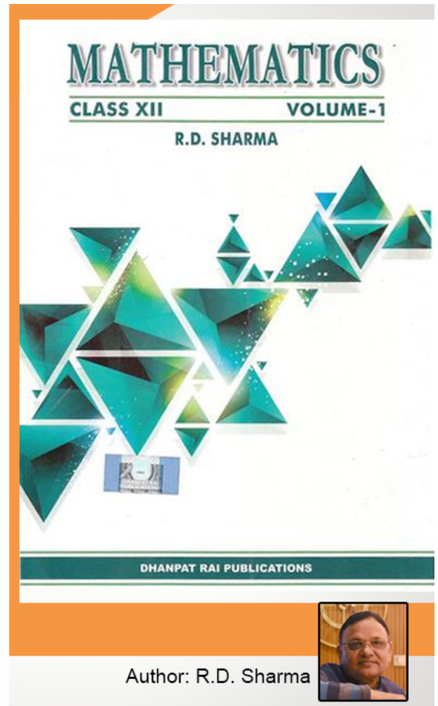
Class 12 - Chapter 3 Binary Operations



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Exercise 3.1 Page No: 3.4

1. Determine whether the following operation define a binary operation on the given set or not:

(i) '*' on N defined by a * b = a^b for all a, b ∈ N.

(ii) 'O' on Z defined by a O b = a^b for all a, b ∈ Z.

(iii) '*' on N defined by a * b = a + b – 2 for all a, b ∈ N

(iv) '$\times_6$' on S = {1, 2, 3, 4, 5} defined by a $\times_6$ b = Remainder when a b is divided by 6.

(v) '$+_6$' on S = {0, 1, 2, 3, 4, 5} defined by a $+_6$ b

$$= \begin{cases} a+b, \ if \ a+b<6 \\ a+b-6, \ if \ a+b \geq 6 \end{cases}$$

(vi) '⊙' on N defined by a ⊙ b= $a^b + b^a$ for all a, b ∈ N

(vii) '*' on Q defined by a * b = (a – 1)/ (b + 1) for all a, b ∈ Q

Solution:

(i) Given '*' on N defined by a * b = a^b for all a, b ∈ N.

Let a, b ∈ N. Then,

a^b ∈ N [∵ $a^b \neq 0$ and a, b is positive integer]

⇒ a * b ∈ N

Therefore,

a * b ∈ N, ∀ a, b ∈ N

Thus, * is a binary operation on N.

(ii) Given 'O' on Z defined by a O b = a^b for all a, b ∈ Z.

Both a = 3 and b = -1 belong to Z.

⇒ a * b = 3^{-1}

= 1/3 ∉ Z

Thus, * is not a binary operation on Z.

(iii) Given '*' on N defined by a * b = a + b – 2 for all a, b ∈ N

If a = 1 and b = 1,

a * b = a + b – 2

= 1 + 1 – 2

= 0 ∉ N

Thus, there exist a = 1 and b = 1 such that a * b ∉ N

So, * is not a binary operation on N.

(iv) Given '$\times_6$' on S = {1, 2, 3, 4, 5} defined by a $\times_6$ b = Remainder when a b is divided by 6.

Consider the composition table,

X_6	1	2	3	4	5
1	1	2	3	4	5
2	2	4	0	2	4
3	3	0	3	0	3
4	4	2	0	4	2
5	5	4	3	2	1

Here all the elements of the table are not in S.

⇒ For a = 2 and b = 3,

a $\times_6$ b = 2 $\times_6$ 3 = remainder when 6 divided by 6 = 0 ≠ S

Thus, $\times_6$ is not a binary operation on S.

(v) Given '$+_6$' on S = {0, 1, 2, 3, 4, 5} defined by a $+_6$ b

$$= \begin{cases} a+b, \ if \ a+b<6 \\ a+b-6, \ if \ a+b \geq 6 \end{cases}$$

Consider the composition table,

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Here all the elements of the table are not in S.

⇒ For a = 2 and b = 3,

a $\times_6$ b = 2 $\times_6$ 3 = remainder when 6 divided by 6 = 0 ≠ Thus, $\times_6$ is not a binary operation on S.

(vi) Given '⊙' on N defined by a ⊙ b= $a^b + b^a$ for all a, b ∈ N

Let a, b ∈ N. Then,

$a^b, b^a \in N$

$\Rightarrow a^b + b^a \in N$ [∵Addition is binary operation on N]

$\Rightarrow a \odot b \in N$

Thus, ⊙ is a binary operation on N.

(vii) Given '*' on Q defined by a * b = (a – 1)/ (b + 1) for all a, b ∈ Q

If a = 2 and b = -1 in Q,

a * b = (a – 1)/ (b + 1)

= (2 – 1)/ (- 1 + 1)

= 1/0 [which is not defined]

For a = 2 and b = -1

a * b does not belongs to Q

So, * is not a binary operation in Q.

2. Determine whether or not the definition of * given below gives a binary operation. In the event that * is not a binary operation give justification of this.

(i) On Z^+, defined * by a * b = a – b

(ii) On Z^+, define * by a*b = ab

(iii) On R, define * by a*b = ab^2

(iv) On Z^+ define * by a * b = |a − b|

(v) On Z^+ define * by a * b = a

(vi) On R, define * by a * b = a + $4b^2$

Here, Z^+ denotes the set of all non-negative integers.

Solution:

(i) Given On Z^+, defined * by a * b = a – b

If a = 1 and b = 2 in Z^+, then

a * b = a – b

= 1 – 2

= -1 ∉ Z^+ [because Z^+ is the set of non-negative integers]

For a = 1 and b = 2,

a * b ∉ Z^+

Thus, * is not a binary operation on Z^+.

(ii) Given Z^+, define * by a*b = a b

Let a, b ∈ Z^+

⇒ a, b ∈ Z^+

⇒ a * b ∈ Z^+

Thus, * is a binary operation on R.

(iii) Given on R, define by a*b = ab^2

Let a, b ∈ R

⇒ a, b^2 ∈ R

⇒ ab^2 ∈ R

⇒ a * b ∈ R

Thus, * is a binary operation on R.

(iv) Given on Z^+ define * by a * b = |a − b|

Let a, b ∈ Z^+

⇒ | a – b | ∈ Z^+

⇒ a * b ∈ Z^+

Therefore,

a * b ∈ Z^+, ∀ a, b ∈ Z^+

Thus, * is a binary operation on Z^+.

(v) Given on Z^+ define * by a * b = a

Let a, b ∈ Z^+

⇒ a ∈ Z^+

⇒ a * b ∈ Z^+

Therefore, a * b ∈ Z^+ ∀ a, b ∈ Z^+

Thus, * is a binary operation on Z^+.

(vi) Given On R, define * by a * b = a + $4b^2$

Let a, b ∈ R

⇒ a, $4b^2$ ∈ R

⇒ a + $4b^2$ ∈ R

⇒ a * b ∈ R

Therefore, a *b ∈ R, ∀ a, b ∈ R

Thus, * is a binary operation on R.

3. Let * be a binary operation on the set I of integers, defined by a * b = 2a + b − 3. Find the value of 3 * 4.

Solution:

Given $a * b = 2a + b - 3$

$3 * 4 = 2(3) + 4 - 3$

$= 6 + 4 - 3$

$= 7$

4. Is * defined on the set {1, 2, 3, 4, 5} by a * b = LCM of a and b a binary operation? Justify your answer.

Solution:

LCM	1	2	3	4	5
1	1	2	3	4	5
2	2	2	6	4	10
3	3	5	3	12	15
4	4	4	12	4	20
5	5	10	15	20	5

In the given composition table, all the elements are not in the set {1, 2, 3, 4, 5}.

If we consider a = 2 and b = 3, a * b = LCM of a and b = 6 ∉ {1, 2, 3, 4, 5}.

Thus, * is not a binary operation on {1, 2, 3, 4, 5}.

5. Let S = {a, b, c}. Find the total number of binary operations on *S*.

Solution:

Number of binary operations on a set with n elements is nn2nn2

Here, S = {a, b, c}

Number of elements in S = 3

Number of binary operations on a set with 3 elements is 332332

Exercise 3.2 Page No: 3.12

1. Let '*' be a binary operation on N defined by a * b = l.c.m. (a, b) for all a, b ∈ N

(i) Find 2 * 4, 3 * 5, 1 * 6.

(ii) Check the commutativity and associativity of '*' on N.

Solution:

(i) Given a * b = 1.c.m. (a, b)

2 * 4 = l.c.m. (2, 4)

= 4

3 * 5 = l.c.m. (3, 5)

= 15

1 * 6 = l.c.m. (1, 6)

= 6

(ii) We have to prove commutativity of *

Let a, b ∈ N

a * b = l.c.m (a, b)

= l.c.m (b, a)

= b * a

Therefore

a * b = b * a ∀ a, b ∈ N

Thus * is commutative on N.

Now we have to prove associativity of *

Let a, b, c ∈ N

a * (b * c) = a * l.c.m. (b, c)

= l.c.m. (a, (b, c))

= l.c.m (a, b, c)

(a * b) * c = l.c.m. (a, b) * c

= l.c.m. ((a, b), c)

= l.c.m. (a, b, c)

Therefore

(a * (b * c) = (a * b) * c, ∀ a, b , c ∈ N

Thus, * is associative on N.

2. Determine which of the following binary operation is associative and which is commutative:

(i) * on N defined by a * b = 1 for all a, b ∈ N

(ii) * on Q defined by a * b = (a + b)/2 for all a, b ∈ Q

Solution:

(i) We have to prove commutativity of *

Let a, b ∈ N

a * b = 1

b * a = 1

Therefore,

a * b = b * a, for all a, b ∈ N

Thus * is commutative on N.

Now we have to prove associativity of *

Let a, b, c ∈ N

Then a * (b * c) = a * (1)

= 1

(a * b) *c = (1) * c

= 1

Therefore a * (b * c) = (a * b) *c for all a, b, c ∈ N

Thus, * is associative on N.

(ii) First we have to prove commutativity of *

Let a, b ∈ N

a * b = (a + b)/2

= (b + a)/2

= b * a

Therefore, a * b = b * a, ∀ a, b ∈ N

Thus * is commutative on N.

Now we have to prove associativity of *

Let a, b, c ∈ N

a * (b * c) = a * (b + c)/2

= [a + (b + c)]/2

= (2a + b + c)/4

Now, (a * b) * c = (a + b)/2 * c

= [(a + b)/2 + c] /2

= (a + b + 2c)/4

Thus, a * (b * c) ≠ (a * b) * c

If a = 1, b= 2, c = 3

1 * (2 * 3) = 1 * (2 + 3)/2

= 1 * (5/2)

= [1 + (5/2)]/2

= 7/4

(1 * 2) * 3 = (1 + 2)/2 * 3

= 3/2 * 3

= [(3/2) + 3]/2

= 4/9

Therefore, there exist a = 1, b = 2, c = 3 ∈ N such that a * (b * c) ≠ (a * b) * c

Thus, * is not associative on N.

3. Let A be any set containing more than one element. Let '*' be a binary operation on A defined by a * b = b for all a, b ∈ A Is '*' commutative or associative on A?

Solution:

Let a, b ∈ A

Then, a * b = b

b * a = a

Therefore a * b ≠ b * a

Thus, * is not commutative on A

Now we have to check associativity:

Let a, b, c ∈ A

a * (b * c) = a * c

= c

Therefore

a * (b * c) = (a * b) * c, ∀ a, b, c ∈ A

Thus, * is associative on A

4. Check the commutativity and associativity of each of the following binary operations:

(i) '*' on Z defined by a * b = a + b + a b for all a, b ∈ Z

(ii) '*' on N defined by a * b = 2^{ab} for all a, b ∈ N

(iii) '*' on Q defined by a * b = a – b for all a, b ∈ Q

(iv) '⊙' on Q defined by a ⊙ b = $a^2 + b^2$ for all a, b ∈ Q

(v) 'o' on Q defined by a o b = (ab/2) for all a, b ∈ Q

(vi) '*' on Q defined by a * b = ab^2 for all a, b ∈ Q

(vii) '*' on Q defined by a * b = a + a b for all a, b ∈ Q

(viii) '*' on R defined by a * b = a + b -7 for all a, b ∈ R

(ix) '*' on Q defined by a * b = $(a – b)^2$ for all a, b ∈ Q

(x) '*' on Q defined by a * b = a b + 1 for all a, b ∈ Q

(xi) '*' on N defined by a * b = a^b for all a, b ∈ N

(xii) '*' on Z defined by a * b = a – b for all a, b ∈ Z

(xiii) '*' on Q defined by a * b = (ab/4) for all a, b ∈ Q

(xiv) '*' on Z defined by a * b = a + b – ab for all a, b ∈ Z

(xv) '*' on Q defined by a * b = gcd (a, b) for all a, b ∈ Q

Solution:

(i) First we have to check commutativity of *

Let a, b ∈ Z

Then a * b = a + b + ab

= b + a + ba

= b * a

Therefore,

a * b = b * a, ∀ a, b ∈ Z

Now we have to prove associativity of *

Let a, b, c ∈ Z, Then,

a * (b * c) = a * (b + c + b c)

= a + (b + c + b c) + a (b + c + b c)

= a + b + c + b c + a b + a c + a b c

$(a * b) * c = (a + b + a b) * c$

$= a + b + a b + c + (a + b + a b) c$

$= a + b + a b + c + a c + b c + a b c$

Therefore,

$a * (b * c) = (a * b) * c,\ \forall\ a, b, c \in Z$

Thus, * is associative on Z.

(ii) First we have to check commutativity of *

Let $a, b \in N$

$a * b = 2^{ab}$

$= 2^{ba}$

$= b * a$

Therefore, $a * b = b * a,\ \forall\ a, b \in N$

Thus, * is commutative on N

Now we have to check associativity of *

Let $a, b, c \in N$

Then, $a * (b * c) = a * (2^{bc})$

=2a∗2bc2a∗2bc

$(a * b) * c = (2^{ab}) * c$

=2ab∗2c2ab∗2c

Therefore, $a * (b * c) \neq (a * b) * c$

Thus, * is not associative on N

(iii) First we have to check commutativity of *

Let a, b $\in$ Q, then

$a * b = a - b$

$b * a = b - a$

Therefore, $a * b \neq b * a$

Thus, * is not commutative on Q

Now we have to check associativity of *

Let a, b, c $\in$ Q, then

$a * (b * c) = a * (b - c)$

$= a - (b - c)$

$= a - b + c$

$(a * b) * c = (a - b) * c$

$= a - b - c$

Therefore,

$a * (b * c) \neq (a * b) * c$

Thus, * is not associative on Q

(iv) First we have to check commutativity of $\odot$

Let a, b $\in$ Q, then

$a \odot b = a^2 + b^2$

$= b^2 + a^2$

$= b \odot a$

Therefore, a ⊙ b = b ⊙ a, ∀ a, b ∈ Q

Thus, ⊙ on Q

Now we have to check associativity of ⊙

Let a, b, c ∈ Q, then

$a \odot (b \odot c) = a \odot (b^2 + c^2)$

$= a^2 + (b^2 + c^2)^2$

$= a^2 + b^4 + c^4 + 2b^2c^2$

$(a \odot b) \odot c = (a^2 + b^2) \odot c$

$= (a^2 + b^2)^2 + c^2$

$= a^4 + b^4 + 2a^2b^2 + c^2$

Therefore,

$(a \odot b) \odot c \neq a \odot (b \odot c)$

Thus, ⊙ is not associative on Q.

(v) First we have to check commutativity of o

Let a, b ∈ Q, then

a o b = (ab/2)

= (b a/2)

= b o a

Therefore, a o b = b o a, ∀ a, b ∈ Q

Thus, o is commutative on Q

Now we have to check associativity of o

Let a, b, c ∈ Q, then

a o (b o c) = a o (b c/2)

= [a (b c/2)]/2

= [a (b c/2)]/2

= (a b c)/4

(a o b) o c = (ab/2) o c

= [(ab/2) c] /2

= (a b c)/4

Therefore a o (b o c) = (a o b) o c, ∀ a, b, c ∈ Q

Thus, o is associative on Q.

(vi) First we have to check commutativity of *

Let a, b ∈ Q, then

a * b = ab^2

b * a = ba^2

Therefore,

a * b ≠ b * a

Thus, * is not commutative on Q

Now we have to check associativity of *

Let a, b, c ∈ Q, then

a * (b * c) = a * (bc^2)

= a $(bc^2)^2$

$= ab^2 c^4$

$(a * b) * c = (ab^2) * c$

$= ab^2c^2$

Therefore a * (b * c) ≠ (a * b) * c

Thus, * is not associative on Q.

(vii) First we have to check commutativity of *

Let a, b ∈ Q, then

a * b = a + ab

b * a = b + ba

= b + ab

Therefore, a * b ≠ b * a

Thus, * is not commutative on Q.

Now we have to prove associativity on Q.

Let a, b, c ∈ Q, then

a * (b * c) = a * (b + b c)

= a + a (b + b c)

= a + ab + a b c

(a * b) * c = (a + a b) * c

= (a + a b) + (a + a b) c

= a + a b + a c + a b c

Therefore a * (b * c) ≠ (a * b) * c

Thus, * is not associative on Q.

(viii) First we have to check commutativity of *

Let a, b ∈ R, then

a * b = a + b – 7

= b + a – 7

= b * a

Therefore,

a * b = b * a, for all a, b ∈ R

Thus, * is commutative on R

Now we have to prove associativity of * on R.

Let a, b, c ∈ R, then

a * (b * c) = a * (b + c – 7)

= a + b + c -7 -7

= a + b + c – 14

(a * b) * c = (a + b – 7) * c

= a + b – 7 + c – 7

= a + b + c – 14

Therefore,

a * (b * c) = (a * b) * c, for all a, b, c ∈ R

Thus, * is associative on R.

(ix) First we have to check commutativity of *

Let a, b ∈ Q, then

$a * b = (a - b)^2$

$= (b - a)^2$

$= b * a$

Therefore,

a * b = b * a, for all a, b ∈ Q

Thus, * is commutative on Q

Now we have to prove associativity of * on Q

Let a, b, c ∈ Q, then

$a * (b * c) = a * (b - c)^2$

$= a * (b^2 + c^2 - 2\ b\ c)$

$= (a - b^2 - c^2 + 2bc)^2$

$(a * b) * c = (a - b)^2 * c$

$= (a^2 + b^2 - 2ab) * c$

$= (a^2 + b^2 - 2ab - c)^2$

Therefore, $a * (b * c) \neq (a * b) * c$

Thus, * is not associative on Q.

(x) First we have to check commutativity of *

Let a, b ∈ Q, then

$a * b = ab + 1$

$= ba + 1$

= b * a

Therefore

a * b = b * a, for all a, b ∈ Q

Thus, * is commutative on Q

Now we have to prove associativity of * on Q

Let a, b, c ∈ Q, then

a * (b * c) = a * (bc + 1)

= a (b c + 1) + 1

= a b c + a + 1

(a * b) * c = (ab + 1) * c

= (ab + 1) c + 1

= a b c + c + 1

Therefore, a * (b * c) ≠ (a * b) * c

Thus, * is not associative on Q.

(xi) First we have to check commutativity of *

Let a, b ∈ N, then

a * b = a^b

b * a = b^a

Therefore, a * b ≠ b * a

Thus, * is not commutative on N.

Now we have to check associativity of *

a * (b * c) = a * (b^c)

=

a^{b^c}

(a * b) * c = (a^b) * c

= $(a^b)^c$

= a^{bc}

Therefore, a * (b * c) ≠ (a * b) * c

Thus, * is not associative on N

(xii) First we have to check commutativity of *

Let a, b ∈ Z, then

a * b = a – b

b * a = b – a

Therefore,

a * b ≠ b * a

Thus, * is not commutative on Z.

Now we have to check associativity of *

Let a, b, c ∈ Z, then

a * (b * c) = a * (b – c)

= a – (b – c)

= a – (b + c)

(a * b) * c = (a – b) – c

= a – b – c

Therefore, a * (b * c) ≠ (a * b) * c

Thus, * is not associative on Z

(xiii) First we have to check commutativity of *

Let a, b ∈ Q, then

a * b = (ab/4)

= (ba/4)

= b * a

Therefore, a * b = b * a, for all a, b ∈ Q

Thus, * is commutative on Q

Now we have to check associativity of *

Let a, b, c ∈ Q, then

a * (b * c) = a * (b c/4)

= [a (b c/4)]/4

= (a b c/16)

(a * b) * c = (ab/4) * c

= [(ab/4) c]/4

= a b c/16

Therefore,

a * (b * c) = (a * b) * c for all a, b, c ∈ Q

Thus, * is associative on Q.

(xiv) First we have to check commutativity of *

Let a, b $\in$ Z, then

a * b = a + b – ab

= b + a – ba

= b * a

Therefore, a * b = b * a, for all a, b $\in$ Z

Thus, * is commutative on Z.

Now we have to check associativity of *

Let a, b, c $\in$ Z

a * (b * c) = a * (b + c – b c)

= a + b + c- b c – ab – ac + a b c

(a * b) * c = (a + b – a b) c

= a + b – ab + c – (a + b – ab) c

= a + b + c – ab – ac – bc + a b c

Therefore,

a * (b * c) = (a * b) * c, for all a, b, c $\in$ Z

Thus, * is associative on Z.

(xv) First we have to check commutativity of *

Let a, b $\in$ N, then

a * b = gcd (a, b)

= gcd (b, a)

= b * a

Therefore, a * b = b * a, for all a, b ∈ N

Thus, * is commutative on N.

Now we have to check associativity of *

Let a, b, c ∈ N

a * (b * c) = a * [gcd (a, b)]

= gcd (a, b, c)

(a * b) * c = [gcd (a, b)] * c

= gcd (a, b, c)

Therefore,

a * (b * c) = (a * b) * c, for all a, b, c ∈ N

Thus, * is associative on N.

5. If the binary operation o is defined by a0b = a + b – ab on the set Q – {-1} of all rational numbers other than 1, show that o is commutative on Q – [1].

Solution:

Let a, b ∈ Q – {-1}.

Then aob = a + b – ab

= b+ a – ba

= boa

Therefore,

aob = boa for all a, b ∈ Q – {-1}

Thus, o is commutative on Q – {-1}

6. Show that the binary operation * on Z defined by a * b = 3a + 7b is not commutative?

Solution:

Let a, b ∈ Z

a * b = 3a + 7b

b * a = 3b + 7a

Thus, a * b ≠ b * a

Let a = 1 and b = 2

1 * 2 = 3 × 1 + 7 × 2

= 3 + 14

= 17

2 * 1 = 3 × 2 + 7 × 1

= 6 + 7

= 13

Therefore, there exist a = 1, b = 2 ∈ Z such that a * b ≠ b * a

Thus, * is not commutative on Z.

7. On the set Z of integers a binary operation * is defined by a 8 b = ab + 1 for all a, b ∈ Z. Prove that * is not associative on Z.

Solution:

Let a, b, c ∈ Z

a * (b * c) = a * (bc + 1)

= a (bc + 1) + 1

$= a\,b\,c + a + 1$

$(a * b) * c = (ab + 1) * c$

$= (ab + 1)\,c + 1$

$= a\,b\,c + c + 1$

Thus, $a * (b * c) \neq (a * b) * c$

Thus, * is not associative on Z.

Exercise 3.3 Page No: 3.15

1. Find the identity element in the set I^+ of all positive integers defined by $a * b = a + b$ for all $a, b \in I^+$.

Solution:

Let e be the identity element in I^+ with respect to * such that

$a * e = a = e * a, \ \forall\ a \in I^+$

$a * e = a$ and $e * a = a, \ \forall\ a \in I^+$

$a + e = a$ and $e + a = a, \ \forall\ a \in I^+$

$e = 0, \ \forall\ a \in I^+$

Thus, 0 is the identity element in I^+ with respect to *.

2. Find the identity element in the set of all rational numbers except – 1 with respect to * defined by $a * b = a + b + ab$

Solution:

Let e be the identity element in I^+ with respect to * such that

$a * e = a = e * a, \ \forall\ a \in Q - \{-1\}$

$a * e = a$ and $e * a = a$, $\forall\ a \in Q - \{-1\}$

$a + e + ae = a$ and $e + a + ea = a$, $\forall\ a \in Q - \{-1\}$

$e + ae = 0$ and $e + ea = 0$, $\forall\ a \in Q - \{-1\}$

$e (1 + a) = 0$ and $e (1 + a) = 0$, $\forall\ a \in Q - \{-1\}$

$e = 0$, $\forall\ a \in Q - \{-1\}$ [because a not equal to -1]

Thus, 0 is the identity element in $Q - \{-1\}$ with respect to *.

Exercise 3.4 Page No: 3.25

1. Let * be a binary operation on Z defined by a * b = a + b – 4 for all a, b ∈ Z.

(i) Show that * is both commutative and associative.

(ii) Find the identity element in Z

(iii) Find the invertible element in Z.

Solution:

(i) First we have to prove commutativity of *

Let a, b ∈ Z. then,

$a * b = a + b - 4$

$= b + a - 4$

$= b * a$

Therefore,

$a * b = b * a$, $\forall\ a, b \in Z$

Thus, * is commutative on Z.

Now we have to prove associativity of Z.

Let a, b, c ∈ Z. then,

a * (b * c) = a * (b + c – 4)

= a + b + c -4 – 4

= a + b + c – 8

(a * b) * c = (a + b – 4) * c

= a + b – 4 + c – 4

= a + b + c – 8

Therefore,

a * (b * c) = (a * b) * c, for all a, b, c ∈ Z

Thus, * is associative on Z.

(ii) Let e be the identity element in Z with respect to * such that

a * e = a = e * a ∀ a ∈ Z

a * e = a and e * a = a, ∀ a ∈ Z

a + e – 4 = a and e + a – 4 = a, ∀ a ∈ Z

e = 4, ∀ a ∈ Z

Thus, 4 is the identity element in Z with respect to *.

(iii) Let a ∈ Z and b ∈ Z be the inverse of a. Then,

a * b = e = b * a

a * b = e and b * a = e

a + b – 4 = 4 and b + a – 4 = 4

$b = 8 – a \in Z$

Thus, 8 – a is the inverse of $a \in Z$

2. Let * be a binary operation on Q_0 (set of non-zero rational numbers) defined by a * b = (3ab/5) for all a, b $\in Q_0$. Show that * is commutative as well as associative. Also, find its identity element, if it exists.

Solution:

First we have to prove commutativity of *

Let a, b $\in Q_0$

a * b = (3ab/5)

= (3ba/5)

= b * a

Therefore, a * b = b * a, for all a, b $\in Q_0$

Now we have to prove associativity of *

Let a, b, c $\in Q_0$

a * (b * c) = a * (3bc/5)

= [a (3 bc/5)] /5

= 3 abc/25

(a * b) * c = (3 ab/5) * c

= [(3 ab/5) c]/ 5

= 3 abc /25

Therefore a * (b * c) = (a * b) * c, for all a, b, c $\in Q_0$

Thus * is associative on Q_0

Now we have to find the identity element

Let e be the identity element in Z with respect to * such that

a * e = a = e * a $\forall$ a $\in$ Q_0

a * e = a and e * a = a, $\forall$ a $\in$ Q_0

3ae/5 = a and 3ea/5 = a, $\forall$ a $\in$ Q_0

e = 5/3 $\forall$ a $\in$ Q_0 [because a is not equal to 0]

Thus, 5/3 is the identity element in Q_0 with respect to *.

3. Let * be a binary operation on Q – {-1} defined by a * b = a + b + ab for all a, b $\in$ Q – {-1}. Then,

(i) Show that * is both commutative and associative on Q – {-1}

(ii) Find the identity element in Q – {-1}

(iii) Show that every element of Q – {-1} is invertible. Also, find inverse of an arbitrary element.

Solution:

(i) First we have to check commutativity of *

Let a, b $\in$ Q – {-1}

Then a * b = a + b + ab

= b + a + ba

= b * a

Therefore,

a * b = b * a, $\forall$ a, b $\in$ Q – {-1}

Now we have to prove associativity of *

Let a, b, c ∈ Q – {-1}, Then,

a * (b * c) = a * (b + c + b c)

= a + (b + c + b c) + a (b + c + b c)

= a + b + c + b c + a b + a c + a b c

(a * b) * c = (a + b + a b) * c

= a + b + a b + c + (a + b + a b) c

= a + b + a b + c + a c + b c + a b c

Therefore,

a * (b * c) = (a * b) * c, ∀ a, b, c ∈ Q – {-1}

Thus, * is associative on Q – {-1}.

(ii) Let e be the identity element in I^+ with respect to * such that

a * e = a = e * a, ∀ a ∈ Q – {-1}

a * e = a and e * a = a, ∀ a ∈ Q – {-1}

a + e + ae = a and e + a + ea = a, ∀ a ∈ Q – {-1}

e + ae = 0 and e + ea = 0, ∀ a ∈ Q – {-1}

e (1 + a) = 0 and e (1 + a) = 0, ∀ a ∈ Q – {-1}

e = 0, ∀ a ∈ Q – {-1} [because a not equal to -1]

Thus, 0 is the identity element in Q – {-1} with respect to *.

(iii) Let a ∈ Q – {-1} and b ∈ Q – {-1} be the inverse of a. Then,

a * b = e = b * a

a * b = e and b * a = e

$a + b + ab = 0$ and $b + a + ba = 0$

$b (1 + a) = – a$ Q – {-1}

$b = -a/1 + a$ Q – {-1} [because a not equal to -1]

Thus, $-a/1 + a$ is the inverse of $a \in Q – \{-1\}$

4. Let $A = R_0 \times R$, where R_0 denote the set of all non-zero real numbers. A binary operation 'O' is defined on A as follows: (a, b) O (c, d) = (ac, bc + d) for all (a, b), (c, d) $\in R_0 \times R$.

(i) Show that 'O' is commutative and associative on A

(ii) Find the identity element in A

(iii) Find the invertible element in A.

Solution:

(i) Let X = (a, b) and Y = (c, d) $\in$ A, $\forall$ a, c $\in R_0$ and b, d $\in$ R

Then, X O Y = (ac, bc + d)

And Y O X = (ca, da + b)

Therefore,

X O Y = Y O X, $\forall$ X, Y $\in$ A

Thus, O is not commutative on A.

Now we have to check associativity of O

Let X = (a, b), Y = (c, d) and Z = (e, f), $\forall$ a, c, e $\in R_0$ and b, d, f $\in$ R

X O (Y O Z) = (a, b) O (ce, de + f)

= (ace, bce + de + f)

(X O Y) O Z = (ac, bc + d) O (e, f)

= (ace, (bc + d) e + f)

= (ace, bce + de + f)

Therefore, X O (Y O Z) = (X O Y) O Z, ∀ X, Y, Z ∈ A

(ii) Let E = (x, y) be the identity element in A with respect to O, ∀ x ∈ R_0 and y ∈ R

Such that,

X O E = X = E O X, ∀ X ∈ A

X O E = X and EOX = X

(ax, bx +y) = (a, b) and (xa, ya + b) = (a, b)

Considering (ax, bx + y) = (a, b)

ax = a

x = 1

And bx + y = b

y = 0 [since x = 1]

Considering (xa, ya + b) = (a, b)

xa = a

x = 1

And ya + b = b

y = 0 [since x = 1]

Therefore (1, 0) is the identity element in A with respect to O.

(iii) Let F = (m, n) be the inverse in A ∀ m ∈ R_0 and n ∈ R

X O F = E and F O X = E

(am, bm + n) = (1, 0) and (ma, na + b) = (1, 0)

Considering (am, bm + n) = (1, 0)

am = 1

m = 1/a

And bm + n = 0

n = -b/a [since m = 1/a]

Considering (ma, na + b) = (1, 0)

ma = 1

m = 1/a

And na + b = 0

n = -b/a

Therefore the inverse of (a, b) ∈ A with respect to O is (1/a, -b/a)

Exercise 3.5 Page No: 3.33

1. Construct the composition table for $\times_4$ on set S = {0, 1, 2, 3}.

Solution:

Given that $\times_4$ on set S = {0, 1, 2, 3}

Here,

1 $\times_4$ 1 = remainder obtained by dividing 1 × 1 by 4

= 1

0 $\times_4$ 1 = remainder obtained by dividing 0 × 1 by 4

= 0

$2 \times_4 3$ = remainder obtained by dividing 2×3 by 4

= 2

$3 \times_4 3$ = remainder obtained by dividing 3×3 by 4

= 1

So, the composition table is as follows:

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

2. Construct the composition table for $+_5$ on set S = {0, 1, 2, 3, 4}

Solution:

$1 +_5 1$ = remainder obtained by dividing 1 + 1 by 5

= 2

$3 +_5 1$ = remainder obtained by dividing 3 + 1 by 5

= 2

$4 +_5 1$ = remainder obtained by dividing 4 + 1 by 5

= 3

So, the composition table is as follows:

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

3. Construct the composition table for $\times_6$ on set S = {0, 1, 2, 3, 4, 5}.

Solution:

Here,

$1 \times_6 1$ = remainder obtained by dividing 1×1 by 6

= 1

$3 \times_6 4$ = remainder obtained by dividing 3×4 by 6

= 0

$4 \times_6 5$ = remainder obtained by dividing 4×5 by 6

= 2

So, the composition table is as follows:

$\times_6$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5

2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

4. Construct the composition table for $\times_5$ on set Z_5 = {0, 1, 2, 3, 4}

Solution:

Here,

$1 \times_5 1$ = remainder obtained by dividing 1×1 by 5

= 1

$3 \times_5 4$ = remainder obtained by dividing 3×4 by 5

= 2

$4 \times_5 4$ = remainder obtained by dividing 4×4 by 5

= 1

So, the composition table is as follows:

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

5. For the binary operation $\times_{10}$ set S = {1, 3, 7, 9}, find the inverse of 3.

Solution:

Here,

$1 \times_{10} 1$ = remainder obtained by dividing 1 × 1 by 10

= 1

$3 \times_{10} 7$ = remainder obtained by dividing 3 × 7 by 10

= 1

$7 \times_{10} 9$ = remainder obtained by dividing 7 × 9 by 10

= 3

So, the composition table is as follows:

$\times_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

From the table we can observe that elements of first row as same as the top-most row.

So, 1 ∈ S is the identity element with respect to $\times_{10}$

Now we have to find inverse of 3

$3 \times_{10} 7 = 1$

So the inverse of 3 is 7.

Chapterwise RD Sharma Solutions for Class 12 Maths :

- Chapter 1–Relation
- Chapter 2–Functions
- Chapter 3–Binary Operations
- Chapter 4–Inverse Trigonometric Functions
- Chapter 5–Algebra of Matrices
- Chapter 6–Determinants
- Chapter 7–Adjoint and Inverse of a Matrix
- Chapter 8–Solution of Simultaneous Linear Equations
- Chapter 9–Continuity
- Chapter 10–Differentiability
- Chapter 11–Differentiation
- Chapter 12–Higher Order Derivatives
- Chapter 13–Derivatives as a Rate Measurer
- Chapter 14–Differentials, Errors and Approximations
- Chapter 15–Mean Value Theorems
- Chapter 16–Tangents and Normals
- Chapter 17–Increasing and Decreasing Functions
- Chapter 18–Maxima and Minima
- Chapter 19–Indefinite Integrals

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.