x) = $\cos^{-1} 2x + \sin^{-1} x$.

Now we have to find the domain of f(x),

We know that domain of $\cos^{-1} x$ lies in the interval [-1, 1]

Also know that domain of $\sin^{-1} x$ lies in the interval [-1, 1]

Therefore, the domain of $\cos^{-1} (2x)$ lies in the interval [-1, 1]

Hence we can write as,

$-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$

Hence, domain of $\cos^{-1}(2x) + \sin^{-1} x$ lies in the interval [- ½, ½]

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Exercise 4.3 Page No: 4.14

1. Find the principal value of each of the following:

(i) $\tan^{-1} (1/\sqrt{3})$

(ii) $\tan^{-1} (-1/\sqrt{3})$

(iii) $\tan^{-1} (\cos (\pi/2))$

(iv) $\tan^{-1} (2 \cos (2\pi/3))$

Solution:

(i) Given $\tan^{-1} (1/\sqrt{3})$

We know that for any $x \in R$, $\tan^{-1}$ represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x.

So, $\tan^{-1} (1/\sqrt{3})$ = an angle in $(-\pi/2, \pi/2)$ whose tangent is $(1/\sqrt{3})$

But we know that the value is equal to $\pi/6$

Therefore $\tan^{-1}(1/\sqrt{3}) = \pi/6$

Hence the principal value of $\tan^{-1}(1/\sqrt{3}) = \pi/6$

(ii) Given $\tan^{-1}(-1/\sqrt{3})$

We know that for any $x \in R$, $\tan^{-1}$ represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x.

So, $\tan^{-1}(-1/\sqrt{3})$ = an angle in $(-\pi/2, \pi/2)$ whose tangent is $(1/\sqrt{3})$

But we know that the value is equal to $-\pi/6$

Therefore $\tan^{-1}(-1/\sqrt{3}) = -\pi/6$

Hence the principal value of $\tan^{-1}(-1/\sqrt{3}) = -\pi/6$

(iii) Given that $\tan^{-1}(\cos(\pi/2))$

But we know that $\cos(\pi/2) = 0$

We know that for any $x \in R$, $\tan^{-1}$ represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x.

Therefore $\tan^{-1}(0) = 0$

Hence the principal value of $\tan^{-1}(\cos(\pi/2)$ is 0.

(iv) Given that $\tan^{-1}(2\cos(2\pi/3))$

But we know that $\cos \pi/3 = 1/2$

So, $\cos(2\pi/3) = -1/2$

Therefore $\tan^{-1}(2\cos(2\pi/3)) = \tan^{-1}(2 \times -\frac{1}{2})$

$= \tan^{-1}(-1)$

$= -\pi/4$

Hence, the principal value of $\tan^{-1}(2\cos(2\pi/3))$ is $-\pi/4$

Exercise 4.4 Page No: 4.18

1. Find the principal value of each of the following:

(i) $\sec^{-1}(-\sqrt{2})$

(ii) $\sec^{-1}(2)$

(iii) $\sec^{-1}(2\sin(3\pi/4))$

(iv) $\sec^{-1}(2\tan(3\pi/4))$

Solution:

(i) Given $\sec^{-1}(-\sqrt{2})$

Now let $y = \sec^{-1}(-\sqrt{2})$

Sec $y = -\sqrt{2}$

We know that $\sec \pi/4 = \sqrt{2}$

Therefore, $-\sec(\pi/4) = -\sqrt{2}$

$= \sec(\pi - \pi/4)$

$= \sec(3\pi/4)$

Thus the range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\pi/2\}$

And $\sec(3\pi/4) = -\sqrt{2}$

Hence the principal value of $\sec^{-1}(-\sqrt{2})$ is $3\pi/4$

(ii) Given $\sec^{-1}(2)$

Let $y = \sec^{-1}(2)$

Sec $y = 2$

= Sec $\pi/3$

Therefore the range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\pi/2\}$ and $\sec \pi/3 = 2$

Thus the principal value of sec^{-1} (2) is π/3

(iii) Given sec^{-1} (2 sin (3π/4))

But we know that sin (3π/4) = 1/√2

Therefore 2 sin (3π/4) = 2 × 1/√2

2 sin (3π/4) = √2

Therefore by substituting above values in sec^{-1} (2 sin (3π/4)), we get

Sec^{-1} (√2)

Let Sec^{-1} (√2) = y

Sec y = √2

Sec (π/4) = √2

Therefore range of principal value of sec^{-1} is [0, π] – {π/2} and sec (π/4) = √2

Thus the principal value of sec^{-1} (2 sin (3π/4)) is π/4.

(iv) Given sec^{-1} (2 tan (3π/4))

But we know that tan (3π/4) = -1

Therefore, 2 tan (3π/4) = 2 × -1

2 tan (3π/4) = -2

By substituting these values in sec^{-1} (2 tan (3π/4)), we get

Sec^{-1} (-2)

Now let y = Sec^{-1} (-2)

Sec y = – 2

– sec (π/3) = -2

= sec (π – π/3)

= sec (2π/3)

Therefore the range of principal value of sec^{-1} is [0, π] – {π/2} and sec (2π/3) = -2

Thus, the principal value of sec^{-1} (2 tan (3π/4)) is (2π/3).

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Exercise 4.5 Page No: 4.21

1. Find the principal values of each of the following:

(i) $cosec^{-1}$ (-√2)

(ii) $cosec^{-1}$ (-2)

(iii) $cosec^{-1}$ (2/√3)

(iv) $cosec^{-1}$ (2 cos (2π/3))

Solution:

(i) Given $cosec^{-1}$ (-√2)

Let y = $cosec^{-1}$ (-√2)

Cosec y = -√2

– Cosec y = √2

– Cosec (π/4) = √2

– Cosec (π/4) = cosec (-π/4) [since –cosec θ = cosec (-θ)]

The range of principal value of $cosec^{-1}$ [-π/2, π/2] – {0} and cosec (-π/4) = – √2

Cosec (-π/4) = – √2

Therefore the principal value of $cosec^{-1}$ (-√2) is – π/4

(ii) Given $cosec^{-1}$ (-2)

Let y = $cosec^{-1}$ (-2)

Cosec y = -2

– Cosec y = 2

– Cosec (π/6) = 2

– Cosec (π/6) = cosec (-π/6) [since –cosec θ = cosec (-θ)]

The range of principal value of $cosec^{-1}$ [-π/2, π/2] – {0} and cosec (-π/6) = – 2

Cosec (-π/6) = – 2

Therefore the principal value of $cosec^{-1}$ (-2) is – π/6

(iii) Given $cosec^{-1}$ (2/√3)

Let y = $cosec^{-1}$ (2/√3)

Cosec y = (2/√3)

Cosec (π/3) = (2/√3)

Therefore range of principal value of $cosec^{-1}$ is [-π/2, π/2] – {0} and cosec (π/3) = (2/√3)

Thus, the principal value of $cosec^{-1}$ (2/√3) is π/3

(iv) Given $cosec^{-1}$ (2 cos (2π/3))

But we know that cos (2π/3) = – ½

Therefore 2 cos (2π/3) = 2 × – ½

2 cos (2π/3) = -1

By substituting these values in $cosec^{-1}$ (2 cos (2π/3)) we get,

$Cosec^{-1}$ (-1)

Let y = $cosec^{-1}$ (-1)

– Cosec y = 1

– Cosec (π/2) = cosec (-π/2) [since –cosec θ = cosec (-θ)]

The range of principal value of $cosec^{-1}$ [-π/2, π/2] – {0} and cosec (-π/2) = – 1

Cosec (-π/2) = – 1

Therefore the principal value of $cosec^{-1}$ (2 cos (2π/3)) is – π/2

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Exercise 4.6 Page No: 4.24

1. Find the principal values of each of the following:

(i) $cot^{-1}(-\sqrt{3})$

(ii) $Cot^{-1}(\sqrt{3})$

(iii) $cot^{-1}(-1/\sqrt{3})$

(iv) cot^{-1}(tan 3π/4)

Solution:

(i) Given $cot^{-1}(-\sqrt{3})$

Let y = $cot^{-1}(-\sqrt{3})$

– Cot (π/6) = √3

= Cot (π – π/6)

= cot (5π/6)

The range of principal value of cot^{-1} is (0, π) and cot (5 π/6) = – √3

Thus, the principal value of cot^{-1} (- √3) is 5π/6

(ii) Given $Cot^{-1}(\sqrt{3})$

Let y = $\cot^{-1}(\sqrt{3})$

Cot $(\pi/6) = \sqrt{3}$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$ and

Thus, the principal value of $\cot^{-1}(\sqrt{3})$ is $\pi/6$

(iii) Given $\cot^{-1}(-1/\sqrt{3})$

Let y = $\cot^{-1}(-1/\sqrt{3})$

Cot y = $(-1/\sqrt{3})$

– Cot $(\pi/3) = 1/\sqrt{3}$

= Cot $(\pi - \pi/3)$

= cot $(2\pi/3)$

The range of principal value of $\cot^{-1}(0, \pi)$ and cot $(2\pi/3) = -1/\sqrt{3}$

Therefore the principal value of $\cot^{-1}(-1/\sqrt{3})$ is $2\pi/3$

(iv) Given $\cot^{-1}(\tan 3\pi/4)$

But we know that tan $3\pi/4 = -1$

By substituting this value in $\cot^{-1}(\tan 3\pi/4)$ we get

$\text{Cot}^{-1}(-1)$

Now, let y = $\cot^{-1}(-1)$

Cot y = (-1)

– Cot $(\pi/4) = 1$

= Cot $(\pi - \pi/4)$

= cot $(3\pi/4)$

The range of principal value of $\cot^{-1}(0, \pi)$ and cot $(3\pi/4) = -1$

Therefore the principal value of $\cot^{-1}(\tan 3\pi/4)$ is $3\pi/4$

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Exercise 4.7 Page No: 4.42

1. Evaluate each of the following:

(i) $\sin^{-1}(\sin \pi/6)$

(ii) $\sin^{-1}(\sin 7\pi/6)$

(iii) $\sin^{-1}(\sin 5\pi/6)$

(iv) $\sin^{-1}(\sin 13\pi/7)$

(v) $\sin^{-1}(\sin 17\pi/8)$

(vi) $\sin^{-1}\{(\sin - 17\pi/8)\}$

(vii) $\sin^{-1}(\sin 3)$

(viii) $\sin^{-1}(\sin 4)$

(ix) $\sin^{-1}(\sin 12)$

(x) $\sin^{-1}(\sin 2)$

Solution:

(i) Given $\sin^{-1}(\sin \pi/6)$

We know that the value of $\sin \pi/6$ is ½

By substituting this value in $\sin^{-1}(\sin \pi/6)$

We get, $\sin^{-1}(1/2)$

Now let $y = \sin^{-1}(1/2)$

$\sin(\pi/6) = ½$

The range of principal value of $\sin^{-1}(-\pi/2, \pi/2)$ and $\sin(\pi/6) = ½$

Therefore $\sin^{-1}(\sin \pi/6) = \pi/6$

(ii) Given $\sin^{-1}(\sin 7\pi/6)$

But we know that $\sin 7\pi/6 = – ½$

By substituting this in $\sin^{-1}(\sin 7\pi/6)$ we get,

$\text{Sin}^{-1}(-1/2)$

Now let $y = \sin^{-1}(-1/2)$

$– \text{Sin } y = ½$

$– \text{Sin } (\pi/6) = ½$

$– \text{Sin } (\pi/6) = \sin(- \pi/6)$

The range of principal value of $\sin^{-1}(-\pi/2, \pi/2)$ and $\sin(- \pi/6) = – ½$

Therefore $\sin^{-1}(\sin 7\pi/6) = – \pi/6$

(iii) Given $\sin^{-1}(\sin 5\pi/6)$

We know that the value of $\sin 5\pi/6$ is ½

By substituting this value in $\sin^{-1}(\sin 5\pi/6)$

We get, $\sin^{-1}(1/2)$

Now let $y = \sin^{-1}(1/2)$

$\text{Sin } (\pi/6) = ½$

The range of principal value of $\sin^{-1}(-\pi/2, \pi/2)$ and $\sin(\pi/6) = ½$

Therefore $\sin^{-1}(\sin 5\pi/6) = \pi/6$

(iv) Given $\sin^{-1}(\sin 13\pi/7)$

Given question can be written as $\sin(2\pi – \pi/7)$

Sin (2π – π/7) can be written as sin (-π/7) [since sin (2π – θ) = sin (-θ)]

By substituting these values in $\sin^{-1}$(sin 13π/7) we get $\sin^{-1}$(sin – π/7)

As $\sin^{-1}$(sin x) = x with x ∈ [-π/2, π/2]

Therefore $\sin^{-1}$(sin 13π/7) = – π/7

(v) Given $\sin^{-1}$(sin 17π/8)

Given question can be written as sin (2π + π/8)

Sin (2π + π/8) can be written as sin (π/8)

By substituting these values in $\sin^{-1}$(sin 17π/8) we get $\sin^{-1}$(sin π/8)

As $\sin^{-1}$(sin x) = x with x ∈ [-π/2, π/2]

Therefore $\sin^{-1}$(sin 17π/8) = π/8

(vi) Given $\sin^{-1}${(sin – 17π/8)}

But we know that – sin θ = sin (-θ)

Therefore (sin -17π/8) = – sin 17π/8

– Sin 17π/8 = – sin (2π + π/8) [since sin (2π – θ) = -sin (θ)]

It can also be written as – sin (π/8)

– Sin (π/8) = sin (-π/8) [since – sin θ = sin (-θ)]

By substituting these values in $\sin^{-1}${(sin – 17π/8)} we get,

Sin^{-1}(sin – π/8)

As $\sin^{-1}$(sin x) = x with x ∈ [-π/2, π/2]

Therefore $\sin^{-1}$(sin -π/8) = – π/8

(vii) Given $\sin^{-1}$(sin 3)

We know that $\sin^{-1}$(sin x) = x with x ∈ [-π/2, π/2] which is approximately equal to [-1.57, 1.57]

But here x = 3, which does not lie on the above range,

Therefore we know that sin (π – x) = sin (x)

Hence sin (π – 3) = sin (3) also π – 3 ∈ [-π/2, π/2]

$Sin^{-1}(sin\ 3) = \pi - 3$

(viii) Given $sin^{-1}(sin\ 4)$

We know that $sin^{-1}(sin\ x) = x$ with x ∈ [-π/2, π/2] which is approximately equal to [-1.57, 1.57]

But here x = 4, which does not lie on the above range,

Therefore we know that sin (π – x) = sin (x)

Hence sin (π – 4) = sin (4) also π – 4 ∈ [-π/2, π/2]

$Sin^{-1}(sin\ 4) = \pi - 4$

(ix) Given $sin^{-1}(sin\ 12)$

We know that $sin^{-1}(sin\ x) = x$ with x ∈ [-π/2, π/2] which is approximately equal to [-1.57, 1.57]

But here x = 12, which does not lie on the above range,

Therefore we know that sin (2nπ – x) = sin (-x)

Hence sin (2nπ – 12) = sin (-12)

Here n = 2 also 12 – 4π ∈ [-π/2, π/2]

$Sin^{-1}(sin\ 12) = 12 - 4\pi$

(x) Given $sin^{-1}(sin\ 2)$

We know that $sin^{-1}(sin\ x) = x$ with x ∈ [-π/2, π/2] which is approximately equal to [-1.57, 1.57]

But here x = 2, which does not lie on the above range,

Therefore we know that sin (π – x) = sin (x)

Hence sin (π – 2) = sin (2) also π – 2 ∈ [-π/2, π/2]

$Sin^{-1}(\sin 2) = \pi - 2$

2. Evaluate each of the following:

(i) $\cos^{-1}\{\cos(-\pi/4)\}$

(ii) $\cos^{-1}(\cos 5\pi/4)$

(iii) $\cos^{-1}(\cos 4\pi/3)$

(iv) $\cos^{-1}(\cos 13\pi/6)$

(v) $\cos^{-1}(\cos 3)$

(vi) $\cos^{-1}(\cos 4)$

(vii) $\cos^{-1}(\cos 5)$

(viii) $\cos^{-1}(\cos 12)$

Solution:

(i) Given $\cos^{-1}\{\cos(-\pi/4)\}$

We know that $\cos(-\pi/4) = \cos(\pi/4)$ [since $\cos(-\theta) = \cos\theta$

Also know that $\cos(\pi/4) = 1/\sqrt{2}$

By substituting these values in $\cos^{-1}\{\cos(-\pi/4)\}$ we get,

$Cos^{-1}(1/\sqrt{2})$

Now let $y = \cos^{-1}(1/\sqrt{2})$

Therefore $\cos y = 1/\sqrt{2}$

Hence range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos(\pi/4) = 1/\sqrt{2}$

Therefore $\cos^{-1}\{\cos(-\pi/4)\} = \pi/4$

(ii) Given $\cos^{-1}(\cos 5\pi/4)$

But we know that $\cos(5\pi/4) = -1/\sqrt{2}$

By substituting these values in $\cos^{-1}\{\cos(5\pi/4)\}$ we get,

$\text{Cos}^{-1}(-1/\sqrt{2})$

Now let $y = \cos^{-1}(-1/\sqrt{2})$

Therefore $\cos y = -1/\sqrt{2}$

$-\text{Cos}(\pi/4) = 1/\sqrt{2}$

$\text{Cos}(\pi - \pi/4) = -1/\sqrt{2}$

$\text{Cos}(3\pi/4) = -1/\sqrt{2}$

Hence range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos(3\pi/4) = -1/\sqrt{2}$

Therefore $\cos^{-1}\{\cos(5\pi/4)\} = 3\pi/4$

(iii) Given $\cos^{-1}(\cos 4\pi/3)$

But we know that $\cos(4\pi/3) = -1/2$

By substituting these values in $\cos^{-1}\{\cos(4\pi/3)\}$ we get,

$\text{Cos}^{-1}(-1/2)$

Now let $y = \cos^{-1}(-1/2)$

Therefore $\cos y = -1/2$

$-\text{Cos}(\pi/3) = 1/2$

$\text{Cos}(\pi - \pi/3) = -1/2$

$\text{Cos}(2\pi/3) = -1/2$

Hence range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos(2\pi/3) = -1/2$

Therefore $\cos^{-1}\{\cos(4\pi/3)\} = 2\pi/3$

(iv) Given $\cos^{-1}(\cos 13\pi/6)$

But we know that $\cos(13\pi/6) = \sqrt{3}/2$

By substituting these values in $\cos^{-1}\{\cos(13\pi/6)\}$ we get,

$\text{Cos}^{-1}(\sqrt{3}/2)$

Now let $y = \cos^{-1}(\sqrt{3}/2)$

Therefore $\cos y = \sqrt{3}/2$

$\text{Cos}(\pi/6) = \sqrt{3}/2$

Hence range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos(\pi/6) = \sqrt{3}/2$

Therefore $\cos^{-1}\{\cos(13\pi/6)\} = \pi/6$

(v) Given $\cos^{-1}(\cos 3)$

We know that $\cos^{-1}(\cos\theta) = \theta$ if $0 \le \theta \le \pi$

Therefore by applying this in given question we get,

$\text{Cos}^{-1}(\cos 3) = 3, 3 \in [0, \pi]$

(vi) Given $\cos^{-1}(\cos 4)$

We have $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi] \approx [0, 3.14]$

And here $x = 4$ which does not lie in the above range.

We know that $\cos(2\pi - x) = \cos(x)$

Thus, $\cos(2\pi - 4) = \cos(4)$ so $2\pi - 4$ belongs in $[0, \pi]$

Hence $\cos^{-1}(\cos 4) = 2\pi - 4$

(vii) Given $\cos^{-1}(\cos 5)$

We have $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi] \approx [0, 3.14]$

And here $x = 5$ which does not lie in the above range.

We know that $\cos(2\pi - x) = \cos(x)$

Thus, $\cos(2\pi - 5) = \cos(5)$ so $2\pi - 5$ belongs in $[0, \pi]$

Hence $\cos^{-1}(\cos 5) = 2\pi - 5$

(viii) Given $\cos^{-1}(\cos 12)$

$\cos^{-1}(\cos x) = x$ if $x \in [0, \pi] \approx [0, 3.14]$

And here x = 12 which does not lie in the above range.

We know $\cos(2n\pi - x) = \cos(x)$

$\cos(2n\pi - 12) = \cos(12)$

Here n = 2.

Also $4\pi - 12$ belongs in $[0, \pi]$

$\therefore \cos^{-1}(\cos 12) = 4\pi - 12$

3. Evaluate each of the following:

(i) $\tan^{-1}(\tan \pi/3)$

(ii) $\tan^{-1}(\tan 6\pi/7)$

(iii) $\tan^{-1}(\tan 7\pi/6)$

(iv) $\tan^{-1}(\tan 9\pi/4)$

(v) $\tan^{-1}(\tan 1)$

(vi) $\tan^{-1}(\tan 2)$

(vii) $\tan^{-1}(\tan 4)$

(viii) $\tan^{-1}(\tan 12)$

Solution:

(i) Given $\tan^{-1}(\tan \pi/3)$

As $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

By applying this condition in the given question we get,

$Tan^{-1}(tan\ \pi/3) = \pi/3$

(ii) Given $tan^{-1}(tan\ 6\pi/7)$

We know that tan 6π/7 can be written as (π – π/7)

Tan (π – π/7) = – tan π/7

We know that $tan^{-1}(tan\ x) = x$ if x ϵ [-π/2, π/2]

$Tan^{-1}(tan\ 6\pi/7) = -\pi/7$

(iii) Given $tan^{-1}(tan\ 7\pi/6)$

We know that tan 7π/6 = 1/√3

By substituting this value in $tan^{-1}(tan\ 7\pi/6)$ we get,

$Tan^{-1}(1/\sqrt{3})$

Now let $tan^{-1}(1/\sqrt{3}) = y$

Tan y = 1/√3

Tan (π/6) = 1/√3

The range of the principal value of tan^{-1} is (-π/2, π/2) and tan (π/6) = 1/√3

Therefore $tan^{-1}(tan\ 7\pi/6) = \pi/6$

(iv) Given $tan^{-1}(tan\ 9\pi/4)$

We know that tan 9π/4 = 1

By substituting this value in $tan^{-1}(tan\ 9\pi/4)$ we get,

$Tan^{-1}(1)$

Now let $tan^{-1}(1) = y$

Tan y = 1

Tan (π/4) = 1

The range of the principal value of $\tan^{-1}$ is $(-\pi/2, \pi/2)$ and $\tan(\pi/4) = 1$

Therefore $\tan^{-1}(\tan 9\pi/4) = \pi/4$

(v) Given $\tan^{-1}(\tan 1)$

But we have $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

By substituting this condition in given question

$\text{Tan}^{-1}(\tan 1) = 1$

(vi) Given $\tan^{-1}(\tan 2)$

As $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

But here $x = 2$ which does not belongs to above range

We also have $\tan(\pi - \theta) = -\tan(\theta)$

Therefore $\tan(\theta - \pi) = \tan(\theta)$

$\text{Tan}(2 - \pi) = \tan(2)$

Now $2 - \pi$ is in the given range

Hence $\tan^{-1}(\tan 2) = 2 - \pi$

(vii) Given $\tan^{-1}(\tan 4)$

As $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

But here $x = 4$ which does not belongs to above range

We also have $\tan(\pi - \theta) = -\tan(\theta)$

Therefore $\tan(\theta - \pi) = \tan(\theta)$

$\text{Tan}(4 - \pi) = \tan(4)$

Now $4 - \pi$ is in the given range

Hence $\tan^{-1}(\tan 2) = 4 - \pi$

(viii) Given $\tan^{-1}(\tan 12)$

As $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

But here x = 12 which does not belongs to above range

We know that $\tan(2n\pi - \theta) = -\tan(\theta)$

$\tan(\theta - 2n\pi) = \tan(\theta)$

Here n = 2

$\tan(12 - 4\pi) = \tan(12)$

Now $12 - 4\pi$ is in the given range

$\therefore \tan^{-1}(\tan 12) = 12 - 4\pi$.

RD Sharma 12th Maths Chapter 4, Class 12 Maths Chapter 4 solutions

Exercise 4.8 Page No: 4.54

1. Evaluate each of the following:

(i) $\sin(\sin^{-1} 7/25)$

(ii) $\sin(\cos^{-1} 5/13)$

(iii) $\sin(\tan^{-1} 24/7)$

(iv) $\sin(\sec^{-1} 17/8)$

(v) $\text{Cosec}(\cos^{-1} 8/17)$

(vi) $\sec(\sin^{-1} 12/13)$

(vii) $\tan(\cos^{-1} 8/17)$

(viii) $\cot(\cos^{-1} 3/5)$

(ix) $\cos(\tan^{-1} 24/7)$

Solution:

(i) Given sin ($\sin^{-1}$ 7/25)

Now let y = $\sin^{-1}$ 7/25

Sin y = 7/25 where y ∈ [0, π/2]

Substituting these values in sin ($\sin^{-1}$ 7/25) we get

Sin ($\sin^{-1}$ 7/25) = 7/25

(ii) Given Sin ($\cos^{-1}$ 5/13)

Let $\cos^{-1}\frac{5}{13} = y$

$\Rightarrow$ $\cos y = \frac{5}{13}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have to find

$$\sin\left(\cos^{-1}\frac{5}{13}\right) = \sin y$$

We know that $\sin^2\theta + \cos^2\theta = 1$

By substituting this trigonometric identity we get

$\Rightarrow \sin y = \pm\sqrt{1-\cos^2 y}$

Where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sin y = \sqrt{1-\cos^2 y}$

Now by substituting cos y value we get

$\Rightarrow \sin y = \sqrt{1-\left(\frac{5}{13}\right)^2}$

$\Rightarrow \sin y = \sqrt{1-\frac{25}{169}}$

$\Rightarrow \sin y = \sqrt{\frac{144}{169}}$

$\Rightarrow \sin y = \frac{12}{13}$ $\Rightarrow \sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right] = \frac{12}{13}$

(iii) Given Sin ($\tan^{-1}$ 24/7)

Let $\tan^{-1}\frac{24}{7}=y$

$\Rightarrow \tan y=\frac{24}{7}$ Where $y\in\left[0,\frac{\pi}{2}\right]$

Now we have to find

$$\sin\left(\tan^{-1}\frac{24}{7}\right)=\sin y$$

We know that $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

$\Rightarrow 1 + \cot^2 y = \mathrm{cosec}^2 y$

Now substituting this trigonometric identity we get,

$\Rightarrow 1+\left(\frac{7}{24}\right)^2=\mathrm{cosec}^2 y$

$\Rightarrow 1+\frac{49}{576}=\frac{1}{\sin^2 y}$

On rearranging we get,

$\Rightarrow \sin^2 y=\frac{576}{625}$

$\Rightarrow \sin y=\frac{24}{25}$ Where $y\in\left[0,\frac{\pi}{2}\right]$

$\Rightarrow \sin\left(\tan^{-1}\frac{24}{7}\right)=\frac{24}{25}$

(iv) Given Sin (sec^{-1} 17/8)

Let $\sec^{-1}\frac{17}{8} = y$

$\Rightarrow \sec y = \frac{17}{8}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have find

$\sin\left(\sec^{-1}\frac{17}{8}\right) = \sin y$

We know that, $\cos y = \frac{1}{\sec y}$

$\Rightarrow \cos y = \frac{8}{17}$

Now, $\sin y = \sqrt{1-\cos^2 y}$ where $y \in \left[0, \frac{\pi}{2}\right]$

By substituting, cos y value we get,

$\Rightarrow \sin y = \sqrt{1-\left(\frac{8}{17}\right)^2}$

$\Rightarrow \sin y = \sqrt{\frac{225}{289}}$

$\Rightarrow \sin y = \frac{15}{17}$

$\Rightarrow \sin\left(\sec^{-1}\frac{17}{8}\right) = \frac{15}{17}$

(v) Given Cosec ($\cos^{-1}$ 8/17)

Let $\cos^{-1}(8/17) = y$

cos y = 8/17 where y ∈ [0, π/2]

Now, we have to find

Cosec ($\cos^{-1}$ 8/17) = cosec y

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

$\sin^2 \theta = \sqrt{(1 - \cos^2 \theta)}$

So,

$\sin y = \sqrt{(1 - \cos^2 y)}$

$= \sqrt{(1 - (8/17)^2)}$

$= \sqrt{(1 - 64/289)}$

$= \sqrt{(289 - 64/289)}$

$= \sqrt{(225/289)}$

= 15/17

Hence,

Cosec y = 1/sin y = 1/ (15/17) = 17/15

Therefore,

Cosec ($\cos^{-1}$ 8/17) = 17/15

(vi) Given Sec ($\sin^{-1}$ 12/13)

Let $\sin^{-1}\frac{12}{13} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sin y = \frac{12}{13}$

Now we have to find

$$\sec\left(\sin^{-1}\frac{12}{13}\right) = \sec y$$

We know that $\sin^2\theta + \cos^2\theta = 1$

According to this identity cos y can be written as

$\Rightarrow \cos y = \sqrt{1-\sin^2 y}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now substituting the value of sin y we get,

$$\Rightarrow \cos y = \sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \cos y = \sqrt{1-\frac{144}{169}}$$

$$\Rightarrow \cos y = \sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos y = \frac{5}{13}$$

$$\Rightarrow \sec y = \frac{1}{\cos y}$$

$$\Rightarrow \sec y = \frac{13}{5}$$

$$\Rightarrow \sec\left(\sin^{-1}\frac{12}{13}\right) = \frac{13}{5}$$

(vii) Given Tan ($\cos^{-1}$ 8/17)

Let $\cos^{-1}\frac{8}{17} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \cos y = \frac{8}{17}$

Now we have to find

$\tan\left(\cos^{-1}\frac{8}{17}\right) = \tan y$

We know that $1+\tan^2\theta = \sec^2\theta$

Rearranging and substituting the value of tan y we get,

$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

We have sec y = 1/cos y

$\Rightarrow \tan y = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$

$\Rightarrow \tan y = \sqrt{\left(\frac{17}{8}\right)^2 - 1}$

$\Rightarrow \tan y = \sqrt{\frac{289}{64} - 1}$

$\Rightarrow \tan y = \sqrt{\frac{225}{64}}$

$\Rightarrow \tan y = \frac{15}{8}$

$\Rightarrow \tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{15}{8}$

(viii) Given cot ($\cos^{-1}$ 3/5)

Let $\cos^{-1}\frac{3}{5}=y$ where $y\in\left[0,\frac{\pi}{2}\right]$

$\Rightarrow \cos y=\frac{3}{5}$

Now we have to find

$\cot\left(\cos^{-1}\frac{3}{5}\right)=\cot y$

We know that $1+\tan^2\theta = \sec^2\theta$

Rearranging and substituting the value of tan y we get,

$\Rightarrow \tan y=\sqrt{\sec^2 y-1}$ Where $y\in\left[0,\frac{\pi}{2}\right]$

We have sec y = 1/cos y, on substitution we get,

$\Rightarrow \frac{1}{\cot y}=\sqrt{\left(\frac{1}{\cos^2 y}\right)-1}$

$\Rightarrow \frac{1}{\cot y}=\sqrt{\left(\frac{5}{3}\right)^2-1}$

$\Rightarrow \frac{1}{\cot y}=\sqrt{\frac{16}{9}}$

$\Rightarrow \cot y=\frac{3}{4}$

$\Rightarrow \cot\left(\cos^{-1}\frac{3}{5}\right)=\frac{3}{4}$

(ix) Given Cos ($\tan^{-1}$ 24/7)

.

Let $\tan^{-1}\frac{24}{7} = y$

$\Rightarrow \tan y = \frac{24}{7}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have to find,

$\cos\left(\tan^{-1}\frac{24}{7}\right) = \cos y$

We know that $1+\tan^2\theta = \sec^2\theta$

$\Rightarrow 1 + \tan^2 y = \sec^2 y$

On rearranging and substituting the value of sec y we get,

$\Rightarrow \sec y = \sqrt{1 + \tan^2 y}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sec y = \sqrt{1 + \left(\frac{24}{7}\right)^2}$

$\Rightarrow \sec y = \sqrt{\frac{625}{49}}$

$\Rightarrow \sec y = \frac{25}{7}$

$\Rightarrow \cos y = \frac{1}{\sec y}$

$\Rightarrow \cos y = \frac{7}{25}$

$\Rightarrow \cos\left(\tan^{-1}\frac{24}{7}\right) = \frac{7}{25}$

Exercise 4.9 Page No: 4.58

1. Evaluate:

(i) Cos {$\sin^{-1}$ (-7/25)}

(ii) Sec {$\cot^{-1}$ (-5/12)}

(iii) Cot {$\sec^{-1}$ (-13/5)}

Solution:

(i) Given Cos {$\sin^{-1}$ (-7/25)}

Let $\sin^{-1}\left(-\frac{7}{25}\right)=x$ Where $x\in\left[-\frac{\pi}{2},0\right]$

$\Rightarrow \sin x=-\frac{7}{25}$

Now we have to find

$$\cos\left[\sin^{-1}\left(-\frac{7}{25}\right)\right]=\cos x$$

We know that $\sin^2x + \cos^2x = 1$
On rearranging and substituting we get,

$\Rightarrow \cos x=\sqrt{1-\sin^2 x}$ Since $x\in\left[-\frac{\pi}{2},0\right]$

$\Rightarrow \cos x=\sqrt{1-\frac{49}{625}}$

$\Rightarrow \cos x=\sqrt{\frac{576}{625}}$

$\Rightarrow \cos x=\frac{24}{25}$

$\Rightarrow \cos\left[\sin^{-1}\left(-\frac{7}{25}\right)\right]=\frac{24}{25}$

(ii) Given Sec {$\cot^{-1}$ (-5/12)}

Let $\cot^{-1}\left(-\frac{5}{12}\right) = x$ where $x \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow \cot x = -\frac{5}{12}$

Now we have to find,

$\sec\left[\cot^{-1}\left(-\frac{5}{12}\right)\right] = \sec x$

We know that $1 + \tan^2 x = \sec^2 x$

On rearranging, we get

$\Rightarrow 1 + \frac{1}{\cot^2 x} = \sec^2 x$

Substituting these values we get,

$\Rightarrow \sec x = -\sqrt{1 + \frac{1}{\cot^2 x}}$ Since $x \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow \sec x = -\sqrt{1 + \left(\frac{12}{5}\right)^2}$

$\Rightarrow \sec x = -\frac{13}{5}$

$\Rightarrow \sec\left[\cot^{-1}\left(-\frac{5}{12}\right)\right] = -\frac{13}{5}$

(iii) Given Cot {sec^{-1} (-13/5)}

Let $\sec^{-1}\left(-\frac{13}{5}\right)=x$ where $x\in\left(\frac{\pi}{2},\pi\right)$

$\Rightarrow \sec x=-\frac{13}{5}$

Now we have find,

$\cot\left[\sec^{-1}\left(-\frac{13}{5}\right)\right]=\cot x$

We know that $1+\tan^2 x=\sec^2 x$

On rearranging, we get

$\Rightarrow \tan x=-\sqrt{\sec^2 x-1}$

Now substitute the value of sec x, we get

$\Rightarrow \tan x=-\sqrt{\left(-\frac{13}{5}\right)^2-1}$

$\Rightarrow \tan x=-\frac{12}{5}$

$\Rightarrow \cot x=-\frac{5}{12}$

$\Rightarrow \cot\left[\sec^{-1}\left(-\frac{13}{5}\right)\right]=-\frac{5}{12}$

Exercise 4.10 Page No: 4.66

1. Evaluate:

(i) Cot (sin^{-1} (3/4) + sec^{-1} (4/3))

(ii) Sin ($\tan^{-1}$ x + $\tan^{-1}$ 1/x) for x < 0

(iii) Sin ($\tan^{-1}$ x + $\tan^{-1}$ 1/x) for x > 0

(iv) Cot ($\tan^{-1}$ a + $\cot^{-1}$ a)

(v) Cos ($\sec^{-1}$ x + $\operatorname{cosec}^{-1}$ x), |x| ≥ 1

Solution:

(i) Given Cot ($\sin^{-1}$ (3/4) + $\sec^{-1}$ (4/3))

$$= \cot\left(\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}\right)$$

$$\left(\because \sec^{-1}x=\cos^{-1}\frac{1}{x}\right)$$

We have

$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$$

By substituting these values in given questions, we get

$$= \cot\frac{\pi}{2}$$

= 0

(ii) Given Sin ($\tan^{-1}$ x + $\tan^{-1}$ 1/x) for x < 0

= $\sin\left(\tan^{-1}x + (\cot^{-1}x - \pi)\right)\left(\because \tan^{-1}\theta = \cot^{-1}\frac{1}{\theta} - \pi \quad \text{for } x < 0\right)$

= $\sin\left(\frac{\pi}{2} - \pi\right)\left(\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right)$

On simplifying, we get

= $\sin\left(-\frac{\pi}{2}\right)$

We know that sin (- θ) = – sin θ

= $-\sin\frac{\pi}{2} = -1$

(iii) Given Sin ($\tan^{-1}$ x + $\tan^{-1}$ 1/x) for x > 0

= $\sin\left(\tan^{-1}x + \cot^{-1}x\right)\left(\because \tan^{-1}\theta = \cot^{-1}\frac{1}{\theta} \quad \text{for } x > 0\right)$

Again we know that,

$\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}$

Now by substituting above identity in given question we get,

= $\sin\frac{\pi}{2}$

= 1

(iv) Given Cot ($\tan^{-1}$ a + $\cot^{-1}$ a)

We know that,

$$\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}$$

Now by substituting above identity in given question we get,

$$= \cot\left(\frac{\pi}{2}\right)$$

= 0

(v) Given Cos ($\sec^{-1}$ x + cosec^{-1} x), |x| ≥ 1

We know that

$$\sec^{-1}\theta = \cos^{-1}\frac{1}{\theta}$$

Again we have

$$\operatorname{cosec}^{-1}\theta = \sin^{-1}\frac{1}{\theta}$$

By substituting these values in given question we get,

$$= \cos\left(\cos^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x}\right)$$

We know that from the identities,

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$$

Now by substituting we get,

$$= \cos\frac{\pi}{2}$$

= 0

2. If $\cos^{-1} x + \cos^{-1} y = \pi/4$, find the value of $\sin^{-1} x + \sin^{-1} y$.

Solution:

Given $\cos^{-1} x + \cos^{-1} y = \pi/4$

We know that

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$$

Now substituting above identity in given question we get,

$$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1}x\right) + \left(\frac{\pi}{2} - \sin^{-1}y\right) = \frac{\pi}{4}$$

Adding and simplifying we get,

$$\Rightarrow \pi - \left(\sin^{-1}x + \sin^{-1}y\right) = \frac{\pi}{4}$$

On rearranging,

$$\Rightarrow \sin^{-1}x + \sin^{-1}y = \pi - \frac{\pi}{4}$$

$$\Rightarrow \sin^{-1}x + \sin^{-1}y = \frac{3\pi}{4}$$

3. If $\sin^{-1} x + \sin^{-1} y = \pi/3$ and $\cos^{-1} x - \cos^{-1} y = \pi/6$, find the values of x and y.

Solution:

Given $\sin^{-1} x + \sin^{-1} y = \pi/3$ ……. Equation (i)

And $\cos^{-1} x - \cos^{-1} y = \pi/6$ ……… Equation (ii)

Subtracting Equation (ii) from Equation (i), we get

$$\Rightarrow (\sin^{-1} x - \cos^{-1} x) + (\sin^{-1} y + \cos^{-1} y) = \frac{\pi}{3} - \frac{\pi}{6}$$

We know that,

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2},$$

By substituting above identity, we get

$$\Rightarrow (\sin^{-1} x - \cos^{-1} x) + \left(\frac{\pi}{2}\right) = \frac{\pi}{6}$$

Replacing $\sin^{-1} x$ by $\pi/2 - \cos^{-1} x$ and rearranging we get,

$$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x\right) - \cos^{-1} x = -\frac{\pi}{3}$$

Now by adding,

$$\Rightarrow 2\cos^{-1} x = \frac{5\pi}{6}$$

$$\Rightarrow \cos^{-1} x = \frac{5\pi}{12}$$

$$\Rightarrow x = \cos\left(\frac{5\pi}{12}\right)$$

$$\Rightarrow x = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

We know that Cos (A + B) = Cos A. Cos B – Sin A. Sin B, substituting this we get,

4. If cot ($\cos^{-1}$ 3/5 + $\sin^{-1}$ x) = 0, find the value of x.

Solution:

Given cot ($\cos^{-1}$ 3/5 + $\sin^{-1}$ x) = 0

On rearranging we get,

$(\cos^{-1} 3/5 + \sin^{-1} x) = \cot^{-1} (0)$

$(\text{Cos}^{-1} 3/5 + \sin^{-1} x) = \pi/2$

We know that $\cos^{-1} x + \sin^{-1} x = \pi/2$

Then $\sin^{-1} x = \pi/2 - \cos^{-1} x$

Substituting the above in $(\cos^{-1} 3/5 + \sin^{-1} x) = \pi/2$ we get,

$(\text{Cos}^{-1} 3/5 + \pi/2 - \cos^{-1} x) = \pi/2$

Now on rearranging we get,

$(\text{Cos}^{-1} 3/5 - \cos^{-1} x) = \pi/2 - \pi/2$

$(\text{Cos}^{-1} 3/5 - \cos^{-1} x) = 0$

Therefore $\text{Cos}^{-1} 3/5 = \cos^{-1} x$

On comparing the above equation we get,

$x = 3/5$

5. If $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$, find x.

Solution:

Given $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$

We know that $\cos^{-1} x + \sin^{-1} x = \pi/2$

Then $\cos^{-1} x = \pi/2 - \sin^{-1} x$

Substituting this in $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$ we get

$(\sin^{-1} x)^2 + (\pi/2 - \sin^{-1} x)^2 = 17 \pi^2/36$

Let $y = \sin^{-1} x$

$y^2 + ((\pi/2) - y)^2 = 17 \pi^2/36$

$y^2 + \pi^2/4 – y^2 – 2y ((\pi/2) – y) = 17 \pi^2/36$

$\pi^2/4 – \pi y + 2 y^2 = 17 \pi^2/36$

On rearranging and simplifying, we get

$2y^2 – \pi y + 2/9 \pi^2 = 0$

$18y^2 – 9 \pi y + 2 \pi^2 = 0$

$18y^2 – 12 \pi y + 3 \pi y + 2 \pi^2 = 0$

$6y (3y – 2\pi) + \pi (3y – 2\pi) = 0$

Now, $(3y – 2\pi) = 0$ and $(6y + \pi) = 0$

Therefore $y = 2\pi/3$ and $y = – \pi/6$

Now substituting $y = – \pi/6$ in $y = \sin^{-1} x$ we get

$\sin^{-1} x = – \pi/6$

$x = \sin (- \pi/6)$

$x = -1/2$

Now substituting $y = -2\pi/3$ in $y = \sin^{-1} x$ we get

$x = \sin (2\pi/3)$

$x = \sqrt{3}/2$

Now substituting $x = \sqrt{3}/2$ in $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$ we get,

$= \pi/3 + \pi/6$

$= \pi/2$ which is not equal to $17 \pi^2/36$

So we have to neglect this root.

Now substituting $x = -1/2$ in $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$ we get,

$= \pi^2/36 + 4 \pi^2/9$

$= 17\pi^2/36$

Hence x = -1/2.

Exercise 4.11 Page No: 4.82

1. Prove the following results:

(i) $\tan^{-1}(1/7) + \tan^{-1}(1/13) = \tan^{-1}(2/9)$

(ii) $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi$

(iii) $\tan^{-1}(1/4) + \tan^{-1}(2/9) = \sin^{-1}(1/\sqrt{5})$

Solution:

(i) Given $\tan^{-1}(1/7) + \tan^{-1}(1/13) = \tan^{-1}(2/9)$

Consider LHS

$\tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{1}{13})$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

According to the formula, we can write as

$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\times\frac{1}{13}}\right)$

$= \tan^{-1}\left(\frac{\frac{13+7}{91}}{\frac{91-1}{91}}\right)$

$= \tan^{-1}\left(\frac{20}{90}\right)$

$= \tan^{-1}\left(\frac{2}{9}\right)$

= RHS

Hence, the proof.

Hence, proved.

(ii) Given $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi$

Consider LHS

$\sin^{-1}(\frac{12}{13}) + \cos^{-1}\frac{4}{5} + \tan^{-1}(\frac{63}{16})$

We know that, Formula

$$\sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Now, by substituting the formula we get,

$$= \tan^{-1}\left(\frac{\frac{12}{13}}{\sqrt{1-\left(\frac{12}{13}\right)^2}}\right) + \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}\right) + \tan^{-1}(\frac{63}{16})$$

$$= \tan^{-1}(\frac{12}{5}) + \tan^{-1}(\frac{3}{4}) + \tan^{-1}(\frac{63}{16})$$

Again we know that,

$$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\frac{x+y}{1-xy}$$

Again by substituting, we get

$$= \pi + \tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}\right) + \tan^{-1}(\frac{63}{16})$$

$$= \pi + \tan^{-1}(-\frac{63}{16}) + \tan^{-1}(\frac{63}{16})$$

We know that,

$\tan^{-1}(-x) = -\tan^{-1} x$

$= \pi - \tan^{-1}(\frac{63}{16}) + \tan^{-1}(\frac{63}{16})$

$= \pi$

So, $\sin^{-1}(\frac{12}{13}) + \cos^{-1}\frac{4}{5} + \tan^{-1}(\frac{63}{16}) = \pi$

Hence, the proof.

Hence, proved.

(iii) Given $\tan^{-1}(1/4) + \tan^{-1}(2/9) = \sin^{-1}(1/\sqrt{5})$

$\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9})$

We know that,

$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy}$

By substituting this formula we get,

$= \tan^{-1} \frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}$

$= \tan^{-1} \frac{\frac{17}{36}}{\frac{34}{36}}$

$= \tan^{-1} \frac{\frac{17}{36}}{\frac{34}{36}}$

$= \tan^{-1} \frac{1}{2}$

Now let, $\tan\theta = \frac{1}{2}$

Therefore, $\sin\theta = \frac{1}{\sqrt{5}}$

So, $\theta = \sin^{-1} \frac{1}{\sqrt{5}}$

$\Rightarrow \tan^{-1}(\frac{1}{2}) = \sin^{-1}(\frac{1}{\sqrt{5}}) = \text{RHS}$

$\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \sin^{-1}(\frac{1}{\sqrt{5}})$

Hence, Proved.

2. Find the value of $\tan^{-1}$ (x/y) – $\tan^{-1}$ {(x-y)/(x + y)}

Solution:

Given $\tan^{-1}$ (x/y) – $\tan^{-1}$ {(x-y)/(x + y)}

We know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Now by substituting the formula, we get

$$= \tan^{-1}\frac{\frac{x}{y}-\left(\frac{x-y}{x+y}\right)}{1+\frac{x}{y}\times\left(\frac{x-y}{x+y}\right)}$$

$$= \tan^{-1}\frac{x(x+y)-y(x-y)}{y(x+y)+x(x-y)}$$

$$= \tan^{-1}\frac{x^2+y^2}{x^2+y^2}$$

$$= \tan^{-1}1$$

$$= \frac{\pi}{4}$$

So,

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \frac{\pi}{4}$$

Exercise 4.12 Page No: 4.89

1. Evaluate: Cos ($\sin^{-1}$ 3/5 + $\sin^{-1}$ 5/13)

Solution:

Given Cos ($\sin^{-1}$ 3/5 + $\sin^{-1}$ 5/13)

We know that,

$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

By substituting this formula we get,

$= \cos\left(\sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]\right)$

$= \cos\left(\sin^{-1}\left[\frac{3}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{4}{5}\right]\right)$

$= \cos\left(\sin^{-1}\left[\frac{56}{65}\right]\right)$

Again, we know that

$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$

Now substituting, we get

$= \cos\left(\cos^{-1}\sqrt{1-\left(\frac{56}{65}\right)^2}\right)$

$= \cos\left(\cos^{-1}\sqrt{\frac{33}{65}}\right)$

$= \frac{33}{65}$

Hence, $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right) = \frac{33}{65}$

$$= \cos\left(\cos^{-1}\sqrt{1-\left(\frac{56}{65}\right)^2}\right)$$

$$= \cos\left(\cos^{-1}\sqrt{\left(\frac{33}{65}\right)^2}\right) = \cos\left(\cos^{-1}\left(\frac{33}{65}\right)\right)$$

$$= \frac{33}{65}$$

Hence, $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right) = \frac{33}{65}$

Exercise 4.13 Page No: 4.92

1. If $\cos^{-1}(x/2) + \cos^{-1}(y/3) = \alpha$, then prove that $9x^2 - 12xy \cos \alpha + 4y^2 = 36 \sin^2 \alpha$

Solution:

Given $\cos^{-1}(x/2) + \cos^{-1}(y/3) = \alpha$

We know that,

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$

Now by substituting, we get

$\Rightarrow \cos^{-1}\left[\frac{x}{2} \times \frac{y}{3} - \sqrt{1-\left(\frac{x}{2}\right)^2}\sqrt{1-\left(\frac{y}{3}\right)^2}\right] = \alpha$

$\Rightarrow \left[\frac{xy}{6} - \frac{\sqrt{4-x^2}}{2} \times \frac{\sqrt{9-y^2}}{3}\right] = \cos\alpha$

$\Rightarrow xy - \sqrt{4-x^2} \times \sqrt{9-y^2} = 6\cos\alpha$

$\Rightarrow xy - 6\cos\alpha = \sqrt{4-x^2}\sqrt{9-y^2}$

On squaring both the sides we get

$\Rightarrow (xy - 6\cos\alpha)^2 = (4-x^2)(9-y^2)$

$\Rightarrow x^2y^2 + 36\cos^2\alpha - 12xy\cos\alpha = 36 - 9x^2 - 4y^2 + x^2y^2$

$\Rightarrow 9x^2 + 4y^2 - 36 + 36\cos^2\alpha - 12xy\cos\alpha = 0$

$\Rightarrow 9x^2 + 4y^2 - 12xy\cos\alpha - 36(1 - \cos^2\alpha) = 0$

$\Rightarrow 9x^2 + 4y^2 - 12xy\cos\alpha - 36\sin^2\alpha = 0$

$\Rightarrow 9x^2 + 4y^2 - 12xy\cos\alpha = 36\sin^2\alpha$

Hence the proof.

Hence, proved.

2. Solve the equation: $\cos^{-1}(a/x) - \cos^{-1}(b/x) = \cos^{-1}(1/b) - \cos^{-1}(1/a)$

Solution:

Given $\cos^{-1}(a/x) - \cos^{-1}(b/x) = \cos^{-1}(1/b) - \cos^{-1}(1/a)$

$\Rightarrow \cos^{-1}\frac{a}{x}+\cos^{-1}\frac{1}{a}=\cos^{-1}\frac{1}{b}+\cos^{-1}\frac{b}{x}$

We know that,

$\cos^{-1}x+\cos^{-1}y = \cos^{-1}\left[xy-\sqrt{1-x^2}\sqrt{1-y^2}\right]$

By substituting this formula we get,

$\Rightarrow \cos^{-1}\left[\frac{1}{x}-\sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2}\right]=\cos^{-1}\left[\frac{1}{x}-\sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}\right]$

$\Rightarrow \frac{1}{x}-\sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2}=\frac{1}{x}-\sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}$

$\Rightarrow \sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2}=\sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}$

Squaring on both the sides, we get

$\Rightarrow \left(1-\left(\frac{a}{x}\right)^2\right)\left(1-\left(\frac{1}{a}\right)^2\right)=\left(1-\left(\frac{b}{x}\right)^2\right)\left(1-\left(\frac{1}{b}\right)^2\right)$

$\Rightarrow 1-\left(\frac{a}{x}\right)^2-\left(\frac{1}{a}\right)^2+\left(\frac{1}{x}\right)^2 = 1-\left(\frac{b}{x}\right)^2-\left(\frac{1}{b}\right)^2+\left(\frac{1}{x}\right)^2$

$\Rightarrow \left(\frac{b}{x}\right)^2-\left(\frac{a}{x}\right)^2 = \left(\frac{1}{a}\right)^2-\left(\frac{1}{b}\right)^2$

On simplifying, we get

$\Rightarrow (b^2 - a^2)\ a^2b^2 = x^2(b^2 - a^2)$

$\Rightarrow x^2 = a^2b^2$

$\Rightarrow x = a\ b$

Exercise 4.14 Page No: 4.115

1. Evaluate the following:

(i) tan {2 $\tan^{-1}$ (1/5) – π/4}

(ii) Tan {1/2 $\sin^{-1}$ (3/4)}

(iii) Sin {1/2 $\cos^{-1}$ (4/5)}

(iv) Sin (2 $\tan^{-1}$ 2/3) + cos ($\tan^{-1}$ √3)

Solution:

(i) Given tan {2 $\tan^{-1}$ (1/5) – π/4}

We know that,

$$2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right), \text{if } |x| < 1$$

And $\frac{\pi}{4}$ can be written as $\tan^{-1}(1)$

Now substituting these values we get,

$$= \tan\left\{\tan^{-1}\left(\frac{2 \times \frac{1}{5}}{1-\frac{1}{25}}\right) - \tan^{-1} 1\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1} 1\right\}$$

Again we know that,

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$$

Now substituting this formula, we get

$$= \tan\left\{\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}\right)\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{-7}{17}\right)\right\}$$

$$= -\frac{7}{17}$$

(ii) Given tan {1/2 $\sin^{-1}$ (3/4)}

Let $\frac{1}{2}\sin^{-1}\frac{3}{4} = t$

Therefore,

$\Rightarrow \sin^{-1}\frac{3}{4} = 2t$

$\Rightarrow \sin 2t = \frac{3}{4}$

Now, by Pythagoras theorem, we have

$\Rightarrow \sin 2t = \frac{3}{4} = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\Rightarrow \cos 2t = \frac{\sqrt{4^2-3^2}}{4} = \frac{\text{Base}}{\text{hypotenuse}}$

$\Rightarrow \cos 2t = \frac{\sqrt{7}}{4}$

By considering, given question

$\tan\left\{\frac{1}{2}\sin^{-1}\frac{3}{4}\right\}$

$= \tan(t)$

We know that,

$\tan(x) = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

$= \sqrt{\frac{1-\cos 2t}{1+\cos 2t}}$

$= \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}}$

$= \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}}$

Now by rationalizing the denominator, we get

$= \sqrt{\frac{(4-\sqrt{7})(4-\sqrt{7})}{(4+\sqrt{7})(4-\sqrt{7})}}$

$= \sqrt{\frac{(4-\sqrt{7})^2}{9}}$

$= \frac{4-\sqrt{7}}{3}$

Hence

$$\tan\left\{\frac{1}{2}\sin^{-1}\frac{3}{4}\right\} = \frac{4-\sqrt{7}}{3}$$

(iii) Given sin {1/2 $\cos^{-1}$ (4/5)}

We know that

$$\cos^{-1}x = 2\sin^{-1}\left(\pm\sqrt{\frac{1-x}{2}}\right)$$

Now by substituting this formula we get,

$$\sin\left(\frac{1}{2}2\sin^{-1}\left(\pm\sqrt{\frac{1-\frac{4}{5}}{2}}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\pm\sqrt{\frac{1}{2\times5}}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\pm\frac{1}{\sqrt{10}}\right)\right)$$

As we know that

$$\sin(\sin^{-1}x) = x \text{ as } n \in [-1, 1]$$

$$= \pm\frac{1}{\sqrt{10}}$$

Hence, $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \pm\frac{1}{\sqrt{10}}$

(iv) Given Sin (2 tan $^{-1}$ 2/3) + cos (tan^{-1} √3)

We know that

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}(x)$;

$\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \tan^{-1}(x)$;

Now by substituting these formulae we get,

$= \sin\left(\sin^{-1}\left(\frac{2\times\frac{2}{3}}{1+\frac{4}{9}}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+3}}\right)\right)$

$= \sin\left(\sin^{-1}\left(\frac{12}{13}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

$= \frac{12}{13} + \frac{1}{2}$

$= \frac{37}{26}$

Hence,

$\sin\left(2\tan^{-1}\left(\frac{2}{3}\right)\right) + \cos\left(\tan^{-1}\sqrt{3}\right) = \frac{37}{26}$

2. Prove the following results:

(i) 2 $\sin^{-1}$ (3/5) = $\tan^{-1}$ (24/7)

(ii) $\tan^{-1}$ ¼ + $\tan^{-1}$ (2/9) = ½ $\cos^{-1}$ (3/5) = ½ $\sin^{-1}$ (4/5)

(iii) $\tan^{-1}$ (2/3) = ½ $\tan^{-1}$ (12/5)

(iv) $\tan^{-1}$ (1/7) + 2 $\tan^{-1}$ (1/3) = π/4

(v) $\sin^{-1}$ (4/5) + 2 $\tan^{-1}$ (1/3) = π/2

(vi) 2 sin^{-1} (3/5) – tan^{-1} (17/31) = π/4

(vii) 2 tan^{-1} (1/5) + tan^{-1} (1/8) = tan^{-1} (4/7)

(viii) 2 tan^{-1} (3/4) – tan^{-1} (17/31) = π/4

(ix) 2 tan^{-1} (1/2) + tan^{-1} (1/7) = tan^{-1} (31/17)

(x) 4 tan^{-1}(1/5) – tan^{-1}(1/239) = π/4

Solution:

(i) Given 2 sin^{-1} (3/5) = tan^{-1} (24/7)

$= \sin\left(\sin^{-1}\left(\frac{12}{13}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

$= \frac{12}{13} + \frac{1}{2}$

$= \frac{37}{26}$

Hence,

$$\sin\left(2\tan^{-1}\left(\frac{2}{3}\right)\right) + \cos\left(\tan^{-1}\sqrt{3}\right) = \frac{37}{26}$$

Consider LHS

$2\sin^{-1}\frac{3}{5}$

We know that

$$\sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Now by substituting the above formula we get,

Hence, proved.

(ii) Given $\tan^{-1} \frac{1}{4} + \tan^{-1} (2/9) = \frac{1}{2} \cos^{-1} (3/5) = \frac{1}{2} \sin^{-1} (4/5)$

Consider LHS

$= \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$

We know that

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Now by substituting this formula, we get

$$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right)$$

$$= \tan^{-1}\left(\frac{17}{34}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

Multiplying and dividing by 2

$$= \frac{1}{2}\left\{2\tan^{-1}\left(\frac{1}{2}\right)\right\}$$

Again we know that

$$2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right)$$

$= \frac{1}{2}\cos^{-1}\left(\frac{\frac{3}{4}}{\frac{5}{4}}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

= RHS

So, $\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

Now,

$= \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

We know that,

$= \cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$

By substituting this, we get

$= \frac{1}{2}\sin^{-1}\sqrt{1-\frac{9}{25}}$

$= \frac{1}{2}\sin^{-1}\sqrt{\frac{16}{25}}$

$= \frac{1}{2}\sin^{-1}\frac{4}{5}$

= RHS

So, $\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right) = \frac{1}{2}\sin^{-1}\frac{4}{5}$

Hence the proof.

Hence, proved.

(iii) Given $\tan^{-1}(2/3) = ½ \tan^{-1}(12/5)$

Consider LHS

$= \tan^{-1}(\frac{2}{3})$

Now, Multiplying and dividing by 2, we get

$= \frac{1}{2}\left\{2\tan^{-1}\left(\frac{2}{3}\right)\right\}$

We know that

$$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

By substituting the above formula we get

$= \frac{1}{2}\tan^{-1}\left(\frac{2\times\frac{2}{3}}{1-\frac{4}{9}}\right)$

$= \frac{1}{2}\tan^{-1}\left(\frac{\frac{4}{3}}{\frac{5}{9}}\right)$

$= \frac{1}{2}tan^{-1}\left(\frac{12}{5}\right)$

= RHS

So, $tan^{-1}(\frac{2}{3}) = \frac{1}{2}tan^{-1}\left(\frac{12}{5}\right)$

Hence the proof.

Hence, proved.

(iv) Given $\tan^{-1}(1/7) + 2\tan^{-1}(1/3) = \pi/4$

Consider LHS

$= \tan^{-1}(\frac{1}{7}) + 2\tan^{-1}(\frac{1}{3})$

We know that,

$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

By substituting the above formula we get,

$= \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}})$

$= \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{\frac{2}{3}}{\frac{8}{9}})$

$= \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{3}{4})$

Again we know that

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}\right)$

$= \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So, $\tan^{-1}(\frac{1}{7}) + 2\tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$

Hence the proof.

Hence, proved.

(v) Given $\sin^{-1}(4/5) + 2\tan^{-1}(1/3) = \pi/2$

Consider LHS

$= \sin^{-1}(\frac{4}{5}) + 2\tan^{-1}(\frac{1}{3})$

We know that,

$$\sin^{-1}(x) = \tan^{-1}(\frac{x}{\sqrt{1-x^2}})$$

And, $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Now by substituting the formula we get,

$$= \tan^{-1}(\frac{\frac{4}{5}}{\sqrt{1-\frac{16}{25}}}) + \tan^{-1}(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}})$$

$$= \tan^{-1}(\frac{\frac{4}{5}}{\sqrt{\frac{9}{25}}}) + \tan^{-1}(\frac{\frac{2}{3}}{\frac{8}{9}})$$

$$= \tan^{-1}(\frac{4}{3}) + \tan^{-1}(\frac{3}{4})$$

We know that,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\times\frac{3}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{25}{12}}{0}\right)$$

$= \tan^{-1}(\infty)$

$= \frac{\pi}{2}$

= RHS

So, $\sin^{-1}(\frac{4}{5}) + 2\tan^{-1}(\frac{1}{3}) = \frac{\pi}{2}$

Hence Proved

(vi) Given $2 \sin^{-1} (3/5) – \tan^{-1} (17/31) = \pi/4$

Consider LHS

$= 2\sin^{-1}(\frac{3}{5}) - \tan^{-1}(\frac{17}{31})$

We know that

$$\sin^{-1}(x) = \tan^{-1}(\frac{x}{\sqrt{1-x^2}})$$

According to the formula we have,

$= 2\tan^{-1}(\frac{\frac{3}{5}}{\sqrt{1-\frac{16}{25}}}) - \tan^{-1}(\frac{17}{31})$

$= 2\tan^{-1}(\frac{\frac{4}{5}}{\sqrt{\frac{9}{25}}}) - \tan^{-1}(\frac{17}{31})$

$= 2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31})$

Again we know that,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

By substituting this formula, we get

$$= 2\tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\frac{9}{25}}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= 2\tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{\frac{16}{25}}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= 2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

Again we know that,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

By substituting this formula, we get

$$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

Again we have,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$

$= \tan^{-1}\left(\frac{\frac{744-119}{217}}{\frac{217+408}{217}}\right)$

$= \tan^{-1}\left(\frac{625}{625}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$ = RHS

So, $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

Hence the proof.

(vii) Given 2 $\tan^{-1}$ (1/5) + $\tan^{-1}$ (1/8) = $\tan^{-1}$ (4/7)

Consider LHS

$= 2\tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{1}{8})$

We know that

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by substituting the formula we get,

$$= \tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

Again from the formula we have,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{1}{8}}{1-\frac{5}{12}\times\frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{10+3}{24}}{\frac{96-5}{96}}\right)$$

$$= \tan^{-1}\left(\frac{13}{24}\times\frac{96}{91}\right)$$

$= \tan^{-1}\left(\frac{4}{7}\right)$

= RHS

So, $2\tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{1}{8}) = \tan^{-1}(\frac{4}{7})$

Hence the proof.

Hence, proved.

(viii) Given $2\tan^{-1}(3/4) - \tan^{-1}(17/31) = \pi/4$

Consider LHS

$= 2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31})$

We know that,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by substituting the formula we get,

$= \tan^{-1}(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}) - \tan^{-1}(\frac{17}{31})$

$= \tan^{-1}(\frac{3}{2}\times\frac{16}{7}) - \tan^{-1}(\frac{17}{31})$

$= \tan^{-1}(\frac{24}{7}) - \tan^{-1}(\frac{17}{31})$

We know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Again by substituting the formula we get,

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$

$= \tan^{-1}\left(\frac{\frac{744-119}{217}}{\frac{217+408}{217}}\right)$

$= \tan^{-1}\left(\frac{625}{625}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So, $2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

Hence the proof.

Hence, proved.

(ix) Given 2 $\tan^{-1}(1/2) + \tan^{-1}(1/7) = \tan^{-1}(31/17)$

Consider LHS

= $2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7})$

We know that,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by substituting the formula we get,

= $\tan^{-1}(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}) + \tan^{-1}(\frac{1}{7})$

= $\tan^{-1}(\frac{\frac{2}{2}}{\frac{3}{4}}) + \tan^{-1}(\frac{1}{7})$

= $\tan^{-1}(\frac{4}{3}) + \tan^{-1}(\frac{1}{7})$

Again by using the formula, we can write as

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

= $\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{1}{7}\times\frac{4}{3}}\right)$

$= \tan^{-1}\left(\frac{\frac{31}{21}}{\frac{17}{21}}\right)$

$= \tan^{-1}\left(\frac{31}{17}\right)$

= RHS

So, $2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \tan^{-1}(\frac{31}{17})$

Hence the proof.

Hence, proved.

(x) Given 4 $\tan^{-1}(1/5) - \tan^{-1}(1/239) = \pi/4$

Consider LHS

$= 4\tan^{-1}(\frac{1}{5}) - \tan^{-1}(\frac{1}{239})$

We know that,

$$4\tan^{-1}x = \tan^{-1}\left(\frac{4x - 4x^3}{1 - 6x^2 + x^4}\right)$$

Now by substituting the formula, we get

$= \tan^{-1}\left(\frac{4\times\frac{1}{5}-4\left(\frac{1}{5}\right)^3}{1-6\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^4}\right) - \tan^{-1}(\frac{1}{239})$

$= \tan^{-1}(\frac{120}{119}) - \tan^{-1}(\frac{1}{239})$

Again we know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x - y}{1 + xy}$$

$= \tan^{-1}\left(\frac{\frac{120}{119}-\frac{1}{239}}{1-\frac{120}{119}\times\frac{1}{239}}\right)$

$= \tan^{-1}\left(\frac{120\times239-119}{119\times239+120}\right)$

$= \tan^{-1}\left(\frac{28561}{28561}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So,

$$4\tan^{-1}(\frac{1}{5}) - \tan^{-1}(\frac{1}{239}) = \frac{\pi}{4}$$

Hence the proof.

Hence, proved.

3. If $\sin^{-1}(2a/1 + a^2) - \cos^{-1}(1 - b^2/1 + b^2) = \tan^{-1}(2x/1 - x^2)$, then prove that x = (a – b)/ (1 + a b)

Solution:

Given $\sin^{-1}(2a/1 + a^2) - \cos^{-1}(1 - b^2/1 + b^2) = \tan^{-1}(2x/1 - x^2)$

Consider,

$\Rightarrow \sin^{-1}(\frac{2a}{1+a^2}) - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}(\frac{2x}{1-x^2})$

We know that,

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by applying these formulae in given equation we get,

$\Rightarrow 2\tan^{-1}(a) - 2\tan^{-1}(b) = 2\tan^{-1}(x)$

$\Rightarrow 2(\tan^{-1}(a) - \tan^{-1}(b)) = 2\tan^{-1}(x)$

$\Rightarrow \tan^{-1}(a) - \tan^{-1}(b) = \tan^{-1}(x)$

Again we know that,

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x - y}{1 + xy}$$

Now by substituting this in above equation we get,

$\Rightarrow \tan^{-1}(\frac{a-b}{1+ab}) = \tan^{-1}(x)$

On comparing we get,

$\Rightarrow x = \frac{a-b}{1+ab}$

Hence the proof.

Hence, proved.

4. Prove that:

(i) $\tan^{-1}\{(1 - x^2)/ 2x)\} + \cot^{-1}\{(1 - x^2)/ 2x)\} = \pi/2$

(ii) $\sin \{\tan^{-1} (1 - x^2)/ 2x) + \cos^{-1} (1 - x^2)/ (1 + x^2)\} = 1$

Solution:

(i) Given $\tan^{-1}\{(1 - x^2)/ 2x)\} + \cot^{-1}\{(1 - x^2)/ 2x)\} = \pi/2$

Consider LHS

$= \tan^{-1}\frac{1-x^2}{2x} + \cot^{-1}\frac{1-x^2}{2x}$

We know that,

$$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$$

Now by applying the above formula we get,

$= \tan^{-1}\left(\frac{1-x^2}{2x}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Again we know

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

By substituting this we get,

$$= \tan^{-1}\left(\frac{\left(\frac{1-x^2}{2x}\right)+\left(\frac{2x}{1-x^2}\right)}{1-\left(\frac{1-x^2}{2x}\right)\times\left(\frac{2x}{1-x^2}\right)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}}\right)$$

$$= \tan^{-1}\left(\frac{1+x^4+2x^2}{0}\right)$$

$= \tan^{-1}(\infty)$

$= \frac{\pi}{2}$ = RHS

$$\tan^{-1}\frac{1-x^2}{2x} + \cot^{-1}\frac{1-x^2}{2x} = \frac{\pi}{2}$$

Hence, proved.

(ii) Given sin {$\tan^{-1}$ $(1 - x^2)/ 2x$) + $\cos^{-1}$ $(1 - x^2)/ (1 + x^2)$}

Consider LHS

$= \sin\left(\tan^{-1}\frac{1-x^2}{2x} + \cos^{-1}\frac{1-x^2}{1+x^2}\right)$

We know that,

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Now by applying the formula in above question we get,

$= \sin\left(\tan^{-1}\frac{1-x^2}{2x} + 2\tan^{-1}x\right)$

Again, we have

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by substituting the formula we get,

$= \sin\left(\tan^{-1}\frac{1-x^2}{2x} + \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)$

Again we know that,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Now by applying the formula,

$= \sin\left(\tan^{-1}\left(\frac{\frac{1-x^2}{2x}+\left(\frac{2x}{1-x^2}\right)}{1-\frac{1-x^2}{2x}\times\left(\frac{2x}{1-x^2}\right)}\right)\right)$

$= \sin\left(\tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}}\right)\right)$

$= \sin\left(\tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{0}\right)\right)$

$= \sin(\tan^{-1}(\infty))$

$= \sin\left(\frac{\pi}{2}\right)$

= 1

= RHS

So,

$$\sin^{-1}\left(\tan^{-1}\frac{1-x^2}{2x} + \cos^{-1}\frac{1-x^2}{1+x^2}\right) = 1$$

Hence the proof.

Hence, proved.

5. If $\sin^{-1}$ (2a/ 1+ a^2) + $\sin^{-1}$ (2b/ 1+ b^2) = 2 $\tan^{-1}$ x, prove that x = (a + b/ 1 – a b)

Solution:

Given $\sin^{-1}$ (2a/ 1+ a^2) + $\sin^{-1}$ (2b/ 1+ b^2) = 2 $\tan^{-1}$ x

Consider

$$\sin^{-1}(\frac{2a}{1+a^2}) + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1}(x)$$

We know that,

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Now by applying the above formula we get,

$$\Rightarrow 2\tan^{-1}(a) + 2\tan^{-1}(b) = 2\tan^{-1}(x)$$

$$\Rightarrow 2(\tan^{-1}(a) + \tan^{-1}(b)) = 2\tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}(x)$$

Again we have,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Now by substituting, we get

$$\Rightarrow \tan^{-1}(\frac{a+b}{1-ab}) = \tan^{-1}(x)$$

On comparing we get,

$$\Rightarrow x = \frac{a+b}{1-ab}$$

Hence the proof.

Hence, proved.

Chapterwise RD Sharma Solutions for Class 12 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.