Class 12 - Chapter 2 Functions

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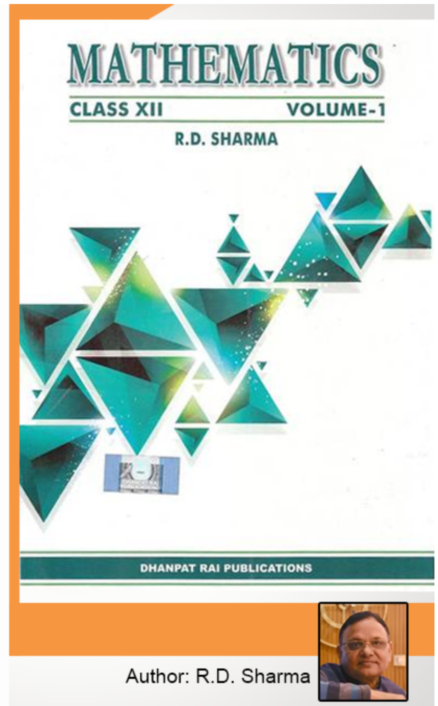

RD Sharma Solutions for Class 12 Maths Chapter 2–Functions

Class 12: Maths Chapter 1 solutions. Complete Class 12 Maths Chapter 1 Notes.

RD Sharma Solutions for Class 12 Maths Chapter 2–Functions

RD Sharma 12th Maths Chapter 1, Class 12 Maths Chapter 1 solutions

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Question 1:

Give an example of a function

(i) which is one-one but not onto

(ii) which is not one-one but onto

(iii) which is neither one-one nor onto

ANSWER:

(i) which is one-one but not onto.

f: $Z \rightarrow Z$ given by $f(x)=3x+2$

Injectivity:

Let x and y be any two elements in the domain (Z), such that $f(x) = f(y)$.

$f(x)=f(y)$

$\Rightarrow 3x + 2 = 3y + 2$

$\Rightarrow 3x = 3y$

$\Rightarrow x = y$

$\Rightarrow f(x) = f(y) \Rightarrow x = y$

So, f is one-one.

Surjectivity:

Let y be any element in the co-domain (Z), such that $f(x) = y$ for some element x in Z (domain).

$f(x) = y$

$\Rightarrow 3x + 2 = y$

$\Rightarrow 3x = y - 2$

$\Rightarrow x = y$-23. It may not be in the domain (Z) because if we take y = 3,$x = y$-23 = 3-23 = 13$\notin$ domain Z.

So, for every element in the co domain there need not be any element in the domain such that $f(x) = y$.

Thus, f is not onto.

(ii) which is not one-one but onto.

$f: Z \rightarrow N \cup \{0\}$ given by $f(x) = |x|$

Injectivity:

Let x and y be any two elements in the domain (Z), such that $f(x) = f(y)$.

$\Rightarrow |x| = |y|$

$\Rightarrow x = \pm y$

So, different elements of domain f may give the same image.

So, f is not one-one.

Surjectivity:

Let y be any element in the co domain (Z), such that $f(x) = y$ for some element x in Z (domain).

$f(x) = y$

$\Rightarrow |x| = y$

$\Rightarrow x = \pm y$, which is an element in Z (domain).

So, for every element in the co-domain, there exists a pre-image in the domain.

Thus, f is onto.

(iii) which is neither one-one nor onto.

f: $Z \rightarrow Z$ given by $f(x) = 2x^2 + 1$

Injectivity:

Let x and y be any two elements in the domain (Z), such that $f(x) = f(y)$.

$f(x) = f(y) \Rightarrow 2x2+1 = 2y2+1 \Rightarrow 2x2 = 2y2 \Rightarrow x2 = y2 \Rightarrow x = \pm y$

So, different elements of domain f may give the same image.

Thus, f is not one-one.

Surjectivity:

Let y be any element in the co-domain (Z), such that $f(x) = y$ for some element x in Z (domain).

$f(x) = y$

$\Rightarrow 2x2+1=y \Rightarrow 2x2=y-1 \Rightarrow x2=y-12 \Rightarrow x=\pm\sqrt{}y-12, \notin Z$ always.For example, if we take, $y = 4, x=\pm\sqrt{}y-12=\pm\sqrt{}4-12=\pm\sqrt{}32, \notin$ ZSo, x may not be in Z (domain).

Thus, f is not onto.

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Question 2:

Which of the following functions from A to B are one-one and onto?

(i) $f_1 = \{(1, 3), (2, 5), (3, 7)\}$; $A = \{1, 2, 3\}$, $B = \{3, 5, 7\}$

(ii) $f_2 = \{(2, a), (3, b), (4, c)\}$; $A = \{2, 3, 4\}$, $B = \{a, b, c\}$

(iii) $f_3 = \{(a, x), (b, x), (c, z), (d, z)\}$; $A = \{a, b, c, d,\}$, $B = \{x, y, z\}$

ANSWER:

(i) $f_1 = \{(1, 3), (2, 5), (3, 7)\}$; $A = \{1, 2, 3\}$, $B = \{3, 5, 7\}$

Injectivity:

$f_1\,(1) = 3$

$f_1\,(2) = 5$

$f_1\,(3) = 7$

$\Rightarrow$Every element of A has different images in B.

So, f_1 is one-one.

Surjectivity:

Co-domain of $f_1 = \{3, 5, 7\}$

Range of f_1 =set of images = $\{3, 5, 7\}$

$\Rightarrow$Co-domain = range

So, f_1 is onto.

(ii) $f_2 = \{(2, a), (3, b), (4, c)\}$; $A = \{2, 3, 4\}$, $B = \{a, b, c\}$

Injectivity:

$f_2\ (2) = a$

$f_2\ (3) = b$

$f_2\ (4) = c$

⇒Every element of A has different images in B.

So, f_2 is one-one.

Surjectivity:

Co-domain of $f_2 = \{a, b, c\}$

Range of f_2 = set of images = $\{a, b, c\}$

⇒Co-domain = range

So, f_2 is onto.

(iii) $f_3 = \{(a, x), (b, x), (c, z), (d, z)\}$; $A = \{a, b, c, d,\}$, $B = \{x, y, z\}$

Injectivity:

$f_3\ (a) = x$

$f_3\ (b) = x$

$f_3\ (c) = z$

$f_3\ (d) = z$

$\Rightarrow a$ and b have the same image x. (Also c and d have the same image z)

So, f_3 is not one-one.

Surjectivity:

Co-domain of $f_1 = \{x, y, z\}$

Range of f_1 =set of images $= \{x, z\}$

So, the co-domain is not same as the range.

So, f_3 is not onto.

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Question 3:

Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$, is one-one but not onto.

ANSWER:

$f : N \to N$, defined by $f(x) = x^2 + x + 1$

Injectivity:

Let x and y be any two elements in the domain (N), such that $f(x) = f(y)$.

$\Rightarrow x2+x+1=y2+y+1 \Rightarrow (x2-y2)+(x-y)=0 \Rightarrow (x+y)(x-y)+(x-y)=0 \Rightarrow (x-y)(x+y+1)=0 \Rightarrow x-y=0$ *[(x+y+1) cannot be zero because x and y are natural numbers]* $\Rightarrow x=y$

So, f is one-one.

Surjectivity:

The minimum number in N is 1.When $x=1, x2+x+1=1+1+1=3 \Rightarrow x2+x+1 \geq 3$, *for every x in N.* $\Rightarrow f(x)$ *will not assume the values 1 and 2.So, f is not onto.*

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Question 4:

Let $A = \{-1, 0, 1\}$ and $f = \{(x, x^2) : x \in A\}$. Show that $f : A \to A$ is neither one-one nor onto.

ANSWER:

$A = \{-1, 0, 1\}$ and $f = \{(x, x^2) : x \in A\}$

Given, $f(x) = x^2$

Injectivity:

$f(1) = 1^2 = 1$ and

$f(-1) = (-1)^2 = 1$

$\Rightarrow$1 and -1 have the same images.

So, f is not one-one.

Surjectivity:

Co-domain of $f = \{-1, 0, 1\}$

$f(1) = 1^2 = 1$,

$f(-1) = (-1)^2 = 1$ and

$f(0) = 0$

$\Rightarrow$Range of $f = \{0, 1\}$

So, both are not same.

Hence, f is not onto.

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Question 5:

Classify the following functions as injection, surjection or bijection :

(i) $f : \mathbf{N} \to \mathbf{N}$ given by $f(x) = x^2$

(ii) $f : \mathbf{Z} \to \mathbf{Z}$ given by $f(x) = x^2$

(iii) $f : \mathbf{N} \to \mathbf{N}$ given by $f(x) = x^3$

(iv) $f : \mathbf{Z} \to \mathbf{Z}$ given by $f(x) = x^3$

(v) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = |x|$

(vi) $f : \mathbf{Z} \to \mathbf{Z}$, defined by $f(x) = x^2 + x$

(vii) $f : \mathbf{Z} \to \mathbf{Z}$, defined by $f(x) = x - 5$

(viii) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = \sin x$

(ix) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = x^3 + 1$

(x) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = x^3 - x$

(xi) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = \sin^2 x + \cos^2 x$

(xii) $f : \mathbf{Q} - \{3\} \to \mathbf{Q}$, defined by $f(x)=2x+3x-3$

(xiii) $f : \mathbf{Q} \to \mathbf{Q}$, defined by $f(x) = x^3 + 1$

(xiv) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = 5x^3 + 4$

(xv) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = 3 - 4x$

(xvi) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = 1 + x^2$

(xvii) f : **R** → **R**, defined by $f(x)$ = $xx2+1$
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ANSWER:

(i) f : **N** → **N**, given by $f(x) = x^2$

Injection test:

Let x and y be any two elements in the domain (**N**), such that $f(x) = f(y)$.

$f(x)=f(y)$

$x2=y2x=y$ (We do not get ± because x and y are in **N**)

So, f is an injection .

Surjection test:

Let y be any element in the co-domain (**N**), such that $f(x) = y$ for some element x in **N** (domain).

$f(x) = y$

$x2=yx=\sqrt{y}$, which may not be in **N**.For example, if $y=3, x=\sqrt{3}$ is not in **N**.

So, f is not a surjection.

So, f is not a bijection.

(ii) $f : \mathbf{Z} \rightarrow \mathbf{Z}$, given by $f(x) = x^2$

Injection test:

Let x and y be any two elements in the domain ($\mathbf{Z}$), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x2=y2x=\pm y$

So, f is not an injection .

Surjection test:

Let y be any element in the co-domain ($\mathbf{Z}$), such that $f(x) = y$ for some element x in $\mathbf{Z}$ (domain).

$f(x) = y$

$x2=yx=\pm\sqrt{y}$ which may not be in **Z**.For example, if $y=3, x=\pm\sqrt{3}$ is not in **Z**.

So, f is not a surjection.

So, f is not a bijection.

(iii) $f : \mathbf{N} \rightarrow \mathbf{N}$, given by $f(x) = x^3$

Injection test:

Let x and y be any two elements in the domain (**N**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x3=y3x=y$

So, f is an injection .

Surjection test:

Let y be any element in the co-domain (**N**), such that $f(x) = y$ for some element x in **N** (domain).

$f(x) = y$

$x3=yx=3\sqrt{y}$which may not be in **N**.For example, if $y=3,x=3\sqrt{3}$ is not in **N**.

So, f is not a surjection and f is not a bijection.

(iv) f : **Z** → **Z**, given by $f(x) = x^3$

Injection test:

Let x and y be any two elements in the domain (**Z**), such that $f(x) = f(y)$

$f(x) = f(y)$

$x3=y3x=y$

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (**Z**), such that $f(x) = y$ for some element x in **Z** (domain).

$f(x) = y$

$x3=yx=3\sqrt{y}$ which may not be in **Z**.For example, if $y=3, x=3\sqrt{3}$ is not in **Z**.

So, f is not a surjection and f is not a bijection.

(v) f : **R** → **R**, defined by $f(x) = |x|$

Injection test:

Let x and y be any two elements in the domain (**R**), such that $f(x) = f(y)$

$f(x) = f(y)$

$|x|=|y|x=\pm y$

So, f is not an injection .

Surjection test:

Let y be any element in the co-domain (**R**), such that $f(x) = y$ for some element x in **R** (domain).

$f(x) = y$

$|x|=yx=\pm y\in$ **Z**

So, f is a surjection and f is not a bijection.

(vi) f : **Z** → **Z**, defined by $f(x) = x^2 + x$

Injection test:

Let x and y be any two elements in the domain (**Z**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x2+x=y2+y$Here, we cannot say that $x = y$.For example, x = 2 and y = - 3Then, $x2+x=22+2=$ 6$y2+y=(-3)2-3=$ 6So, we have two numbers 2 and -3 in the domain **Z** whose image is same as 6.

So, f is not an injection .

Surjection test:

Let y be any element in the co-domain (**Z**), such that $f(x) = y$ for some element x in **Z** (domain).

$f(x) = y$

$x2+x=y$Here, we cannot say $x\in$**Z**.For example, y =-4.$x2+x=-4x2+x+4=0x=-1\pm\sqrt{}-152=-1\pm i\sqrt{}152$ which is not in **Z**.

So, f is not a surjection and f is not a bijection.

(vii) f : **Z** → **Z**, defined by $f(x) = x - 5$

Injection test:

Let x and y be any two elements in the domain (**Z**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x - 5 = y - 5$

$x = y$

So, f is an injection .

Surjection test:

Let y be any element in the co-domain (**Z**), such that $f(x) = y$ for some element x in **Z** (domain).

$f(x) = y$

$x - 5 = y$

$x = y + 5$, which is in **Z**.

So, f is a surjection and f is a bijection.

(viii) f : **R** → **R**, defined by $f(x) = \sin x$

Injection test:

Let x and y be any two elements in the domain (**R**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$\sin x=\sin y$Here, x may not be equal to y because $\sin 0=\sin \pi$.So, 0 and π have the same image 0.

So, f is not an injection .

Surjection test:

Range of f = [-1, 1]

Co-domain of f = **R**

Both are not same.

So, f is not a surjection and f is not a bijection.

(ix) f : **R** → **R**, defined by $f(x) = x^3 + 1$

Injection test:

Let x and y be any two elements in the domain (**R**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x3+1=y3+1x3=y3x=y$

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (**R**), such that $f(x) = y$ for some element x in **R** (domain).

$f(x) = y$

$x3+1=yx=3\sqrt{y}\text{-}1\in\mathbf{R}$

So, f is a surjection.

So, f is a bijection.

(x) $f : \mathbf{R} \to \mathbf{R}$, defined by $f(x) = x^3 - x$

Injection test:

Let x and y be any two elements in the domain (**R**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x3\text{-}x=y3\text{-}y$Here, we cannot say $x=y$.For example, $x=1$ and $y=-1x3\text{-}x=1\text{-}1=0y3\text{-}y=(-1)3\text{-}(-1)\text{-}1+1=0$So, 1 and -1 have the same image 0.

So, f is not an injection.

Surjection test:

Let y be any element in the co-domain (**R**), such that $f(x) = y$ for some element x in **R** (domain).

$f(x) = y$

$x3\text{-}x=y$By observation we can say that there exist some x in **R**, such that $x3\text{-}x=y$.

So, f is a surjection and f is not a bijection.

(xi) f : **R** → **R**, defined by $f(x) = \sin^2 x + \cos^2 x$

$f(x) = \sin^2 x + \cos^2 x = 1$

So, $f(x) = 1$ for every x in **R**.

So, for all elements in the domain, the image is 1.

So, f is not an injection.

Range of $f = \{1\}$

Co-domain of f = **R**

Both are not same.

So, f is not a surjection and f is not a bijection.

(xii) f : **Q** − {3} → **Q**, defined by $f(x)=2x+3x-3$

Injection test:

Let x and y be any two elements in the domain (**Q** − {3}), such that $f(x) = f(y)$.

$f(x) = f(y)$

$2x+3x-3=2y+3y-3(2x+3)(y-3)=(2y+3)(x-3)2xy-6x+3y-9=2xy-6y+3x-99x=9yx=y$

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (**Q** − {3}), such that $f(x) = y$ for some element x in **Q** (domain).

$f(x) = y$

$2x+3x-3=y2x+3=xy-3y2x-xy=-3y-3x(2-y)=-3(y+1)x=3(y+1)y-2$, which is not defined at $y=2$.

So, f is not a surjection and f is not a bijection.

(xiii) f : **Q** → **Q**, defined by $f(x) = x^3 + 1$

Injection test:

Let x and y be any two elements in the domain (**Q**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x3+1=y3+1x3=y3x=y$

So, f is an injection .

Surjection test:

Let y be any element in the co-domain (**Q**), such that $f(x) = y$ for some element x in **Q** (domain).

$f(x) = y$

$x3+1=yx=3\sqrt{y}$-1, which may not be in **Q**.For example, if y= 8,$x3+1$= 8$x3=7x=3\sqrt{7}$, which is not in **Q**.

So, f is not a surjection and f is not a bijection.

So, f is a surjection and f is a bijection.

(xiv) $f : \mathbf{R} \rightarrow \mathbf{R}$, defined by $f(x) = 5x^3 + 4$

Injection test:

Let x and y be any two elements in the domain (**R**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$5x3+4 = 5y3+45x3= 5y3x3= y3x=y$

So, f is an injection .

Surjection test:

Let y be any element in the co-domain (**R**), such that $f(x) = y$ for some element x in **R** (domain).

$f(x) = y$

$5x3+4=y5x3=y-4x3=y-45x=3\sqrt{}y-45\in\mathbf{R}$

So, f is a surjection and f is a bijection.

(xv) $f : \mathbf{R} \rightarrow \mathbf{R}$, defined by $f(x) = 3 - 4x$

Injection test:

Let x and y be any two elements in the domain (**R**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$3-4x=3-4y-4x=-4yx= y$

So, f is an injection .

Surjection test:

Let y be any element in the co-domain (**R**), such that $f(x) = y$ for some element x in **R** (domain).

$f(x) = y$

$3-4x=y4x=3-yx=3-y4\in\mathbf{R}$

So, f is a surjection and f is a bijection.

(xvi) f : **R** → **R**, defined by $f(x) = 1 + x^2$

Injection test:

Let x and y be any two elements in the domain (**R**), such that $f(x) = f(y)$.

$f(x) = f(y)$

1+x2=1+y2x2=y2x= ±y

So, f is not an injection.

Surjection test:

Let y be any element in the co-domain (**R**), such that $f(x) = y$ for some element x in **R** (domain).

$f(x) = y$

$1+x^2=y$ $x^2=y-1$ $x=\pm\sqrt{y-1}$ which may not be in **R** For example, if $y=0$, $x=\pm\sqrt{-1}=\pm i$ is not in **R**.

So, *f* is not a surjection and *f* is not a bijection.

(xvii) f : **R** → **R**, defined by $f(x) = \frac{x}{x^2+1}$

Injection test:

Let *x* and *y* be any two elements in the domain (**R**), such that $f(x) = f(y)$.

$f(x) = f(y)$

$\frac{x}{x^2+1}=\frac{y}{y^2+1}$ $xy^2+x=x^2y+y$ $xy^2-x^2y+x-y=0$ $-xy(-y+x)+1(x-y)=0$ $(x-y)(1-xy)=0$ $x=y$ or $x=\frac{1}{y}$

So, *f* is not an injection.

Surjection test:

Let *y* be any element in the co-domain (**R**), such that $f(x) = y$ for some element *x* in **R** (domain).

$f(x) = y$

$xx2+1=yyx2-x+y=0x=-(-1)±\sqrt{1-4y22y}$, if $y≠0=1±\sqrt{1-4y22y}$, which may not be in **R**For example, if $y=1$, then$x=1±\sqrt{1-42=1±i\sqrt{32}}$, which is not in **R**So, *f* is not surjection and *f* is not bijection.

So, *f* is not a surjection and *f* is not a bijection.

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Question 6:

If $f : A \to B$ is an injection, such that range of $f = \{a\}$, determine the number of elements in A.

ANSWER:

Range of $f = \{a\}$

So, the number of images of $f = 1$

Since, f is an injection, there will be exactly one image for each element of f .

So, number of elements in $A = 1$.

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Question 7:

Show that the function $f : R − \{3\} \to R − \{2\}$ given by $f(x)=x-2x-3$ is a bijection.

ANSWER:

$f : R − \{3\} \to R − \{2\}$ given by

$f(x)=x-2x-3$

Injectivity:

Let x and y be any two elements in the domain $(R − \{3\})$, such that $f(x) = f(y)$.

f(x) = f(y)

$\Rightarrow x\text{-}2x\text{-}3=y\text{-}2y\text{-}3 \Rightarrow (x-2)(y-3)=(y-2)(x-3) \Rightarrow xy-3x-2y+6=xy-3y-2x+6 \Rightarrow x=y$

So, *f* is one-one.

Surjectivity:

Let *y* be any element in the co-domain ($R - \{2\}$), such that *f(x)* = *y* for some element *x* in $R - \{3\}$ (domain).

f(x) = *y*

$\Rightarrow x\text{-}2x\text{-}3=y \Rightarrow x-2=xy-3y \Rightarrow xy-x=3y-2 \Rightarrow x(y-1)=3y-2 \Rightarrow x=3y\text{-}2y\text{-}1$, which is in R-{3}

So, for every element in the co-domain, there exists some pre-image in the domain.

$\Rightarrow f$ is onto.

Since, *f* is both one-one and onto, it is a bijection.

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Question 8:

Let *A* = [-1, 1]. Then, discuss whether the following functions from *A* to itself are one-one, onto or bijective:

(i) $f(x) = x2$ (ii) $g(x) = |x|$ (iii) $h(x) = x^2$
[NCERT EXEMPLAR]

ANSWER:

(i) $f : A \to A$, given by $f(x) = x2$

Injection test:

Let x and y be any two elements in the domain (A), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x2 = y2$

$x = y$

So, f is one-one.

Surjection test:

Let y be any element in the co-domain (A), such that $f(x) = y$ for some element x in A (domain)

$f(x) = y$

$x2 = y$

$x = 2y$, which may not be in A.

For example, if $y = 1$, then

$x = 2$, which is not in A.

So, f is not onto.

So, f is not bijective.

(ii) $g(x) = |x|$

Injection test:

Let x and y be any two elements in the domain (A), such that $f(x) = f(y)$.

$f(x) = f(y)$

$|x| = |y|$

$x = \pm y$

So, f is not one-one.

Surjection test:

For $y = -1$, there is no value of x in A.

So, f is not onto.

So, f is not bijective.

(iii) $h(x) = x^2$

Injection test:

Let x and y be any two elements in the domain (A), such that $f(x) = f(y)$.

$f(x) = f(y)$

$x^2 = y^2$

$x = \pm y$

So, f is not one-one.

Surjection test:

For y = -1, there is no value of x in A.

So, f is not onto.

So, f is not bijective.

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Question 9:

Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective:

(i) {(x, y) : x is a person, y is the mother of x}

(ii) {(a, b) : a is a person, b is an ancestor of a}
[NCERT EXEMPLAR]

ANSWER:

(i) f = {(x, y) : x is a person, y is the mother of x}

As, for each element x in domain set, there is a unique related element y in co-domain set.

So, *f* is the function.

Injection test:

As, *y* can be mother of two or more persons

So, *f* is not injective.

Surjection test:

For every mother *y* defined by (*x*, *y*), there exists a person *x* for whom *y* is mother.

So, *f* is surjective.

Therefore, *f* is surjective function.

(ii) *g* = {(*a*, *b*) : *a* is a person, *b* is an ancestor of *a*}

Since, the ordered map (*a*, *b*) does not map '*a*' - a person to a living person.

So, *g* is not a function.

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Question 10:

Let A = {1, 2, 3}. Write all one-one from A to itself.

ANSWER:

A ={1, 2, 3}

Number of elements in A = 3

Number of one-one functions = number of ways of arranging 3 elements = 3! = 6

(i) {(1, 1), (2, 2), (3, 3)}

(ii) {(1, 1), (2, 3), (3, 2)}

(iii) {(1, 2), (2, 2), (3, 3)}

(iv) {(1, 2), (2, 1), (3, 3)}

(v) {(1, 3), (2, 2), (3, 1)}

(vi) {(1, 3), (2, 1), (3,2)}

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Question 11:

If $f : R \to R$ be the function defined by $f(x) = 4x^3 + 7$, show that f is a bijection.

ANSWER:

Injectivity:

Let x and y be any two elements in the domain (R), such that $f(x) = f(y)$

$\Rightarrow 4x3+7=4y3+7 \Rightarrow 4x3=4y3 \Rightarrow x3=y3 \Rightarrow x=y$

So, f is one-one.

Surjectivity:

Let y be any element in the co-domain (R), such that $f(x) = y$ for some element x in R (domain).

$f(x) = y$

$\Rightarrow 4x3+7=y \Rightarrow 4x3=y-7 \Rightarrow x3=y-74 \Rightarrow x=3\sqrt{y}-74 \in R$

So, for every element in the co-domain, there exists some pre-image in the domain.

$\Rightarrow f$ is onto.

Since, f is both one-to-one and onto, it is a bijection.

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Question 12:

Show that the exponential function $f : R \to R$, given by $f(x) = e^x$, is one-one but not onto. What happens if the co-domain is replaced by $R+0$ (set of all positive real numbers)?

ANSWER:

$f : R \to R$, given by $f(x) = e^x$

Injectivity:

Let x and y be any two elements in the domain (R), such that $f(x) = f(y)$

$f(x)=f(y)$

$\Rightarrow ex=ey \Rightarrow x=y$

So, f is one-one.

Surjectivity:

We know that range of e^x is $(0, \infty) = R^+$

Co-domain = R

Both are not same.

So, f is not onto.

If the co-domain is replaced by R^+, then the co-domain and range become the same and in that case, f is onto and hence, it is a bijection.

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Question 13:

Show that the logarithmic function $f : R+0 \to R$ given by $f(x)=\log a\ x$, $a>0$ is a bijection.

ANSWER:

$f:R^+ \to R$ given by $f(x)= \log a\ x$, $a>0$

Injectivity:

Let x and y be any two elements in the domain (N), such that $f(x) = f(y)$.

$f(x) = f(y)$

$\log a\ x=\log a\ y \Rightarrow x=y$

So, f is one-one.

Surjectivity:

Let y be any element in the co-domain (R), such that $f(x) = y$ for some element x in R^+ (domain).

$f(x) = y$

$\log a\ x=y \Rightarrow x=ay \in R+$

So, for every element in the co-domain, there exists some pre-image in the domain.

$\Rightarrow f$ is onto.

Since f is one-one and onto, it is a bijection.

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Question 14:

If A = {1, 2, 3}, show that a one-one function $f : A \to A$ must be onto.

ANSWER:

A ={1, 2, 3}

Number of elements in A = 3

Number of one - one functions = number of ways of arranging 3 elements = 3! = 6

So, the possible one -one functions can be the following:

(i) {(1, 1), (2, 2), (3, 3)}

(ii) {(1, 1), (2, 3), (3, 2)}

(iii) {(1, 2), (2, 2), (3, 3)}

(iv) {(1, 2), (2, 1), (3, 3)}

(v) {(1, 3), (2, 2), (3, 1)}

(vi) {(1, 3), (2, 1), (3,2)}

Here, in each function, range = {1, 2, 3}, which is same as the co-domain.

So, all the functions are onto.

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Question 15:

If A = {1, 2, 3}, show that a onto function $f : A \to A$ must be one-one.

ANSWER:

$A = \{1, 2, 3\}$

Possible onto functions from A to A can be the following:

(i) {(1, 1), (2, 2), (3, 3)}

(ii) {(1, 1), (2, 3), (3, 2)}

(iii) {(1, 2), (2, 2), (3, 3)}

(iv) {(1, 2), (2, 1), (3, 3)}

(v) {(1, 3), (2, 2), (3, 1)}

(vi) {(1, 3), (2, 1), (3,2)}

Here, in each function, different elements of the domain have different images.

So, all the functions are one-one.

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Question 16:

Find the number of all onto functions from the set $A = \{1, 2, 3, ..., n\}$ to itself.

ANSWER:

We know that every onto function from A to itself is one-one.

So, the number of one-one functions = number of bijections = $n!$

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Question 17:

Give examples of two one-one functions f_1 and f_2 from R to R, such that $f_1 + f_2 : R \rightarrow R$. defined by $(f_1 + f_2)(x) = f_1(x) + f_2(x)$ is not one-one.

ANSWER:

We know that $f_1: R \to R$, given by $f_1(x)=x$, and $f_2(x)=-x$ are one-one.

Proving f_1 is one-one:

Let $f1(x)=f1(y) \Rightarrow x=y$

So, f_1 is one-one.

Proving f_2 is one-one:

Let $f2(x)=f2(y) \Rightarrow -x=-y \Rightarrow x=y$

So, f_2 is one-one.

Proving $(f_1 + f_2)$ is not one-one:

Given:

$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x + (-x) = 0$

So, for every real number x, $(f_1 + f_2)(x)=0$

So, the image of ever number in the domain is same as 0.

Thus, $(f_1 + f_2)$ is not one-one.

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Question 18:

Give examples of two surjective functions f_1 and f_2 from Z to Z such that $f_1 + f_2$ is not surjective.

ANSWER:

We know that $f_1: R \to R$, given by $f_1(x) = x$, and $f_2(x) = -x$ are surjective functions.

Proving f_1 is surjective :

Let y be an element in the co-domain (R), such that $f_1(x) = y$.

$f_1(x) = y$

$\Rightarrow x = y$, which is in R.

So, for every element in the co-domain, there exists some pre-image in the domain.1(x)=f1(y)x=y

So, $f_1$is surjective .

Proving f_2 is surjective :*Let f*2(x)=f2(y)−x=−yx=y

Let y be an element in the co domain (R) such that $f_2(x) = y$.

$f_2(x) = y$

$\Rightarrow x = y$, which is in R.

So, for every element in the co-domain, there exists some pre-image in the domain.1(x)=f1(y)x=y

So, f_2 is surjective .

Proving (f_1 + f_2) is not surjective :

Given:

$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x + (-x) = 0$

So, for every real number x, $(f_1 + f_2)(x) = 0$

So, the image of every number in the domain is same as 0.

$\Rightarrow$Range = {0}

Co-domain = R

So, both are not same.

So, $f_1 + f_2$ is not surjective.

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Question 19:

Show that if f_1 and f_2 are one-one maps from R to R, then the product $f1×f2 : R→R$ defined by $(f1×f2)(x)=f1(x)f2(x)$ need not be one-one.

ANSWER:

We know that f_1: $R \to R$, given by $f_1(x) = x$, and $f_2(x) = x$ are one-one.

Proving f_1 is one-one:

Let x and y be two elements in the domain R, such that

$f_1(x) = f_1(y)$

$\Rightarrow x = y$ et $f1(x)=f1(y)x=y$

So, f_1 is one-one.

Proving f_2 is one-one:

Let x and y be two elements in the domain R, such that

$f_2(x) = f_2(y)$

$\Rightarrow x = y$ et $f1(x)=f1(y)x=y$

So, f_2 is one-one.

Proving $f1 \times f2$ is not one-one:

Given:

$(f1 \times f2)(x)=f1\ (x) \times f2\ (x)=x \times x=x2$Let x and y be two elements in the domain R, such that$(f1 \times f2)(x)=(f1 \times f2)(y)\Rightarrow x2 = y2\Rightarrow x=\pm\ y$So, $(f1 \times f2)$ is not one-one.$f1\times f2$

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Question 20:

Suppose f_1 and f_2 are non-zero one-one functions from R to R. Is $f1f2$ necessarily one-one? Justify your answer. Here, $f1f2:R\to R$ is given by $(f1f2)\ (x)=f1\ (x)f2\ (x)$ for all $x\in R$.

ANSWER:

We know that f_1: $R \to R$, given by $f_1(x)=x^3$ and $f_2(x)=x$ are one-one.

Injectivity of f_1:

Let x and y be two elements in the domain R, such that

$f1(x)=f2(y)\Rightarrow x3=y\Rightarrow x=3\sqrt{y}\in R$Let $f1(x)=f1(y)x=y$

So, f_1 is one-one.

Injectivity of f_2:

Let x and y be two elements in the domain R, such that

$f2(x)=f2(y)\Rightarrow x=y\ \Rightarrow x\in R$.Let $f2(x)=f2(y)-x=-yx=y$

So, f_2 is one-one.

Proving $f1f2$is not one-one:

Given that $f1f2(x)=f1(x)f2(x)=x3x=x2$

Let x and y be two elements in the domain R, such that

$f1f2(x)=f1f2(y) \Rightarrow x2=y2 \Rightarrow x=\pm y$

So, $f1f2$ is not one-one.

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Question 21:

Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following:

(i) an injective map from A to B

(ii) a mapping from A to B which is not injective

(iii) a mapping from A to B.

ANSWER:

(i) {(2, 7), (3, 6), (4, 5)}

(ii) {(2, 2), (3, 2), (4, 5)}

(iii) {(2, 5), (3, 6), (4, 7)}

Disclaimer: There are many more possibilities of each case.

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Question 22:

Show that $f : R \rightarrow R$, given by $f(x) = x - [x]$, is neither one-one nor onto.

ANSWER:

We have, $f(x) = x - [x]$

Injection test:

$f(x) = 0$ for all $x \in \mathbf{Z}$

So, f is a many-one function.

Surjection test:

Range (f) = $[0, 1) \neq \mathbf{R}$.

So, f is an into function.

Therefore, f is neither one-one nor onto.

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Question 23:

Let $f : \mathbf{N} \to \mathbf{N}$ be defined by

$f(n)=\{n+1$, if n is odd$n-1$, if n is even

Show that f is a bijection.
[CBSE 2012, NCERT]

ANSWER:

We have,$f(n)=\{n+1$, if n is odd$n-1$, if n is evenInjection test:Case I: If n is odd,Let x, $y\in\mathbf{N}$ such that $f(x)=f(y)$As, $f(x)=f(y)\Rightarrow x+1=y+1\Rightarrow x=y$Case II: If n is even,Let x, $y\in\mathbf{N}$ such that $f(x)=f(y)$As, $f(x)=f(y)\Rightarrow x-1=y-1\Rightarrow x=y$So, f is injective.Surjection test:Case I: If n is odd,As, for every $n\in\mathbf{N}$, there exists $y=n-1$ in $\mathbf{N}$ such that$f(y)=f(n-1)=n-1+1=n$Case II: If n is even,As, for every $n\in\mathbf{N}$, there exists $y=n+1$ in $\mathbf{N}$ such that$f(y)=f(n+1)=n+1-1=n$So, f is surjective.So, f is a bijection.

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Question 1:

Find *gof* and *fog* when $f : R \to R$ and $g : R \to R$ are defined by

(i) $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

(ii) $f(x) = 2x + x^2$ and $g(x) = x^3$

(iii) $f(x) = x^2 + 8$ and $g(x) = 3x^3 + 1$

(iv) $f(x) = x$ and $g(x) = |x|$

(v) $f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 4$

(vi) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

ANSWER:

Given, $f : R \to R$ and $g : R \to R$

So, $gof : R \to R$ and $fog : R \to R$

(i) $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

Now, $(gof)(x)$

$= g(f(x))$

$= g(2x + 3)$

$= (2x + 3)^2 + 5$

$= 4x^2 + 9 + 12x + 5$

$= 4x^2 + 12x + 14$

(fog) (x)

=f (g (x))

$= f(x^2 + 5)$

$= 2(x^2 + 5) + 3$

$= 2x^2 + 10 + 3$

$= 2x^2 + 13$

(ii) $f(x) = 2x + x^2$ and $g(x) = x^3$

(*gof*) (*x*)=*g* (*f* (*x*))=*g* (2*x*+*x*2)=(2*x*+*x*2)3(*fog*) (*x*)=*f* (*g* (*x*))=*f* (*x*3)=2 (*x*3)+(*x*3)2=2*x*3+*x*6

(iii) $f(x) = x^2 + 8$ and $g(x) = 3x^3 + 1$

(*gof*) (*x*)=*g* (*f*(*x*))=*g* (*x*2+8)=3 (*x*2+8)3+1(*fog*) (*x*)=*f* (*g* (*x*))=*f* (3*x*3+1)=(3*x*3+1)2+8=9*x*6+6*x*3+1+8=9*x*6+6*x*3+9

(iv) $f(x) = x$ and $g(x) = |x|$

(*gof*) (*x*)=*g* (*f*(*x*))=*g* (*x*)=|*x*|(*fog*) (*x*)=*f* (*g* (*x*))=*f* (|*x*|)=|*x*|

(v) $f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 4$

(*gof*) (*x*)=*g* (*f*(*x*))=*g* (*x*2+2*x*-3)=3 (*x*2+2*x*-3)-4=3*x*2+6*x*-9-4=3*x*2+6*x*-13(*fog*) (*x*)=*f* (*g* (*x*))=*f* (3*x*-4)=(3*x*-4)2+2 (3*x*-4)-3=9*x*2+16-24*x*+6*x*-8-3=9*x*2-18*x*+5

(vi) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

(*gof*) (*x*)=*g* (*f* (*x*))=*g* (8*x*3)=(8*x*3)13=[(2*x*)3]13=2*x*(*fog*) (*x*)=*f* (*g* (*x*))=*f* (*x*13)=8 (*x*13)3=8*x*

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Question 2:

Let *f* = {(3, 1), (9, 3), (12, 4)} and *g* = {(1, 3), (3, 3) (4, 9) (5, 9)}. Show that *gof* and *fog* are both defined. Also, find *fog* and *gof*.

ANSWER:

f = {(3, 1), (9, 3), (12, 4)} and *g* = {(1, 3), (3, 3) (4, 9) (5, 9)}

f : {3, 9, 12} → {1, 3,4} and *g* : {1, 3, 4, 5} → {3, 9}

Co-domain of *f* is a subset of the domain of *g.*

So, *gof* exists and *gof* : {3, 9, 12} → {3, 9}

(*gof*) (3)=*g* (*f* (3))=*g* (1)=3(*gof*) (9)=*g* (*f* (9))=*g* (3)=3(*gof*) (12)=*g* (*f* (12))=*g* (4)=9⇒*gof* ={(3, 3), (9, 3), (12, 9)}

Co-domain of *g* is a subset of the domain of *f.*

So, *fog* exists and *fog* : {1, 3, 4, 5} → {3, 9, 12}

(*fog*) (1)=*f* (*g* (1))=*f* (3)=1(*fog*) (3)=*f* (*g* (3))=*f* (3)=1(*fog*) (4)=*f* (*g* (4))=*f* (9)=3(*fog*) (5)=*f* (*g* (5))=*f* (9)=3⇒*fog*={(1, 1), (3, 1), (4, 3), (5, 3)}

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Question 3:

Let f = {(1, −1), (4, −2), (9, −3), (16, 4)} and g = {(−1, −2), (−2, −4), (−3, −6), (4, 8)}. Show that gof is defined while fog is not defined. Also, find gof.

ANSWER:

f = {(1, −1), (4, −2), (9, −3), (16, 4)} and g = {(−1, −2), (−2, −4), (−3, −6), (4, 8)}

f : {1, 4, 9, 16} → {-1, -2, -3, 4} and g : {-1, -2, -3, 4} → {-2, -4, -6, 8}

Co-domain of f = domain of g

So, gof exists and gof : {1, 4, 9, 16} → {-2, -4, -6, 8}

(gof) (1)=g (f (1))=g (-1)=-2(gof) (4)=g (f (4))=g (-2)=-4(gof) (9)=g (f (9))=g (-3)=-6(gof) (16)=g (f (16))=g (4)=8So, gof={(1, -2), (4, -4), (9, -6), (16, 8)}

But the co-domain of g is not same as the domain of f.

So, fog does not exist.

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Question 4:

Let A = {a, b, c}, B = {u v, w} and let f and g be two functions from A to B and from B to A, respectively, defined as :

f = {(a, v), (b, u), (c, w)}, g = {(u, b), (v, a), (w, c)}.

Show that f and g both are bijections and find fog and gof.

ANSWER:

Proving f is a bijection:

f = {(a, v), (b, u), (c, w)} and f : A → B

Injectivity of *f*: No two elements of *A* have the same image in B.

So, *f* is one-one.

Surjectivity of *f*: Co-domain of $f = \{u\ v, w\}$

Range of $f = \{u\ v, w\}$

Both are same.

So, *f* is onto.

Hence, *f* is a bijection.

Proving *g* is a bijection:

$g = \{(u, b), (v, a), (w, c)\}$ and $g : B \to A$

Injectivity of *g*: No two elements of *B* have the same image in A.

So, *g* is one-one.

Surjectivity of g: Co-domain of $g = \{a, b, c\}$

Range of $g = \{a, b, c\}$

Both are the same.

So, *g* is onto.

Hence, *g* is a bijection.

Finding *fog*:

Co-domain of *g* is same as the domain of *f*.

So, *fog* exists and $fog : \{u\ v, w\} \to \{u\ v, w\}$

$(fog)\ (u)=f\ (g\ (u))=f\ (b)=u(fog)\ (v)=f\ (g\ (v))=f\ (a)=v(fog)\ (w)=f\ (g\ (w))=f\ (c)=w$So, $fog=\{(u, u), (v, v), (w, w)\}$

Finding *gof*:

Co-domain of *f* is same as the domain of *g*.

So, *fog* exists and $gof: \{a, b, c\} \to \{a, b, c\}$

$(gof)\ (a)=g\ (f\ (a))=g\ (v)=a(gof)\ (b)=g\ (f\ (b))=g\ (u)=b(gof)\ (c)=g\ (f\ (c))=g\ (w)=c$So, $gof=\{(a, a), (b, b), (c, c)\}$

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Question 5:

Find *fog* (2) and *gof* (1) when : $f: R \to R$; $f(x) = x^2 + 8$ and $g: R \to R$; $g(x) = 3x^3 + 1$.

ANSWER:

$(fog)\ (2)=f\ (g\ (2))=f(3\times23+1)=f(25)=252+8=633(gof)\ (1)=g\ (f\ (1))=g\ (12+8)=g\ (9)=3\times93+1=2188$

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Question 6:

Let R^+ be the set of all non-negative real numbers. If $f: R^+ \to R^+$ and $g: R^+ \to R^+$ are defined as $f(x)=x2$ and $g(x)=+\sqrt{x}$, find *fog* and *gof*. Are they equal functions?

ANSWER:

Given, $f: R^+ \to R^+$ and $g: R^+ \to R^+$

So, $fog: R^+ \to R^+$ and $gof: R^+ \to R^+$

Domains of *fog* and *gof* are the same.

$(fog)\ (x)=f\ (g\ (x))=f\ (\sqrt{x})=(\sqrt{x})2=x(gof)\ (x)=g\ (f\ (x))=g\ (x2)=\sqrt{x2}=x$So, $(fog)\ (x)=(gof)\ (x), \forall x\in R+$

Hence, $fog = gof$

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Question 7:

Let $f : R \rightarrow R$ and $g : R \rightarrow R$ be defined by $f(x) = x^2$ and $g(x) = x + 1$. Show that $fog \neq gof$.

ANSWER:

Given, $f : R \rightarrow R$ and $g : R \rightarrow$ R.

So, the domains of f and g are the same.

$(fog)\ (x)=f\ (g\ (x))=f\ (x+1)=(x+1)2=x2+1+2x(gof)\ (x)=g\ (f\ (x))=g\ (x2)=x2+1$

So, $fog \neq gof$

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Question 8:

Let $f : R \rightarrow R$ and $g : R \rightarrow R$ be defined by $f(x) = x + 1$ and $g(x) = x - 1$. Show that $fog = gof = I_R$.

ANSWER:

Given, $f : R \rightarrow R$ and $g : R \rightarrow$ R

$\Rightarrow fog : R \rightarrow$ R and $gof : R \rightarrow$ R (Also, we know that $I_R : R \rightarrow$ R)

So, the domains of all fog, gof and I_R are the same.

$(fog)\ (x)=f\ (g\ (x))=f\ (x-1)=x-1+1=x=IR\ (x)$ ... (1)$(gof)\ (x)=g\ (f\ (x))=g\ (x+1)=x+1-1=x=IR\ (x)$ ... (2)From (1) and (2), $(fog)\ (x)=(gof)\ (x)=IR\ (x), \forall x\in R$Hence, $fog=gof=IR$

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Question 9:

Verify associativity for the following three mappings : $f : N \to Z_0$ (the set of non-zero integers), $g : Z_0 \to Q$ and $h : Q \to R$ given by $f(x) = 2x$, $g(x) = 1/x$ and $h(x) = e^x$.

ANSWER:

Given that $f : N \to Z_0$, $g : Z_0 \to Q$ and $h : Q \to R$.

$gof : N \to Q$ and $hog : Z_0 \to R$

$\Rightarrow h\ o\ (gof) : N \to R$ and $(hog)\ o\ f: N \to R$

So, both have the same domains.

$(gof)\ (x)=g\ (f\ (x))=g\ (2x)=12x$ ...(1)$(hog)\ (x)=h\ (g\ (x))=h\ (1x)=e1x$ ...(2)Now,$(h\ o(gof))\ (x)=h((gof)\ (x))=h\ (12x)=e12x$ [from (1)]$((hog)\ o\ f)(x)=(hog)\ (f\ (x))= (hog)\ (2x)=e12x$ [from (2)]$\Rightarrow(h\ o(gof))\ (x)=((hog)\ o\ f)(x),\ \forall x\in N$So, $h\ o(gof)=(hog)\ o\ f$

Hence, the associative property has been verified.

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Question 10:

Consider $f : N \to N$, $g : N \to N$ and $h : N \to R$ defined as $f(x) = 2x$, $g(y) = 3y + 4$ and $h(z) = \sin z$ for all $x, y, z \in N$. Show that $ho\ (gof) = (hog)\ of$.

ANSWER:

Given, $f : N \to N$, $g : N \to N$ and $h : N \to R$

$\Rightarrow gof : N \to N$ and $hog : N \to R$

$\Rightarrow ho\ (gof) : N \to R$ and $(hog)\ of : N \to R$

So, both have the same domains.

$(gof)\ (x)=g\ (f\ (x))=g\ (2x)=3\ (2x)+4=6x+4$ ...(1)$(hog)\ (x)=h(g\ (x))=h\ (3x+4)=\sin\ (3x+4)$ ... (2)Now,$(h\ o\ (gof))\ (x)=h\ ((gof)\ (x))=h(6x+4) = \sin\ (6x+4)$ [from (1)]$((hog)\ o\ f)\ (x)=(hog)\ (f\ (x))=(hog)\ (2x)=\sin\ (6x+4)$ [from (2)]So, $(h\ o\ (gof))\ (x)=((hog)\ o\ f)\ (x),\ \forall x\in N$Hence, $h\ o\ (gof)=(hog)\ o\ f$

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Question 11:

Give examples of two functions $f : N \to N$ and $g : N \to N$, such that *gof* is onto but f is not onto.

ANSWER:

Let us consider a function $f : N \to N$ given by $f(x) = x + 1$, which is not onto.

[This not onto because if we take 0 in N (co-domain), then,

$0=x+1$

$\Rightarrow x=-1 \notin N$]

Let us consider $g : N \to N$ given by

$g(x)=\{x-1$, if $x>11$, if $x=1$Now, let us find $(gof)(x)$Case 1: $x>1$$(gof)(x)=g(f(x))=g(x+1)=x+1-1=x$Case 2: $x=1$$(gof)(x)=g(f(x))=g(x+1)=1$From case-1 and case-2, $(gof)(x)=x$, $\forall x \in N$, which is an identity function and, hence, it is onto.

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Question 12:

Give examples of two functions $f : N \to Z$ and $g : Z \to Z$, such that *gof* is injective but g is not injective.

ANSWER:

Let $f : N \to Z$ be given by $f(x) = x$, which is injective.

(If we take $f(x) = f(y)$, then it gives $x = y$)

Let $g : Z \to Z$ be given by $g(x) = |x|$, which is not injective.

If we take $f(x) = f(y)$, we get:

$|x| = |y|$

$\Rightarrow x = \pm\, y$

Now, $gof : N \to Z$.

$(gof)\,(x) = g\,(f\,(x)) = g\,(x) = |x|$

Let us take two elements x and y in the domain of gof , such that

$(gof)\,(x) = (gof)\,(y) \Rightarrow |x| = |y| \Rightarrow x = y$ (We don't get ± here because x, y $\in$N)

So, gof is injective.

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Question 13:

If $f : A \to B$ and $g : B \to C$ are one-one functions, show that gof is a one-one function.

ANSWER:

Given, $f : A \to B$ and $g : B \to C$ are one - one.

Then, $gof : A \to B$

Let us take two elements x and y from A, such that

$(gof)\,(x) = (gof)\,(y) \Rightarrow g\,(f\,(x)) = g\,(f\,(y)) \Rightarrow f\,(x) = f\,(y)$ (As, g is one-one)$\Rightarrow x = y$ (As, f is one-one)

Hence, gof is one-one.

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Question 14:

If $f : A \to B$ and $g : B \to C$ are onto functions, show that gof is a onto function.

ANSWER:

Given, $f : A \to B$ and $g : B \to C$ are onto.

Then, $gof : A \to C$

Let us take an element z in the co-domain (C).

Now, z is in C and $g : B \to C$ is onto.

So, there exists some element y in B, such that $g(y) = z$ ... (1)

Now, y is in B and $f : A \to B$ is onto.

So, there exists some x in A, such that $f(x) = y$ ... (2)

From (1) and (2),

$z = g(y) = g(f(x)) = (gof)(x)$

So, $z = (gof)(x)$, where x is in A.

Hence, gof is onto.

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Question 1:

Find fog and gof if

(i) $f(x)=ex$, $g(x)=\text{loge } x$

(ii) $f(x)=x2$, $g(x)=\cos x$

(iii) $f(x)=|x|$, $g(x)=\sin x$

(iv) $f(x)=x+1$, $g(x)=ex$

(v) $f(x)=\sin\text{-}1\ x$, $g(x)=x2$

(vi) $f(x)=x+1$, $g(x)=\sin x$

(vii) $f(x)=x+1$, $g(x)=2x+3$

(viii) $f(x)=c$, $c \in R$, $g(x)=\sin x2$

(ix) $f(x)=x2+2$, $g(x)=1-11-x$

ANSWER:

(i) $f(x)=ex$, $g(x)=\text{loge } x f:R\to(0,\infty)$; $g:(0,\infty)\to R$Computing fog:Clearly, the range of g is a subset of the domain of f.$fog : (0,\infty)\to R (fog)(x)=f(g(x))=f(\text{loge } x)=\text{loge } ex=x$Computing gof:Clearly, the range of f is a subset of the domain of g.$\Rightarrow fog : R\to R (gof)(x)=g(f(x))=g(ex)=\text{loge } ex=x$

(ii) $f(x)=x2$, $g(x)=\cos x f:R\to[0, \infty)$; $g:R\to[-1, 1]$Computing fog:Clearly, the range of g is not a subset of the domain of f.$\Rightarrow$Domain $(fog)=\{x: x\in$domain of g and $g(x)\in$domain of $f\}\Rightarrow$Domain $(fog)=x: x\in R$ and $\cos x \in R\}\Rightarrow$Domain of $(fog)=R fog: R\to R (fog)(x)=f(g(x))=f(\cos x)=\cos 2x$Computing gof:Clearly, the range of f is a subset of the domain of g.$\Rightarrow fog : R\to R (gof)(x)=g(f(x))=g(x2)=\cos(x2)$

(iii) $f(x)=|x|$, $g(x)=\sin x f:R\to(0, \infty)$; $g:R\to[-1, 1]$Computing fog:Clearly, the range of g is a subset of the domain of f.$\Rightarrow fog : R\to R (fog)(x)=f(g(x))=f(\sin x)=|\sin x|$Computing gof:Clearly, the range of f is a subset of the domain of g.$\Rightarrow fog : R\to R (gof)(x)=g(f(x))=g(|x|)=\sin|x|$

(iv) $f(x)=x+1$, $g(x)=ex f:R\to R$; $g:R\to[1, \infty)$Computing fog:Clearly, range of g is a subset of domain of f.$\Rightarrow fog : R\to R (fog)(x)=f(g(x))=f(ex)=ex+1$Computing gof:Clearly, range of f is a subset of domain of g.$\Rightarrow fog : R\to R (gof)(x)=g(f(x))=g(x+1)=ex+1$

(v) $f(x)=\sin-1x$, $g(x)=x2 f:[-1,1]\to[-\pi2,\pi2]$; $g:R\to[0, \infty)$**Computing *fog*:**Clearly, the range of g is not a subset of the domain of f.Domain $(fog)=\{x: x\in$domain of g and $g(x)\in$domain of $f\}$Domain $(fog)=\{x: x\in R$ and $x2\in[-1,1]\}$Domain $(fog)=\{x: x\in R$ and $x\in[-1,1]\}$Domain of $(fog)=[-1,1] fog: [-1,1]\to R (fog)(x)=f(g(x))=f(x2)=\sin-1$

(x2)**Computing gof:**Clearly, the range of f is a subset of the domain of g.fog : [-1,1]→R(gof) (x)=g (f (x))=g (sin-1x)=(sin-1 x)2

(vi) f(x)=x+1, g(x)=sin xf:R→R ; g:R→[-1, 1]Computing fog:Clearly, the range of g is a subset of the domain of f.⇒fog: R→R(fog) (x)=f (g (x))=f (sin x)=sin x+1Computing gof:Clearly, the range of f is a subset of the domain of g.⇒fog : R→R(gof) (x)=g (f (x))=g (x+1)=sin (x+1)

(vii) f (x)=x+1, g(x)=2x+3f:R→R ; g:R→RComputing fog:Clearly, the range of g is a subset of the domain of f.⇒fog: R→R(fog) (x)=f (g (x))=f (2x+3)=2x+3+1=2x+4Computing gof:Clearly, the range of f is a subset of the domain of g.⇒fog : R→R(gof) (x)=g (f (x))=g (x+1)=2 (x+1)+3=2x+5

(viii) f (x)=c, g(x)=sin x2f:R→{c} ; g:R→[0, 1]Computing fog:Clearly, the range of g is a subset of the domain of f.fog: R→R(fog) (x)=f (g (x))=f (sin x2)=cComputing gof:Clearly, the range of f is a subset of the domain of g.⇒fog : R→R(gof) (x)=g (f (x))=g (c)=sin c2

(ix) f(x)=x2+2f:R→[2,∞) g(x)=1-11-xFor domain of g: 1-x≠0 ⇒x≠1⇒Domain of g=R-{1}g(x)=1-11-x=1-x-11-x=-x1-xFor range of g:y=-x1-x⇒y-xy=-x⇒y=xy-x⇒y=x(y-1)⇒x=yy-1Range of g =R-{1}So, g: R-{1}→R-{1}Computing fog:Clearly, the range of g is a subset of the domain of f.⇒fog: R-{1}→R(fog) (x)=f (g (x))=f (-xx-1)=(-xx-1)2+2=x2+2x2+2-4x(1-x)2=3x2-4x+2(1-x)2Computing gof:Clearly, the range of f is a subset of the domain of g.⇒gof : R→R(gof) (x)=g (f (x))=g (x2+2)=1-11-(x2+2)=1-1-(x2+1)=x2+2x2+1

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Question 2:

Let $f(x) = x^2 + x + 1$ and $g(x) = \sin x$. Show that $fog \neq gof$.

ANSWER:

$(fog)\ (x)=f\ (g\ (x))=f(\sin x)=\sin 2x+\sin x+1$ and $(gof)\ (x)=g\ (f\ (x))=g\ (x2+x+1)= \sin (x2+x+1)$ So, $fog \neq gof$.

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Question 3:

If $f(x) = |x|$, prove that $fof = f$.

ANSWER:

Domains of f and fof are same as R.

$(fof)\ (x)=f\ (f\ (x))=f\ (|x|)=|\ |x|\ |=|x|=f\ (x)$ So, $(fof)\ (x)=f\ (x),\ \forall x \in R$ Hence, $fof=f$

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Question 4:

If $f(x) = 2x + 5$ and $g(x) = x^2 + 1$ be two real functions, then describe each of the following functions:

(i) fog

(ii) gof

(iii) fof

(iv) f^2

Also, show that $fof \neq f^2$

ANSWER:

$f(x)$ and $g(x)$ are polynomials.

$\Rightarrow f : R \rightarrow R$ and $g : R \rightarrow R$.

So, $fog : R \rightarrow R$ and $gof : R \rightarrow R$.

(i) $(fog)\ (x)=f\ (g\ (x))=f\ (x2+1)=2\ (x2+1)+5=2x2+2+5=2x2+7$

(ii) (gof) (x)=g (f (x))=g (2x+5)=(2x+5)2+1=4x2+20x+26

(iii) (fof) (x)=f (f (x))=f (2x+5)=2 (2x+5)+5=4x+10+5=4x+15

(iv) f2 (x)=f(x)×f(x)=(2x+5)(2x+5)=(2x+5)2=4x2+20x+25

→→ →

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Question 5:

If $f(x)$ = sin x and $g(x)$ = 2x be two real functions, then describe *gof* and *fog.* Are these equal functions?

ANSWER:

We know that$f$:R→[-1, 1] and g: R→RClearly, the range of f is a subset of the domain of g.*gof*:R→R(*gof*) (x)=g (f (x))=g (sin x)=2sin x

Clearly, the range of g is a subset of the domain of f.*fog*:R→RSo, (*fog*) (x)=f (g (x))=f (2x)=sin (2x)

Clearly, *fog≠gof*

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Question 6:

Let *f*, *g*, *h* be real functions given by $f(x)$ = sin *x*, $g(x) = 2x$ and $h(x)$ = cos *x*. Prove that *fog* = *go* (*fh*).

ANSWER:

We know that $f:R\to[-1, 1]$ and $g:R\to R$Clearly, the range of *g* is a subset of the domain of *f*.$fog:R\to R$Now, $(fh)(x)=f(x)h(x)=(\sin x)(\cos x)=12\sin(2x)$Domain of *fh* is *R*.Since range of sin *x* is [-1,1],$-1\le\sin 2x\le1\Rightarrow-12\le\sin x2\le12$Range of *fh* =[-12, 12]So, $(fh):R\to[-12, 12]$Clearly, range of *fh* is a subset of *g*.$\Rightarrow go(fh):R\to R\Rightarrow$domains of *fog* and *go*(*fh*) are the same.So, $(fog)(x)=f(g(x))=f(2x)=\sin(2x)$and $(go(fh))(x)=g((fh)(x))=g(\sin x\cos x)=2\sin x\cos x=\sin(2x)\Rightarrow(fog)(x)=(go(fh))(x),\ \forall x\in R$Hence, *fog* = *go*(*fh*)

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Question 7:

Let *f* be any real function and let *g* be a function given by $g(x) = 2x$. Prove that *gof* = *f* + *f*.

ANSWER:

Given, $f:R\to R$Since $g(x)=2x$ is a polynomial, $g:R\to R$Clearly, $gof:R\to R$ *and* $f+f:R\to R$So, domains of gof and f+f are the same.$(gof)(x)=g(f(x))=2f(x)(f+f)(x)=f(x)+f(x)=2f(x)\Rightarrow(gof)(x)=(f+f)(x),\ \forall x\in R$Hence, *gof* =*f*+*f*

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Question 8:

If $f(x)=\sqrt{1-x}$ and $g(x)=\log_e x$ are two real functions, then describe functions *fog* and *gof*.

ANSWER:

$f(x)=\sqrt{1-x}$For domain, $1-x\ge0\Rightarrow x\le1\Rightarrow$domain of $f=(-\infty, 1]\Rightarrow f:(-\infty, 1]\to(0,\infty)$ $g(x)=\log_e x$Clearly, $g:(0, \infty)\to R$Computation of *fog*:Clearly, the range of *g* is not a subset of the domain of *f*.So,we need to compute the domain of *fog*.$\Rightarrow$Domain $(fog)=\{x : x\in$Domain (g) and $g(x)\in$Domain of f$\}\Rightarrow$Domain $(fog)=\{x: x\in(0, \infty)$ and $\log_e x\in(-\infty, 1]\}\Rightarrow$Domain $(fog)=\{x:x\in(0, \infty)$ and $x\in(0, e]\}\Rightarrow$Domain $(fog)=\{x: x\in(0, e]\}\Rightarrow$Domain $(fog)=(0, e]\Rightarrow fog: (0, e)\to R$So, $(fog)(x)=f(g(x))=f(\log_e x)=\sqrt{1-\log_e x}$ Computation of *gof*:Clearly, the range of f is a subset of the domain of g.⇒gof:(-∞,1]→R⇒(gof) (x)=g (f (x))=g (√1-x)=loge√1-x=loge (1-x)12=12loge (1-x)

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Question 9:

If f:(-π2,π2)→R and g:[-1, 1]→R be defined as

f(x)=tan x and g(x)=√1-x2 respectively, describe fog and gof.

ANSWER:

g (x)=√1-x2⇒x2≥0, ∀x∈[-1, 1]⇒-x2≤0, ∀x∈[-1, 1]⇒1-x2≤1, ∀x∈[-1, 1]We know that 1-x2≥0⇒0≤1-x2≤1⇒Range of g(x)=[0, 1]So, f:(-π2, π2)→R and g:[-1, 1]→ [0, 1]Computation of fog:Clearly, the range of g is a subset of the domain of f.So, fog: [-1, 1]→R(fog) (x)=f (g (x))=f (√1-x2)=*tan* √1-x2Computation of gof:Clearly, the range of f is not a subset of the domain of g.⇒Domain (gof)={x∈domain of f and f(x)∈domain of g}⇒Domain (gof)={x∈(-π2, π2) and tan x ∈[-1,1]}⇒Domain (gof)={x∈(-π2, π2) and x ∈(-π4, π4)}⇒Domain (gof)={x∈(-π4, π4)}Now, gof:(-π4, π4)→RSo, (gof) (x)=g (f (x))=g (tan x)=√1-tan2x

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Question 10:

If f(x)=√x+3 and g(x)=x2+1 be two real functions, then find fog and gof.

ANSWER:

f(x)=√x+3For domain,x+3≥0⇒x≥-3Domain of f =[-3, ∞)Since f is a square root function, range of f =[0, ∞)f: [-3, ∞)→[0, ∞)g(x)=x2+1 is a polynomial.⇒g:R→RComputation of fog:Range of g is not a subset of the domain of f.and domain (fog)={x: x∈domain of g and g(x)∈domain of f(x)}⇒Domain (fog)={x:x∈R and x2+1∈[-3, ∞)}⇒Domain (fog)={x:x∈R and x2+1≥-3}⇒Domain (fog)={x:x∈R and x2+4≥0}⇒Domain (fog)={x:x∈R and x∈R}⇒Domain (fog)=Rfog:R→R(fog) (x)=f(g (x))=f (x2+1)=√x2+1+3=√x2+4Computation of gof:Range of f is a subset of the domain of g.gof: [-3, ∞)→R⇒(gof) (x)=g (f (x))=g (√x+3)=(√x+3)2+1=x+3+1=x+4

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Question 11:

Let f be a real function given by $f(x)=\sqrt{x-2}$.

Find each of the following:

(i) fof

(ii) $fofof$

(iii) $(fofof)$ (38)

(iv) f^2

Also, show that $fof \neq f^2$.

ANSWER:

$f(x)=\sqrt{x-2}$ For domain, $x-2\geq 0 \Rightarrow x\geq 2$ Domain of $f=[2,\infty)$ Since f is a square-root function, range of $f=(0,\infty)$ So, $f:[2,\infty)\rightarrow(0,\infty)$ (i) fof Range of f is not a subset of the domain of f. $\Rightarrow$ Domain$(fof)=\{x: x \in$ domain of f and $f(x) \in$ domain of f$\}\Rightarrow$ Domain$(fof)=\{x: x \in[2,\infty)$ and $\sqrt{x-2}\in[2,\infty)\}\Rightarrow$ Domain$(fof)=\{x: x \in[2,\infty)$ and $\sqrt{x-2}\geq 2\}\Rightarrow$ Domain$(fof)=\{x: x \in[2,\infty)$ and $x-2\geq 4\}\Rightarrow$ Domain$(fof)=\{x: x \in[2,\infty)$ and $x\geq 6\}\Rightarrow$ Domain$(fof)=\{x: x\geq 6\}\Rightarrow$ Domain$(fof)=[6, \infty)$ $fof :[6, \infty)\rightarrow R$ $(fof)(x)=f(f(x))=f(\sqrt{x-2})=\sqrt{\sqrt{x-2}-2}$

(ii) $fofof= (fof) of$ We have, $f:[2,\infty)\rightarrow(0,\infty)$ and $fof : [6, \infty)\rightarrow R \Rightarrow$ Range of f is not a subset of the domain of fof. Then, domain$((fof)of)=\{x: x \in$ domain of f and $f(x)\in$ domain of $fof\}\Rightarrow$ Domain$((fof)of)=\{x: x \in[2,\infty)$ and $\sqrt{x-2}\in[6,\infty)\}\Rightarrow$ Domain$((fof)of)=\{x: x \in[2,\infty)$ and $\sqrt{x-2}\geq 6\}\Rightarrow$ Domain$((fof)of)=\{x: x \in[2,\infty)$ and $x-2\geq 36\}\Rightarrow$ Domain$((fof)of)=\{x: x \in[2,\infty)$ and $x\geq 38\}\Rightarrow$ Domain$((fof)of)=\{x: x\geq 38\}\Rightarrow$ Domain$((fof)of)=[38, \infty)$ $fof :[38,\infty)\rightarrow R$ So, $((fof)of)(x)=(fof)(f(x))=(fof)(\sqrt{x-2})=\sqrt{\sqrt{\sqrt{x-2}-2}-2}$

(*iii*) We have, $(fofof)(x)=\sqrt{\sqrt{\sqrt{x-2}-2}-2}$ So, $(fofof)(38)=\sqrt{\sqrt{\sqrt{38-2}-2}-2}=\sqrt{\sqrt{\sqrt{36}-2}-2}=\sqrt{\sqrt{6-2}-2}=\sqrt{2-2}=0$

(iv) We have, $fof=\sqrt{\sqrt{x-2}-2}\Rightarrow f2(x)=f(x)\times f(x)=\sqrt{x-2}\times\sqrt{x-2}=x-2$ So, $fof \neq f2$

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Question 12:

Let $f(x)$={1+x,0≤x≤23-x,2<x≤3. Find fof.

ANSWER:

$f(x)$={1+x,0≤x≤23-x,2<x≤3It can be written as,$f(x)$={ 1+x,0≤x≤11+x,1<x≤23-x,2<x≤3 When,0≤x≤1Then, $f(x)$=1+xNow when ,0≤x≤1 then ,1≤x+1≤2Then, $f(f(x))$=1+(1+x)=2+x [∵1≤$f(x)$<2]When ,1<x≤2Then, $f(x)$=1+xNow when ,1<x≤2 then,2<x+1≤3Then, $f(f(x))$=3-(1+x)=2-x [∵2≤$f(x)$<3]When ,2<x≤3Then, $f(x)$=3-xNow when ,2<x≤3 then ,0≤3-x<1Then, $f(f(x))$=1+(3-x)=4-x [∵0≤$f(x)$<1]$f(f(x))$={ 2+x,0≤x≤12-x,1<x≤24-x,2<x≤3

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Question 13:

If f, g : R → R be two functions defined as $f(x) = |x| + x$ and g(x) = $|x| – x$, ∀ x∈R. Then find fog and gof. Hence find fog(–3), fog(5) and gof (–2).

ANSWER:

Given: $f(x) = |x| + x$

and $g(x) = |x| – x$, ∀ x∈R

$fog = f(g(x)) = |g(x)| + g(x)$ =||x| - x|+(|x| - x)

Therefore,

$f(g(x))$ = {0 x≥04x x<0fog = {4x x<00 x≥0

$gof = g(f(x))$=|$f(x)$|-$f(x)$ =||x|+x|-(|x|+x)$g(f(x))$={0 x≥00 x<0

Therefore, $g(f(x)) = gof = 0$

Now, $fog(-3) = (4)(-3) = -12$ (since, $fog = 4x$ for $x < 0$)

$fog(5) = 0$ (since, $fog = 0$ for $x \geq 0$)

$gof(-2) = 0$ (since, $gof = 0$ for $x < 0$)

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Question 1:

State with reasons whether the following functions have inverse:

(i) $f : \{1, 2, 3, 4\} \to \{10\}$ with $f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

(ii) $g : \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ with $g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii) $h : \{2, 3, 4, 5\} \to \{7, 9, 11, 13\}$ with $h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

ANSWER:

(i) $f : \{1, 2, 3, 4\} \to \{10\}$ with $f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

We have:

$f(1) = f(2) = f(3) = f(4) = 10$

$\Rightarrow f$ is not one-one.

$\Rightarrow f$ is not a bijection.

So, f does not have an inverse.

(ii) $g : \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ with $g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

$g(5) = g(7) = 4$

⇒ f is not one-one.

⇒ f is not a bijection.

So, f does not have an inverse.

(iii) h : {2, 3, 4, 5} → {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Here, different elements of the domain have different images in the co-domain.

⇒ h is one-one.

Also, each element in the co-domain has a pre-image in the domain.

⇒ h is onto.

⇒ h is a bijection.

⇒ h has an inverse and it is given by

h^{-1}={(7, 2), (9, 3), (11, 4), (13, 5)}

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Question 2:

Find f^{-1} if it exists : $f : A \to B$, where

(i) $A = \{0, -1, -3, 2\}$; $B = \{-9, -3, 0, 6\}$ and $f(x) = 3x$.

(ii) $A = \{1, 3, 5, 7, 9\}$; $B = \{0, 1, 9, 25, 49, 81\}$ and $f(x) = x^2$

ANSWER:

(i) $A = \{0, -1, -3, 2\}$; $B = \{-9, -3, 0, 6\}$ and $f(x) = 3x$.

Given: $f(x) = 3x$

So, f = {(0, 0), (-1, -3), (-3, -9), (2, 6)}

Clearly, this is one-one.

Range of f = Range of f =B

So, f is a bijection and, thus, f^{-1} exists.

Hence, f^{-1}= {(0, 0), (-3, -1), (-9, -3), (6, 2)}

(ii) A = {1, 3, 5, 7, 9}; B = {0, 1, 9, 25, 49, 81} and $f(x) = x^2$

Given: $f(x) = x^2$

So, f = {(1, 1), (3, 9), (5, 25), (7,49), (9, 81)}

Clearly, f is one-one.

But this is not onto because the element 0 in the co-domain (B) has no pre-image in the domain (A) .

⇒f is not a bijection.

So, f^{-1}does not exist.

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Question 3:

Consider f : {1, 2, 3} → {a, b, c} and g : {a, b, c} → {apple, ball, cat} defined as f (1) = a, f (2) = b, f (3) = c, g (a) = apple, g (b) = ball and g (c) = cat. Show that f, g and gof are invertible. Find f^{-1}, g^{-1} and gof^{-1} and show that $(gof)^{-1} = f^{-1} o\ g^{-1}$.

ANSWER:

f={(1, a), (2, b), (3, c)} and g={(a, apple), (b, ball), (c, cat)}Clearly, f and g are bijections.So, f and g are invertible.Now,f-1={(a, 1), (b, 2), (c, 3)} and g-1={(apple, a), (ball, b), (cat, c)}So, f-1o g-1={(apple, 1), (ball, 2), (cat, 3)}   ...(1)f:{1, 2, 3}→{a, b, c} and g:{a, b, c}→{apple, ball, cat}So, gof:{1, 2, 3}→{apple, ball, cat}⇒(gof) (1)=g (f (1))=g (a)=apple(gof) (2)=g (f (2))=g (b)=ball,and (gof) (3)=g (f (3))=g (c)=cat∴gof ={(1, apple), (2, ball), (3, cat)}Clearly, gofis a bijection.So, gof is invertible.(gof)-1={(apple, 1), (ball, 2), (cat, 3)}   ...(2)From (1) and (2), we get:(gof)-1=f-1o g-1

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Question 4:

Let $A = \{1, 2, 3, 4\}$; $B = \{3, 5, 7, 9\}$; $C = \{7, 23, 47, 79\}$ and $f : A \rightarrow B$, $g : B \rightarrow C$ be defined as $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Express $(gof)^{-1}$ and $f^{-1}\ og^{-1}$ as the sets of ordered pairs and verify that $(gof)^{-1} = f^{-1}\ og^{-1}$.

ANSWER:

f(*x*)=2*x*+1⇒*f*={(1, 2(1)+1), (2, 2(2)+1), (3, 2(3)+1), (4, 2(4)+1)}={(1, 3), (2, 5), (3, 7), (4, 9)}*g*(*x*)=*x*2-2⇒*g*={(3, 32-2), (5, 52-2), (7, 72-2), (9, 92-2)}={(3, 7), (5, 23), (7, 47), (9, 79)}Clearly *f* and *g* are bijections and, hence, *f*-1:*B*→*A* and *g*-1: *C*→*B* exist.So, *f*-1={(3, 1), (5, 2), (7, 3), (9, 4)} and *g*-1={(7, 3), (23, 5), (47, 7), (79, 9)}Now, (*f*-1 *o g*-1):*C*→*Af*-1 *o g*-1={(7, 1), (23, 2), (47, 3), (79, 4)} ...(1)Also, *f*:*A*→*B* and *g*:*B*→*C*,⇒*gof*:*A*→*C*, (*gof*)-1:*C*→*A*So, *f*-1 *o g*-1and (*gof*)-1 have same domains.(*gof*)(*x*)=*g* (*f* (*x*))=*g* (2*x*+1)=(2*x*+1)2-2⇒ (*gof*)(*x*)=4*x*2+4*x*+1-2⇒ (*gof*)(*x*)=4*x*2+4*x*-1Then, (*gof*)(1)=*g* (*f* (1))=4+4-1=7,(*gof*)(2)=*g* (*f* (2))=4+4-1=23,(*gof*)(3)=*g* (*f* (3))=4+4-1=47 and (*gof*)(4)=*g* (*f* (4))=4+4-1=79So, *gof*={(1, 7), (2, 23), (3, 47), (4, 79)}⇒(*gof*)-1={(7, 1), (23, 2), (47, 3), (79, 4)} ...(2)From (1) and (2), we get: (*gof*)-1=*f*-1 *o g*-1

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Question 5:

Show that the function $f : Q \rightarrow Q$, defined by $f(x) = 3x + 5$, is invertible. Also, find f^{-1}

ANSWER:

Injectivity of *f*:

Let *x* and *y* be two elements of the domain (*Q*), such that

$f(x)=f(y)$

$\Rightarrow 3x + 5 = 3y + 5$

$\Rightarrow 3x = 3y$

$\Rightarrow x = y$

So, *f* is one-one.

Surjectivity of *f*:

Let *y* be in the co-domain (*Q*), such that *f(x)* = *y*

$\Rightarrow 3x+5=y \Rightarrow 3x=y-5 \Rightarrow x=y-53 \in Q$ *(domain)*

$\Rightarrow f$ is onto.

So, *f* is a bijection and, hence, it is invertible.

Finding f^{-1}:

Let $f\text{-}1(x)=y$ ...(1)$\Rightarrow x=f(y) \Rightarrow x=3y+5 \Rightarrow x-5=3y \Rightarrow y=x-53$So, $f\text{-}1(x)=x-53$ [from (1)]

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Question 6:

Consider $f : R \rightarrow R$ given by $f(x) = 4x + 3$. Show that *f* is invertible. Find the inverse of *f*.

ANSWER:

Injectivity of *f* :

Let *x* and *y* be two elements of domain (*R*), such that

$f(x) = f(y)$

$\Rightarrow 4x + 3 = 4y + 3$

$\Rightarrow 4x = 4y$

$\Rightarrow x = y$

So, *f* is one-one.

Surjectivity of f :

Let y be in the co-domain (R), such that $f(x) = y$.

$\Rightarrow 4x+3=y \Rightarrow 4x=y-3 \Rightarrow x=y-34 \in R(Domain)$

$\Rightarrow f$ is onto.

So, f is a bijection and, hence, is invertible.

Finding f^{-1}:

Let f-1(x)=y ...(1)$\Rightarrow x=f(y) \Rightarrow x=4y+3 \Rightarrow x-3=4y \Rightarrow y=x-34$So, f-1(x)=x-34 [from (1)]

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Question 7:

Consider $f : R \rightarrow R_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with inverse f^{-1} of f given by $f^{-1}(x)=\sqrt{x}-4$, where R^+ is the set of all non-negative real numbers.

ANSWER:

Injectivity of f :

Let x and y be two elements of the domain (Q), such that

$f(x)=f(y)$

$\Rightarrow x2+4=y2+4 \Rightarrow x2=y2 \Rightarrow x=y$ (as co-domain as R+)

So, f is one-one.

Surjectivity of *f* :

Let *y* be in the co-domain (*Q*), such that *f(x)* = *y*

⇒*x2+4=y*⇒*x2=y-4*⇒*x=√y-4∈R*

⇒*f* is onto.

So, *f* is a bijection and, hence, it is invertible.

Finding f^{-1}:

Let *f*-1(*x*)=*y* ...(1)⇒*x*=*f*(*y*)⇒*x*=*y*2+4⇒*x*-4=*y*2⇒*y*=√*x*-4So, *f*-1(*x*)=√*x*-4 [from (1)]

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Question 8:

If *f*(*x*)=4*x*+36*x*-4, *x*≠23, show that *fof*(*x*) = *x* for all *x*≠23. What is the inverse of *f*?

ANSWER:

(*fof*)(*x*)=*f*(*f*(*x*))=*f* (4*x*+36*x*-4)=4(4*x*+36*x*-4)+36(4*x*+36*x*-4)-4=16*x*+12+18*x*-1224*x*+18-24*x*+16=34*x*34=*x*⇒(*fof*)(*x*)=*x*=*IX*, where *I* is an identity function.So, *f*=*f*-1 Hence, *f*-1=4*x*+36*x*-4

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Question 9:

Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that *f* is invertible with *f*-1(*x*)=√*x*+6-13.

ANSWER:

Injectivity of *f* :

Let x and y be two elements of domain (R^+), such that

$f(x)=f(y)$

$\Rightarrow 9x2+6x-5=9y2+6y-5\Rightarrow 9x2+6x=9y2+6y\Rightarrow x=y$ (As, x, $y\in R+$)

So, f is one-one.

Surjectivity of f:

Let y is in the co domain (Q) such that $f(x) = y$

$\Rightarrow 9x2+6x-5=y\Rightarrow 9x2+6x=y+5\Rightarrow 9x2+6x+1=y+6$ *(Adding 1 on both sides)* $\Rightarrow (3x+1)2=y+6\Rightarrow 3x+1=\sqrt{}y+6\Rightarrow 3x=\sqrt{}y+6-1\Rightarrow x=\sqrt{}y+6-13\in R+$*(domain)*

$\Rightarrow f$ is onto.

So, f is a bijection and hence, it is invertible.

Finding f^{-1}:

Let $f\text{-}1(x)=y$ ...(1) $\Rightarrow x=f(y)\Rightarrow x=9y2+6y-5\Rightarrow x+5=9y2+6y\Rightarrow x+6=9y2+6y+1$ (adding 1 on both sides) $\Rightarrow x+6=(3y+1)2\Rightarrow 3y+1=\sqrt{}x+6\Rightarrow 3y=\sqrt{}x+6-1\Rightarrow y=\sqrt{}x+6-13$So, $f\text{-}1(x)=\sqrt{}x+6-13$ [from (1)]

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Question 10:

If $f : R \rightarrow R$ be defined by $f(x) = x^3 - 3$, then prove that f^{-1} exists and find a formula for f^{-1}. Hence, find f^{-1} (24) and f^{-1} (5).

ANSWER:

Injectivity of f :

Let x and y be two elements in domain (R),

such that, $x3-3=y3-3$ $\Rightarrow x3=y3$ $\Rightarrow x=y$

So, f is one-one.

Surjectivity of f :

Let y be in the co-domain (R) such that $f(x) = y$

$\Rightarrow x3-3=y \Rightarrow x3=y+3 \Rightarrow x=3\sqrt{}y+3 \in R$

$\Rightarrow f$ is onto.

So, f is a bijection and, hence, it is invertible.

Finding f^{-1}:

Let $f\text{-}1(x)=y$ ...(1)$\Rightarrow x=f(y) \Rightarrow x=y3-3 \Rightarrow x+3=y3 \Rightarrow y=3\sqrt{}x+3 = f\text{-}1(x)$ [from (1)]So, $f\text{-}1(x)=3\sqrt{}x+3$ Now, $f\text{-}1(24)=3\sqrt{}24+3=3\sqrt{}27=3\sqrt{}33=3$ and $f\text{-}1(5)=3\sqrt{}5+3=3\sqrt{}8=3\sqrt{}23=2$

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Question 11:

A function $f : R \rightarrow R$ is defined as $f(x) = x^3 + 4$. Is it a bijection or not? In case it is a bijection, find f^{-1} (3).

ANSWER:

Injectivity of f:

Let x and y be two elements of domain (R), such that

$f(x)=f(y) \Rightarrow x3+4=y3+4 \Rightarrow x3=y3 \Rightarrow x=y$

So, f is one-one.

Surjectivity of f:

Let y be in the co-domain (R), such that $f(x) = y$.

$\Rightarrow x3+4=y \Rightarrow x3=y-4 \Rightarrow x=3\sqrt{}y-4 \in R$ *(domain)*

$\Rightarrow$ f is onto.

So, f is a bijection and, hence, is invertible.

Finding f^{-1}:

Let $f\text{-}1(x)=y$ ...(1)$\Rightarrow x=f(y) \Rightarrow x=y3+4 \Rightarrow x\text{-}4=y3 \Rightarrow y=3\sqrt{}x\text{-}4$So, $f\text{-}1(x)=3\sqrt{}x\text{-}4$ [from (1)]$f\text{-}1(3)=3\sqrt{}3\text{-}4=3\sqrt{}\text{-}1=\text{-}1$

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Question 12:

If $f : Q \to Q$, $g : Q \to Q$ are two functions defined by $f(x) = 2x$ and $g(x) = x + 2$, show that f and g are bijective maps. Verify that $(gof)^{-1} = f^{-1} og^{-1}$.

ANSWER:

Injectivity of f:

Let x and y be two elements of domain (Q), such that

$f(x) = f(y)$

$\Rightarrow 2x = 2y$

$\Rightarrow x = y$

So, *f* is one-one.

Surjectivity of *f*:

Let *y* be in the co-domain (*Q*), such that *f(x)* = *y.*

$\Rightarrow 2x = y \Rightarrow x = y2 \in Q$ *(domain)*

$\Rightarrow$ *f* is onto.

So, *f* is a bijection and, hence, it is invertible.

Finding f^{-1}:

Let $f\text{-}1(x)=y$ ...(1)$\Rightarrow x=f(y)\Rightarrow x=2y\Rightarrow y=x2$ So, $f\text{-}1(x)=x2$ (from (1))

Injectivity of *g*:

Let *x* and *y* be two elements of domain (*Q*), such that

g(x) = *g(y)*

$\Rightarrow x + 2 = y + 2$

$\Rightarrow x = y$

So, *g* is one-one.

Surjectivity of g:

Let y be in the co domain (Q), such that $g(x) = y$.

$\Rightarrow x+2=y \Rightarrow x= 2-y \in Q$ (domain)

$\Rightarrow$ g is onto.

So, g is a bijection and, hence, it is invertible.

Finding g^{-1}:

Let g-1(x)=y ...(2)⇒x=g(y)⇒x=y+2⇒y=x-2So, g-1(x)=x-2   (From (2))

Verification of $(gof)^{-1} = f^{-1}\ og^{-1}$:

***f*(*x*)=2*x*; *g*(*x*)=*x*+2and *f*-1(*x*)=*x*2; *g*-1(*x*)=*x*-2Now, (*f*-1*o* *g*-1)(*x*)=*f*-1(*g*-1(*x*))⇒(*f*-1*o* *g*-1)(*x*)=*f*-1(*x*-2)⇒(*f*-1*o* *g*-1)(*x*)=*x*-22   ...(3)(*gof*)(*x*)=*g* (*f* (*x*))=*g* (2*x*)=2*x*+2Let (*gof*)-1(*x*)=y ....   (4)*x*=(*gof*)(*y*)⇒*x*=2*y*+2⇒2*y*=*x*-2⇒*y*=*x*-22 ⇒(*gof*)-1(*x*)=*x*-22   [from (4)   ... (5)]From (3) and (5), (*gof*)-1=*f*-1*o* *g*-1**

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Question 13:

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \rightarrow B$ defined by $f(x)$ = x-2x-3. Show that f is one-one and onto and

hence find f^{-1}.
[CBSE 2012, 2014]

ANSWER:

We have,

A = R - {3} and B = R - {1}

The function f : $A \to B$ defined by $f(x)$ = x-2x-3

Let $x,y \in A$ such that $f(x)=f(y)$.
Then,x-2x-3=y-2y-3⇒xy-3x-2y+6=xy-2x-3y+6⇒-x=-y⇒x=y.∴ f is one-one.Let $y \in B$. Then, $y \neq 1$.The function f is onto if there exists $x \in A$ such that $f(x)=y$.Now,$f(x)=y$⇒x-2x-3=y⇒x-2=xy-3y⇒x-xy=2-3y⇒x(1-y)=2-3y⇒x=2-3y1-$y \in A$ [$y \neq 1$]Thus, for any $y \in B$, there exists 2-3y1-$y \in A$ such that$f$(2-3y1-y)=(2-3y1-y)-2(2-3y1-y)-3=2-3y-2+2y2-3y-3+3y=-y-1=y.∴ f is onto.So, f is one-one and onto fucntion.Now,As, x=(2-3y1-y)So, f-1(x)=(2-3x1-x)=3x-2x-1

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Question 14:

Consider the function f : $\mathbf{R}^+ \to [-9,\infty]$ given by $f(x)$ = $5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y)$ = √54+5y-35.   [CBSE 2015]

ANSWER:

We have,$f(x)$=5x2+6x-9Let y=5x2+6x-9=5(x2+65x-95)=5(x2+2×x×35+925-925-95)=5((x+35)2-925-95)=5(x+35)2-95-9=5(x+35)2-545⇒y+545=5(x+35)2⇒5y+5425=(x+35)2⇒√5y+5425=x+35⇒x=√5y+545-35⇒x=√5y+54-35Let $g(y)$=√5y+54-35Now,$fog(y)=f(g(y))$=f(√5y+54-35)=5(√5y+54-35)2+6(√5y+54-35)-9=5(5y+54+9-6√5y+5425)+(6√5y+54-185)-9=5y+63-6√5y+545+6√5y+54-185-9=5y+63-18-455=y=I_Y, Identity functionAlso, $gof(x)=g(f(x))$=g(5x2+6x-9)=√5(5x2+6x-9)+54-35=√25x2+30x-45+54-35=√25x2+30x+9-35=√(5x+3)2-35=5x+3-35=x=I_X, Identity functionSo, f is invertible.Also, f-1(y)=$g(y)$=√5y+54-35

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Question 15:

Let $f : N \to N$ be a function defined as $f(x)=9x2+6x-5$. Show that $f : N \to S$, where S is the range of f, is invertible. find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

ANSWER:

We have,

$f : N \to N$ is a function defined as $f(x) = 9x^2 + 6x - 5$.

Let $y = f(x) = 9x^2 + 6x - 5$

$\Rightarrow y=9x2+6x-5 \Rightarrow y=9x2+6x+1-1-5 \Rightarrow y=(9x2+6x+1)-6 \Rightarrow y=(3x+1)2-6 \Rightarrow y+6=(3x+1)2$

$\Rightarrow \sqrt{}y+6=3x+1$ $(\because y \in N) \Rightarrow \sqrt{}y+6-1=3x \Rightarrow x=\sqrt{}y+6-13 \Rightarrow g(y)=\sqrt{}y+6-13$ [Let $x=g(y)$]

Now,

$fog(y)=f[g(y)]=f(\sqrt{}y+6-13)=9(\sqrt{}y+6-13)2+6(\sqrt{}y+6-13)-5=9(y+6-2\sqrt{}y+6+19)+2(\sqrt{}y+6-1)-5=y+6-2\sqrt{}y+6+1+2\sqrt{}y+6-2-5=y=IY$, Identity function

$gof(x)=g[f(x)]=g(9x2+6x-5)=\sqrt{}(9x2+6x-5)+6-13=\sqrt{}(9x2+6x+1)-13=\sqrt{}(3x+1)2-13=(3x+1)-13=3x3=x=IX$, Identity function

Since, $fog(y)$ and $gof(x)$ are identity function.

Thus, f is invertible.

So, f-1(x)=g(x)=√x+6-13.

Now,

f^{-1}(43) = √43+6-13=√49-13=7-13=63=2

And f^{-1}(163) = √163+6-13=√169-13=13-13=123=4

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Question 16:

Let f : R -{-43} →R be a function defined as $f(x)$ =4x3x+4 . Show that

f : R -{-43} → Rang (f) is one-one and onto. Hence, find f^{-1}.

ANSWER:

The function f:**R**-{-43}→**R**-{43} is given by f(x)=4x3x+4.

Injectivity: Let x, y∈**R**-{-43} be such that

f(x)=f(y)⇒4x3x+4=4y3y+4⇒4x(3y+4)=4y(3x+4)⇒12xy+16x=12xy+16y⇒16x=16y⇒x=y

Hence, f is one-one function.

Surjectivity: Let y be an arbitrary element of **R**-{43}. Then,

f(x) = y

⇒4x3x+4=y⇒4x=3xy+4y⇒4x-3xy=4y⇒x=4y4-3y

As y∈**R**-{43}, 4y4-3y∈**R**.

Also, $4y4\text{-}3y \neq \text{-}43$ because $4y4\text{-}3y=\text{-}43 \Rightarrow 12y=\text{-}16+12y \Rightarrow 0=\text{-}16$, which is not possible.

Thus,

$x=4y4\text{-}3y \in \mathbf{R}\text{-}\{\text{-}43\}$ such that

$f(x)=f(4x3x+4)=4(4y4\text{-}3y)3(4y4\text{-}3y)+4=16y12y+16\text{-}12y=16y16=y$, so every element in $\mathbf{R}\text{-}\{43\}$ has pre-image in $\mathbf{R}\text{-}\{\text{-}43\}$.

Hence, f is onto.

Now,

$x=4y4\text{-}3y$

Replacing x by $f\text{-}1(x)$ and y by x, we have

$f\text{-}1(x)=4x4\text{-}3x$

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Question 17:

Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x)=x\text{-}1x\text{-}2$, show that f is one-one and onto. Find f^{-1}.

ANSWER:

Given: $f(x)=x\text{-}1x\text{-}2$

To show f is one-one:

Let $f(x1)=f(x2) \Rightarrow x1\text{-}1x1\text{-}2=x2\text{-}1x2\text{-}2 \Rightarrow (x1\text{-}1)(x2\text{-}2)=(x2\text{-}1)(x1\text{-}2) \Rightarrow x1x2\text{-}2x1\text{-}x2+2=x1x2\text{-}2x2\text{-}x1+2 \Rightarrow \text{-}2x1\text{-}x2=\text{-}2x2\text{-}x1 \Rightarrow \text{-}2x1+x1=\text{-}2x2+x2 \Rightarrow \text{-}x1=\text{-}x2 \Rightarrow x1=x2$Hence, f is one-one.To show f is onto: Let $y \in B \therefore$ $y=f(x) \Rightarrow y=x\text{-}1x\text{-}2 \Rightarrow y(x\text{-}2)=x\text{-}1 \Rightarrow xy\text{-}2y=x\text{-}1 \Rightarrow xy\text{-}x=2y\text{-}1 \Rightarrow x(y\text{-}1)=2y\text{-}1 \Rightarrow x=2y\text{-}1y\text{-}1$Thus, for every value of y in R-{1}, there exists a pre-image x=2y-1y-1 in R-{2}.Hence, f is onto.

Since, f is one-one and onto

Therefore, f is invertible with f-1(y)=2y-1y-1.

Hence, f-1(x)=2x-1x-1.

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Question 18:

Show that the function $f : N \rightarrow N$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find the inverse of $f : N \rightarrow S$, where S is range of f.

ANSWER:

Given: The function $f : N \rightarrow N$ defined by $f(x) = x^2 + x + 1$

To show f is one-one:

Let
f(x1)=f(x2)⇒x21+x1+1=x22+x2+1⇒x21+x1=x22+x2⇒x21+x1-x22-x2=0⇒x21-x22+x1-x2=0⇒(x1-x2)(x1+x2)+(x1-x2)=0⇒(x1-x2)(x1+x2+1)=0⇒x1-x2=0 or x1+x2+1=0⇒x1=x2 or x1=-(x2+1)⇒x1=x2 (∵ x1, x2∈N)Hence, f is one-one.To show f is not onto:
Since f(x)=x2+x+1∴ f(1)=3f(2)=7f(3)=13and so onThus, Range of f={3, 7, 13, ...}≠NHence, f is not onto.Now, Let f:N→Range of $f$$y$=$x$2+$x$+1⇒$x$2+$x$+1-$y$=0⇒$x$2+$x$+(1-$y$)=0⇒$x$=-1±√12-4(1)(1-$y$)2(1)⇒$x$=-1±√1-4+4$y$2⇒$x$=

-1±√4y-32⇒x=-1+√4y-32 or x=-1-√4y-32⇒x=-1+√4y-32 (∵ x∈N)Hence, f-1(x)=-1+√4x-32.

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Question 19:

Let $f : [-1, \infty) \to [-1, \infty)$ be given by $f(x) = (x + 1)^2 - 1$, $x \geq -1$. Show that f is invertible. Also, find the set $S = \{x : f(x) = f^{-1}(x)\}$.

ANSWER:

Injectivity: Let x and y ∈[-1, ∞), such that f(x)=f(y)⇒(x+1)2-1=(y+1)2-1⇒(x+1)2=(y+1)2⇒(x+1)=(y+1)⇒x=ySo, f is a injection.Surjectivity: Let y ∈[-1, ∞). Then, f(x)=y⇒(x+1)2-1=y⇒x+1=√y+1⇒x=√y+1-1Clearly, x=√y+1-1 is real for all y≥-1.Thus, every element y ∈[-1, ∞) has its pre-image x∈[-1, ∞) given by x=√y+1-1.⇒f is a surjection.So, f is a bijection.Hence, f is invertible.Let f-1(x)=y ...(1)⇒f(y)=x⇒(y+1)2-1=x⇒(y+1)2=x+1⇒y+1=√x+1⇒y=±√x+1-1⇒f-1(x)=±√x+1-1 [from (1)]f(x)=f-1(x)⇒(x+1)2-1=±√x+1-1⇒(x+1)2=±√x+1⇒(x+1)4=x+1⇒(x+1)[(x+1)3-1]=0⇒x+1=0 or (x+1)3-=0⇒x=-1 or (x+1)3=1⇒x=-1 or x+1=1⇒x=-1 or x=0⇒S={0, -1}

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Question 20:

Let $A = \{x$ &epsis; $R \mid -1 \leq x \leq 1\}$ and let $f : A \to A$, $g : A \to A$ be two functions defined by $f(x) = x^2$ and $g(x) = \sin(\pi x/2)$. Show that g^{-1} exists but f^{-1} does not exist. Also, find $g-1$.

ANSWER:

f is not one-one because

f(-1)=(-1)2=1and f(1)=12=1

⇒ -1 and 1 have the same image under *f*.

⇒ *f* is not a bijection.

So, f^{-1} does not exist.

Injectivity of g:

Let *x* and *y* be any two elements in the domain (*A*), such that

$g(x)=g(y)\Rightarrow\sin(\pi x2)=\sin(\pi y2)\Rightarrow(\pi x2)=(\pi y2)\Rightarrow x=y$

So, *g* is one-one.

Surjectivity of g:

Range of *g* = [sin (π(-1)2), sin (π(1)2)] = [sin (-π2), sin (π2)] = [-1, 1] = *A* (co-domain of *g*)

⇒*g* is onto.

⇒*g* is a bijection.

So, g^{-1} exists.

Also,

let $g-1(x)=y$ ...(1)$\Rightarrow g(y)=x\Rightarrow\sin(\pi y2)=x\Rightarrow(\pi y2)=\sin-1\ x\Rightarrow y=2\pi\sin-1\ x\Rightarrow g-1(x)=2\pi\sin-1\ x$ [from (1)]

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Question 21:

Let *f* be a function from *R* to *R,* such that *f*(*x*) = cos (*x* + 2). Is *f* invertible? Justify your answer.

ANSWER:

Injectivity:

Let x and y be two elements in the domain (R), such that

$f(x)=f(y)\Rightarrow\cos(x+2)=\cos(y+2)\Rightarrow x+2=y+2$ or $x+2=2\pi-(y+2)\Rightarrow x=y$ or $x+2=2\pi-y-2\Rightarrow x=y$ or $x=2\pi-y-4$So, we cannot say that x=yFor example,$\cos\frac{\pi}{2}=\cos\frac{3\pi}{2}=0$So,$\frac{\pi}{2}$ and $\frac{3\pi}{2}$ have the same image 0.

⇒ f is not one-one.

⇒ f is not a bijection.

Thus, f is not invertible.

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Question 22:

If A = {1, 2, 3, 4} and B = {a, b, c, d}, define any four bijections from A to B. Also give their inverse functions.

ANSWER:

$f_1=\{(1, a), (2, b), (3, c), (4, d)\}\Rightarrow f_1^{-1}=\{(a, 1), (b, 2), (c, 3), (d, 4)\}$$f_2=\{(1, b), (2, a), (3, c), (4, d)\}\Rightarrow f_2^{-1}=\{(b, 1), (a, 2), (c, 3), (d, 4)\}$$f_3=\{(1, a), (2, b), (4, c), (3, d)\}\Rightarrow f_3^{-1}=\{(a, 1), (b, 2), (c, 4), (d, 3)\}$$f_4=\{(1, b), (2, a), (4, c), (3, d)\}\Rightarrow f_4^{-1}=\{(b, 1), (a, 2), (c, 4), (d, 3)\}$

Clearly, all these are bijections because they are one-one and onto.

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Question 23:

Let A and B be two sets, each with a finite number of elements. Assume that there is an injective map from A to B and that there is an injective map from B to A. Prove that there is a bijection from A to B.

ANSWER:

A and B are two non empty sets. Let f be a function from A to B .It is given that there is injective map from A to B. That means f is one-one function .It is also given that there is injective map from B to A .That means every element of set B has its image in set A.⇒f is onto function or surjective.∴ f is bijective.(If a function is both injective and surjective, then the function is bijective.)

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Question 24:

If $f : A \to A$, $g : A \to A$ are two bijections, then prove that

(i) fog is an injection

(ii) fog is a surjection

ANSWER:

Given: $A \to A$, $g : A \to A$ are two bijections.

Then, $fog : A \to A$

(i) Injectivity of fog:

Let x and y be two elements of the domain (A), such that

$(fog)(x)=(fog)(y) \Rightarrow f(g(x))=f(g(y)) \Rightarrow g(x)=g(y)$ (As, f is one-one)$\Rightarrow x=y$ (As, g is one-one)

So, fog is an injection.

(ii) Surjectivity of fog:

Let z be an element in the co-domain of fog (A).

Now, $z \in A$ (co-domain of f) and f is a surjection.So, $z=f(y)$, where $y \in A$ (domain of f) ...(1)Now, $y \in A$ (co-domain of g) and g is a surjection.So, $y=g(x)$, where $x \in A$ (domain of g) ...(2)From (1) and (2),$z=f(y)=f(g(x))=(fog)(x)$, where $x \in A$(domain of fog)

So, fog is a surjection.

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Question 1:

Let $A=\{x \in R : -1 \leq x \leq 1\}=B$ and $C=\{x \in R : x \geq 0\}$ and

let $S=\{(x, y) \in A \times B : x2+y2=1\}$ and $S0=\{(x, y) \in A \times C : x2+y2=1\}$. Then,

(a) S defines a function from A to B

(b) S_0 defines a function from A to C

(c) S_0 defines a function from A to B

(d) S defines a function from A to C

ANSWER:

(a) S defines a function from A to B

Let $x \in A \Rightarrow -1 \leq x \leq 1$Now, $x2+y2=1 \Rightarrow y2=1-x2 \Rightarrow y=\pm\sqrt{}1-x2 \Rightarrow -1 \leq y \leq 1 \therefore\ y \in B$Thus, S defines a function from A to B.

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Question 2:

$f : R \rightarrow R$ given by $f(x)=x+\sqrt{}x2$ is

(a) injective

(b) surjective

(c) bijective

(d) None of these

ANSWER:

$f(x)=x+\sqrt{}x2=x \pm x=0$ or $2x \Rightarrow$ Each element of the domain has 2 images.

$\Rightarrow f$ is not a function.

So, the answer is (d).

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Question 3:

If $f : A \rightarrow B$ given by $3f(x)+2^{-x}=4$ is a bijection, then

(a) $A=\{x \in R : -1 < x < \infty\}$, $B=\{x \in R : 2 < x < 4\}$

(b) $A=\{x \in R : -3 < x < \infty\}$, $B=\{x \in R : 2 < x < 4\}$

(c) $A=\{x \in R : -2 < x < \infty\}$, $B=\{x \in R : 2 < x < 4\}$

(d) None of these

ANSWER:

(d) None of these

$f:A\rightarrow B$ $3f(x)+2^{-x}=4$ $\Rightarrow 3f(x)=4-2^{-x}$ Taking log on both the sides , $f(x)\log 3=\log(4-2^{-x}) \Rightarrow f(x)=\frac{\log(4-2^{-x})}{\log 3}$ Logaritmic function will only be defined if $4-2^{-x}>0 \Rightarrow 4>2^{-x} \Rightarrow 2^2>2^{-x} \Rightarrow 2>-x \Rightarrow -2<x \Rightarrow x\in(-2,\infty)$ That means $A=\{x\in R:-2<x<\infty\}$ As we know that, $f(x)=\frac{\log(4-2^{-x})}{\log 3}$ We take $x=0\in(-2,\infty) \Rightarrow f(x)=1$ which does not belong to any of the options .

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Question 4:

The function $f : R \rightarrow R$ defined by $f(x)=2^x+2^{|x|}$ is

(a) one-one and onto

(b) many-one and onto

(c) one-one and into

(d) many-one and into

ANSWER:

(d) many-one and into

Graph for the given function is as follows.

A line parallel to X axis is cutting the graph at two different values.

Therefore, for two different values of *x* we are getting the same value of *y*.

That means it is many one function.

From the given graph we can see that the range is $[2,\infty)$

and R is the co-domain of the given function.

Hence, Co-domain≠Range

Therefore, the given function is into.

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Question 5:

Let the function $f: R-\{-b\} \rightarrow R-\{1\}$ be defined by

$f(x)=x+ax+b$, $a\neq b$. Then,

(a) f is one-one but not onto

(b) f is onto but not one-one

(c) f is both one-one and onto

(d) None of these

ANSWER:

(c) f is both one-one and onto

Injectivity:

Let x and y be two elements in the domain R- $\{-b\}$, such that

$f(x)=f(y)\Rightarrow x+ax+b=y+ay+b\Rightarrow(x+a)(y+b)=(x+b)(y+a)\Rightarrow xy+bx+ay+ab=xy+ax+by+ab\Rightarrow bx+ay=ax+by\Rightarrow(a-b)x=(a-b)y\Rightarrow x=y$

So, f is one-one.

Surjectivity:

Let y be an element in the co-domain of f, i.e. R-$\{1\}$, such that $f(x)=y$

$f(x)=y\Rightarrow x+ax+b=y\Rightarrow x+a=yx+yb\Rightarrow x-yx=yb-a\Rightarrow x(1-y)=yb-a\Rightarrow x=yb-a1-y\in R$-$\{-b\}$

So, f is onto.

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Question 6:

The function $f : A \to B$ defined by $f(x) = -x2+6x-8$ is a bijection if

(a) $A=(-\infty, 3]$ and $B=(-\infty, 1]$

(b) $A=[-3, \infty)$ and $B=(-\infty, 1]$

(c) $A=(-\infty, 3]$ and $B=[1, \infty)$

(d) $A=[3, \infty)$ and $B=[1, \infty)$

ANSWER:

(a) $A=(-\infty, 3]$ and $B=(-\infty, 1]$

$f(x)=-x2+6x-8$, is a polynomial functionAnd the domain of polynomial function is real number.$\therefore x \in R$

$f(x) = -x2+6x-8 \quad = -(x2-6x+8) \quad = -(x2-6x+9-1) \quad = -(x-3)2+1$Maximum value of $-(x-3)2$ woud be 0$\therefore$Maximum value of $-(x-3)2+1$ woud be 1$\therefore f(x) \in (-\infty, 1]$

We can see from the given graph that function is symmetrical about $x=3$& the given function is bijective .So, x would be either $(-\infty,3]$ or $[3,\infty)$The correct option which satisfy A and B both is:

$A=(-\infty, 3]$ and $B=(-\infty, 1]$

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Question 7:

Let $A=\{x \in R : -1 \le x \le 1\}=B$. Then, the mapping $f : A \to B$ given by $f(x)=x|x|$ is

(a) injective but not surjective

(b) surjective but not injective

(c) bijective

(d) none of these

ANSWER:

Injectivity:

Let x and y be any two elements in the domain A.

Case-1: Let x and y be two positive numbers, such that

$f(x)=f(y) \Rightarrow x|x|=y|y| \Rightarrow x(x)=y(y) \Rightarrow x2=y2 \Rightarrow x=y$

Case-2: Let x and y be two negative numbers, such that

$f(x)=f(y) \Rightarrow x|x|=y|y| \Rightarrow x(-x)=y(-y) \Rightarrow -x2=-y2 \Rightarrow x2=y2 \Rightarrow x=y$

Case-3: Let x be positive and y be negative.

Then, $x \ne y \Rightarrow f(x)=x|x|$ is positive and $f(y)=y|y|$ is negative $\Rightarrow f(x) \ne f(y)$ So, $x \ne y \Rightarrow f(x) \ne f(y)$

From the 3 cases, we can conclude that f is one-one.

Surjectivity:

Let y be an element in the co-domain, such that $y = f(x)$

Case-1: Let $y>0$. Then, $0<y\leq1\Rightarrow y=f(x)=x|x|>0\Rightarrow x>0\Rightarrow|x|=xf(x)=y\Rightarrow x|x|=y\Rightarrow x(x)=y\Rightarrow x2=y\Rightarrow x=\sqrt{y}\in A$ (We do not get ± because $x>0$)Case-2: Let $y<0$. Then, $-1\leq y<0\Rightarrow y=f(x)=x|x|<0\Rightarrow x<0\Rightarrow|x|=-xf(x)=y\Rightarrow x|x|=y\Rightarrow x(-x)=y\Rightarrow-x2=y\Rightarrow x2=-y\Rightarrow x=-\sqrt{-y}\in A$ (We do not get ± because $x>0$)

$\Rightarrow f$ is onto.

$\Rightarrow f$ is a bijection.

So, the answer is (c).

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Question 8:

Let $f : R\to R$ be given by $f(x)=[x2]+[x+1]-3$ where $[x]$ denotes the greatest integer less than or equal to x. Then, $f(x)$ is

(a) many-one and onto

(b) many-one and into

(c) one-one and into

(d) one-one and onto

ANSWER:

(b) many-one and into

$f : R\to R$

$f(x)=[x2]+[x+1]-3$

It is many one function because in this case for two different values of x

we would get the same value of $f(x)$.

For x=1.1, 1.2 $\in R f(1.1)=[(1.1)2]+[1.1+1]-3 \quad =[1.21]+[2.1]-3 \quad =1+2-3$
$=0 f(1.1)=[(1.2)2]+[1.2+1]-3 \quad =[1.44]+[2.2]-3 \quad =1+2-3 \quad =0$

It is into function because for the given domain we would only get the integral values of

$f(x)$.

but R is the codomain of the given function.

That means , Codomain≠Range

Hence, the given function is into function.

Therefore, $f(x)$ is many one and into

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Question 9:

Let M be the set of all 2 × 2 matrices with entries from the set R of real numbers. Then, the function $f : M \rightarrow R$ defined by $f(A) = |A|$ for every $A \in M$, is

(a) one-one and onto

(b) neither one-one nor onto

(c) one-one but-not onto

(d) onto but not one-one

ANSWER:

$M=\{A=[abcd]: a, b, c, d \in R\} f: M \rightarrow R$ is given by $f(A)=|A|$

Injectivity:

$f([0000])=|0000|=0$and $f([1000])=|1000|=0 \Rightarrow f([0000])=f([1000])=0$

So, f is not one-one.

Surjectivity:

Let y be an element of the co-domain, such that

$f(A)=-y$, $A=[abcd] \Rightarrow |abcd|=y \Rightarrow ad-bc=y \Rightarrow a$, b, c, $d \in R \Rightarrow A=[abcd] \in M$

$\Rightarrow f$ is onto.

So, the answer is (d).

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Question 10:

The function $f:[0, \infty) \to R$ given by $f(x)=xx+1$ is

(a) one-one and onto

(b) one-one but not onto

(c) onto but not one-one

(d) onto but not one-one

ANSWER:

Injectivity:

Let x and y be two elements in the domain, such that

$f(x)=f(y) \Rightarrow xx+1=yy+1 \Rightarrow xy+x=xy+y \Rightarrow x=y$

So, f is one-one.

Surjectivity:

Let y be an element in the co domain R, such that

$y=f(x) \Rightarrow y=xx+1 \Rightarrow xy+y=x \Rightarrow x(y-1)=-y \Rightarrow x=-yy-1$Range of $f=R-\{1\} \neq$co domain (R)

$\Rightarrow f$ is not onto.

So, the answer is (b).

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Question 11:

The range of the function $f(x)=7-xPx-3$ is

(a) {1, 2, 3, 4, 5}

(b) {1, 2, 3, 4, 5, 6}

(c) {1, 2, 3, 4}

(d) {1, 2, 3}

ANSWER:

We know that

$7-x>0$; $x-3 \geq 0$ and $7-x \geq x-3 \Rightarrow x<7$; $x \geq 3$ and $2x \leq 10 \Rightarrow x<7$; $x \geq 3$ and $x \leq 5$So, $x=\{3, 4, 5\}$Range of $f=\{P(3-3)(7-3), P(4-3)(7-4), P(7-5)(5-3)\}=\{4P0, 3P1, 2P2\}=\{1, 3, 2\}=\{1, 2, 3\}$

So, the answer is (d).

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Question 12:

A function f from the set of natural numbers to integers defined by

$f(n)=\{n-12$, when n is odd$-n2$, when n is evenis

(a) neither one-one nor onto

(b) one-one but not onto

(c) onto but not one-one

(d) one-one and onto both

ANSWER:

(d) one-one and onto both

Injectivity:

Let *x* and *y* be any two elements in the domain (*N*).

Case-1: Both *x* and *y* are even.Let *f*(*x*)=*f*(*y*)⇒-*x*2=-*y*2⇒-*x*=-*y*⇒*x*=*y*Case-2: Both *x* and *y* are odd.Let *f*(*x*)=*f*(*y*)⇒*x*-12=*y*-12⇒*x*-1=*y*-1⇒*x*=*y*Case-3: Let *x* be even and *y* be odd. Then, *f*(*x*)=-*x*2and *f*(*y*)=*y*-12Then, clearly *x*≠*y* ⇒*f*(*x*)≠*f*(*y*)From all the cases, *f* is one-one.

Surjectivity:

Co-domain of *f*=*Z*={...,-3, -2, -1, 0, 1, 2, 3, ...}Range of *f*={..., -3-12, -(-2)2,-1-12, 02, 1-12, -22, 3-12, ...}⇒Range of *f*={...,-2, 1, -1, 0, 0, -1, 1,...}⇒Range of *f*={..., -2, -1, 0, 1, 2, ...}⇒Co-domain of *f*=Range of *f*

⇒ *f* is onto.

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Question 13:

Let f be an injective map with domain {x, y, z} and range {1, 2, 3}, such that exactly one of the following statements is correct and the remaining are false.

$f(x)=1$, $f(y)\neq 1$, $f(z)\neq 2$.

The value of f-1 (1) is

(a) x

(b) y

(c) z

(d) none of these

ANSWER:

Case-1: Let $f(x)=1$ be true.Then, $f(y)\neq 1$ and $f(z)\neq 2$ are false.So, $f(y)=1$ and $f(z)=2\Rightarrow$ $f(x)=1$, $f(y)=1\Rightarrow x$ and y have the same images.This contradicts the fact that $f$is one-one.Case-2: Let $f(y)\neq 1$ be true.Then, $f(x)=1$ and $f(z)\neq 2$ are false.So, $f(x)\neq 1$ and $f(z)=2\Rightarrow$ $f(x)\neq 1$, $f(y)\neq 1$ and $f(z)=2\Rightarrow$There is no pre-image for 1.This contradicts the fact that range is {1, 2, 3}.Case-3: Let $f(z)\neq 2$ be true.Then, $f(x)=1$ and $f(y)\neq 1$ are false.So, $f(x)\neq 1$ and $f(y)=1\Rightarrow f(x)=2$, $f(y)=1$ and $f(z)=3\Rightarrow f(y)=1\Rightarrow f$-1(1)=$y$

So, the answer is (b).

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Question 14:

Which of the following functions form Z to itself are bijections?

(a) $f(x)=x3$

(b) $f(x)=x+2$

(c) $f(x)=2x+1$

(d) $f(x)=x2+x$

ANSWER:

(a) f is not onto because for y = 3∈Co-domain(Z), there is no value of x∈Domain(Z)x3=3⇒x=3√3∉Z⇒f is not onto.So, $f$is not a bijection.

(b) Injectivity:

Let x and y be two elements of the domain (Z), such that

$x+2=y+2 \Rightarrow x=y$

So, f is one-one.

Surjectivity:

Let y be an element in the co-domain (Z), such that

$y=f(x) \Rightarrow y=x+2 \Rightarrow x=y-2 \in Z$ (Domain)

⇒f is onto.

So, f is a bijection.

(c) $f(x)=2x+1$ is not onto because if we take 4 ∈ Z(co domain), then 4=f(x)⇒4=2x+1⇒2x=3⇒x=32∉ZSo, f is not a bijection.(d) f(0)=02+0=0and f(-1)=(-1)2+(-1)=1-1=0⇒0 and -1 have the same image.⇒f is not one-one.So, f is not a bijection.

So, the answer is (b).

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Question 15:

Which of the following functions from A={x : -1≤x≤1} to itself are bijections?

(a) $f(x)=x2$

(b) $g(x)=\sin(\pi x2)$

(c) $h(x)=|x|$

(d) $k(x)=x2$

ANSWER:

(*a*) Range of $f=[-12, 12] \neq A$So, f is not a bijection.(*b*) Range $=[\sin(-\pi 2), \sin(\pi 2)]=[-1,1]=A$So, g is a bijection.(*c*) $h(-1)=|-1|=1$and $h(1)=|1|=1\Rightarrow-1$ and 1 have the same imagesSo, h is not a bijection.(*d*) $k(-1)=(-1)2=1$and $k(1)=(1)2=1\Rightarrow-1$ and 1 have the same imagesSo, k is not a bijection.

So, the answer is (b).

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Question 16:

Let $A=\{x : -1\leq x\leq 1\}$ *and* $f : A\rightarrow A$ such that $f(x)=x|x|$, then f is

(a) a bijection

(b) injective but not surjective

(c) surjective but not injective

(d) neither injective nor surjective

ANSWER:

Injectivity:

Let x and y be any two elements in the domain A.

Case-1: Let x and y be two positive numbers, such that

$f(x)=f(y)\Rightarrow x|x|=y|y|\Rightarrow x(x)=y(y)\Rightarrow x2=y2\Rightarrow x=y$

Case-2: Let x and y be two negative numbers, such that

$f(x)=f(y)\Rightarrow x|x|=y|y|\Rightarrow x(-x)=y(-y)\Rightarrow -x2=-y2\Rightarrow x2=y2\Rightarrow x=y$

Case-3: Let x be positive and y be negative.

Then, $x\neq y\Rightarrow f(x)=x|x|$ is positive and $f(y)=y|y|$ is negative$\Rightarrow f(x)\neq f(y)$So, $x\neq y\Rightarrow f(x)\neq f(y)$

So, f is one-one.

Surjectivity:

Let y be an element in the co-domain, such that $y = f(x)$

Case-1: Let $y>0$. Then,
$0<y\leq 1y=f(x)=x|x|>0\Rightarrow x>0\Rightarrow |x|=x\Rightarrow f(x)=y\Rightarrow x|x|=y\Rightarrow x(x)=y\Rightarrow x2=y\Rightarrow x=\sqrt{y} \in A$ (We do not get ±, as $x>0$)Case-2: Let $y<0$. Then,
$-1\leq y<0y=f(x)=x|x|<0\Rightarrow x<0\Rightarrow |x|=-x\Rightarrow f(x)=y\Rightarrow x|x|=y\Rightarrow x(-x)=y\Rightarrow -x2=y\Rightarrow x2=-y\Rightarrow x=-\sqrt{-y} \in A$ (We do not get ±, as $x>0$)

$\Rightarrow f$ is onto

$\Rightarrow f$ is a bijection.

So, the answer is (a).

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Question 17:

If the function $f : R\rightarrow A$ *given by* $f(x)=x2x2+1$ is a surjection, then A =

(a) R

(b) [0, 1]

(c) [0, 1)

(d) [0, 1)

ANSWER:

As f is surjective, range of f=co-domain of f⇒A= range of f ∵$f(x)$= $x2x2+1$, $y=x2x2+1⇒y(x2+1)= x2⇒(y-1)x2+y= 0⇒x2= -y(y-1)⇒x=\sqrt{y(1-y)}⇒y(1-y)≥0⇒y∈[0, 1)$⇒Range of f= [0, 1)⇒A= [0, 1)

So, the answer is (d).

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Question 18:

If a function f : [2, ∞) → B defined by $f(x)=x2-4x+5$ is a bijection, then B =

(a) R

(b) [1, ∞)

(c) [4, ∞)

(d) [5, ∞)

ANSWER:

Since f is a bijection, co-domain of f = range of f

⇒B = range of f

Given: $f(x)=x2-4x+5$Let $f(x)=y⇒y=x2-4x+5⇒x2-4x+(5-y)=0$∵Discrimant, $D=b2-4ac≥0,(-4)2-4×1×(5-y)≥0⇒16-20+4y≥0⇒4y≥4⇒y≥1⇒y∈[1, ∞)$⇒Range of f=[1, ∞)⇒B=[1, ∞)

So, the answer is (b).

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Question 19:

The function f : R→R defined by

$f(x)=(x-1)\ (x-2)\ (x-3)$ is

(a) one-one but not onto

(b) onto but not one-one

(c) both one and onto

(d) neither one-one nor onto

ANSWER:

$f(x)=(x-1)(x-2)(x-3)$

Injectivity:

$f(1)=(1-1)(1-2)(1-3)=0 f(2)=(2-1)(2-2)(2-3)=0 f(3)=(3-1)(3-2)(3-3)=0 \Rightarrow f(1)=f(2)=f(3)=0$

So, f is not one-one.

Surjectivity:

Let y be an element in the co domain R, such that

$y=f(x) \Rightarrow y=(x-1)(x-2)(x-3)$Since $y \in R$ and $x \in R$, f is onto.

So, the answer is (b).

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Question 20:

The function f : [-1/2, 1/2, 1/2]→[-π/2, π/2] , defined by $f(x)$=sin-1 (3x-4x3), is

(a) bijection

(b) injection but not a surjection

(c) surjection but not an injection

(d) neither an injection nor a surjection

ANSWER:

$f(x)$=sin-1(3x-4x3)⇒$f(x)$=3sin-1x

Injectivity:

Let x and y be two elements in the domain [-12, 12] , such that

$f(x)$=$f(y)$⇒3sin-1x=3sin-1y⇒sin-1x=sin-1y⇒x=y

So, f is one-one.

Surjectivity:

Let y be any element in the co-domain, such that

$f(x)$=y⇒3sin-1(x)=y⇒sin-1(x)=y3⇒x=sin$y$3∈[-12, 12]

⇒f is onto.

⇒f is a bijection.

So, the answer is (a).

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Question 21:

Let f : R→R be a function defined by $f(x)$=e|x|-e-$x$$e$$x$+$e$-$x$. Then,

(a) f is a bijection

(b) f is an injection only

(c) f is surjection on only

(d) *f* is neither an injection nor a surjection

ANSWER:

(d) *f* is neither an injection nor a surjection

$f : R \to R$

$f(x) = e|x| - e - xex + e - x$For $x = -2$ and $-3 \in R$ $f(-2) = e|-2| - e2e - 2 + e2$ $= e2 - e2e - 2 + e2$ $= 0$& $f(-3) = e|-3| - e3e - 3 + e3$ $= e3 - e3e - 3 + e3$ $= 0$Hence, for different values of x we are getting same values of $f(x)$That means , the given function is many one .

Therefore, this function is not injective.

For $x < 0$ $f(x) = 0$For $x > 0$ $f(x) = ex - e - xex + e - x$ $= ex + e - xex + e - x - 2e - xex + e - x$ $= 1 - 2e - xex + e - x$The value of $2e - xex + e - x$ is always positive.Therefore, the value of $f(x)$ is always less than 1Numbers more than 1 are not included in the range but they are included in codomain.As the codomain is R.$\therefore$ Codomain$\neq$RangeHence, the given function is not onto .

Therefore, this function is not surjective .

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Question 22:

Let $f : R - \{n\} \to R$ be a function defined by

$f(x) = x - mx - n$, where $m \neq n$. Then,

(a) *f* is one-one onto

(b) *f* is one-one into

(c) *f* is many one onto

(d) *f* is many one into

ANSWER:

Injectivity:

Let *x* and *y* be two elements in the domain *R*-{*n*}, such that

$f(x)=f(y)\Rightarrow x\text{-}mx\text{-}n=y\text{-}my\text{-}n\Rightarrow(x-m)(y-n)=(x-n)(y-m)\Rightarrow xy-nx-my+mn=xy-mx-ny+mn\Rightarrow(m-n)x=(m-n)y\Rightarrow x=y$

So, *f* is one-one.

Surjectivity:

Let *y* be an element in the co domain *R,* such that

$f(x)=y\Rightarrow x\text{-}mx\text{-}n=y\Rightarrow x-m=xy-ny\Rightarrow ny-m=xy-x\Rightarrow ny-m=x(y-1)\Rightarrow x=ny\text{-}my\text{-}1$, which is not defined for $y=1$So, $1\in R$ (co domain) has no pre image in *R*-{*n*}

$\Rightarrow$*f* is not onto.

Thus, the answer is (b).

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Question 23:

Let $f: R\rightarrow R$ be a function defined by $f(x)=x2\text{-}8x2+2$. Then, *f* is

(a) one-one but not onto

(b) one-one and onto

(c) onto but not one-one

(d) neither one-one nor onto

ANSWER:

Injectivity:

Let x and y be two elements in the domain (R), such that

$f(x)=f(y)\Rightarrow x2-8x2+2=y2-8y2+2\Rightarrow(x2-8)(y2+2)=(x2+2)(y2-8)\Rightarrow x2y2+2x2-8y2-16=x2y2-8x2+2y2-16\Rightarrow 10x2=10y2\Rightarrow x2=y2\Rightarrow x=\pm y$

So, f is not one-one.

Surjectivity:

$f(-1)=(-1)2-8(-1)2+2=1-81+2=-73$ and $f(1)=(1)2-8(1)2+2=1-81+2=-73\Rightarrow f(-1)=f(1)=-73$

$\Rightarrow f$ is not onto.

The correct answer is (d).

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Question 24:

$f : R\rightarrow R$ is defined by $f(x)=ex2-e-x2ex2+e-x2$ is

(a) one-one but not onto

(b) many-one but onto

(c) one-one and onto

(d) neither one-one nor onto

ANSWER:

(d) neither one-one nor onto

We have,$f(x)=ex2-e-x2ex2+e-x2$Here, -2, 2$\in R$Now, 2≠-2But, $f(2)=f(-2)$Therefore, function is not one-one.And,The minimum value of the function is 0 and maximum value is 1That is range of the function is [0, 1] but the co-domain of the function is given R.Therefore, function is not onto.∴function is neither one-one nor onto.

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Question 25:

The function $f : R \to R$, $f(x)=x2$ is

(a) injective but not surjective

(b) surjective but not injective

(c) injective as well as surjective

(d) neither injective nor surjective

ANSWER:

Injectivity:

Let x and y be any two elements in the domain (R), such that $f(x) = f(y)$. Then,

$x2=y2 \Rightarrow x=\pm y$

So, f is not one-one.

Surjectivity:

As $f(-1)=(-1)2=1$and $f(1)=12=1$, $f(-1)=f(1)$

So, both -1 and 1 have the same images.

$\Rightarrow f$ is not onto.

So, the answer is (d).

x2+x+1=y2+y+1(x2−y2)+(x−y)=0(x+y)(x−y)+(x−y)=0(x−y)(x+y+1)=0x−y=0 (x+y+1) cannot be zero because x and y are natural numbers)x=y

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Question 26:

A function *f* from the set of natural numbers to the set of integers defined by

f(*n*){*n*-12,when *n* is odd-*n*2,when *n* is evenis

(a) neither one-one nor onto

(b) one-one but not onto

(c) onto but not one-one

(d) one-one and onto

ANSWER:

Injectivity:

Let *x* and *y* be any two elements in the domain (*N*).

Case-1: Both *x* and *y* are even.Let *f*(*x*)=*f*(*y*)⇒-*x*2=-*y*2⇒-*x*=-*y*⇒*x*=*y*Case-2: Both *x* and *y* are odd.Let *f*(*x*)=*f*(*y*)⇒*x*-12=*y*-12⇒*x*-1=*y*-1⇒*x*=*y*Case-3: Let *x* be even and *y* be odd. Then, *f*(*x*)=-*x*2and *f*(*y*)=*y*-12Then, clearly *x*≠*y* ⇒*f*(*x*)≠*f*(*y*)From all the cases, *f* is one-one.

Surjectivity:

Co-domain of *f*=*Z*={...,-3, -2, -1, 0, 1, 2, 3, ...}Range of *f*={..., -3-12, -(-2)2,-1-12, 02, 1-12, -22, 3-12, ...}⇒Range of *f*={...,-2, 1, -1, 0, 0, -1, 1,...}⇒Range of *f*={..., -2, -1, 0, 1, 2, ...}⇒Co-domain of *f*=Range of *f*

⇒ *f* is onto.

So, the answer is (d).

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Question 27:

Which of the following functions from $A=\{x \in R : -1 \le x \le 1\}$ to itself are bijections?

(a) $f(x)=|x|$

(b) $f(x)=\sin \pi x 2$

(c) $f(x)=\sin \pi x 4$

(d) None of these

ANSWER:

(b) $f(x)=\sin \pi x 2$

It is clear that $f(x)$ is one-one.

Range of $f=[\sin\pi(-1)2, \sin\pi(1)2]=[\sin -\pi 2, \sin\pi 2]=[-1,1]= A=$Co domain of f

$\Rightarrow$ f is onto.

So, f is a bijection.

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Question 28:

Let $f : Z\to Z$ be given by $f(x)=\{x2$, if x is even0, if x is odd

Then, f is

(a) onto but not one-one

(b) one-one but not onto

(c) one-one and onto

(d) neither one-one nor onto

ANSWER:

Injectivity:

Let x and y be two elements in the domain (Z), such that

$f(x)=f(y)$Case-1: Let both x and y be even.Then,$f(x)=f(y)\Rightarrow x2=y2\Rightarrow x=y$Case-2: Let both x and y be odd.Then,$f(x)=f(y)\Rightarrow 0=0$Here, we cannot determine whether $x=y$.

So, f is not one-one.

Surjectivity:

Let y be an element in the co-domain (Z), such that

Co-domain of $f=Z=\{0, \pm1, \pm2, \pm3, \pm4, ...\}$Range of $f=\{0, 0, \pm22, 0, \pm42, ...\}=\{0, \pm1, \pm2, ...\}\Rightarrow$Co-domain of f=Range of f

$\Rightarrow f$ is onto.

So, the answer is (a).

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Question 29:

The function $f: R\to R$ defined by $f(x)=6x+6|x|$ is

(a) one-one and onto

(b) many one and onto

(c) one-one and into

(d) many one and into

ANSWER:

(d) many one and into

Graph of the given function is as follows :

A line parallel to X axis is cutting the graph at two different values.

Therefore, for two different values of x we are getting the same value of y .

That means it is many one function .

From the given graph we can see that the range is $[2,\infty)$

and R is the codomain of the given function .

Hence, Codomain$\neq$Range

Therefore, the given function is into .

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Question 30:

Let $f(x)=x2$ and $g(x)=2x$. Then, the solution set of the equation $fog\ (x)=gof\ (x)$ is

(a) R

(b) {0}

(c) {0, 2}

(d) none of these

ANSWER:

Since $(fog)(x)=(gof)(x)$,
$f(g(x))=g(f(x))\Rightarrow f(2x)=g(x2)\Rightarrow(2x)2=2x2\Rightarrow 22x=2x2\Rightarrow x2=2x\Rightarrow x2-2x=0\Rightarrow x(x-2)=0\Rightarrow x=0,$
$2\Rightarrow x\in\{0, 2\}$

So, the answer is (c).

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Question 31:

If $f : R\rightarrow R$ is given by $f(x)=3x-5$, then $f\text{-}1(x)$

(a) is given by $13x-5$

(b) is given by $x+53$

(c) does not exist because f is not one-one

(d) does not exist because f is not onto

ANSWER:

Clearly, f is a bijection.

So, f^{-1} exists.

Let $f\text{-}1(x)=y$ ...$(1)\Rightarrow f(y)=x\Rightarrow 3y-5=x\Rightarrow 3y=x+5\Rightarrow y=x+53\Rightarrow f\text{-}1(x)=x+53$
[from (1)]

So, the answer is (b).

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Question 32:

If $g(f(x))=|\sin x|$ and $f(g(x))=(\sin \sqrt{x})2$, then

(a) $f(x)=\sin 2x$, $g(x)=\sqrt{x}$

(b) $f(x)=\sin x$, $g(x)=|x|$

(c) $f(x)=x2$, $g(x)=\sin \sqrt{x}$

(d) f and g cannot be determined.

ANSWER:

If we solve it by the trial-and-error method, we can see that (a) satisfies the given condition.

From (a):

$f(x)=\sin 2x$ and $g(x)=\sqrt{x} \Rightarrow f(g(x))=f(\sqrt{x})=\sin 2\sqrt{x}=(\sin \sqrt{x})2$

So, the answer is (a).

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Question 33:

The inverse of the function $f: R \to \{x \in R : x < 1\}$ given by $f(x)=ex\text{-}e\text{-}xex+e\text{-}x$ is

(a) 12 log $1+x1-x$

(b) 12 log $2+x2-x$

(c) 12 log $1-x1+x$

(d) none of these

ANSWER:

Let f-1(x)=y
...(1)⇒f(y)=x⇒ey-e-yey+e-y=x⇒e-y(e2y-1)e-y(e2y+1)=x⇒(e2y-1)=x(e2y+1)⇒e2y-1=xe2y+x⇒e2y(1-x)=x+1⇒e2y=1+x1-x⇒2y=loge (1+x1-x)⇒y=12loge (1+x1-x)⇒f-1(x)=12loge (1+x1-x)   [from (1)]

So, the answer is (a).

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Question 34:

Let A={x ∈ R : x ≥ 1}. The inverse of the function, f : A→A given by f(x)=2x (x-1), is

(a) (12)x (x-1)

(b) 12 {1+√1+4 log2 x}

(c) 12 {1-√1+4 log2 x}

(d) not defined

ANSWER:

Let f-1(x)=y...(1)⇒f(y)=x⇒2y(y-1)=x⇒2y2-y=x⇒y2-y=log2 x⇒y2-y+14=log2 x+14⇒(y-12)2=4log2 x+14⇒y-12=±√4log2 x+12⇒y=12±√4log2 x+12⇒y=12+√4log2 x+12   (∵ y ≥1)So, f-1(x)=12(1+√1+4log2 x)   [from (1)]

So, the answer is (b).

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Question 35:

Let A={x ∈ R : x ≤1} and f : A→A be defined as f(x)=x (2-x). Then, f-1 (x) is

(a) $1+\sqrt{1-x}$

(b) $1-\sqrt{1-x}$

(c) $\sqrt{1-x}$

(d) $1\pm\sqrt{1-x}$

ANSWER:

Let y be the element in the codomain R such that $f-1(x)=y$ ...(1)$\Rightarrow f(y)=x$ and y $\le 1 \Rightarrow y(2-y)=x \Rightarrow 2y-y2=x \Rightarrow y2-2y+x=0 \Rightarrow y2-2y=-x \Rightarrow y2-2y+1=1-x \Rightarrow (y-1)2=1-x \Rightarrow y-1=\pm\sqrt{1-x}$ $\Rightarrow y=1\pm\sqrt{1-x} \Rightarrow y=1-\sqrt{1-x}$ ($\because$ y ≤ 1)

The correct answer is (b).

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Question 36:

Let $f(x)=11-x$. Then, $\{f\ o\ (fof)\}\ (x)$

(a) x for all $x \in R$

(b) x for all $x \in R-\{1\}$

(c) x for all $x \in R-\{0, 1\}$

(d) none of these

ANSWER:

Domain of f:$1-x\neq 0 \Rightarrow x\neq 1$Domain of $f=R-\{1\}$Range of f:$y=11-x \Rightarrow 1-x=1y \Rightarrow x=1-1y \Rightarrow x=y-1y \Rightarrow y\neq 0$Range of $f=R-\{0\}$So, f:$R-\{1\}\rightarrow R-\{0\}$ and f:$R-\{1\}\rightarrow R-\{0\}$ Range of f is not a subset of the domain of f.Domain $(fof)=\{x$: $x\in$domain of f and $f(x)\in$domain of $f\}$Domain $(fof)=\{x$: $x\in R-\{1\}$ and $11-x\in R-\{1\}\}$ Domain $(fof)=\{x$: $x \neq 1$ and $11-x\neq 1\}$Domain $(fof)=\{x$: $x \neq 1$ and $1-x\neq 1\}$Domain $(fof)=\{x$: $x \neq 1$ and $x\neq 0\}$Domain $(fof)=R-\{0, 1\}(fof)(x)=f(f(x))=f(11-x)=11-11-x=1-x1-x-1=1-x-x=x-1x$For range of fof, $x\neq 0$Now, fof:$R-\{0, 1\}\rightarrow R$ $-\{0\}$ and f:$R-\{1\}\rightarrow R-\{0\}$Range of fof is not a subset of domain of f.Domain$(f\ o\ (fof))=\{x$: $x\in$domain of fof and $(fof)(x)\in$domain of $f\}$Domain $(f\ o\ (fof))=\{x$: $x\in R-\{0, 1\}$ and $x-1x\in R-\{1\}\}$ Domain$(f\ o\ (fof))=\{x$: $x \neq 0$, 1 and $x-1x\neq 1\}$Domain $(f\ o\ (fof))=\{x$: $x \neq 0$, 1 and $x-1\neq x\}$Domain $(f\ o\ (fof))=\{x$: $x \neq 0$, 1 and $x\in R\}$Domain $(f\ o$

$(fof))=R-\{0, 1\}(fo(fof))(x)=f((fof)(x))=f(x-1x)=11-x-1x=xx-x+1=x$So, $(fo(fof))(x)=x$, where $x\neq 0,1$

So, the answer is (c).

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Question 37:

If the function $f : R\to R$ be such that $f(x)=x-[x]$, where $[x]$ denotes the greatest integer less than or equal to x, then f-1 (x) is

(a) $1x-[x]$

(b) $[x] - x$

(c) not defined

(d) none of these

ANSWER:

$f(x) = x - [x]$

We know that the range of f is [0, 1).

Co-domain of $f = R$

As range of $f \neq$Co-domain of f, f is not onto.

⇒ f is not a bijective function.

So, f^{-1} does not exist.

Thus, the answer is (c).

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Question 38:

If F : [1, ∞)→[2, ∞) is given by $f(x)=x+1x$, then f-1 (x) equals

(a) $x+\sqrt{x2-42}$

(b) $x1+x2$

(c) $x-\sqrt{x2-42}$

(d) $1+\sqrt{x2-4}$

ANSWER:

Let f-1(x) = $y \Rightarrow f(y)=x \Rightarrow y+1y=x \Rightarrow y2+1=xy \Rightarrow y2-xy+1=0 \Rightarrow y2-2\times y\times x2+(x2)2-(x2)2+1=0 \Rightarrow y2-2\times y\times x2+(x2)2=x2-14 \Rightarrow (y-x2)2=x2-14 \Rightarrow y-x2=\sqrt{x2-42} \Rightarrow y=x2+\sqrt{x2-42} \Rightarrow y=x+\sqrt{x2-42} \Rightarrow f\text{-}1(x)=x+\sqrt{x2-42}$

So, the answer is (a).

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Question 39:

Let $g(x)=1+x-[x]$ and $f(x)=\{-1,x<00,x=0, 1,x>0$, where $[x]$ denotes the greatest integer less than or equal to x. Then for all x, $f(g(x))$ is equal to

(a) x

(b) 1

(c) $f(x)$

(d) $g(x)$

ANSWER:

(b) 1

When, $-1<x<0$Then, $g(x)=1+x-[x]$ $=1+x-(-1)=2+x$∵$f(g(x))=1$ When, $x=0$Then, $g(x)=1+x-[x]$ $=1+x-0=1+x$∵$f(g(x))=1$When, $x>1$Then, $g(x)=1+x-[x]$ $=1+x-1=x$∵$f(g(x))=1$

Therefore, for each interval $f(g(x))=1$

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Question 40:

Let $f(x)=\alpha\ xx+1$, $x\neq-1$. Then, for what value of α is $f\ (f(x))=x$?

(a) √2

(b) -√2

(c) 1

(d) −1

ANSWER:

(d) −1

$f(f(x))=x\Rightarrow$ $f(\alpha xx+1)=x\Rightarrow\alpha(\alpha xx+1)(\alpha xx+1)+1=x\Rightarrow\alpha 2x\alpha x+x+1=x\Rightarrow\alpha 2x=\alpha x2+x2+x\Rightarrow\alpha 2x-\alpha x2-x2-x=0\Rightarrow$ $\alpha 2x-\alpha x2-(x2+x)=0$Solving for the α we get,$\alpha=-(-x2)\pm\sqrt{}(-x2)2-4\times x\times[-(x2+x)]2x$ $=x2\pm\sqrt{}x4+4x3+4x22x$ $=x+1$, -1Here, -1 is independent of x,∴for, $\alpha=-1$, $f(f(x))=x$

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Question 41:

The distinct linear functions that map [−1, 1] onto [0, 2] are

(a) $f(x)=x+1$, $g(x)=-x+1$

(b) $f(x)=x-1$, $g(x)=x+1$

(c) $f(x)=-x-1$, $g(x)=x-1$

(d) None of these

ANSWER:

Let us substitute the end-points of the intervals in the given functions. Here, domain = [-1, 1] and range =[0, 2]

By substituting -1 or 1 in each option, we get:

Option (a):

$f(-1)=-1+1=0$ and $f(1)=1+1=2g(-1)=1+1=2$ and $g(1)=-1+1=0$

So, option (a) is correct.

Option (b):

$f(-1)=-1-1=-2$ and $f(1)=1-1=0g(-1)=-1+1=0$ and $g(1)=1+1=2$

Here, f(-1) gives $-2\notin[0, 2]$

So, (b) is not correct.

Similarly, we can see that (c) is also not correct.

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Question 42:

Let f : [2, ∞)→X be defined by $f(x)=4x-x2$. Then, f is invertible if X =

(a) [2, ∞)

(b) (-∞, 2]

(c) (-∞, 4]

(d) [4, ∞)

ANSWER:

Since f is invertible, range of f = co domain of f = X

So, we need to find the range of f to find X.

For finding the range, let

$f(x)=y\Rightarrow 4x-x2=y\Rightarrow x2-4x=-y\Rightarrow x2-4x+4=4-y\Rightarrow (x-2)2=4-y\Rightarrow x-2=\pm\sqrt{}4-y\Rightarrow x=2\pm\sqrt{}4-y$This is defined only when $4-y\geq 0\Rightarrow y\leq 4X$=Range of f=(-∞,4]

So, the answer is (c).

Page No 2.76:

Question 43:

If f : R→ (-1, 1) is defined by $f(x)=-x|x|1+x2$, then f-1 (x) equals

(a) $\sqrt{}|x|1-|x|$

(b) *Sgn* (x) $\sqrt{}|x|1-|x|$

(c) $-\sqrt{}x1-x$

(d) None of these

ANSWER:

(b) -Sgn (x) √|x|1-|x|

We have, f(x)=-x|x|1+x2 x∈(-1, 1)Case-(I)When, x<0,Then, |x|=-xAnd f(x)>0Now,f(x)=-x(-x)1+x2⇒y=x21+x2⇒y1=x21+x2⇒y+1y-1=x2+1+x2x2-1-x2 [Using Componendo and dividendo]⇒y+1y-1=2x2+1-1⇒-y+1y-1=2x2+1⇒2y1-y=2x2⇒y1-y=x2⇒x=-√y1-y (As x<0)⇒x=-√|y|1-|y| [As y>0]To find the inverse interchanging x and y we get,f-1(x)=-√|x|1-|x| ...(i)Case-(II)When, x>0,Then, |x|=xAnd f(x)<0Now,f(x)=-x(x)1+x2⇒y=-x21+x2⇒y1=-x21+x2⇒y+1y-1=-x2+1+x2-x2-1-x2 [Using Componendo and dividendo]⇒y+1y-1=1-2x2-1⇒1+y1-y=12x2+1⇒1-y1+y=2x2+1⇒-2y1+y=2x2⇒x2=-y1+y⇒x=√-y1+y (As x>0)⇒x=√|y|1-|y| [As y<0]To find the inverse interchanging x and y we get,f-1(x)=√|x|1-|x| ...(ii)Case-(III)When, x=0,Then, f(x)=0Hence, f-1(x)=0   ...(iii)Combinig equation (i) , (ii) and (iii) we get,f-1(x)=-Sgn(x)√|x|1-|x|

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Question 44:

Let [x] denote the greatest integer less than or equal to x. If f(x)=sin-1x, g(x)=[x2] and h(x)=2x,12≤x≤1√2, then

(a) $fogoh$(x)=π2

(b) $fogoh$(x)=π

(c) $hofog$=$hogof$

(d) $hofog$≠$hogof$

ANSWER:

(c) $hofog$=$hogof$

We have, $g(x)=[x^2] \quad =0 \quad (\text{As } \frac{1}{2}\le x\le \frac{1}{\sqrt{2}} \therefore \frac{1}{4}\le x^2\le \frac{1}{2}) fog(x)=f(g(x))=\sin^{-1}(0) =0 \; hofog(x)=h(f(g(x)))=2\times 0=0$

And $f(x)=\sin^{-1}x$ Now, for, $x\in[\frac{1}{2}, \frac{1}{\sqrt{2}}]$ $f(x)\in[\frac{\pi}{6}, \frac{\pi}{4}]$ $f(x)\in[0.52, 0.78]$ $gof(x)=0 \quad (\text{As, } f(x)\in[0.52, 0.78]) \quad =0$ $hogof(x)=h(g(f(x)))=2\times 0=0$

$\therefore hofog=hogof=0$

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Question 45:

If $g(x)=x^2+x-2$ and $\frac{1}{2} gof(x)=2x^2-5x+2$, then $f(x)$ is equal to

(a) $2x-3$

(b) $2x+3$

(c) $2x^2+3x+1$

(d) $2x^2-3x-1$

ANSWER:

We will solve this problem by the trial-and-error method.

Let us check option (a) first.

If $f(x) = 2x-3$ $\frac{1}{2}(gof)(x) = g(f(x))= \frac{1}{2}g(2x-3)= \frac{1}{2}[(2x-3)^2+(2x-3)-2]= \frac{1}{2}[4x^2+9-12x+2x-3-2]= \frac{1}{2}[4x^2-10x+4]= 2x^2-5x+2$

The given condition is satisfied by (a).

So, the answer is (a).

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Question 46:

If $f(x)$=sin2 x and the composite function $g(f(x))$=|sin x|, then $g(x)$ is equal to

(a) √x-1

(b) √x

(c) √x+1

(d) -√x

ANSWER:

(b)

If we take $g(x)$ = √x, then$g(f(x))$ = g(sin2x) = √sin2x = ±sin x = |sin x|

So, the answer is (b).

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Question 47:

If f : R→R is given by $f(x)$=x3+3, then f-1(x) is equal to

(a) x1/3-3

(b) x1/3+3

(c) (x-3)1/3

(d) x+31/3

ANSWER:

(c)

Let $f^{-1}(x) = y$ $f(y) = x \Rightarrow y^3+3 = x \Rightarrow y^3 = x-3 \Rightarrow y = \sqrt[3]{x-3} \Rightarrow y = (x-3)^{\frac{1}{3}}$

So, the answer is (c).

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Question 48:

Let $f(x)=x^3$ be a function with domain {0, 1, 2, 3}. Then domain of f^{-1} is

(a) {3, 2, 1, 0}

(b) {0, −1, −2, −3}

(c) {0, 1, 8, 27}

(d) {0, −1, −8, −27}

ANSWER:

(c) {0, 1, 8, 27}

$f(x)=x^3$ Domain = {0, 1, 2, 3} Range = $\{0^3, 1^3, 2^3, 3^3\}$ = {0, 1, 8, 27} So, f = {(0, 0), (1, 1), (2, 8), (3, 27)} f^{-1} = {(0, 0), (1, 1), (8, 2), (27, 3)} Domain of f^{-1} = {0, 1, 8, 27}

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Question 49:

Let $f : R \to R$ be given by $f(x)=x^2-3$. Then, f^{-1} is given by

(a) $\sqrt{x+3}$

(b) $\sqrt{x}+3$

(c) $x+\sqrt{3}$

(d) None of these

ANSWER:

(d)

Let $f^{-1}(x)=y$ $f(y)=x$ $y^2-3=x$ $y^2=x+3$ $y=\pm\sqrt{x+3}$

So, the answer is (d).

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Question 50:

Mark the correct alternative in the following question:

Let f : **R** → **R** be given by $f(x)$ = tan$x$. Then, $f^{-1}(1)$ is

(a) $\frac{\pi}{4}$ (b) $\{n\pi+\frac{\pi}{4}:n\in\mathbf{Z}\}$ (c) does not exist
(d) none of these

ANSWER:

We have, $f:\mathbf{R}\to\mathbf{R}$ is given by $f(x)=\tan x \Rightarrow f^{-1}(x)=\tan^{-1}x$ $\therefore$ $f^{-1}(1)=\tan^{-1}1=\{n\pi+\frac{\pi}{4}:n\in\mathbf{Z}\}$

Hence, the correct alternative is option (b).

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Question 51:

Mark the correct alternative in the following question:

Let f : **R** → **R** be defined as $f(x) = \begin{cases} 2x, & \text{if } x>3 \\ x^2, & \text{if } 1<x\le 3 \\ 3x, & \text{if } x\le 1 \end{cases}$

Then, find $f(-1) + f(2) + f(4)$

(a) 9 (b) 14 (c) 5 (d) none of these

ANSWER:

We have, $f(x)=\{2x$, if $x>3$ $x2$, if $1<x\le 33x$, if $x\le 1$
Now, $f(-1)+f(2)+f(4)=3(-1)+22+2(4)=-3+4+8=9$

Hence, the correct alternative is option (a).

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Question 52:

Mark the correct alternative in the following question:

Let $A = \{1, 2, ... , n\}$ and $B = \{a, b\}$. Then the number of subjections from A into B is

(a) ${}^nP_2$ (b) $2^n - 2$ (c) $2^n - 1$ (d) ${}^nC_2$

ANSWER:

As, the number of surjections from A to B is equal to the number of functions from A to B minus the number of functions from A to B whose images are proper subsets of B.

And, the number of functions from a set with n number of elements into a set with m number of elements = m^n

So, the number of subjections from A into B where $A = \{1, 2, \ldots, n\}$ and $B = \{a, b\}$ is $2^n - 2$. (As, two functions can be many-one into functions)

Hence, the correct alternative is option (b).

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Question 53:

Mark the correct alternative in the following question:

If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is

(a) 720 (b) 120 (c) 0 (d) none of these

ANSWER:

As, the number of bijection from A into B can only be possible when provided $n(A) \geq n(B)$But here $n(A) < n(B)$So, the number of bijection i.e. one-one and onto mappings from A to B=0

Hence, the correct alternative is option (c).

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Question 54:

Mark the correct alternative in the following question:

If the set A contains 7 elements and the set B contains 10 elements, then the number one-one functions from A to B is

(a) $^{10}C_7$ (b) $^{10}C_7 \times 7!$ (c) 7^{10}
(d)10^7

ANSWER:

As, the number of one-one functions from A to B with m and n elements, respectively = $^{n}P_{m} = {}^{n}C_{m} \times m!$

So, the number of one-one functions from A to B with 7 and 10 elements, respectively = $^{10}P_7 = {}^{10}C_7 \times 7!$

Hence, the correct alternative is option (b).

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Question 55:

Mark the correct alternative in the following question:

Let f : **R** - {35} → **R** be defined by $f(x)$ = 3x+25x-3. Then,

(a) $f^{-1}(x) = f(x)$ (b) $f^{-1}(x) = -f(x)$ (c) $fof(x) = -x$
(d) $f^{-1}(x) = 119f(x)$

ANSWER:

We have,

f : **R** - {35} → **R** is defined by $f(x)$ = 3x+25x-3

$fof(x)=f(f(x))=f(3x+25x-3)=3(3x+25x-3)+25(3x+25x-3)-3=(9x+65x-3)+2(15x+105x-3)-3=(9x+6+10x-65x-3)(15x+10-15x+95x-3)=19x19=x$Let $y=3x+25x-3\Rightarrow 5xy-3y=3x+2\Rightarrow 5xy-3x=3y+2\Rightarrow x(5y-3)=3y+2\Rightarrow x=3y+25y-3\Rightarrow f\text{-}1(y)=3y+25y-3$So, $f\text{-}1(x)=3x+25x-3=f(x)$

Hence, the correct alternative is option (a).

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Question 56:

Let $f : R \to R$ be defined by $f(x)=1x$. Then, f is

(a) one-one

(b) onto

(e) bijective

(d) not defined

ANSWER:

Given: The function $f : R \to R$ be defined by $f(x)=1x$.

To check f is one-one:

Let $f(x1)=f(x2)\Rightarrow 1x1=1x2\Rightarrow x1=x2$Hence, f is one-one.To check f is onto: Since, $y=1x\Rightarrow x=1y\Rightarrow y\in R\text{-}\{0\}\neq R$There is no pre-image of $y=0$.Hence, f is not onto.

Hence, the correct option is (a).

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Question 57:

Let *f* : *R* → *R* be defined by *f*(*x*) = 3x^2 – 5 and *g* : *R* → *R* by *g*(*x*)=*xx*2+1. Then (*gof*) (*x*) is

(a) 3*x*2−59*x*4−30*x*2+26

(b) 3*x*2−59*x*4−6*x*2+26

(c) 3*x*2*x*4+2*x*2−4

(d) 3*x*29*x*4+30*x*2−2

ANSWER:

Given: *f*(*x*) = 3x^2 – 5 and *g*(*x*)=*xx*2+1

(*gof*)(*x*)=*g*(*f*(*x*)) =*g*(3*x*2-5) =3*x*2-5(3*x*2-5)2+1
=3*x*2-5(3*x*2)2+52-2(3*x*2)(5)+1 =3*x*2-59*x*4+25-30*x*2+1 =3*x*2-59*x*4-30*x*2+26

Hence, the correct option is (a).

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Question 58:

Which of the following functions from *Z* to *Z* are bijections?

(a) $f(x) = x^3$

(b) $f(x) = x + 2$

(c) $f(x) = 2x + 1$

(d) $f(x) = x^2 + 1$

ANSWER:

Given: $f : Z \to Z$

(a) $f(x) = x^3$

It is one-one but not onto.

Thus, it is not bijective.

(b) $f(x) = x + 2$

It is one-one and onto.

Thus, it is bijective.

(c) $f(x) = 2x + 1$

It is one-one but not onto.

Thus, it is not bijective.

(d) $f(x) = x^2 + 1$

It is neither one-one nor onto.

Thus, it is not bijective.

Hence, the correct option is (b).

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Question 59:

Let $f : A \to B$ and $g : B \to C$ be the bijective functions. Then, $(gof)^{-1}$ =

(a) $f^{-1} o\ g^{-1}$

(b) fog

(c) $g^{-1} o f^{-1}$

(d) gof

ANSWER:

Given: $f : A \to B$ and $g : B \to C$ be the bijective functions

Since, $f: A \to B$ Thus, $f^{-1}: B \to A$ ...(1) Since, $g: B \to C$ Thus, $g^{-1}: C \to B$ ...(2) From (1) and (2), we get $f^{-1} o g^{-1}: C \to A$ ...(3) Also, $gof: A \to C \Rightarrow (gof)^{-1}: C \to A$ ...(4) Therefore, $(gof)^{-1} = f^{-1} o g^{-1}$

Hence, the correct option is (a).

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Question 60:

Let $f : N \to R$ be the function defined by $f(x)=2x-12$ and $g : Q \to R$ be another function defined by $g(x) = x + 2$. Then, (gof) (3/2) is

(a) 1

(b) 2

(c) 72

(d) none of these

ANSWER:

Given: $f(x)=2x-12$ and $g(x) = x + 2$

$(gof)(x)=g(f(x))$ $=g(2x\text{-}12)$ $=2x\text{-}12+2$ $=2x\text{-}1+42$ $=2x+32(gof)(32)=2(32)+32$ $=3+32$ $=62$ $=3$

Hence, the correct option is (d).

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Question 1:

The total number of functions from the set A = (1, 2, 3, 4 to the set B = a, b, c) is __________.

ANSWER:

Given: $f{:}A \rightarrow B$ where A={1, 2, 3, 4} and B={a, b, c}

Number of elements in A = 4

Number of elements in B = 3

Each element of A have 3 options to form an image.

Thus, Number of functions that can be formed = 3 × 3 × 3 × 3 = 81

Hence, the total number of functions from the set A = {1, 2, 3, 4} to the set B = {a, b, c} is 81.

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Question 2:

The total number of one-one functions from the set A = {a, b, c} to the set B = {x, y, z, t} is __________.

ANSWER:

Given: $f:A \to B$ where $A=\{a, b, c\}$ and $B=\{x, y, z, t\}$

Number of elements in A = 3

Number of elements in B = 4

To form a one-one function,

Element $a \in A$ have 4 options to form an image.

Element $b \in A$ have 3 options to form an image.

Element $c \in A$ have 2 options to form an image.

Thus, Number of one-one functions that can be formed = 4 × 3 × 2 = 24

Hence, the total number of one-one functions from the set A = {a, b, c} to the set $B = \{x, y, z, t\}$ is 24.

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Question 3:

The total number of onto functions from the set A = (1, 2, 3, 4, 5) to the set $B = \{x, y\}$ is __________.

ANSWER:

Given: $f:A \to B$ where $A=\{1, 2, 3, 4, 5\}$ and $B=\{x, y\}$

Number of elements in A = 5

Number of elements in B = 2

Each Element of A have 2 options to form an image.

Thus, Total number of functions that can be formed = 2 × 2 × 2 × 2 × 2 = 32

Number of functions having only one image i.e., $\{x\}$ = 1

Number of functions having only one image i.e., $\{y\}$ = 1

Thus, Number of onto functions that can be formed = 32 − 1 − 1 = 30

Hence, the total number of onto functions from the set $A = \{1, 2, 3, 4, 5\}$ to the set $B = \{x, y\}$ is 30.

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Question 4:

The domain of the real function $f(x)=\sqrt{16-x^2}$ is _________.

ANSWER:

Given: $f(x)=\sqrt{16-x^2}$

To find the domain, we find the real values of x for which the function is defined.

$16-x^2 \geq 0 \Rightarrow 16 \geq x^2 \Rightarrow x^2 \leq 16 \Rightarrow x \leq 4$ and $x \geq -4 \Rightarrow -4 \leq x \leq 4 \Rightarrow x \in [-4, 4]$

Hence, the domain of the real function $f(x)=\sqrt{16-x^2}$ is $[-4, 4]$.

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Question 5:

The domain of the real function $f(x)=\frac{x}{\sqrt{9-x^2}}$ is ___________.

ANSWER:

Given: $f(x)=\frac{x}{\sqrt{9-x^2}}$

To find the domain, we find the real values of x for which the function is defined.

$x \in R$ and $9-x^2>0 \Rightarrow x \in R$ and $9>x^2 \Rightarrow x \in R$ and $x^2<9 \Rightarrow x \in R$ and $-3<x<3 \Rightarrow -3<x<3 \Rightarrow x \in (-3, 3)$

Hence, the domain of the real function $f(x)=\frac{x}{\sqrt{9-x^2}}$ is $(-3, 3)$.

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Question 6:

The range of the function $f : R \rightarrow R$ given by $f(x)=x+\sqrt{x^2}$ is __________.

ANSWER:

Given: $f(x)=x+\sqrt{x^2}$

$f(x)=x+\sqrt{x2}$ $=x+|x|$ $=\{x+x$ $,x\geq0x-x$ $,x<0$ $=\{2x$ $,x\geq00$ $,x<0$

To find the range, we find the real values of y obtained.

$y=2x$ when $x\geq0\Rightarrow x=y2\geq0\Rightarrow y\geq0\Rightarrow y\in[0, \infty)$ ...(1)$y=0$ when $x<0$ ...(2)Thus, from (1) and (2),$y\in[0, \infty)$

Hence, the range of the function $f: R \to R$ given by $f(x)=x+\sqrt{x2}$ is $[0, \infty)$.

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Question 7:

The range of the function $f: R -\{-2\} \to R$ given by $f(x)=x+2|x+2|$ is __________.

ANSWER:

Given: $f(x)=x+2|x+2|$

$f(x)=x+2|x+2|$ $=\{x+2x+2$ $, x+2\geq0x+2\text{-}(x+2)$ $, x+2<0$ $=\{1$ $, x+2\geq0\text{-}1$ $, x+2<0$

To find the range, we find the real values of y obtained.

Hence, the range of the function $f: R -\{-2\} \to R$ given by $f(x)=x+2|x+2|$ is $\{-1, 1\}$.

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Question 8:

If $f : C \rightarrow C$ is defined by $f(x) = 8x^3$, then $f^{-1}(8)$ = . __________.

ANSWER:

Given: $f(x) = 8x^3$

$f(x)=8x3 \Rightarrow y=8x3 \Rightarrow x3=y8 \Rightarrow x=(y8)13$Thus, $f\text{-}1(x)=(x8)13f\text{-}1(8)=(88)13$ $=113$ $=1$, ω, $\omega 2$ $(\because f{:}C \rightarrow C)$where, ω is the cube root of unity.

Hence, if $f : C \rightarrow C$ is defined by $f(x) = 8x^3$, then $f^{-1}(8) = 1, \omega, \omega 2$.

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Question 9:

If $f : R \rightarrow R$ is defined by $f(x) = 8x^3$ then, $f^{-1}(8)$ = __________.

ANSWER:

Given: $f(x) = 8x^3$

$f(x)=8x3 \Rightarrow y=8x3 \Rightarrow x3=y8 \Rightarrow x=(y8)13$Thus, $f\text{-}1(x)=(x8)13f\text{-}1(8)=(88)13$ $=113$ $=1$ $(\because f{:}R \rightarrow R)$

Hence, if $f : R \rightarrow R$ is defined by $f(x) = 8x^3$ then $f^{-1}(8) = 1$.

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Question 10:

If $f : R - \{0\} \rightarrow R - \{0\}$ is defined as $f(x)=23x$, then $f^{-1}(x)$ = ____________.

ANSWER:

Given: A function $f : R - \{0\} \rightarrow R - \{0\}$ is defined as $f(x)=23x$

$f(x)=23x \Rightarrow y=23x \Rightarrow 3x=2y \Rightarrow x=23y$Thus, f-1$(x)=23x$

Hence, if $f : R - \{0\} \rightarrow R - \{0\}$ is defined as $f(x)$=23x, then $f^{-1}(x)$ = 23x.

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Question 11:

If $f : R \rightarrow R$ is defined by $f(x) = 6 - (x - 9)^3$, then $f^{-1}(x)$ = ___________.

ANSWER:

Given: A function $f : R \rightarrow R$ is defined by $f(x) = 6 - (x - 9)^3$

$f(x)$=6-(x-9)3⇒y=6-(x-9)3⇒(x-9)3=6-y⇒x-9=(6-y)13⇒x=9+(6-y)13Thus,
f-1(x)=9+(6-x)13

Hence, if $f : R \rightarrow R$ is defined by $f(x) = 6 - (x - 9)^3$, then $f^{-1}(x)$ = 9+(6-x)13.

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Question 12:

Let $A = \{1, 2, 3, 4\}$ and $f : A \rightarrow A$ be given by $f = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$. Then f^{-1} = ____________.

ANSWER:

Given: A function $f : A \rightarrow A$ be given by $f = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

f={(1, 4), (2, 3), (3, 2), (4, 1)}⇒f-1={(4, 1), (3, 2), (2, 3), (1, 4)}Thus, f-1={(4, 1), (3, 2), (2, 3), (1, 4)}

Hence, *f*-1={(4, 1), (3, 2), (2, 3), (1, 4)}.

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Question 13:

If *f* : *R* → *R* be defined by *f*(*x*) = $(2 – x^5)^{1/5}$, then *fof*(*x*) = ___________.

ANSWER:

Given: *f*(*x*) = $(2 – x^5)^{1/5}$

fof(*x*)=*f*(*f*(*x*))   =*f*((2-*x*5)15)   =[2-((2-*x*5)15)5]15   =[2-(2-*x*5)15×5]15
=[2-2+*x*5]15   =[*x*5]15   =*x*5×15   =*x*

Hence, if *f* : *R* → *R* be defined by *f*(*x*) = $(2 – x^5)^{1/5}$, then *fof*(*x*) = *x*.

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Question 14:

Let *A* = {1, 2, 3, 4, 5, ..., 10} and *f* : *A* → *A* be an invertible function. Then, ∑10*r*=1(*f*−1*of*)(*r*)=___________.

ANSWER:

Given: *f* : *A* → *A* is an invertible function, where *A* = {1, 2, 3, 4, 5, ..., 10}

Since, *f* is invertibleTherefore, *f*-1*of*(*x*)=*x*
...(1)Now,∑10*r*=1(*f*-1*of*)(*r*)=*f*-1*of*(1)+*f*-1*of*(2)+*f*-1*of*(3)+....+*f*-1*of*(10)
=1+2+3+....+10   (From (1))   =10(10+1)2   (∵
1+2+3+...+*n*=*n*(*n*+1)2)   =5(11)   =55

Hence, ∑10r=1(f−1of) (r)= 55.

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Question 15:

Let A = {1, 2, 3, 4, 5, 6) and B = (2, 4, 6, 8, 10, 12). If $f : A \to B$ is given by $f(x) = 2x$, then f^{-1} as set of ordered pairs, is ___________.

ANSWER:

Given: A function $f : A \to B$ defined as $f(x) = 2x$, where A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8, 10, 12}

Since, $f(x)$=2xTherefore,f={(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12)}Hence,f-1={(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6)}

Hence, if $f : A \to B$ is given by $f(x) = 2x$, then f^{-1} as set of ordered pairs, is {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6)}.

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Question 16:

Let f = {(0, –1), (–1, 3), (2, 3), (3, 5)} be a function from Z to Z defined by $f(x) = ax + b$. Then, (a, b) = ___________.

ANSWER:

Given: f = {(0, –1), (–1, –3), (2, 3), (3, 5)} is a function from Z to Z defined by $f(x) = ax + b$

f = {(0, –1), (–1, –3), (2, 3), (3, 5)} defined by $f(x) = ax + b$

$f(0)=-1 \Rightarrow a(0)+b=-1 \Rightarrow 0+b=-1 \Rightarrow b=-1$ ...(1) $f(2)=3 \Rightarrow a(2)+b=3 \Rightarrow 2a+b=3 \Rightarrow 2a-1=3$ (From (1)) $\Rightarrow 2a=3+1 \Rightarrow 2a=4 \Rightarrow a=2$ ...(2) Thus, $a=2$ and $b=-1$

Hence, $(a, b) = (2, –1)$.

Disclaimer: The function f must be equal to f = {(0, –1), (–1, –3), (2, 3), (3, 5)}.

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Question 17:

Let $f : R \to R$ and $g : R \to R$ be functions defined by $f(x) = 5 – x^2$ and $g(x) = 3x – 4$. Then the value of fog (–1) is ___________.

ANSWER:

Given: $f(x) = 5 – x^2$ and $g(x) = 3x – 4$

$fog(-1)=f(g(-1))$ $=f(3(-1)-4)$ $=f(-3-4)$ $=f(-7)$ $=5-(-7)2$ $=5-49$ $=-44$

Hence, the value of *fog* $(–1)$ is $–44$.

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Question 18:

Let f be the greatest integer function defined as $f(x) = [x]$ and g be the modules function defined as $g(x) = |x|$, then the value of $gof(−54)$ is ___________.

ANSWER:

Given: $f(x) = [x]$ and $g(x) = |x|$

$gof(-54)=g(f(-54))$ $=g([-54])$ $=g(-2)$ $=|-2|$ $=2$

Hence, the value of $gof(−54)$ is 2.

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Question 19:

If $f(x) = \cos [e]\, x + \cos [–e]\, x$, then $f(\pi)$ = ___________.

ANSWER:

Given: $f(x) = \cos[e]x + \cos[–e]x$

$f(x)=\cos[e]x+\cos[-e]x \qquad =\cos 2x+\cos(-3x) \qquad (\because e=2.718 \text{ approx})$
$=\cos 2x+\cos 3x \Rightarrow f(\pi)=\cos 2\pi+\cos 3\pi \qquad =1-1 \qquad =0$

Hence, $f(\pi) = 0$.

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Question 20:

Let A = {1, 2, 3} and B = {a, b} be two sets. Then the number of constant functions from A to B is ___________.

ANSWER:

Given: Sets A = {1, 2, 3} and B = {a, b}

Two constant functions can be formed from A to B.

i.e., one is $f(x) = a$ and other is $f(x) = b$

Hence, the number of constant functions from A to B is 2.

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Question 21:

If $f(x) = \cos [\pi^2]\, x + \cos [-\pi^2]\, x$, then $f(\pi 2)$=______________.

ANSWER:

Given: $f(x) = \cos [\pi^2]\, x + \cos [-\pi^2]\, x$

$f(x)=\cos[\pi 2]x+\cos[-\pi 2]x$ $=\cos 9x+\cos(-10x)$ $(\because \pi 2=9.85 \text{ approx})$ $=\cos 9x+\cos 10x \Rightarrow f(\pi 2)=\cos 9(\pi 2)+\cos 10(\pi 2)$ $=\cos 9\pi 2+\cos 5\pi$ $=0-1$ $=-1$

Hence, $f(\pi 2)= -1$.

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Question 22:

The number of onto functions from A = {a, b, c} to B = {1, 2, 3, 4} is ___________.

ANSWER:

Given: A function from A = {a, b, c} to B = {1, 2, 3, 4}

If a function from A to B is onto, then number of elements of $A \geq$ number of elements of B.

But here, number of elements of A < number of elements of B

Thus, no onto function exist from A = {a, b, c} to B = {1, 2, 3, 4}.

Hence, the number of onto functions from A = {a, b, c} to B = {1, 2, 3, 4} is 0.

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Question 23:

If $f(0, \infty) \to R$ is given by $f(x) = \log_{10} x$, then $f^{-1}(x) =$ ___________.

ANSWER:

Given: $f(x) = \log_{10}x$

$f(x)=\log 10x \Rightarrow y=\log 10x \Rightarrow x=10y$Thus, f-1(y)=10y.

Hence, $f^{-1}(x) = 10^x$.

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Question 24:

If $f : R^+ \to R$ is defined as $f(x) = \log_3 x$, then $f^{-1}(x) =$ ______________.

ANSWER:

Given: $f(x) = \log_3x$

$f(x)=\log 3x \Rightarrow y=\log 3x \Rightarrow x=3y$Thus, f-1(y)=3y.

Hence, $f^{-1}(x) = 3^x$.

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Question 25:

If $f : R \to R$, $g : R \to R$ are defined by $f(x) = 5x - 3$, $g(x) = x^2 + 3$, then $(gof^{-1})(3) =$ _______________.

ANSWER:

Given: $f(x) = 5x - 3$ and $g(x) = x^2 + 3$

$f(x)=5x-3 \Rightarrow y=5x-3 \Rightarrow 5x=y+3 \Rightarrow x=y+35$Thus, f-1$(y)=y+35$.Now,gof-1(3)=$g(f$-1(3))
$=g(3+35)$ $=g(65)$ $=(65)2+3$ $=3625+3$ $=36+7525$
$=11125$

Hence, $gof^{-1}(3)$ = 11125.

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Question 26:

If $f : R \to R$ is given by $f(x) = 2x + |x|$, then $f(2x) + f(-x) + 4x$ = ________________.

ANSWER:

Given: $f(x) = 2x + |x|$

$f(x)=2x+|x|f(2x)=2(2x)+|2x| \Rightarrow f(2x)=4x+2|x|$ ...(1)$f(-x)=2(-x)+|-x| \Rightarrow f(-x)=-2x+|x|$
...(2)Now,$f(2x)+f(-x)+4x=4x+2|x|-2x+|x|+4x$ $=6x+3|x|$
$=3(2x+|x|)$ $=3f(x)$

Hence, $f(2x) + f(-x) + 4x = 3f(x)$.

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Question 27:

If $f(x)=1-x1+x$, then $fof(\cos 2\theta)$ = _______________.

ANSWER:

Given: $f(x)=1-x1+x$

$fof(\cos2\theta)=f(f(\cos2\theta))$ $=f(1-\cos2\theta1+\cos2\theta)$ $=f(2\sin2\theta2\cos2\theta)$
$=f(\tan2\theta)$ $=1-\tan2\theta1+\tan2\theta$ $=\cos2\theta$

Hence, *fof*(cos 2θ) = cos2*θ*.

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Question 28:

Let $f(x)=xx-1$and$f(\alpha)f(\alpha+1)=f(\alpha k)$, then k = ______________.

ANSWER:

Given: $f(x)=xx-1$and$f(\alpha)f(\alpha+1)=f(\alpha k)$

$f(a)f(a+1)=aa\text{-}1a+1a+1\text{-}1$ $=aa\text{-}1a+1a$ $=a2(a\text{-}1)(a+1)$ $=a2a2\text{-}1$ ...(1) It is given that,$f(a)f(a+1)=f(ak)\Rightarrow a2a2\text{-}1=akak\text{-}1\Rightarrow k=2$

Hence, k = 2.

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Question 29:

If $f\,(f(x)) = x + 1$ for all $x \in R$ and if $f(0)=12$, then $f(1)$ = ____________.

ANSWER:

Given: $f\,(f(x)) = x + 1$ for all $x \in R$ and $f(0)=12$

$f(f(x))=x+1\Rightarrow f(f(0))=0+1\Rightarrow f(12)=1$ $(\because\ f(0)=12)$ ....(1)Now,$f(f(12))=12+1\Rightarrow f(1)=1+22$ $(\because\ f(12)=1)\Rightarrow f(1)=32$

Hence, $f(1)$ = 32.

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Question 30:

If $f(x) = 3x + 10$ and $g(x) = x^2 - 1$, then $(fog)^{-1}$ is equal to ___________.

ANSWER:

Given: $f(x) = 3x + 10$ and $g(x) = x^2 - 1$

fog(*x*)=*f*(*g*(*x*))   =*f*(*x*2-1)   =3(*x*2-1)+10   =3*x*2-3+10   =3*x*2+7Thus, *fog*(*x*)=3*x*2+7⇒*y*=3*x*2+7⇒3*x*2=*y*-7⇒*x*2=*y*-73⇒*x*=±√*y*-73*fog*-1(*x*)=±√*x*-73

Hence, $(fog)^{-1}$ is equal to ±√*x*-73.

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Question 31:

Let *f* = {(1, 2), (3, 5), (4, 1)) and *g* = {(2, 3), (5, 1), (1, 3)}. Then, *gof* = __________ and *fog* = __________.

ANSWER:

Given: *f* = {(1, 2), (3, 5), (4, 1)) and *g* = {(2, 3), (5, 1), (1, 3)}

fog(*x*)=*f*(*g*(*x*))*fog*(1)=*f*(*g*(1))   =*f*(3)   =5*fog*(2)=*f*(*g*(2))   =*f*(3)   =5*fog*(5)=*f*(*g*(5))   =*f*(1)   =2Hence, *fog*={(1, 5), (2, 5), (5, 2)}*gof*(*x*)=*g*(*f*(*x*))*gof*(1)=*g*(*f*(1))   =*g*(2)   =3*gof*(3)=*g*(*f*(3))   =*g*(5)   =1*gof*(4)=*g*(*f*(4))   =*g*(1)   =3Hence, *gof*={(1, 3), (3, 1), (4, 3)}.

Hence, *gof* = {(1, 3), (3, 1), (4, 3)} and *fog* = {(1, 5), (2, 5), (5, 2)}.

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Question 1:

Which one of the following graphs represents a function?

ANSWER:

In graph (b), 0 has more than one image, whereas every value of *x* in graph (a) has a unique image.

Thus, graph (a) represents a function.

So, the answer is (a).

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Question 2:

Which of the following graphs represents a one-one function?

ANSWER:

In the graph of (b), different elements on the *x*-axis have different images on the *y*-axis.But in (a), the graph cuts the *x*-axis at 3 points, which means that 3 points on the *x*-axis have the same image as 0 and hence, it is not one-one.

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Question 3:

If *A* = {1, 2, 3} and *B* = {*a*, *b*}, write the total number of functions from *A* to *B*.

ANSWER:

Formula:

If set *A* has *m* elements and set *B* has *n* elements, then the number of functions from *A* to *B* is *nm*.

Given:

$A = \{1, 2, 3\}$ and $B = \{a, b\}$

$\Rightarrow n(A) = 3$ and $n(B) = 2$

$\therefore$ Number of functions from A to $B = 2^3 = 8$

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Question 4:

If $A = \{a, b, c\}$ and $B = \{-2, -1, 0, 1, 2\}$, write the total number of one-one functions from A to B.

ANSWER:

Let $f: A \rightarrow B$ be a one-one function.

Then, f(a) can take 5 values, f(b) can take 4 values and f(c) can take 3 values.

Then, the number of one-one functions $= 5 \times 4 \times 3 = 60$

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Question 5:

Write the total number of one-one functions from set $A = \{1, 2, 3, 4\}$ to set $B = \{a, b, c\}$.

ANSWER:

A has 4 elements and B has 3 elements.

Also, one-one function is only possible from A to B if $n(A) \leq n(B)$.

But, here $n(A) > n(B)$.

So, the number of one-one functions from A to B is 0.

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Question 6:

If $f: R \to R$ is defined by $f(x) = x^2$, write f^{-1} (25).

ANSWER:

Let f-1(25)=x ...
(1)⇒f(x)=25⇒x2=25⇒x2-25=0⇒(x-5)(x+5)=0⇒x=±5⇒f-1(25)={-5, 5} [from (1)]

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Question 7:

If $f: C \to C$ is defined by $f(x) = x^2$, write f^{-1} (−4). Here, C denotes the set of all complex numbers.

ANSWER:

Let f-1(-4)=x ... (1)⇒f(x)=-4⇒x2=-4⇒x2+4=0⇒(x+2i)(x-2i)=0 [using the identity: a2+b2=(a-ib)(a+ib)]⇒x=±2i [as x∈C]⇒f-1(25)={-2i, 2i}
[from (1)]

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Question 8:

If $f: R \to R$ is given by $f(x) = x^3$, write f^{-1} (1).

ANSWER:

Let f-1(1)= x ... (1)⇒f(x)= 1⇒x3= 1⇒x3-1= 0⇒(x-1)(x2+x+1)= 0 [using the identity:a3-b3=(a-b)(a2+ab+b2)]⇒x=1 (as x∈R) ⇒f-1(1)= {1} [from (1)]

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Question 9:

Let C denote the set of all complex numbers. A function $f: C \to C$ is defined by $f(x) = x^3$. Write f^{-1} (1).

ANSWER:

Let f-1(1)=x ... (1)⇒f(x)=1⇒x3=1⇒x3-1=0⇒(x-1)(x2+x+1)=0 [Using identity: a3-b3=(a-b)(a2+ab+b2)]⇒(x-1)(x-ω)(x-ω2)=0, where ω=1±i√32⇒x=1, ω or ω2 (as x∈C)⇒f-1(1)={1, ω, ω2} [from (1)]

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Question 10:

Let f be a function from C (set of all complex numbers) to itself given by $f(x) = x^3$. Write f^{-1} (−1).

ANSWER:

Let f-1(-1)=x ... (1)⇒f(x)=-1⇒x3=-1⇒x3+1=0⇒(x+1)(x2-x+1)=0
[using the identity: a3+b3=(a+b)(a2-ab+b2)]⇒(x+1)(x+ω)(x+ω2)=0, where ω= 1±i√32
⇒x=-1, -ω, -ω2 (as x∈C)⇒f-1(-1)={-1, -ω, -ω2} [from (1)]

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Question 11:

If $f : R \to R$ be defined by $f(x) = x^4$, write f^{-1} (1).

ANSWER:

Let f-1(1)=x ... (1)⇒f(x)=1⇒x4=1⇒x4-1=0⇒(x2-1)(x2+1)=0 [using identity: a2-b2=(a-b)(a+b)]⇒(x-1)(x+1)(x2+1)=0 [using identity: a2-b2=(a-b)(a+b)]⇒x=±1 [as x∈R]⇒f-1(1)={-1, 1} [from (1)]

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Question 12:

If $f : C \to C$ is defined by $f(x) = x^4$, write f^{-1} (1).

ANSWER:

Let f-1(1)=x ... (1)⇒f(x)=1⇒x4=1⇒x4-1=0⇒(x2-1)(x2+1)=0
[using identity: a2-b2=(a-b)(a+b)]⇒(x-1)(x+1)(x-i)(x+i)=0, where i=√-1 [using identity: a2-b2=(a-b)(a+b)]⇒x=±1, ±i ⇒f-1(1)={-1, 1, i,-i} [from (1)]

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Question 13:

If $f : R \to R$ is defined by $f(x) = x^2$, find f^{-1} (−25).

ANSWER:

Let f-1(-25)=x⇒f(x)=-25⇒x2=-25We cannot find x∈R, such that x2=-25 (as x2≥0 for all x∈R)So, f-1(-25)=ϕ

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Question 14:

If f : $C \to C$ is defined by $f(x) = (x - 2)^3$, write f^{-1} (−1).

ANSWER:

Let f-1(-1)=x ... (1)⇒f(x)=-1⇒(x-2)3=-1⇒x-2=-1 or -ω or -ω2 (as the roots of (-1)13are -1, -ω and -ω2, where ω=1±i√32)⇒x=-1+2 or 2-ω or 2-ω2=1, 2-ω, 2-ω⇒f-1(-1)={1, 2-ω, 2-ω2} [from (1)]

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Question 15:

If f : $R \to R$ is defined by $f(x) = 10\,x - 7$, then write f^{-1} (x).

ANSWER:

Let f-1(x)=y ... (1)⇒f(y)=x⇒10y-7=x⇒10y=x+7⇒y=x+710⇒f-1(x)=x+710 (From (1))

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Question 16:

Let f : (-π2, π2)→R be a function defined by $f(x) = \cos\,[x]$. Write range (f).

ANSWER:

Domain =(-π2, π2)=(-1.57, 1.57) (as π=227)So, cos [x]=cos (-2)=cos 2 ∀x∈(-1.57, 0)Also, cos 0=1 for x=0And cos [x]=cos 1 ∀x∈(0, 1.57)∴Range={1, cos 1, cos 2}

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Question 17:

If $f : R \to R$ defined by $f(x) = 3x - 4$ is invertible, then write $f^{-1}(x)$.

ANSWER:

Let f-1(x)=y ... (1)⇒f(y)=x⇒3y-4=x⇒3y=x+4⇒y=x+43⇒f-1(x)=x+43 [from (1)]

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Question 18:

If $f : R \to R$, $g : R \to$ are given by $f(x) = (x + 1)^2$ and $g(x) = x^2 + 1$, then write the value of $fog\ (-3)$.

ANSWER:

(fog)(-3)=f (g (-3))=f((-3)2+1)=f(10)=(10+1)2=121

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Question 19:

Let $A = \{x \in R : -4 \leq x \leq 4 \text{ and } x \neq 0\}$ and $f : A \to R$ be defined by f(x)=|x|x. Write the range of f.

ANSWER:

∵ f(x)=|x|x=±xx=±1 ∀ x∈A, range of f={-1, 1}.

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Question 20:

Let f : [-π2, π2]→ A be defined by $f(x) = \sin x$. If f is a bijection, write set A.

ANSWER:

∵ f is a bijection,

co-domain of f = range of f

As $-1 \leq \sin x \leq 1$,

$-1 \leq y \leq 1$

So, A = [-1, 1]

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Question 21:

Let $f : R \to R^+$ be defined by $f(x) = a^x$, $a > 0$ and $a \neq 1$. Write $f^{-1}(x)$.

ANSWER:

Let f-1(x)=y ... (1)⇒f(y)=x⇒ay=x⇒y=log$a$ x⇒f-1(x)=log a x [from (1)]

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Question 22:

Let $f : R - \{-1\} \to R - \{1\}$ be given by f(x)=xx+1. Write f-1 (x).

ANSWER:

Let f-1(x)=y ...
(1)⇒f(y)=x⇒yy+1=x⇒y=xy+x⇒y-xy=x⇒y(1-x)=x⇒y=x1-x⇒f-1(x)=x1-x [from (1)]

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Question 23:

Let f : R-{-35}→R be a function defined as f(x)=2x5x+3.

Write f-1 : Range of f→R-{-35}.

ANSWER:

Let f-1(x)=y ...
(1)⇒f(y)=x⇒2y5y+3=x⇒2y=5xy+3x⇒2y-5xy=3x⇒y(2-5x)=3x⇒y=3x2-5x⇒f-1(x)=3x2-5
x [from (1)]

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Question 24:

Let $f : R \to R$, $g : R \to R$ be two functions defined by $f(x) = x^2 + x + 1$ and $g(x) = 1 - x^2$. Write $fog\ (-2)$.

ANSWER:

$(fog)(-2)=f\,(g\,(-2))=f(1-(-2)2)=f(-3)=(-3)2+(-3)+1=9-3+1=7$

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Question 25:

Let $f : R \to R$ be defined as $f(x)=2x-34$. Write fof-1 (1).

ANSWER:

Let f-1$(x)=y$
...(1)$\Rightarrow f(y)=x\Rightarrow 2y-34=x\Rightarrow 2y-3=4x\Rightarrow 2y=4x+3\Rightarrow y=4x+32\Rightarrow f-1(x)=4x+32$ [from (1)]$\Rightarrow f$-1$(x)=4x+32 \because (fof$-1$)(1)=f(4(1)+32)=f(72)=2(72)-34=7-34=44=1$

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Question 26:

Let f be an invertible real function. Write

(f-1 of) (1) +(f-1 of) (2)+...+(f-1of) (100).

ANSWER:

Given that f is an invertible real function.

f-1$o\ f=I$,where I is an identity function.So,(f-1$o\ f$)(1)+(f-1$o\ f$)(2)+...+(f-1o f)(100)=I(1)+I(2)+... +I(100)=1+2+...+100 (As $I(x)=x,\ \forall x\in R$)=100(100+1)2[Sum of first n natural numbers=$n(n+1)2$]=5050

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Question 27:

Let A = {1, 2, 3, 4} and B = {a, b} be two sets. Write the total number of onto functions from A to B.

ANSWER:

Formula:

When two sets *A* and *B* have *m* and *n* elements respectively, then the number of onto functions from *A* to *B* is

$\{\sum nr=1\ (-1)r\ nCr\ rm$, if $m\geq no$, if $m<n$

Here, number of elements in *A* = 4 = *m*

Number of elements in *B* = 2 = *n*

So, *m* > n

Number of onto functions

$=\sum 2r=1\ (-1)r\ 2Cr\ r4=(-1)1\ 2C1\ 14+(-1)2\ 2C2\ 24=-2+16=14$

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Question 28:

Write the domain of the real function $f(x)=\sqrt{}x-[x]$.

ANSWER:

[x] is the greatest integral function.

So, $0\leq x-[x]<1\Rightarrow\sqrt{}x-[x]$ exists for every $x\in R.\Rightarrow$Domain $=R$

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Question 29:

Write the domain of the real function $f(x)=\sqrt{}[x]-x$.

ANSWER:

[*x*] is the greatest integer function.

$[x] \le x,\ \forall x \in R \Rightarrow [x]-x \le 0, \forall x \in R \Rightarrow \sqrt{[x]-x}$ does not exist for any $x \in R$.Domain $=\phi$

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Question 30:

Write the domain of the real function $f(x)=1\sqrt{|x|-x}$.

ANSWER:

Case-1: When $x>0 |x|=x \Rightarrow 1\sqrt{|x|-x}=1\sqrt{x-x}=10=\infty$Case-2: When $x<0 |x|=-x \Rightarrow 1\sqrt{|x|-x}=1\sqrt{-x-x}=1\sqrt{-2x}$ (exists because when x<0, -2x>0)$\Rightarrow f(x)$ is defined when $x<0$So, domain $=(-\infty,0)$

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Question 31:

Write whether $f: R \to R$, given by $f(x)=x+\sqrt{x2}$, is one-one, many-one, onto or into.

ANSWER:

$f(x)=x+\sqrt{x2}=x \pm x=0$ *or* $2x$So, each element x in the domain may contain 2 images.For example,$f(0)=0+\sqrt{02}=0 f(-1)=-1+\sqrt{(-1)2}=-1+\sqrt{1}=-1+1=0$Here, the image of 0 and -1 is 0.

Hence, f is may-one.

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Question 32:

If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, write fog (7).

ANSWER:

$(fog)(7)=f(g(7))=f(7-7)=f(0)=0+7=7$

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Question 33:

What is the range of the function $f(x)=|x-1|x-1$?

ANSWER:

$f(x)=|x-1|x-1=\pm(x-1)x-1=\pm1$Range of $f=\{-1, 1\}$

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Question 34:

If $f: R \rightarrow R$ be defined by $f(x) = (3 - x^3)1/3$, then find $fof\,(x)$.

ANSWER:

$(fof)\,(x)=f\,(f\,(x))=f\,((3-x3)13)=[3-((3-x3)13)3]13=[3-(3-x3)]13=(x3)13=x$

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Question 35:

If $f: R \rightarrow R$ is defined by $f(x) = 3x + 2$, find $f\,(f\,(x))$.

ANSWER:

$f(f\,(x))=f\,(3x+2)=3\,(3x+2)+2=9x+6+2=9x+8$

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Question 36:

Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. State whether f is one-one or not.

ANSWER:

$f = \{(1, 4), (2, 5), (3, 6)\}$

Here, different elements of the domain have different images in the co-domain.

So, f is one-one.

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Question 37:

If f : {5, 6} → {2, 3} and g : {2, 3} → {5, 6} are given by f = {(5, 2), (6, 3)} and g = {(2, 5), (3, 6)}, then find fog. [NCERT EXEMPLAR]

ANSWER:

We have,

f : {5, 6} → {2, 3} and g : {2, 3} → {5, 6} are given by f = {(5, 2), (6, 3)} and g = {(2, 5), (3, 6)}

As,

$fog(2) = f(g(2)) = f(5) = 2$,

$fog(3) = f(g(3)) = f(6) = 3$,

So,

fog : {2, 3} → {2, 3} is defined as

fog = {(2, 2), (3, 3)}

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Question 38:

Let f : **R** → **R** be the function defined by $f(x) = 4x - 3$ for all $x \in$ **R**. Then write f^{-1}. [NCERT EXEMPLAR]

ANSWER:

We have,

f : **R** → **R** is the function defined by $f(x) = 4x - 3$ for all $x \in$ **R**

Let $f(x)=y$. Then,$y=4x-3 \Rightarrow 4x=y+3 \Rightarrow x=y+34$So, f-1(y)=y+34or, f-1(x)=x+34

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Question 39:

Which one the following relations on A = {1, 2, 3} is a function?

f = {(1, 3), (2, 3), (3, 2)}, g = {(1, 2), (1, 3), (3, 1)}
[NCERT EXEMPLAR]

ANSWER:

As, each element of the domain set has unique image in the relation f = {(1, 3), (2, 3), (3, 2)}

So, f is a function.

Also, the element 1 of the domain set has two images 2 and 3 of the range set in the relation g = {(1, 2), (1, 3), (3, 1)}

So, g is not a function.

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Question 40:

Write the domain of the real function f defined by $f(x) = \sqrt{25-x^2}$.
[NCERT EXEMPLAR]

ANSWER:

We have, $f(x)=\sqrt{25-x^2}$ The function is defined only when $25-x^2\geq0 \Rightarrow x^2-25\leq0 \Rightarrow (x+5)(x-5)\leq0 \Rightarrow x\in[-5, 5]$ So, the domain of the given function is [-5, 5].

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Question 41:

Let $A = \{a, b, c, d\}$ and $f : A \to A$ be given by $f = \{(a, b), (b, d), (c, a), (d, c)\}$. Write f^{-1}. [NCERT EXEMPLAR]

ANSWER:

We have,

$A = \{a, b, c, d\}$ and $f : A \to A$ be given by $f = \{(a, b), (b, d), (c, a), (d, c)\}$

Since, the elements of a function when interchanged gives inverse function.

So, $f^{-1} = \{(b, a), (d, b), (a, c), (c, d)\}$

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Question 42:

Let $f, g : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 2x + l$ and $g(x) = x^2 - 2$ for all $x \in \mathbf{R}$, respectively. Then, find gof. [NCERT EXEMPLAR]

ANSWER:

We have,

$f, g : \mathbf{R} \to \mathbf{R}$ are defined by $f(x) = 2x + l$ and $g(x) = x^2 - 2$ for all $x \in \mathbf{R}$, respectively

Now,$gof(x)=g(f(x))=g(2x+1)=(2x+1)2-2=4x2+4x+1-2=4x2+4x-1$

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Question 43:

If the mapping $f : \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 3\}$, given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, then write *fog*.
[NCERT EXEMPLAR]

ANSWER:

We have,

$f : \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 3\}$, are given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, respectively

As,

$fog(2)=f(g(2))=f(3)=5, fog(5)=f(g(5))=f(1)=2, fog(1)=f(g(1))=f(3)=5$, So, $fog : \{1,2,5\}\rightarrow\{1,2,5\}$ is given by $fog=\{(2,5),(5,2),(1,5)\}$

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Question 44:

If a function $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is described by $g(x) = \alpha x+\beta$, then find the values of α and β. [NCERT EXEMPLAR]

ANSWER:

We have,

A function $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is described by $g(x) = \alpha x+\beta$

As, $g(1)=1$ and $g(2)=3$ So, $\alpha(1)+\beta=1\Rightarrow\alpha+\beta=1$ .....(i) and $\alpha(2)+\beta=3\Rightarrow2\alpha+\beta=3$ .....(ii) (ii)-(i), we get $2\alpha-\alpha=2\Rightarrow\alpha=2$ Substituting $\alpha=2$ in (i), we get $2+\beta=1\Rightarrow\beta=-1$

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Question 45:

If $f(x) = 4 - (x - 7)^3$, then write $f^{-1}(x)$.
[NCERT EXEMPLAR]

ANSWER:

We have, $f(x)=4-(x-7)^3$ Let $y=4-(x-7)^3 \Rightarrow (x-7)^3=4-y \Rightarrow x-7=\sqrt[3]{4-y} \Rightarrow x=7+\sqrt[3]{4-y} \Rightarrow f^{-1}(y)=7+\sqrt[3]{4-y}$ $\therefore f^{-1}(x)=7+\sqrt[3]{4-x}$

Chapterwise RD Sharma Solutions for Class 12 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.