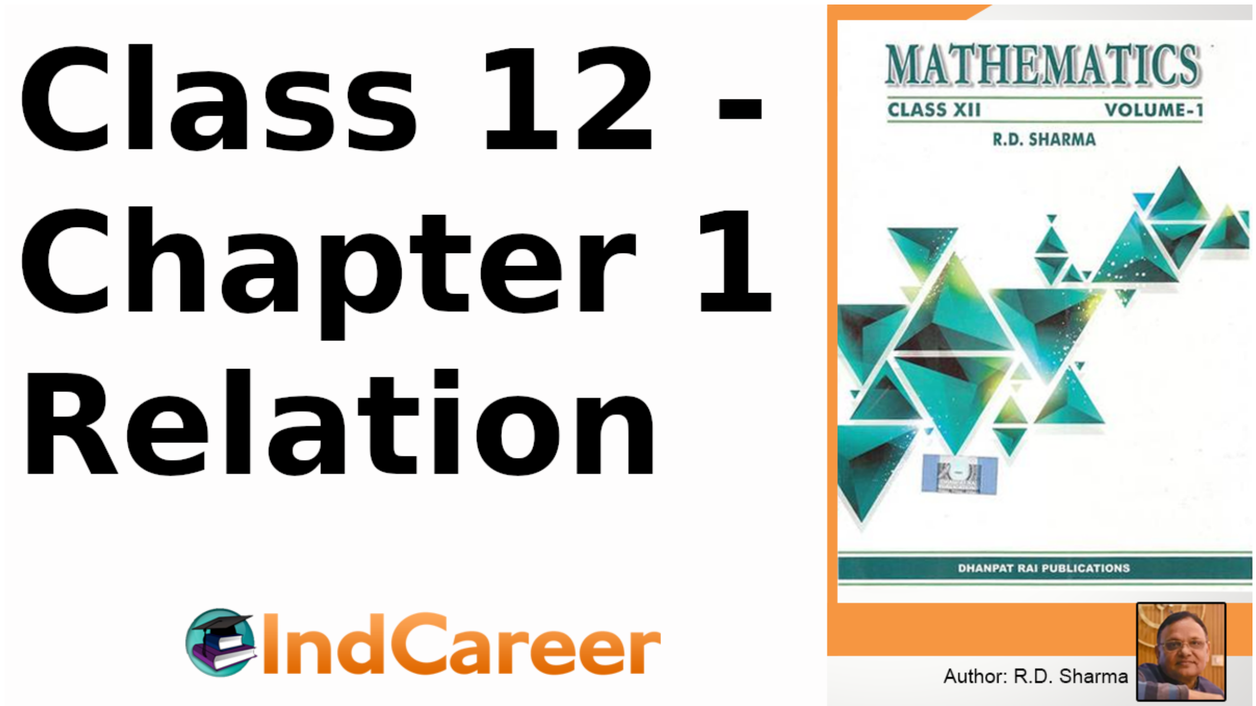

RD Sharma Solutions for Class 12 Maths Chapter 1–Relation

Class 12: Maths Chapter 1 solutions. Complete Class 12 Maths Chapter 1 Notes.

RD Sharma Solutions for Class 12 Maths Chapter 1–Relation

RD Sharma 12th Maths Chapter 1, Class 12 Maths Chapter 1 solutions

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Question 1:

Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) R = {(x, y) : x and y work at the same place}

(ii) R = {(x, y) : x and y live in the same locality}

(iii) R = {(x, y) : x is wife of y}

(iv) R = {(x, y) : x is father of and y}

ANSWER:

(i) Reflexivity:

Let x be an arbitrary element of R. Then,x∈R ⇒x and x work at the same place is true since they are the same.⇒(x, x)∈RSo, R is a reflexive relation.Let x be an arbitrary element of R. Then,x∈R ⇒x and x work at the same place is true since they are the same.⇒x, x∈RSo, R is a reflexive relation.

Symmetry:

Let (x, y)∈R⇒x and y work at the same place ⇒y and x work at the same place⇒(y, x)∈RSo, R is a symmetric relation.Let x, y∈R⇒x and y work at the same place ⇒y and x work at the same place⇒y, x∈RSo, R is a symmetric relation.

Transitivity:

Let (x, y)∈R and (y, z)∈R. Then,x and y work at the same place.y and z also work at the same place.⇒x , y and z all work at the same place.⇒x and z work at the same place.⇒(x, z)∈RSo, R is a transitive relation.Let x, y∈R and y, z∈R. Then,x and y work at the same place.y and z also work at the same place.⇒x , y and z all work at the same place.⇒x and z work at the same place.⇒x, z∈RSo, R is a transitive relation.

(ii) Reflexivity:

Let x be an arbitrary element of R. Then,x∈R ⇒x and x live in the same locality is true since they are the same.So, R is a reflexive relation.Let x be an arbitrary element of R. Then,x∈R ⇒x and x live in the same locality is true since they are the same.So, R is a reflexive relation.

Symmetry:

Let (x, y)∈R⇒x and y live in the same locality⇒y and x live in the same locality⇒(y, x)∈R So, R is a symmetric relation.Let x, y∈R⇒x and y live in the same locality⇒y and x live in the same locality⇒y, x∈R So, R is a symmetric relation.

Transitivity:

Let (x, y)∈R and (y, z)∈R. Then,x and y live in the same locality and y and z live in the same locality⇒x, y and z all live in the same locality⇒x and z live in the same locality ⇒(x, z) ∈RSo, R is a transitive relation.Let x, y∈R and y, z∈R. Then,x and y live in the same locality and y and z live in the same locality⇒x, y and z all live in the same locality⇒x and z live in the same locality ⇒x, z ∈RSo, R is a transitive relation.

(iii)

Reflexivity:

Let x be an element of R. Then,x is wife of x cannot be true.⇒(x, x)∉RSo, R is not a reflexive relation.Let x be an element of R. Then,x is wife of x cannot be true.⇒x, x∉RSo, R is not a reflexive relation.

Symmetry:

Let (x, y)∈R⇒x is wife of y ⇒x is female and y is male⇒y cannot be wife of x as y is husband of x⇒(y, x)∉R So, R is not a symmetric relation.Let x, y∈R⇒x is wife of y ⇒x is female and y is male⇒y cannot be wife of x as y is husband of x⇒y, x∉R So, R is not a symmetric relation.

Transitivity:

Let (x, y)∈R, but (y, z)∉RSince x is wife of y, but y cannot be the wife of z, y is husband of x.⇒x is not the wife of z⇒(x, z)∈RSo, R is a transitive relation.Let x, y∈R, but y, z∉RSince x is wife of y, but y cannot be the wife of z, y is husband of x.⇒x is not the wife of z⇒x, z∈RSo, R is a transitive relation.

(iv)

Reflexivity:

Let x be an arbitrary element of R. Then,x is father of x cannot be true since no one can be father of himself.So, R is not a reflexive relation.Let x be an arbitrary element of R. Then,x is father of x cannot be true since no one can be father of himself.So, R is not a reflexive relation.

Symmetry:

Let (x, y)∈R⇒x is father of y⇒y is son/daughter of x⇒(y, x)∉R So, R is not a symmetric relation.Let x, y∈R⇒x is father of y⇒y is son/daughter of x⇒y, x∉R So, R is not a symmetric relation.

Transitivity:

Let (x, y)∈R and (y, z)∈R. Then, x is father of y and y is father of z⇒x is grandfather of z⇒(x, z)∉RSo, R is not a transitive relation.Let x, y∈R and y, z∈R. Then, x is father of y and y is father of z⇒x is grandfather of z⇒x, z∉RSo, R is not a transitive relation.

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Question 2:

Three relations R_1, R_2 and R_3 are defined on a set $A = \{a, b, c\}$ as follows:

$R_1 = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, a), (c, b), (c, c)\}$

$R_2 = \{(a, a)\}$

$R_3 = \{(b, c)\}$

$R_4 = \{(a, b), (b, c), (c, a)\}$.

Find whether or not each of the relations R_1, R_2, R_3, R_4 on A is (i) reflexive (ii) symmetric and (iii) transitive.

ANSWER:

(i) R_1

Reflexive:

Clearly, (a, a), (b, b) and (c, c)∈∈R_1

So, R_1 is reflexive.

Symmetric:

We see that the ordered pairs obtained by interchanging the components of R_1 are also in R_1.

So, R_1 is symmetric.

Transitive:

Here,

(a, b)∈R1, (b, c)∈R1 and also (a, c)∈R1a, b∈R1, b, c∈R1 and also a, c∈R1

So, R_1 is transitive.

(ii) R_2

Reflexive: Clearly (a,a)∈R2a,a∈R2. So, R_2 is reflexive.

Symmetric: Clearly (a,a)∈R⇒(a,a)∈Ra,a∈R⇒a,a∈R. So, R_2 is symmetric.

Transitive: R_2 is clearly a transitive relation, since there is only one element in it.

(iii) R_3

Reflexive:

Here,

(b, b)∉R3 neither (c, c)∉R3b, b∉R3 neither c, c∉R3

So, R_3 is not reflexive.

Symmetric:

Here,

(b, c)∈R3, but (c,b)∉R3So, R3 is not symmetric.b, c∈R3, but c,b∉R3So, R3 is not symmetric.

Transitive:

Here, R_3 has only two elements. Hence, R_3 is transitive.

(iv) R_4

Reflexive:

Here,

(a, a)∉R4, (b, b)∉ R4 (c, c)∉ R4So, R4 is not reflexive.a, a∉R4, b, b∉ R4 c, c∉ R4So, R4 is not reflexive.

Symmetric:

Here,

(a, b)∈R4, but (b,a)∉R4.So, R4 is not symmetric.a, b∈R4, but b,a∉R4.So, R4 is not symmetric.

Transitive:

Here,

(a, b)∈R4, (b, c)∈R4, but (a, c)∉R4So, R4 is not transitive.a, b∈R4, b, c∈R4, but a, c∉R4So, R4 is not transitive.

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Question 3:

Test whether the following relations R_1, R_2, and R_3 are (i) reflexive (ii) symmetric and (iii) transitive:

(i) R_1 on Q_0 defined by $(a, b) \in R_1 \Leftrightarrow a = 1/b$.

(ii) R_2 on Z defined by $(a, b) \in R_2 \Leftrightarrow |a - b| \leq 5$

(iii) R_3 on R defined by $(a, b) \in R_3 \Leftrightarrow a^2 - 4ab + 3b^2 = 0$.

ANSWER:

(i) Reflexivity:

Let a be an arbitrary element of R_1. Then, a∈R1⇒a≠1a for all a∈Q0So, R1 is not reflexive.a∈R1⇒a≠1a for all a∈Q0So, R1 is not reflexive.

Symmetry:

Let (*a*, *b*) ∈R1∈R1. Then,

(a, b)∈R1⇒a=1b⇒b=1a⇒(b, a)∈R1So, R1 is symmetric.a, b∈R1⇒a=1b⇒b=1a⇒b, a∈R1So, R1 is symmetric.

Transitivity:

Here,

(a, b)∈R1 and (b, c)∈R2⇒a=1b and b=1c⇒a=11c=c⇒a≠1c⇒(a, c)∉R1 So, R1 is not transitive.a, b∈R1 and b, c∈R2⇒a=1b and b=1c⇒a=11c=c⇒a≠1c⇒a, c∉R1 So, R1 is not transitive.

(ii)

Reflexivity:

Let *a* be an arbitrary element of R_2. Then,

a∈R2 ⇒|a−a|=0≤5So, R1 is reflexive.   a∈R2 ⇒a-a=0≤5So, R1 is reflexive.

Symmetry:

Let (a, b)∈R2⇒|a−b|≤5⇒|b−a|≤5   [Since, |a−b| = |b−a|]⇒(b, a)∈R2So, R2 is symmetric.Let a, b∈R2⇒a-b≤5⇒b-a≤5   Since, a-b = b-a⇒b, a∈R2So, R2 is symmetric.

Transitivity:

Let (1, 3)∈R2 and (3, 7)∈R2⇒|1−3|≤5 and |3−7|≤5But |1−7|≰5 ⇒(1,7)∉R2So, R2 is not transitive.Let 1, 3∈R2 and 3, 7∈R2⇒1-3≤5 and 3-7≤5But 1-7≰5 ⇒1,7∉R2So, R2 is not transitive.

(iii)

Reflexivity: Let *a* be an arbitrary element of R_3. Then,

a∈R3⇒a2−4a×a+3a2=0 So, R3 is reflexive. a∈R3⇒a2-4a×a+3a2=0 So, R3 is reflexive.

Symmetry:

Let (a, b)∈R3⇒a2−4ab+3b2=0But b2−4ba+3a2≠0 for all a, b ∈RSo, R3 is not symmetric.Let a, b∈R3⇒a2-4ab+3b2=0But b2-4ba+3a2≠0 for all a, b ∈RSo, R3 is not symmetric.

Transitivity:

(1, 2)∈R3 and (2, 3)∈R3⇒1−8+6=0 and 4−24+27=0But 1−12+9≠0So, R3 is not transitive.1, 2∈R3 and 2, 3∈R3⇒1-8+6=0 and 4-24+27=0But 1-12+9≠0So, R3 is not transitive.

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Question 4:

Let *A* = {1, 2, 3}, and let R_1 = {(1, 1), (1, 3), (3, 1), (2, 2), (2, 1), (3, 3)}, R_2 = {(2, 2), (3, 1), (1, 3)}, R_3 = {(1, 3), (3, 3)}. Find whether or not each of the relations R_1, R_2, R_3 on A is (i) reflexive (ii) symmetric (iii) transitive.

ANSWER:

(1)1 R_1

Reflexivity:

Here,

(1, 1), (2, 2), (3, 3)∈RSo, R1 is reflexive.1, 1, 2, 2, 3, 3∈RSo, R1 is reflexive.

Symmetry:

Here,(2, 1)∈R1, but (1, 2)∉R1 So, R1 is not symmetric.Here,2, 1∈R1, but 1, 2∉R1 So, R1 is not symmetric.

Transitivity:

Here, (2, 1)∈R1 and (1, 3)∈R1, but (2, 3)∉R1So, R1 is not transitive.Here, 2, 1∈R1 and 1, 3∈R1, but 2, 3∉R1So, R1 is not transitive.

(2)2 *R_2*

Reflexivity:

Clearly, (1, 1) and (3, 3)∉R2 So, R2 is not reflexive.Clearly, 1, 1 and 3, 3∉R2 So, R2 is not reflexive.

Symmetry:

Here, (1, 3)∈R2 and (3, 1)∈R2So, R2 is symmetric.Here, 1, 3∈R2 and 3, 1∈R2So, R2 is symmetric.

Transitivity:

Here, (1, 3)∈R2 and (3, 1)∈R2 But (3, 3)∉R2So, R2 is not transitive.Here, 1, 3∈R2 and 3, 1∈R2 But 3, 3∉R2So, R2 is not transitive.

(3)3 R_3

Reflexivity:

Clearly, (1, 1)∉R3 So, R3 is not reflexive.Clearly, 1, 1∉R3 So, R3 is not reflexive.

Symmetry:

Here, (1, 3)∈R3, but (3, 1)∉R3So, R3 is not symmetric.Here, 1, 3∈R3, but 3, 1∉R3So, R3 is not symmetric.

Transitivity:

Here, (1, 3)∈R3 and (3, 3)∈R3 Also, (1, 3)∈R3So, R3 is transitive.Here, 1, 3∈R3 and 3, 3∈R3 Also, 1, 3∈R3So, R3 is transitive.

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Question 5:

The following relations are defined on the set of real numbers.

(i) aRb if $a – b > 0$

(ii) aRb if $1 + ab > 0$

(iii) aRb if $|a| \leq b$

Find whether these relations are reflexive, symmetric or transitive.

ANSWER:

(i)

Reflexivity: Let *a* be an arbitrary element of *R*. Then,

a∈RBut a−a = 0 ≯0So, this relation is not reflexive. a∈RBut a-a = 0 ≯0So, this relation is not reflexive.

Symmetry:

Let (a, b)∈R⇒a−b>0⇒−(b−a)>0⇒b−a<0So, the given relation is not symmetric. Let a, b∈R⇒a-b>0⇒-(b-a)>0⇒b-a<0So, the given relation is not symmetric.

Transitivity:

Let (a, b)∈R and (b, c)∈R. Then,a−b>0 and b−c>0Adding the two, we geta−b+b−c>0⇒a−c>0 ⇒(a, c)∈R. So, the given relation is transitive. Let a, b∈R and b, c∈R. Then,a-b>0 and b-c>0Adding the two, we geta-b+b-c>0⇒a-c>0 ⇒a, c∈R. So, the given relation is transitive.

(ii)

Reflexivity: Let *a* be an arbitrary element of *R*. Then, a∈R⇒1+a×a>0i.e. 1+a2>0 [Since, square of any number is positive]So, the given relation is reflexive. a∈R⇒1+a×a>0i.e. 1+a2>0 Since, square of any number is positiveSo, the given relation is reflexive.

Symmetry:

Let (a, b)∈R⇒1+ab>0⇒1+ba>0⇒(b, a)∈RSo, the given relation is symmetric. Let a, b∈R⇒1+ab>0⇒1+ba>0⇒b, a∈RSo, the given relation is symmetric.

Transitivity:

Let (a, b)∈R and (b, c)∈R⇒1+ab>0 and 1+bc>0But 1+ac≯0⇒(a, c)∉RSo, the given relation is not transitive. Let a, b∈R and b, c∈R⇒1+ab>0 and 1+bc>0But 1+ac≯0⇒a, c∉RSo, the given relation is not transitive.

(iii)

Reflexivity: Let *a* be an arbitrary element of *R*. Then,

a∈R ⇒|a|≮a [Since, |a|=a]So, R is not reflexive. a∈R ⇒a≮a Since, a=aSo, R is not reflexive.

Symmetry:

Let (a, b)∈R⇒|a|≤b ⇒ |b|≮a for all a, b∈R⇒(b, a)∉R So, R is not symmetric. Let a, b∈R⇒a≤b ⇒ b≮a for all a, b∈R⇒b, a∉R So, R is not symmetric.

Transitivity:

Let (a, b)∈R and (b, c)∈R⇒|a|≤b and |b|≤cMultiplying the corresponding sides, we get |a| |b|≤bc⇒|a|≤c⇒(a, c)∈RThus, R is transitive. Let a, b∈R and b, c∈R⇒a≤b and b≤cMultiplying the corresponding sides, we get a b≤bc⇒a≤c⇒a, c∈RThus, R is transitive.

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Question 6:

Check whether the relation *R* defined on the set *A* = {1, 2, 3, 4, 5, 6} as *R* = {(*a, b*) : *b* = *a* + 1} is reflexive, symmetric or transitive.

ANSWER:

Reflexivity:

Letabeanarbitraryelementof R.Then,a=a+1 cannot be true for all a∈A.⇒(a, a)∉R So, R is not reflexive on A.Letabeanarbitraryelementof R.Then,a=a+1 cannot be true for all a∈A.⇒a, a∉R So, R is not reflexive on A.

Symmetry:

Let (a, b)∈R⇒b=a+1⇒−a=−b+1⇒a=b−1Thus, (b, a)∉RSo, R is not symmetric on A.Let a, b∈R⇒b=a+1⇒-a=-b+1⇒a=b-1Thus, b, a∉RSo, R is not symmetric on A.

Transitivity:

Let (1, 2) and (2, 3)∈R⇒2=1+1 and 3 2+1 is true.But 3 ≠ 1+1⇒(1, 3)∉RSo, R is not transitive on A.Let 1, 2 and 2, 3∈R⇒2=1+1 and 3 2+1 is true.But 3 ≠ 1+1⇒1, 3∉RSo, R is not transitive on A.

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Question 7:

Check whether the relation *R* on **R** defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

ANSWER:

Reflexivity:

Since 12>(12)3,(12, 12)∉RSo, R is not reflexive.Since 12>123,12, 12∉RSo, R is not reflexive.

Symmetry:

Since (12, 2)∈R,12<23But 2>(12)3⇒(2, 12)∈RSo, R is not symmetric.Since 12, 2∈R,12<23But 2>123⇒2, 12∈RSo, R is not symmetric.

Transitivity:

Since (7, 3)∈R and (3, 313)∈R,7<33 and 3=(313)3But 7>(313)3⇒(7, 313)∉RSo, R is not transitive.Since 7, 3∈R and 3, 313∈R,7<33 and 3=3133But 7>3133⇒7, 313∉RSo, R is not transitive.

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Question 8:

Prove that every identity relation on a set is reflexive, but the converse is not necessarily true.

ANSWER:

Let *A* be a set. Then,

Identity relation IA=IA is reflexive, since (a, a)∈A∀aIdentity relation IA=IA is reflexive, since a, a∈A∀a

The converse of it need not be necessarily true.

Consider the set *A* = {1, 2, 3}

Here,

Relation *R* = {(1, 1), (2, 2) , (3, 3), (2, 1), (1, 3)} is reflexive on *A.*

However, *R* is not an identity relation.

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Question 9:

If *A* = {1, 2, 3, 4} define relations on *A* which have properties of being

(i) reflexive, transitive but not symmetric

(ii) symmetric but neither reflexive nor transitive

(iii) reflexive, symmetric and transitive.

ANSWER:

(i) The relation on *A* having properties of being reflexive, transitive, but not symmetric is

R = {(1, 1), (2, 2), (3, 3), (4, 4), (2, 1)}

Relation R satisfies reflexivity and transitivity.⇒(1, 1), (2, 2), (3, 3)∈R and (1, 1), (2, 1)∈R ⇒(1, 1)∈RHowever, (2, 1)∈R, but (1, 2)∉RRelation R satisfies reflexivity and transitivity.⇒1, 1, 2, 2, 3, 3∈R and 1, 1, 2, 1∈R ⇒1, 1∈RHowever, 2, 1∈R, but 1, 2∉R

(ii) The relation on A having properties of being symmetric, but neither reflexive nor transitive is

R = {(1, 2), (2, 1)}

The relation R on A is neither reflexive nor transitive, but symmetric.

(iii) The relation on A having properties of being symmetric, reflexive and transitive is

R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)}

The relation R is an equivalence relation on A.

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Question 10:

Let R be a relation defined on the set of natural numbers N as

$R = \{(x, y) : x, y \in N, 2x + y = 41\}$

Find the domain and range of R. Also, verify whether R is (i) reflexive, (ii) symmetric (iii) transitive.

ANSWER:

Domain of R is the values of x and range of R is the values of y that together should satisfy $2x+y = 41$.

So,

Domain of R = {1, 2, 3, 4, ... , 20}

Range of R = {1, 3, 5, ... , 37, 39}

Reflexivity: Let x be an arbitrary element of R. Then,

x∈R⇒2x+x=41 cannot be true.⇒(x, x)∉R So, R is not reflexive.x∈R⇒2x+x=41 cannot be true.⇒x, x∉R So, R is not reflexive.

Symmetry:

Let (x, y)∈R. Then, 2x+y=41⇒ 2y+x = 41 ⇒(y, x)∉RSo, R is not symmetric.Let x, y∈R. Then, 2x+y=41⇒ 2y+x = 41 ⇒y, x∉RSo, R is not symmetric.

Transitivity:

Let (x, y) and (y, z)∈R⇒2x+y=41 and 2y +z=41⇒2x+z=2x+41−2y 41−y−2y=41−3y⇒(x, z)∉RThus, R is not transitive.Let x, y and y, z∈R⇒2x+y=41 and 2y +z=41⇒2x+z=2x+41-2y 41-y-2y=41-3y⇒x, z∉RThus, R is not transitive.

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Question 11:

Is it true that every relation which is symmetric and transitive is also reflexive? Give reasons.

ANSWER:

No, it is not true.

Consider a set A = {1, 2, 3} and relation R on A such that R = {(1, 2), (2, 1), (2, 3), (1, 3)}

The relation *R* on *A* is symmetric and transitive. However, it is not reflexive.

(1, 1), (2, 2) and (3, 3)∉ R1, 1, 2, 2 and 3, 3∉ R

Hence, *R* is not reflexive.

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Question 12:

An integer m is said to be related to another integer *n* if m is a multiple of *n*.Check if the relation is symmetric, reflexive and transitive.

ANSWER:

R={(m, n) : m, n∈Z, m=kn, where k∈N}Reflexivity:Let m be an arbitrary element of R. Then,m=km is true for k=1⇒(m, m)∈RThus, R is reflexive.Symmetry: Let (m, n)∈R⇒m=kn for some k∈N→n=1km⇒(n, m)∉R Thus, R is not symmetric.Transitivity: Let (m, n) and (n, o)∈R⇒m=kn and n=lo for some k, l ∈N⇒m=(kl) oHere, kl∈R⇒(m, o)∈RThus, R is transitive.R=m, n : m, n∈Z, m=kn, where k∈NReflexivity:Let m be an arbitrary element of R. Then,m=km is true for k=1⇒m, m∈RThus, R is reflexive.Symmetry: Let m, n∈R⇒m=kn for some k∈N→n=1km⇒n, m∉R Thus, R is not symmetric.Transitivity: Let m, n and n, o∈R⇒m=kn and n=lo for some k, l ∈N⇒m=(kl) oHere, kl∈R⇒m, o∈RThus, R is transitive.

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Question 13:

Show that the relation '≥' on the set *R* of all real numbers is reflexive and transitive but not symmetric.

ANSWER:

Let *R* be the set such that *R* = {(*a*, *b*) : *a*, *b*∈R; a≥b∈R; a≥b}

Reflexivity:

Let a be an arbitrary element of R. ⇒a∈R⇒a=a ⇒a≥a is true for a=a⇒(a, a)∈R Hence, R is reflexive.Let a be an arbitrary element of R. ⇒a∈R⇒a=a ⇒a≥a is true for a=a⇒a, a∈R Hence, R is reflexive.

Symmetry:

Let (a, b)∈R⇒a≥b is same as b≤a, but not b≥aThus, (b, a)∉R Hence, R is not symmetric.Let a, b∈R⇒a≥b is same as b≤a, but not b≥aThus, b, a∉R Hence, R is not symmetric.

Transitivity:

Let (a, b) and (b, c)∈R⇒a≥b and b≥c⇒a≥b≥c⇒a≥c⇒(a, c)∈RHence, R is transitive.Let a, b and b, c∈R⇒a≥b and b≥c⇒a≥b≥c⇒a≥c⇒a, c∈RHence, R is transitive.

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Question 14:

Give an example of a relation which is

(i) reflexive and symmetric but not transitive;

(ii) reflexive and transitive but not symmetric;

(iii) symmetric and transitive but not reflexive;

(iv) symmetric but neither reflexive nor transitive.

(v) transitive but neither reflexive nor symmetric.

ANSWER:

Suppose *A* be the set such that *A* = {1, 2, 3}

(i) Let *R* be the relation on *A* such that

R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3)}

Thus,

R is reflexive and symmetric, but not transitive.

(ii) Let *R* be the relation on *A* such that

R = {(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 3)}

Clearly, the relation *R* on *A* is reflexive and transitive, but not symmetric.

(iii) Let *R* be the relation on *A* such that

R = {(1, 2), (2, 1), (1, 3), (3, 1), (2, 3)}

We see that the relation *R* on *A* is symmetric and transitive, but not reflexive.

(iv) Let *R* be the relation on *A* such that

R = {(1, 2), (2, 1), (1, 3), (3, 1)}

The relation *R* on *A* is symmetric, but neither reflexive nor transitive.

(v) Let *R* be the relation on *A* such that

R = {(1, 2), (2, 3), (1, 3)}

The relation *R* on *A* is transitive, but neither symmetric nor reflexive.

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Question 15:

Given the relation R = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, add a minimum number of ordered pairs so that the enlarged relation is symmeteric, transitive and reflexive.

ANSWER:

We have,

R = {(1, 2), (2, 3)}

R can be a transitive only when the elements (1, 3) is added

R can be a reflexive only when the elements (1, 1), (2, 2), (3, 3) are added

R can be a symmetric only when the elements (2, 1), (3, 1) and (3, 2) are added

So, the required enlarged relation, R' = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} = A ×× A

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Question 16:

Let A = {1, 2, 3} and R = {(1, 2), (1, 1), (2, 3)} be a relation on A. What minimum number of ordered pairs may be added to R so that it may become a transitive relation on A.

ANSWER:

We have,

A = {1, 2, 3} and R = {(1, 2), (1, 1), (2, 3)}

To make R a transitive relation on A, (1, 3) must be added to it.

So, the minimum number of ordered pairs that may be added to R to make it a transitive relation is 1.

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Question 17:

Let A = {a, b, c} and the relation R be defined on A as follows: R = {(a, a), (b, c), (a, b)}. Then, write minimum number of ordered pairs to be added in R to make it reflexive and transitive.

ANSWER:

We have,

A = {a, b, c} and R = {(a, a), (b, c), (a, b)}

R can be a reflexive relation only when elements (b, b) and (c, c) are added to it

R can be a transitive relation only when the element (a, c) is added to it

So, the minmum number of ordered pairs to be added in R is 3.

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Question 18:

Each of the following defines a relation on **N**:

(i) $x > y$, $x, y \in\in$ **N**

(ii) $x + y = 10$, $x, y \in\in$ **N**

(iii) xy is square of an integer, $x, y \in\in$ **N**

(iv) $x + 4y = 10$, $x, y \in\in$ **N**

Determine which of the above relations are reflexive, symmetric and transitive.

ANSWER:

(i) We have,

$R = \{(x, y) : x > y, x, y \in\in \mathbf{N}\}$

As, x=x ∀x∈N⇒(x,x)∉RSo, R is not a reflexive relationLet (x,y)∈R⇒x>ybut y<x⇒(y,x)∉RSo, R is not a symmeteric relationLet (x,y)∈R and (y,z)∈R⇒x>y and y>z⇒x>z⇒(x,z)∈RSo, R is a transitive relationAs, x=x ∀x∈N⇒x,x∉RSo, R is not a reflexive relationLet x,y∈R⇒x>ybut y<x⇒y,x∉RSo, R is not a symmeteric relationLet x,y∈R and y,z∈R⇒x>y and y>z⇒x>z⇒x,z∈RSo, R is a transitive relation

(ii) We have,

$R = \{(x, y) : x + y = 10, x, y \in\in \mathbf{N}\}$

R={(1,9),(2,8),(3,7),(4,6),(5,5),(6,4),(7,3),(8,2),(9,1)}As, (1,1)∉RSo, R is not a reflexive relationLet (x,y)∈R⇒x+y=10⇒y+x=10⇒(y,x)∈RSo, R is a symmeteric relationAs, (1,9)∈R and (9,1)∈R but (1,1)∉RSo, R is not a transitive relationR=1,9,2,8,3,7,4,6,5,5,6,4,7,3,8,2,9,1As, 1,1∉RSo, R is not a reflexive relationLet x,y∈R⇒x+y=10⇒y+x=10⇒y,x∈RSo, R is a symmeteric relationAs, 1,9∈R and 9,1∈R but 1,1∉RSo, R is not a transitive relation

(iii) We have,

R = {(x, y) : xy is square of an integer, x, y ∈∈ **N**}

As, x×x=x2, which is a square of an integer x⇒(x,x)∈RSo, R is a reflexive relationLet (x,y)∈R⇒xy is square of an integer⇒yx is also a square of an integer⇒(y,x)∈RSo, R is a symmeteric relationLet (x,y)∈R and (y,z)∈R⇒xy is square of an integer and yz is also a square of an interger⇒xz must be a square of an integer⇒(x,z)∈RSo, R is a transitive relationAs, x×x=x2, which is a square of an integer x⇒x,x∈RSo, R is a reflexive relationLet x,y∈R⇒xy is square of an integer⇒yx is also a square of an integer⇒y,x∈RSo, R is a symmeteric relationLet x,y∈R and y,z∈R⇒xy is square of an integer and yz is also a square of an interger⇒xz must be a square of an integer⇒x,z∈RSo, R is a transitive relation

(iv) We have,

R = {(x, y) : x + 4y = 10, x, y ∈∈ **N**}

R={(2,4),(6,1)}As, (2,2)∉RSo, R is not a reflexive relationAs, (2,4)∈R but (4,2)∉RSo, R is not a symmeteric relationAs, (2, 4)∈R but 4 is not related to any natural numberSo, R is a transitive relationR=2,4,6,1As, 2,2∉RSo, R is not a reflexive relationAs, 2,4∈R but 4,2∉RSo, R is not a symmeteric relationAs, 2, 4∈R but 4 is not related to any natural numberSo, R is a transitive relation

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Question 1:

Show that the relation *R* defined by *R* = {(*a, b*) : *a* – *b* is divisible by 3; *a, b* ∈ *Z*} is an equivalence relation.

ANSWER:

We observe the following relations of relation *R*.

Reflexivity:

Let a be an arbitrary element of R. Then,a−a=0=0 × 3⇒a−a is divisible by 3⇒(a, a)∈R for all a∈ZSo, R is reflexive on Z. Let a be an arbitrary element of R. Then,a-a=0=0 × 3⇒a-a is divisible by 3⇒a, a∈R for all a∈ZSo, R is reflexive on Z.

Symmetry:

Let (a, b)∈R⇒a−b is divisible by 3⇒a−b 3p for some p∈Z⇒b−a=3 (−p) Here, −p∈Z⇒b−a is divisible by 3⇒(b, a)∈R for all a, b∈ZSo, R is symmetric on Z.Let a, b∈R⇒a-b is divisible by 3⇒a-b 3p for some p∈Z⇒b-a=3 -p Here, -p∈Z⇒b-a is divisible by 3⇒b, a∈R for all a, b∈ZSo, R is symmetric on Z.

Transitivity:

Let (a, b) and (b, c)∈R⇒a−b and b−c are divisible by 3⇒a−b=3p for some p∈Zand b−c=3q for some q∈ZAdding the above two, we get   a−b+b−c=3p+3q⇒a−c=3 (p+q)Here, p+q∈Z⇒a−c is divisible by 3⇒(a, c)∈R for all a, c ∈ZSo, R is transitive on Z.Let a, b and b, c∈R⇒a-b and b-c are divisible by 3⇒a-b=3p for some p∈Zand b-c=3q for some q∈ZAdding the above two, we get a-b+b-c=3p+3q⇒a-c=3 p+qHere, p+q∈Z⇒a-c is divisible by 3⇒a, c∈R for all a, c ∈ZSo, R is transitive on Z.

Hence, *R* is an equivalence relation on *Z*.

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Question 2:

Show that the relation *R* on the set *Z* of integers, given by

R = {(*a, b*) : 2 divides *a* – *b*}, is an equivalence relation.

ANSWER:

We observe the following properties of relation *R.*

Reflexivity:

Let a be an arbitrary element of the set Z. Then,a∈R⇒a−a=0=0 × 2⇒2 divides a−a⇒(a, a)∈R for all a∈ZSo, R is reflexive on Z.Let a be an arbitrary element of the set Z. Then,a∈R⇒a-a=0=0 × 2⇒2 divides a-a⇒a, a∈R for all a∈ZSo, R is reflexive on Z.

Symmetry:

Let (a, b)∈R⇒2 divides a−b⇒a−b2=p for some p∈Z⇒b−a2=−p Here, −p∈Z⇒2 divides b−a⇒(b, a)∈R for all a, b ∈ZSo, R is symmetric on Z.Let a, b∈R⇒2 divides a-b⇒a-b2=p for some p∈Z⇒b-a2=-p Here, -p∈Z⇒2 divides b-a⇒b, a∈R for all a, b ∈ZSo, R is symmetric on Z.

Transitivity:

Let (a, b) and (b, c)∈R⇒2 divides a−b and 2 divides b−c⇒a−b2=p and b−c2=q for some p, q∈ZAdding the above two, we geta−b2+b−c2=p+q⇒a−c2=p+q   Here, p+q∈Z⇒2 divides a−c⇒(a, c)∈R for all a, c ∈ZSo, R is transitive on Z.Let a, b and b, c∈R⇒2 divides a-b and 2 divides b-c⇒a-b2=p and b-c2=q for some p, q∈ZAdding the above two, we geta-b2+b-c2=p+q⇒a-c2=p+q   Here, p+q∈Z⇒2 divides a-c⇒a, c∈R for all a, c ∈ZSo, R is transitive on Z.

Hence, *R* is an equivalence relation on Z.

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Question 3:

Prove that the relation *R* on *Z* defined by

(*a*, *b*) ∈ *R* ⇔ *a* − *b* is divisible by 5

is an equivalence relation on *Z*.

ANSWER:

We observe the following properties of relation *R*.

Reflexivity:

Let a be an arbitrary element of R. Then,⇒a−a = 0 = 0 × 5⇒a−a is divisible by 5⇒(a, a)∈R for all a∈ZSo, R is reflexive on Z.Let a be an arbitrary element of R. Then,⇒a-a = 0 = 0 × 5⇒a-a is divisible by 5⇒a, a∈R for all a∈ZSo, R is reflexive on Z.

Symmetry:

Let (a, b)∈R⇒a−b is divisible by 5⇒a−b = 5p for some p∈Z⇒b−a = 5 (−p)   Here, −p∈Z   [Since p∈Z]⇒b−a is divisible by 5⇒(b, a)∈R for all a, b∈ZSo, R is symmetric on Z.Let a, b∈R⇒a-b is divisible by 5⇒a-b = 5p for some p∈Z⇒b-a = 5 -p

Here, -p∈Z [Since p∈Z]⇒b-a is divisible by 5⇒b, a∈R for all a, b∈ZSo, R is symmetric on Z.

Transitivity:

Let (a, b) and (b, c)∈R⇒a−b is divisible by 5⇒a−b = 5p for some ZAlso, b−c is divisible by 5⇒b−c = 5q for some ZAdding the above two, we geta−b+b−c = 5p+5q⇒a−c = 5 (p+q)⇒a−c is divisible by 5Here, p+q∈Z⇒(a, c)∈R for all a, c∈ZSo, R is transitive on Z.Let a, b and b, c∈R⇒a-b is divisible by 5⇒a-b = 5p for some ZAlso, b-c is divisible by 5⇒b-c = 5q for some ZAdding the above two, we geta-b+b-c = 5p+5q⇒a-c = 5 (p+q)⇒a-c is divisible by 5Here, p+q∈Z⇒a, c∈R for all a, c∈ZSo, R is transitive on Z.

Hence, *R* is an equivalence relation on *Z*.

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Question 4:

Let *n* be a fixed positive integer. Define a relation *R* on *Z* as follows:

$(a, b) \in R \Leftrightarrow a - b$ is divisible by *n*.

Show that *R* is an equivalence relation on *Z*.

ANSWER:

We observe the following properties of *R.* Then,

Reflexivity:

Let a∈NHere,a−a=0=0 × n⇒a−a is divisible by n⇒(a, a)∈R⇒(a, a)∈R for all a∈ZSo, R is reflexive on Z.Let a∈NHere,a-a=0=0 × n⇒a-a is divisible by n⇒a, a∈R⇒a, a∈R for all a∈ZSo, R is reflexive on Z.

Symmetry:

Let (a, b)∈RHere,a−b is divisible by n⇒a−b=np for some p∈Z⇒b−a=n (−p)⇒b−a is divisible by n [p∈Z⇒−p∈Z]⇒(b, a)∈R So, R is symmetric on Z.Let a, b∈RHere,a-b is divisible by n⇒a-b=np for some p∈Z⇒b-a=n -p⇒b-a is divisible by n [p∈Z⇒-p∈Z]⇒b, a∈R So, R is symmetric on Z.

Transitivity:

Let (a, b) and (b, c)∈RHere, a−b is divisible by n and b−c is divisible by n.⇒a−b=np for some p∈Zand b−c=nq for some q∈ZAdding the above two, we geta−b+b−c=np+nq⇒a−c=n (p+q)Here, p+q∈Z⇒(a, c)∈R for all a, c∈ZSo, R is transitive on Z.Let a, b and b, c∈RHere, a-b is divisible by n and b-c is divisible by n.⇒a-b=np for some p∈Zand b-c=nq for some q∈ZAdding the above two, we geta-b+b-c=np+nq⇒a-c=n (p+q)Here, p+q∈Z⇒a, c∈R for all a, c∈ZSo, R is transitive on Z.

Hence, *R* is an equivalence relation on *Z*.

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Question 5:

Let *Z* be the set of integers. Show that the relation

$R = \{(a, b) : a, b \in Z \text{ and } a + b \text{ is even}\}$

is an equivalence relation on *Z*.

ANSWER:

We observe the following properties of *R.*

Reflexivity:

Let a be an arbitrary element of Z. Then, a∈RClearly, a+a=2a is even for all a∈Z.⇒(a, a)∈R for all a∈ZSo, R is reflexive on Z.Let a be an arbitrary element of Z. Then, a∈RClearly, a+a=2a is even for all a∈Z.⇒a, a∈R for all a∈ZSo, R is reflexive on Z.

Symmetry:

Let (a, b)∈R⇒a+b is even⇒b+a is even⇒(b, a)∈R for all a, b∈ZSo, R is symmetric on Z.Let a, b∈R⇒a+b is even⇒b+a is even⇒b, a∈R for all a, b∈ZSo, R is symmetric on Z.

Transitivity:

Let (a, b) and (b, c)∈R⇒a+b and b+c are evenNow, let a+b=2x for some x∈Zand b+c=2y for some y∈ZAdding the above two, we get a+2b+c=2x+2y⇒a+c=2(x+y−b), which is even for all x, y, b∈ZThus, (a, c)∈RSo, R is transitive on Z.Let a, b and b, c∈R⇒a+b and b+c are evenNow, let a+b=2x for some x∈Zand b+c=2y for some y∈ZAdding the above two, we get a+2b+c=2x+2y⇒a+c=2(x+y-b), which is even for all x, y, b∈ZThus, a, c∈RSo, R is transitive on Z.

Hence, *R* is an equivalence relation on *Z*.

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Question 6:

m is said to be related to *n* if *m* and *n* are integers and *m* − *n* is divisible by 13. Does this define an equivalence relation?

ANSWER:

We observe the following properties of relation *R*.

Let R={(m, n) : m, n∈Z : m−n is divisible by 13}Relexivity: Let m be an arbitrary element of Z. Then,m∈R⇒m−m=0=0 × 13⇒m−m is divisible by 13⇒(m, m) is reflexive on Z.Symmetry: Let (m, n)∈R. Then,m−n is divisible by 13⇒m−n=13pHere, p∈Z⇒n−m=13 (−p) Here, −p∈Z⇒n−m is divisible by 13⇒(n, m)∈R for all m, n∈ZSo, R is symmetric on Z.Transitivity: Let (m, n) and (n, o)∈R⇒m−n and n−o are divisible by 13⇒m−n=13p and n−o=13q for some p, q∈ZAdding the above two, we get m−n+n−o=13p+13q⇒m−o=13 (p+q)Here, p+q∈Z⇒m−o is divisible by 13⇒(m, o)∈R for all m, o∈ZSo, R is transitive on Z.Let R={m, n : m, n∈Z : m-n is divisible by 13}Relexivity: Let m be an arbitrary element of Z. Then,m∈R⇒m-m=0=0 × 13⇒m-m is divisible by 13⇒m, m is reflexive on Z.Symmetry: Let m, n∈R. Then,m-n is divisible by 13⇒m-n=13pHere, p∈Z⇒n-m=13 -p Here, -p∈Z⇒n-m is divisible by 13⇒n, m∈R for all m, n∈ZSo, R is symmetric on Z.Transitivity: Let m, n and n, o∈R⇒m-n and n-o are divisible by 13⇒m-n=13p and n-o=13q for some p, q∈ZAdding the above two, we get m-n+n-o=13p+13q⇒m-o=13 p+qHere, p+q∈Z⇒m-o is divisible by 13⇒m, o∈R for all m, o∈ZSo, R is transitive on Z.

Hence, *R* is an equivalence relation on *Z*.

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Question 7:

Let *R* be a relation on the set *A* of ordered pair of integers defined by (*x*, *y*) *R* (*u*, *v*) if *xv* = *yu*. Show that *R* is an equivalence relation.

ANSWER:

We observe the following properties of *R*.

Reflexivity: Let (a, b) be an arbitrary element of the set A. Then,   (a, b)∈A⇒ab=ba ⇒(a, b) R (a, b)Thus, R is reflexive on A.Symmetry: Let (x, y) and (u, v)∈A such that (x, y) R (u, v). Then,   xv=yu⇒vx=uy⇒uy=vx⇒(u, v) R (x, y)So, R is symmetric on A.Transitivity: Let (x, y), (u, v) and (p, q)∈R such that (x, y) R (u, v) and (u, v) R (p, q).⇒xv=yu and uq=vpMultiplying the corresponding sides, we getxv × uq=yu × vp⇒xq=yp⇒(x, y) R (p, q)So, R is transitive on A.Reflexivity: Let a, b be an arbitrary element of the set A. Then, a, b∈A⇒ab=ba ⇒a, b R a, bThus, R is reflexive on A.Symmetry: Let x, y and u, v∈A such that x, y R u, v. Then, xv=yu⇒vx=uy⇒uy=vx⇒u, v R x, ySo, R is symmetric on A.Transitivity: Let x, y, u, v and p, q∈R such that x, y R u, v and u, v R p, q.⇒xv=yu and uq=vpMultiplying the corresponding sides, we getxv × uq=yu × vp⇒xq=yp⇒x, y R p, qSo, R is transitive on A.

Hence, *R* is an equivalence relation on *A.*

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Question 8:

Show that the relation *R* on the set $A = \{x \in Z ; 0 \le x \le 12\}$, given by $R = \{(a, b) : a = b\}$, is an equivalence relation. Find the set of all elements related to 1.

ANSWER:

We observe the following properties of *R*.

Reflexivity: Let *a* be an arbitrary element of *A*. Then,

a∈R⇒a=a [Since, every element is equal to itself]⇒(a, a)∈R for all a∈ASo, R is reflexive on A.Symmetry: Let (a, b) ∈R⇒a b⇒b=a⇒(b, a)∈R for all a, b∈ASo, R is symmetric on A.Transitivity: Let (a, b) and (b, c)∈R⇒a=b and b=c⇒a=b c⇒a=c⇒(a, c)∈RSo, R is transitive on A.a∈R⇒a=a Since, every element is equal to itself⇒a, a∈R for all a∈ASo, R is reflexive on A.Symmetry: Let a, b ∈R⇒a b⇒b=a⇒b, a∈R for all a, b∈ASo, R is symmetric on A.Transitivity: Let a, b and b, c∈R⇒a=b and b=c⇒a=b c⇒a=c⇒a, c∈RSo, R is transitive on A.

Hence, *R* is an equivalence relation on *A.*

The set of all elements related to 1 is {1}.

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Question 9:

Let *L* be the set of all lines in *XY*-plane and *R* be the relation in *L* defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that *R* is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

ANSWER:

We observe the following properties of *R*.

Reflexivity: Let L1 be an arbitrary element of the set L. Then,L1∈L⇒L1 is parallel to L1 [Every line is parallel to itself]⇒(L1, L1)∈R for all L1∈LSo, R is reflexive on L.Symmetry: Let (L1, L2)∈R⇒L1 is parallel to L2⇒L2 is parallel to L1⇒(L2, L1)∈R for all L1 and L2∈LSo, R is symmetric on L.Transitivity: Let (L1, L2) and (L2, L3)∈R⇒L1 is parallel to L2 and L2 is parallel to L3⇒L1, L2 and L3 are all parallel to each other⇒L1 is parallel to L3⇒(L1, L3)∈RSo, R is transitive on L.Reflexivity: Let L1 be an arbitrary element of the set L. Then,L1∈L⇒L1 is parallel to L1 Every line is parallel to itself⇒L1, L1∈R for all L1∈LSo, R is reflexive on L.Symmetry: Let L1, L2∈R⇒L1 is parallel to L2⇒L2 is parallel to L1⇒L2, L1∈R for all L1 and L2∈LSo, R is symmetric on L.Transitivity: Let L1, L2 and L2, L3∈R⇒L1 is parallel to L2 and L2 is parallel to L3⇒L1, L2 and L3 are all parallel to each other⇒L1 is parallel to L3⇒L1, L3∈RSo, R is transitive on L.

Hence, *R* is an equivalence relation on *L*.

Set of all the lines related to $y = 2x+4$

$= L' = \{(x, y) : y = 2x+c, \text{ where } c \in \in R\}$

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Question 10:

Show that the relation *R*, defined on the set *A* of all polygons as

$R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\}$,

is an equivalence relation. What is the set of all elements in *A* related to the right angle triangle *T* with sides 3, 4 and 5?

ANSWER:

We observe the following properties on *R*.

Reflexivity: Let P1 be an arbitrary element of A.Then, polygon P1 and P1 have the same number of sides, since they are one and the same.⇒(P1, P1)∈R for all P1∈ASo, R is reflexive on A.Symmetry: Let (P1, P2)∈R⇒P1 and P2 have the same number of sides.⇒P2 and P1 have the same number of sides.⇒(P2, P1)∈R for all P1, P2∈ASo, R is symmetric on A.Transitivity: Let (P1, P2), (P2, P3)∈R⇒P1 and P2 have the same number of sides and P2 and P3 have the same number of sides.⇒P1, P2 and P3 have the same number of sides.⇒P1 and P3 have the same number of sides.⇒(P1, P3)∈R for all P1, P3 ASo, R is transitive on A.Reflexivity: Let P1 be an arbitrary element of A.Then, polygon P1 and P1 have the same number of sides, since they are one and the same.⇒P1, P1∈R for all P1∈ASo, R is reflexive on A.Symmetry: Let P1, P2∈R⇒P1 and P2 have the same number of sides.⇒P2 and P1 have the same number of sides.⇒P2, P1∈R for all P1, P2∈ASo, R is symmetric on A.Transitivity: Let P1, P2, P2, P3∈R⇒P1 and P2 have the same number of sides and P2 and P3 have the same number of sides.⇒P1, P2 and P3 have the same number of sides.⇒P1 and P3 have the same number of sides.⇒P1, P3∈R for all P1, P3 ASo, R is transitive on A.

Hence, *R* is an equivalence relation on the set *A.*

Also, the set of all the triangles∈∈*A* is related to the right angle triangle *T* with the sides 3, 4, 5.

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Question 11:

Let *O* be the origin. We define a relation between two points *P* and *Q* in a plane if *OP* = *OQ*. Show that the relation, so defined is an equivalence relation.

ANSWER:

Let *A* be the set of all points in a plane such that

A={P : P is a point in the plane}Let R be the relation such that R={(P, Q) : P, Q∈A and OP=OQ, where O is the origin}A={P : P is a point in the plane}Let R be the relation such that R=P, Q : P, Q∈A and OP=OQ, where O is the origin

We observe the following properties of *R*.

Reflexivity: Let *P* be an arbitrary element of *R*.

The distance of a point P will remain the same from the origin.

So, OP = OP

⇒(P, P)∈RSo, R is reflexive on A.Symmetry: Let (P, Q)∈R⇒OP=OQ⇒OQ=OP⇒(Q, P)∈RSo, R is symmetric on A.Transitivity: Let (P, Q), (Q, R)∈R⇒OP=OQ and OQ=OR⇒OP=OQ=OR⇒OP=OR⇒(P, R)∈RSo, R is transitive on A.⇒P, P∈RSo, R is reflexive on A.Symmetry: Let P, Q∈R⇒OP=OQ⇒OQ=OP⇒Q, P∈RSo, R is symmetric on A.Transitivity: Let P, Q, Q, R∈R⇒OP=OQ and OQ=OR⇒OP=OQ=OR⇒OP=OR⇒P, R∈RSo, R is transitive on A.

Hence, *R* is an equivalence relation on *A*.

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Question 12:

Let *R* be the relation defined on the set *A* = {1, 2, 3, 4, 5, 6, 7} by *R* = {(*a*, *b*) : both *a* and *b* are either odd or even}. Show that *R* is an equivalence relation. Further, show that all the elements of the subset {1, 3, 5, 7} are related to each other and all the elements of the subset {2, 4, 6} are related to each other, but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.

ANSWER:

We observe the following properties of *R*.

Reflexivity:

Let a be an arbitrary element of R. Then,a∈R⇒(a, a)∈R for all a∈ASo, R is reflexive on A.Symmetry: Let (a, b)∈R⇒Both a and b are either even or odd.⇒Both b and a are either even or odd.⇒(b, a)∈R for all a, b∈ASo, R is symmetric on A.Transitivity: Let (a, b) and (b, c)∈R⇒Both a and b are either even or odd and both b and c are either even or odd.⇒a, b and c are either even or odd.⇒a and c both are either even or odd.⇒(a, c) ∈R for all a, c∈ASo, R is transitive on A.Let a be an arbitrary element of R. Then,a∈R⇒a, a∈R for all a∈ASo, R is reflexive on A.Symmetry: Let a, b∈R⇒Both a and b are either even or odd.⇒Both b and a are either even or odd.⇒b, a∈R for all a, b∈ASo, R is symmetric on A.Transitivity: Let a, b and b, c∈R⇒Both a and b are either even or odd and both b and c are either even or odd.⇒a, b and c are either even or odd.⇒a and c both are either even or odd.⇒a, c ∈R for all a, c∈ASo, R is transitive on A.

Thus, *R* is an equivalence relation on *A.*

We observe that all the elements of the subset {1, 3, 5, 7} are odd. Thus, they are related to each other.

This is because the relation *R* on *A* is an equivalence relation.

Similarly, the elements of the subset {2, 4, 6} are even. Thus, they are related to each other because every element is even.

Hence proved.

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Question 13:

Let *S* be a relation on the set *R* of all real numbers defined by

$S = \{(a, b) \in R \times R : a^2 + b^2 = 1\}$

Prove that *S* is not an equivalence relation on *R*.

ANSWER:

We observe the following properties of *S*.

Reflexivity:Let a be an arbitrary element of R. Then, a∈R⇒a2+a2≠1∀a∈R⇒(a, a)∉SSo, S is not reflexive on R.Symmetry: Let (a, b)∈R⇒a2+b2=1⇒b2+a2=1⇒(b, a)∈S for all a, b∈RSo, S is symmetric on R.Transitivity: Let (a, b) and (b, c)∈S⇒a2+b2=1 and b2+c2=1Adding the above two, we geta2+c2=2−2b2≠1 for all a, b, c∈RSo, S is not transitive on R.Reflexivity:Let a be an arbitrary element of R. Then, a∈R⇒a2+a2≠1∀a∈R⇒a, a∉SSo, S is not reflexive on R.Symmetry: Let a, b∈R⇒a2+b2=1⇒b2+a2=1⇒b, a∈S for all a, b∈RSo, S is symmetric on R.Transitivity:

Let a, b and b, c∈S⇒a2+b2=1 and b2+c2=1Adding the above two, we geta2+c2=2-2b2≠1 for all a, b, c∈RSo, S is not transitive on R.

Hence, *S* is not an equivalence relation on *R*.

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Question 14:

Let *Z* be the set of all integers and Z_0 be the set of all non-zero integers. Let a relation *R* on $Z \times Z_0$ be defined as

$(a, b)\ R\ (c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in Z \times Z_0$,

Prove that *R* is an equivalence relation on $Z \times Z_0$.

ANSWER:

We observe the following properties of *R*.

Reflexivity:

Let (a, b) be an arbitrary element of Z × Z0. Then,(a, b)∈Z × Z0⇒a, b∈Z, Z0⇒ab=ba⇒(a, b)∈R for all (a, b)∈Z × Z0So, R is reflexive on Z × Z0.Let a, b be an arbitrary element of Z × Z0. Then,a, b∈Z × Z0⇒a, b∈Z, Z0⇒ab=ba⇒a, b∈R for all a, b∈Z × Z0So, R is reflexive on Z × Z0.

Symmetry:

Let (a, b), (c, d)∈Z×Z0 such that (a, b) R (c, d). Then,(a, b) R (c, d)⇒ad=bc⇒cb=da⇒(c, d) R (a, b)Thus, (a, b) R (c, d)⇒(c, d) R (a, b) for all (a, b), (c, d)∈Z×Z0So, R is symmetric on Z×Z0.Let a, b, c, d∈Z×Z0 such that a, b R c, d. Then,a, b R c, d⇒ad=bc⇒cb=da⇒c, d R a, bThus, a, b R c, d⇒c, d R a, b for all a, b, c, d∈Z×Z0So, R is symmetric on Z×Z0.

Transitivity:

Let (a, b), (c, d), (e, f)∈N×N0 such that (a, b) R (c, d) and (c, d) R (e, f). Then,(a, b) R (c, d)⇒ad=bc(c, d) R (e, f)⇒cf=de}⇒(ad) (cf)=(bc) (de)⇒af=be⇒(a, b) R (e, f)Thus, (a, b) R (c, d) and (c, d) R (e, f)⇒(a, b) R (e, f)⇒(a, b) R (e, f) for all values (a, b), (c, d), (e, f)∈N×N0So, R is transitive on N×N0.Let a, b, c, d, e, f∈N×N0 such that a, b R c, d and c, d R e, f. Then,a, b R c, d⇒ad=bcc, d R e, f⇒cf=de⇒ad cf=bc de⇒af=be⇒a, b R e, fThus, a, b R c, d and c, d R e, f⇒a, b R e, f⇒a, b R e, f for all values a, b, c, d, e, f∈N×N0So, R is transitive on N×N0.

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Question 15:

If *R* and *S* are relations on a set *A*, then prove that

(i) *R* and *S* are symmetric ⇒ *R* ∩ *S* and *R* ∪ *S* are symmetric

(ii) *R* is reflexive and *S* is any relation ⇒ *R* ∪ *S* is reflexive.

ANSWER:

(i) *R* and *S* are symmetric relations on the set *A*.

⇒R⊂A×A and S⊂A×A⇒R∩S⊂A×AThus, R∩S is a relation on A.Let a, b∈A such that (a, b)∈R∩S. Then,(a, b)∈R∩S⇒(a, b)∈R and (a, b)∈S⇒(b, a)∈R and (b, a)∈S [Since R and S are symmetric]⇒(b, a)∈R∩SThus, (a, b)∈R∩S⇒(b, a)∈R∩S for all a, b∈ASo, R∩S is symmetric on A.⇒R⊂A×A and S⊂A×A⇒R∩S⊂A×AThus, R∩S is a relation on A.Let a, b∈A such that a, b∈R∩S. Then,a, b∈R∩S⇒a, b∈R and a, b∈S⇒b, a∈R and b, a∈S   Since R and S are symmetric⇒b, a∈R∩SThus, a, b∈R∩S⇒b, a∈R∩S for all a, b∈ASo, R∩S is symmetric on A.

Also,

Let a, b∈A such that (a, b)∈R∪S⇒(a, b)∈R or (a, b)∈S⇒(b, a)∈R or (b, a)∈S [Since R and S are symmetric]⇒(b, a)∈R∪SSo, R∪S is symmetric on A.Let a, b∈A such that a, b∈R∪S⇒a, b∈R or a, b∈S⇒b, a∈R or b, a∈S Since R and S are symmetric⇒b, a∈R∪SSo, R∪S is symmetric on A.

(ii) *R* is reflexive and *S* is any relation.

Suppose a∈A. Then, (a, a)∈R [Since R is reflexive]⇒(a, a)∈R∪S⇒R∪S is reflexive on A.Suppose a∈A. Then, a, a∈R Since R is reflexive⇒a, a∈R∪S⇒R∪S is reflexive on A.

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Question 16:

If *R* and *S* are transitive relations on a set *A*, then prove that *R* ∪ *S* may not be a transitive relation on *A*.

ANSWER:

Let *A* = {*a*, *b*, *c*} and *R* and *S* be two relations on *A,* given by

R = {(*a*, *a*), (*a*, *b*), (*b*, *a*), (*b*, *b*)} and

S = {(*b*, *b*), (*b*, *c*), (*c*, *b*), (*c*, *c*)}

Here, the relations *R* and *S* are transitive on *A*.

(a, b)∈R∪S and (b, c)∈R∪SBut (a, c)∉R∪S a, b∈R∪S and b, c∈R∪SBut a, c∉R∪S

Hence, *R*∪ ∪*S* is not a transitive relation on *A*.

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Question 17:

Let *C* be the set of all complex numbers and C_0 be the set of all no-zero complex numbers. Let a relation *R* on C_0 be defined as

z1z1 R z2 ⇔ z1−z2z1+z2z2 ⇔ z1-z2z1+z2 is real for all z1, z2 ∈z1, z2 ∈C_0.

Show that *R* is an equivalence relation.

ANSWER:

(i) Test for reflexivity:

Since, z1−z1z1+z1=0z1-z1z1+z1=0, which is a real number.

So, (z1, z1)∈Rz1, z1∈R

Hence, *R* is relexive relation.

(ii) Test for symmetric:

Let (z1, z2)∈Rz1, z2∈R.

Then, z1−z2z1+z2=xz1-z2z1+z2=x, where *x* is real

⇒−(z1−z2z1+z2)=−x⇒(z2−z1z2+z1)=−x, is also a real number⇒-z1-z2z1+z2=-x⇒z2-z1z2+z1=-x, is also a real number

So, (z2, z1)∈Rz2, z1∈R

Hence, *R* is symmetric relation.

(iii) Test for transivity:

Let (z1, z2)∈R and (z2, z3)∈Rz1, z2∈R and z2, z3∈R.

Then,

z1−z2z1+z2=x, where x is a real number.⇒z1−z2=xz1+xz2⇒z1−xz1=z2+xz2⇒z1(1−x)=z2(1+x)⇒z1z2=(1+x)(1−x) ...(1)z1-z2z1+z2=x, where x is a real number.⇒z1-z2=xz1+xz2⇒z1-xz1=z2+xz2⇒z11-x=z21+x⇒z1z2=1+x1-x ...1

Also,

z2−z3z2+z3=y, where y is a real number.⇒z2−z3=yz2+Yz3⇒z2−yz2=z3+yz3⇒z2(1−y)=z3(1+y)⇒z2z3=(1+y)(1−y) ...(2)z2-z3z2+z3=y, where y is a real number.⇒z2-z3=yz2+Yz3⇒z2-yz2=z3+yz3⇒z21-y=z31+y⇒z2z3=1+y1-y ...2

Dividing (1) and (2), we get

z1z3=(1+x1−x)×(1−y1+y)=z, where z is a real number.⇒z1−z3z1+z3=z−1z+1, which is real⇒(z1, z3)∈Rz1z3=1+x1-x×1-y1+y=z, where z is a real number.⇒z1-z3z1+z3=z-1z+1, which is real⇒z1, z3∈R

Hence, *R* is transitive relation.

From (i), (ii), and (iii),

R is an equivalenve relation.

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Question 1:

Let *R* be a relation on the set *N* given by

$R = \{(a, b) : a = b - 2, b > 6\}$. Then,

(a) $(2, 4) \in R$

(b) $(3, 8) \in R$

(c) $(6, 8) \in R$

(d) $(8, 7) \in R$

ANSWER:

(c) $(6, 8) \in R$

(6, 8)∈R Then, a=b−2⇒6=8−2and b=8 > 6Hence, (6, 8)∈R6, 8∈R Then, a=b-2⇒6=8-2and b=8 > 6Hence, 6, 8∈R

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Question 2:

If a relation *R* is defined on the set Z of integers as follows:

(a, b) ∈ R ⇔ $a^2 + b^2 = 25$. Then, domain (R) is

(a) {3, 4, 5}

(b) {0, 3, 4, 5}

(c) {0, ± 3, ± 4, ± 5}

(d) none of these

ANSWER:

(c) {0, ± 3, ± 4, ± 5}

R={(a, b) : a2+b2=25, a, b∈Z}⇒a∈{−5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5} and b∈{−5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5} R=a, b : a2+b2=25, a, b∈Z⇒a∈-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 and b∈-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5

So, domain (R)={0, ± 3,± 4, ±5}So, domain (R)=0, ± 3,± 4, ±5

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Question 3:

R is a relation on the set *Z* of integers and it is given by

$(x, y) \in R \Leftrightarrow |x - y| \leq 1$. Then, R is

(a) reflexive and transitive

(b) reflexive and symmetric

(c) symmetric and transitive

(d) an equivalence relation

ANSWER:

(b) reflexive and symmetric

Reflexivity: Let x∈R. Then,x−x=0 < 1⇒|x−x|≤1⇒(x, x)∈R for all x∈ZSo, R is reflexive on Z.Symmetry: Let (x, y)∈R. Then,|x−y| ≤ 0⇒|−(y−x)| ≤ 1⇒|y−x| ≤ 1 [Since |x−y|=|y−x|]⇒(y, x)∈R for all x, y∈ZSo, R is symmetric on Z.Transitivity: Let (x, y)∈R and (y, z)∈R. Then,|x−y| ≤ 1 and |y−z| ≤ 1⇒It is not always true that |x−y| ≤ 1.⇒(x, z)∉RSo, R is not transitive on Z.Reflexivity: Let x∈R. Then,x-x=0 < 1⇒x-x≤1⇒x, x∈R for all x∈ZSo, R is reflexive on Z.Symmetry: Let x, y∈R. Then,x-y ≤ 0⇒-(y-x) ≤ 1⇒y-x ≤ 1 Since x-y=y-x⇒y, x∈R for all x, y∈ZSo, R is symmetric on Z.Transitivity: Let x, y∈R and y, z∈R. Then,x-y ≤ 1 and y-z ≤ 1⇒It is not always true that x-y ≤ 1.⇒x, z∉RSo, R is not transitive on Z.

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Question 4:

The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by

$R = \{(a, b) : |a^2 - b^2| < 16\}$ is given by

(a) {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}

(b) {(2, 2), (3, 2), (4, 2), (2, 4)}

(c) {(3, 3), (4, 3), (5, 4), (3, 4)}

(d) none of these

ANSWER:

(d) none of these

R is given by {(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (1, 3), (3, 1), (1, 4), (4, 1) ,(2, 4), (4, 2)}, which is not mentioned in (a), (b) or (c).

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Question 5:

Let *R* be the relation over the set of all straight lines in a plane such that $l_1 R l_2 \Leftrightarrow l_1 \perp l_2$. Then, *R* is

(a) symmetric

(b) reflexive

(c) transitive

(d) an equivalence relation

ANSWER:

(a) symmetric

A = Set of all straight lines in the plane

R={(l1, l2) : l1, l2∈A : l1⊥l2}Reflexivity: l1 is not ⊥ l1⇒(l1, l1)∉RSo, R is not reflexive on A.Symmetry: Let (l1, l2)∈R⇒l1⊥l2⇒l2⊥l1⇒(l2, l1)∈RSo, R is symmetric on A.Transitivity: Let (l1, l2)∈R, (l2, l3)∈R⇒l1⊥ l2 and l2⊥ l3But l1 is not ⊥ l3⇒(l1, l3)∉RSo, R is not transitive on A.R=l1, l2 : l1, l2∈A : l1⊥l2Reflexivity: l1 is not ⊥ l1⇒l1, l1∉RSo, R is not reflexive on A.Symmetry: Let l1, l2∈R⇒l1⊥l2⇒l2⊥l1⇒l2, l1∈RSo, R is symmetric on A.Transitivity: Let l1, l2∈R, l2, l3∈R⇒l1⊥ l2 and l2⊥ l3But l1 is not ⊥ l3⇒l1, l3∉RSo, R is not transitive on A.

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Question 6:

If A = {a, b, c}, then the relation R = {(b, c)} on A is

(a) reflexive only

(b) symmetric only

(c) transitive only

(d) reflexive and transitive only

ANSWER:

(c) transitive only

The relation R = {(b,c)} is neither reflexive nor symmetric because every element of A is not related to itself. Also, the ordered pair of R obtained by interchanging its elements is not contained in R.

We observe that R is transitive on A because there is only one pair.

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Question 7:

Let A = {2, 3, 4, 5, ..., 17, 18}. Let '≃' be the equivalence relation on $A \times A$, cartesian product of A with itself, defined by (a, b) ≃ (c, d) if $ad = bc$. Then, the number of ordered pairs of the equivalence class of (3, 2) is

(a) 4

(b) 5

(c) 6

(d) 7

ANSWER:

(c) 6

The ordered pairs of the equivalence class of (3, 2) are {(3, 2), (6, 4), (9, 6), (12, 8), (15, 10), (18, 12)}.

We observe that these are 6 pairs.

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Question 8:

Let A = {1, 2, 3}. Then, the number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(a) 1

(b) 2

(c) 3

(d) 4

ANSWER:

(a) 1

The required relation is R.

R = {(1, 2), (1, 3), (1, 1), (2, 2), (3, 3), (2, 1), (3, 1)}

Hence, there is only 1 such relation that is reflexive and symmetric, but not transitive.

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Question 9:

The relation '*R*' in $N \times N$ such that

$(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c$ is

(a) reflexive but not symmetric

(b) reflexive and transitive but not symmetric

(c) an equivalence relation

(d) none of the these

ANSWER:

(c) an equivalence relation

We observe the following properties of relation *R*.

Reflexivity: Let (a, b)∈N×N⇒a, b∈N⇒a+b=b+a⇒(a, b)∈R So, R is reflexive on N×N.Symmetry: Let (a, b), (c, d)∈N×N such that (a, b) R (c, d)⇒a+d=b+c⇒d+a=c+b⇒(d, c), (b, a)∈R So, R is symmetric on N×N.Transitivity: Let (a, b), (c, d), (e, f)∈N×N such that (a, b) R (c, d) and (c, d) R (e, f)⇒a+d=b+c and c+f=d+e⇒a+d+c+f=b+c+d+e⇒a+f=b+e⇒(a, b) R (e, f)So, R is transitive on N×N.Reflexivity: Let (a, b)∈N×N⇒a, b∈N⇒a+b=b+a⇒a, b∈R So, R is reflexive on N×N.Symmetry: Let a, b, c, d∈N×N such that a, b R c, d⇒a+d=b+c⇒d+a=c+b⇒d, c, b, a∈R So, R is symmetric on N×N.Transitivity: Let a, b, c, d, e, f∈N×N such that a, b R c, d and c, d R e, f⇒a+d=b+c and c+f=d+e⇒a+d+c+f=b+c+d+e⇒a+f=b+e⇒a, b R e, fSo, R is transitive on N×N.

Hence, *R* is an equivalence relation on *N*.

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Question 10:

If *A* = {1, 2, 3}, *B* = {1, 4, 6, 9} and *R* is a relation from *A* to *B* defined by '*x* is greater than *y*'. The range of *R* is

(a) {1, 4, 6, 9}

(b) {4, 6, 9}

(c) {1}

(d) none of these

ANSWER:

(c) {1}

Here,

R={(x, y) : x∈A and y∈B : x > y}⇒R={(2, 1), (3, 1)}R=x, y : x∈A and y∈B : x > y⇒R=2, 1, 3, 1

Thus,

Range of *R* = {1}

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Question 11:

A relation *R* is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by : *x R y* ⇔ *x* is relatively prime to *y*. Then, domain of *R* is

(a) {2, 3, 5}

(b) {3, 5}

(c) {2, 3, 4}

(d) {2, 3, 4, 5}

ANSWER:

(d) {2, 3, 4, 5}

The relation *R* is defined as

R = {(x, y) : x∈{2, 3, 4, 5}, y∈{3, 6, 7, 10} : x is relatively prime to y}⇒R= {(2, 3), (2, 7), (3, 7), (3, 10), (4, 7), (5, 3), (5, 7)} R = x, y : x∈2, 3, 4, 5, y∈3, 6, 7, 10 : x is relatively prime to y⇒R= 2, 3, 2, 7, 3, 7, 3, 10, 4, 7, 5, 3, 5, 7

Hence, the domain of *R* includes all the values of *x*, i.e. {2, 3, 4, 5}.

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Question 12:

A relation ϕ from *C* to *R* is defined by *x* ϕ *y* ⇔ | *x* | = *y*. Which one is correct?

(a) (2 + 3 *i*) ϕ 13

(b) 3 ϕ (−3)

(c) (1 + *i*) ϕ 2

(d) *i* ϕ 1

ANSWER:

(d) *i* ϕ 1

∵ |2+3i|=13−−√≠13 |3|≠−3 |1+i|=2–√≠2and | i |=1So, (i, 1)∈ϕ∵ 2+3i=13≠13 3≠-3 1+i=2≠2and i =1So, i, 1∈ϕ

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Question 13:

Let *R* be a relation on *N* defined by $x + 2y = 8$. The domain of *R* is

(a) {2, 4, 8}

(b) {2, 4, 6, 8}

(c) {2, 4, 6}

(d) {1, 2, 3, 4}

ANSWER:

(c) {2,4,6}

The relation *R* is defined as

R={(x, y) : x, y∈N and x+2y = 8}⇒R={(x, y) : x, y∈N and y = (8−x)2} R=x, y : x, y∈N and x+2y = 8⇒R=x, y : x, y∈N and y = 8-x2

Domain of *R* is all values of *x*∈∈*N* satisfying the relation *R.* Also, there are only three values of *x* that result in *y*, which is a natural number. These are {2, 6, 4}.

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Question 14:

R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$. Then, R^{-1} is

(a) {(8, 11), (10, 13)}

(b) {(11, 8), (13, 10)}

(c) {(10, 13), (8, 11)}

(d) none of these

ANSWER:

(a) {(8, 11), (10, 13)}

The relation *R* is defined by

R={(x, y) : x∈{11, 12, 13}, y∈{8, 10, 12} : y = x−3}⇒R={(11, 8), (13, 10)}So, R−1={(8, 11), (10, 13)} R=x, y : x∈11, 12, 13, y∈8, 10, 12 : y = x-3⇒R=11, 8, (13, 10)So, R-1=8, 11, 10, 13

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Question 15:

Let *R* = {(*a*, *a*), (*b*, *b*), (*c*, *c*), (*a*, *b*)} be a relation on set *A* = *a*, *b*, *c*. Then, *R* is

(a) identify relation

(b) reflexive

(c) symmetric

(d) antisymmetric

ANSWER:

(b) reflexive

Reflexivity: Since (a, a)∈R∀ a∈A, R is reflexive on A.Symmetry: Since (a, b)∈R but (b, a)∉R, R is not symmetric on A.⇒R is not antisymmetric on A.Also, R is not an identity relation on A.Reflexivity: Since a, a∈R∀ a∈A, R is reflexive on A.Symmetry: Since a, b∈R but b, a∉R, R is not symmetric on A.⇒R is not antisymmetric on A.Also, R is not an identity relation on A.

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Question 16:

Let *A* = {1, 2, 3} and *B* = {(1, 2), (2, 3), (1, 3)} be a relation on *A*. Then, *R* is

(a) neither reflexive nor transitive

(b) neither symmetric nor transitive

(c) transitive

(d) none of these

ANSWER:

(c) transitive

Reflexivity: Since (1, 1)∉B, B is not reflexive on A.Symmetry: Since (1, 2)∈B but (2, 1)∉B, B is not symmetric on A.Transitivity: Since (1, 2)∈B, (2, 3)∈B and (1, 3)∈B, B is transitive on A.Reflexivity: Since (1, 1)∉B, B is not reflexive on A.Symmetry: Since 1, 2∈B but 2, 1∉B, B is not symmetric on A.Transitivity: Since 1, 2∈B, 2, 3∈B and 1, 3∈B, B is transitive on A.

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Question 17:

If R is the largest equivalence relation on a set A and S is any relation on A, then

(a) $R \subset S$

(b) $S \subset R$

(c) $R = S$

(d) none of these

ANSWER:

(b) $S \subset R$

Since R is the largest equivalence relation on set A,

$R \subseteq A \times AR \subseteq A \times A$

Since S is any relation on A,

$S \subset A \times A S \subset A \times A$

So, $S \subset R$

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Question 18:

If R is a relation on the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} given by $x\ R\ y \Leftrightarrow y = 3\ x$, then R =

(a) {(3, 1), (6, 2), (8, 2), (9, 3)}

(b) {(3, 1), (6, 2), (9, 3)}

(c) {(3, 1), (2, 6), (3, 9)}

(d) none of these

ANSWER:

(d) none of these

The relation R is defined as

R = {(x, y) : x, y∈A : y = 3x}⇒R = {(1, 3), (2, 6), (3, 9)}   R = x, y : x, y∈A : y = 3x⇒R = 1, 3, 2, 6, 3, 9

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Question 19:

If R is a relation on the set A = {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3)}, then R is

(a) reflexive

(b) symmetric

(c) transitive

(d) all the three options

ANSWER:

(d) all the three options

R={(a, b) : a=b and a, b∈A}Reflexivity: Let a∈A. Then,a=a⇒(a, a)∈R for all a∈ASo, R is reflexive on A.Symmetry: Let a, b∈A such that (a, b)∈R. Then,(a, b)∈R⇒a=b⇒b=a⇒(b, a)∈R for all a∈ASo, R is symmetric on A.Transitivity: Let a, b, c∈A such that (a, b)∈R and (b, c)∈R. Then,(a, b)∈R⇒a=band (b, c)∈R⇒b=c⇒a=c⇒(a, c)∈R for all a∈ASo, R is transitive on A.R=a, b : a=b and a, b∈AReflexivity: Let a∈A. Then,a=a⇒a, a∈R for all a∈ASo, R is reflexive on A.Symmetry: Let a, b∈A such that a, b∈R. Then,a, b∈R⇒a=b⇒b=a⇒b, a∈R for all a∈ASo, R is symmetric on A.Transitivity: Let a, b, c∈A such that a, b∈R and b, c∈R. Then,a, b∈R⇒a=band b, c∈R⇒b=c⇒a=c⇒a, c∈R for all a∈ASo, R is transitive on A.

Hence, *R* is an equivalence relation on *A*.

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Question 20:

If $A = \{a, b, c, d\}$, then a relation $R = \{(a, b), (b, a), (a, a)\}$ on A is

(a) symmetric and transitive only

(b) reflexive and transitive only

(c) symmetric only

(d) transitive only

ANSWER:

(a) symmetric and transitive only

Reflexivity: Since (b, b)∉R, R is not reflexive on A.Symmetry: Since (a, b)∈R and (b, a)∈R, R is symmetric on A.Transitivity: Since (a, b)∈R, (b, a)∈R and (a, a)∈R, R is transitive on A.Reflexivity: Since b, b∉R, R is not reflexive on A.Symmetry: Since a, b∈R and b, a∈R, R is symmetric on A.Transitivity: Since a, b∈R, b, a∈R and a, a∈R, R is transitive on A.

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Question 21:

If *A* = {1, 2, 3}, then a relation *R* = {(2, 3)} on *A* is

(a) symmetric and transitive only

(b) symmetric only

(c) transitive only

(d) none of these

ANSWER:

(c) transitive only

The relation R is not reflexive because every element of A is not related to itself. Also, R is not symmetric since on interchanging the elements, the ordered pair in R is not contained in it.

R is transitive by default because there is only one element in it.

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Question 22:

Let *R* be the relation on the set *A* = {1, 2, 3, 4} given by

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Then,

(a) *R* is reflexive and symmetric but not transitive

(b) *R* is reflexive and transitive but not symmetric

(c) *R* is symmetric and transitive but not reflexive

(d) *R* is an equivalence relation

ANSWER:

(b) R is reflexive and transitive but not symmetric.

Reflexivity: Clearly, (a, a)∈R ∀ a∈ASo, R is reflexive on A.Symmetry: Since (1, 2)∈R, but (2, 1)∉R,R is not symmetric on A.Transitivity: Since, (1, 3), (3, 2)∈R and (1, 2)∈R,R is transitive on A.Reflexivity: Clearly, (a, a)∈R ∀ a∈ASo, R is reflexive on A.Symmetry: Since 1, 2∈R, but 2, 1∉R,R is not symmetric on A.Transitivity: Since, 1, 3, 3, 2∈R and 1, 2∈R,R is transitive on A.

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Question 23:

Let *A* = {1, 2, 3}. Then, the number of equivalence relations containing (1, 2) is

(a) 1

(b) 2

(c) 3

(d) 4

ANSWER:

(b) 2

There are 2 equivalence relations containing {1, 2}.

R = {(1, 2)}

S = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, 1)}

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Question 24:

The relation *R* = {(1, 1), (2, 2), (3, 3)} on the set {1, 2, 3} is

(a) symmetric only

(b) reflexive only

(c) an equivalence relation

(d) transitive only

ANSWER:

(c) an equivalence relation

R={(a, b) : a=b and a, b∈A}Reflexivity: Let a∈A Here,a=a⇒(a, a)∈R for all a∈ASo, R is reflexive on A.Symmetry: Let a, b∈A such that (a, b)∈R. Then,(a, b)∈R⇒a=b⇒b=a⇒(b, a)∈R for all a∈ASo, R is symmetric on A.Transitive: Let a, b, c∈A such that (a, b)∈R and (b, c)∈R. Then, (a, b)∈R⇒a=band (b, c)∈R⇒b=c⇒a=c⇒(a, c)∈R for all a∈ASo, R is transitive on A.R=a, b : a=b and a, b∈AReflexivity: Let a∈A Here,a=a⇒a, a∈R for all a∈ASo, R is reflexive on A.Symmetry: Let a, b∈A such that a, b∈R. Then,a, b∈R⇒a=b⇒b=a⇒b, a∈R for all a∈ASo, R is symmetric on A.Transitive: Let a, b, c∈A such that a, b∈R and b, c∈R. Then, a, b∈R⇒a=band b, c∈R⇒b=c⇒a=c⇒a, c∈R for all a∈ASo, R is transitive on A.

Hence, *R* is an equivalence relation on *A.*

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Question 25:

S is a relation over the set *R* of all real numbers and it is given by

$(a, b) \in S \Leftrightarrow ab \geq 0$. Then, S is

(a) symmetric and transitive only

(b) reflexive and symmetric only

(c) antisymmetric relation

(d) an equivalence relation

ANSWER:

(d) an equivalence relation

Reflexivity: Let $a \in \in R$

Then,

aa=a2>0⇒(a, a)∈R ∀ a∈Raa=a2>0⇒a, a∈R ∀ a∈R

So, S is reflexive on R.

Symmetry: Let $(a, b) \in \in S$

Then,

(a, b)∈S⇒ab≥0 ⇒ba≥0 ⇒(b, a)∈S∀ a, b∈Ra, b∈S⇒ab≥0 ⇒ba≥0 ⇒b, a∈S∀ a, b∈R

So, S is symmetric on R.

Transitivity:

If (a, b), (b, c)∈S⇒ab≥0 and bc≥0⇒ab×bc≥0⇒ac≥0 [∵ b2 ≥ 0]⇒(a, c)∈S for all a, b, c∈set RIf a, b, b, c∈S⇒ab≥0 and bc≥0⇒ab×bc≥0⇒ac≥0 ∵ b2 ≥ 0⇒a, c∈S for all a, b, c∈set R

Hence, *S* is an equivalence relation on *R*.

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Question 26:

In the set *Z* of all integers, which of the following relation *R* is not an equivalence relation?

(a) $x\ R\ y$: if $x \le y$

(b) $x\ R\ y$: if $x = y$

(c) $x\ R\ y$: if $x - y$ is an even integer

(d) $x\ R\ y$: if $x \equiv y$ (mod 3)

ANSWER:

(a) $x\ R\ y$: if $x \le y$

Clearly, *R* is not symmetric because $x < y$ does not imply y < x.

Hence, (a) is not an equivalence relation.

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Question 27:

Mark the correct alternative in the following question:

Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}. Then, R is

(a) reflexive but not symmetric
(b) reflexive but not transitive

(c) symmetric and transitive
(d) neither symmetric nor transitive

ANSWER:

We have,

R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

As, (a,a)∈R ∀a∈ASo, R is reflexive relationAlso, (1,2)∈R but (2,1)∉RSo, R is not symmetric relationAnd, (1,2)∈R,(2,3)∈R and (1,3)∈RSo, R is transitive relationAs, a,a∈R ∀a∈ASo, R is reflexive relationAlso, 1,2∈R but 2,1∉RSo, R is not symmetric relationAnd, 1,2∈R,2,3∈R and 1,3∈RSo, R is transitive relation

Hence, the correct alternative is option (a).

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Question 28:

Mark the correct alternative in the following question:

The relation *S* defined on the set **R** of all real number by the rule *aSb* iff *a* ≥≥ *b* is

(a) an equivalence relation

(b) reflexive, transitive but not symmetric

(c) symmetric, transitive but not reflexive

(d) neither transitive nor reflexive but symmetric

ANSWER:

We have,

S = {(*a*, *b*) : *a* ≥≥ *b*; *a*, *b* ∈∈ **R**}

As, a=a ∀a∈R⇒(a,a)∈SSo, S is reflexive relationLet (a,b)∈S⇒a≥bBut b≤a⇒(b,a)∉SSo, S is not symmetric relationLet (a,b)∈S and (b,c)∈S⇒a≥b and b≥c⇒a≥c⇒(a,c)∈SSo, S is transitive relationAs, a=a ∀a∈R⇒a,a∈SSo, S is reflexive relationLet a,b∈S⇒a≥bBut b≤a⇒b,a∉SSo, S is not symmetric relationLet a,b∈S and b,c∈S⇒a≥b and b≥c⇒a≥c⇒a,c∈SSo, S is transitive relation

Hence, the correct alternative is option (b).

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Question 29:

Mark the correct alternative in the following question:

The maximum number of equivalence relations on the set *A* = {1, 2, 3} is

(a) 1 (b) 2 (c) 3 (d) 5

ANSWER:

Consider the relation R1={(1,1)}It is clearly reflexive, symmetric and transitiveSimilarly, R2={(2,2)} and R3={(3,3)} are reflexive, symmetric and transitiveAlso, R4={(1,1),(2,2),(3,3),(1,2),(2,1)}It is reflexive as (a,a)∈R4 for all a∈{1,2,3}It is symmetric as (a,b)∈R4⇒(b,a)∈R4 for all a∈{1,2,3}Also, it is transitive as (1,2)∈R4,(2,1)∈R4⇒(1,1)∈R4The relation defined by R5={(1,1),(2,2),(3,3),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)} is reflexive, symmetric and transitive as well.Thus, the maximum number of equivalence relation on set A={1,2,3} is 5.Consider the relation R1=1,1It is clearly reflexive, symmetric and transitiveSimilarly, R2=2,2 and R3=3,3 are reflexive, symmetric and transitiveAlso, R4=1,1,2,2,3,3,1,2,2,1It is reflexive as a,a∈R4 for all a∈1,2,3It is symmetric as a,b∈R4⇒b,a∈R4 for all a∈1,2,3Also, it is transitive as 1,2∈R4,2,1∈R4⇒(1,1)∈R4The relation defined by R5=1,1,2,2,3,3,1,2,1,3,2,1,2,3,3,1,3,2 is reflexive, symmetric and transitive as well.Thus, the maximum number of equivalence relation on set A={1,2,3} is 5.

Hence, the correct alternative is option (d).

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Question 30:

Mark the correct alternative in the following question:

Let *R* be a relation on the set **N** of natural numbers defined by *nRm* iff *n* divides *m*. Then, *R* is

(a) Reflexive and symmetric (b) Transitive and symmetric

(c) Equivalence (d) Reflexive, transitive but not symmetric [NCERT EXEMPLAR]

ANSWER:

We have,

R = {(m, n) : n divides m; m, n ∈∈ **N**}

As, m divides m⇒(m,m)∈R ∀m∈NSo, R is reflexiveSince, (2,1)∈R i.e. 1 divides 2but 2 cannot divide 1 i.e. (2,1)∉RSo, R is not symmetricLet (m,n)∈R and (n,p)∈R. Then,n divides m and p divides n⇒p divides m⇒(m,p)∈RSo, R is transitiveAs, m divides m⇒m,m∈R ∀m∈NSo, R is reflexiveSince, 2,1∈R i.e. 1 divides 2but 2 cannot divide 1 i.e. 2,1∉RSo, R is not symmetricLet m,n∈R and n,p∈R. Then,n divides m and p divides n⇒p divides m⇒m,p∈RSo, R is transitive

Hence, the correct alternative is option (d).

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Question 31:

Mark the correct alternative in the following question:

Let L denote the set of all straight lines in a plane. Let a relation R be defined by lRm iff l is perpendicular to m for all l, m ∈∈ L. Then, R is

(a) reflexive (b) symmetric (c) transitive
(d) none of these

[NCERT EXEMPLAR]

ANSWER:

We have,R={(l,m):l is perpendicular to m; l,m∈L}As, l is not perpencular to l⇒(l,l)∉RSo, R is not reflexive relationLet (l,m)∈R⇒l is perpendicular to m⇒m is also perpendicular to l⇒(m,l)∈RSo, R is symmetric relationLet (l,m)∈R and (m,n)∈R⇒l is perpendicular to m and m is perpendicular to n⇒l is parallel to n (Lines perpendicular to same line are parallel)⇒(m,l)∉RSo, R is not transitive relationWe have,R=l,m:l is perpendicular to m; l,m∈LAs, l is not perpencular to l⇒l,l∉RSo, R is not reflexive relationLet l,m∈R⇒l is perpendicular to m⇒m is also perpendicular to l⇒m,l∈RSo, R is symmetric relationLet l,m∈R and m,n∈R⇒l is perpendicular to m and m is perpendicular to n⇒l is parallel to n Lines perpendicular to same line are parallel⇒m,l∉RSo, R is not transitive relation

Hence, the correct alternative is option (b).

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Question 32:

Mark the correct alternative in the following question:

Let *T* be the set of all triangles in the Euclidean plane, and let a relation *R* on *T* be defined as *aRb* if *a* is congruent to *b* for all *a*, *b* ∈∈ *T*. Then, *R* is

a) reflexive but not symmetric (b) transitive but not symmetric

c) equivalence (d) none of these

ANSWER:

We have,R={(a,b):a is congruent to b; a,b∈T}As, a≅a⇒(a,a)∈RSo, R is reflexive relationLet (a,b)∈R. Then,a≅b⇒b≅a⇒(b,a)∈RSo, R is symmetric relationLet (a,b)∈R and (b,c)∈R. Then,a≅b and b≅c⇒a≅c⇒(a,c)∈RSo, R is transitive relation∴ R is an equivalence relationWe have,R=a,b:a is congruent to b; a,b∈TAs, a≅a⇒a,a∈RSo, R is reflexive relationLet a,b∈R. Then,a≅b⇒b≅a⇒b,a∈RSo, R is symmetric relationLet a,b∈R and b,c∈R. Then,a≅b and b≅c⇒a≅c⇒a,c∈RSo, R is transitive relation∴ R is an equivalence relation

Hence, the correct alternative is option (c).

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Question 33:

Mark the correct alternative in the following question:

Consider a non-empty set consisting of children in a family and a relation *R* defined as *aRb* if *a* is brother of *b*. Then, *R* is

(a) symmetric but not transitive (b) transitive but not symmetric

(c) neither symmetric nor transitive (d) both symmetric and transitive

ANSWER:

We have,R={(a,b):a is brother of b}Let (a,b)∈R. Then,a is brother of bbut b is not necessary brother of a (As, b can be sister of a)⇒(b,a)∉RSo, R is not symmetricAlso,Let (a,b)∈R and (b,c)∈R⇒a is brother of b and b is brother of c⇒a is brother of c⇒(a,c)∈RSo, R is transitiveWe have,R=a,b:a is brother of bLet a,b∈R. Then,a is brother of bbut b is not necessary brother of a As, b can be sister of a⇒b,a∉RSo, R is not symmetricAlso,Let a,b∈R and b,c∈R⇒a is brother of b and b is brother of c⇒a is brother of c⇒a,c∈RSo, R is transitive

Hence, the correct alternative is option (b).

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Question 34:

Mark the correct alternative in the following question:

For real numbers *x* and *y*, define *xRy* iff x−y+2–√x-y+2 is an irrational number. Then the relation *R* is

(a) reflexive (b) symmetric (c) transitive
(d) none of these

ANSWER:

We have,R={(x,y):x−y+2–√ is an irrational number; x,y∈R}As, x−x+2–√=2–√, which is an irrational number⇒(x,x)∈RSo, R is reflexive relationSince, (2–√,2)∈Ri.e. 2–√−2+2–√=22–√−2, which is an irrational numberbut 2−2–√+2–√=2, which is a rational number⇒(2,2–√)∉RSo, R is not symmetric relationAlso, (2–√,2)∈R and (2,22–√)∈Ri.e. 2–√−2+2–√=22–√−2, which is an irrational number and 2−22–√+2–√=2−2–√, which is also an irrational numberBut 2–√−22–√+2–√=0, which is a rational number⇒(2–√,22–√)∉RSo, R is not transitive relationWe have,R=x,y:x-y+2 is an irrational number; x,y∈RAs, x-x+2=2, which is an irrational number⇒x,x∈RSo, R is reflexive relationSince, 2,2∈Ri.e. 2-2+2=22-2, which is an irrational numberbut 2-2+2=2, which is a rational number⇒2,2∉RSo, R is not symmetric relationAlso, 2,2∈R and 2,22∈Ri.e. 2-2+2=22-2, which is an irrational number and 2-22+2=2-2, which is also an irrational numberBut 2-22+2=0, which is a rational number⇒2,22∉RSo, R is not transitive relation

Hence, the correct alternative is option (a).

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Question 35:

If a relation *R* on the set (1, 2, 3) be defined by *R* = {(1, 2)}, then *R* is

(a) reflexive (b) transitive (c) symmetric (d) none of these

ANSWER:

Given: A relation *R* on the set {1, 2, 3} be defined by *R* = {(1, 2)}.

R = {(1, 2)}

Since, (1, 1) ∉ *R*

Therefore, It is not reflexive.

Since, (1, 2) ∈ *R* but (2, 1) ∉ *R*

Therefore, It is not symmetric.

But there is no counter example to disapprove transitive condition.

Therefore, it is transitive.

Hence, the correct option is (b).

Page No 1.32:

Question 1:

If *R* = {(x, y):x2+y2≤4, x, y∈Z}x, y:x2+y2≤4, x, y∈Z is a relation in Z, then the domain of *R* is ________________________.

ANSWER:

Given: *R* = {(x, y):x2+y2≤4, x, y∈Z}x, y:x2+y2≤4, x, y∈Z

R = {(−2, 0), (2, 0), (0, 2), (0, −2), (−1, 1), (−1, −1), (1, −1), (1, 1), (0, 1), (1, 0), (−1, 0), (0, −1), (0, 0)}

Therefore, Domain of R = {−2, −1, 0, 1, 2}

Hence, if R = {(x, y):x2+y2≤4, x, y∈Z}x, y:x2+y2≤4, x, y∈Z is a relation in Z, then the domain of R is {−2, −1, 0, 1, 2}.

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Question 2:

Let R be a relation in N defined by R ={(x, y): $x + 2y = 8$}, then the range of R is ____________________.

ANSWER:

Given: R = {(x, y): $x + 2y = 8$} where $x, y \in N$

R = {(6, 1), (4, 2), (2, 3)}

Therefore, Range of R = {1, 2, 3}

Hence, the range of R is {1, 2, 3}.

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Question 3:

The number of relations on a finite set having 5 elements is ___________________.

ANSWER:

Let R be a relation on A, where A contains 5 elements.

R is a subset of $A \times A$.

Number of elements in $A \times A = 5 \times 5 = 25$

Number of relations = Number of subsets of $A \times A = 2^{25}$

Hence, the number of relations on a finite set having 5 elements is 2^{25}.

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Question 4:

Let $A = \{1, 2, 3, 4\}$ and R be the relation on A defined by $\{(a, b): a, b \in A, a \times b$ is an even number$\}$, then the range of R is __________________.

ANSWER:

Given: $R = \{(a, b): a, b \in A, a \times b$ is an even number$\}$, where $A = \{1, 2, 3, 4\}$.

$R = \{(1, 2), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

Therefore, Range of R = $\{1, 2, 3, 4\}$

Hence, the range of R is $\{1, 2, 3, 4\}$.

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Question 5:

Let A = {1, 2, 3, 4, 5} The domain of the relation on A defined by R ={(x,y): y = 2x-1},is___________________.

ANSWER:

Given: $R = \{(x, y): y = 2x - 1\}$, where $A = \{1, 2, 3, 4, 5\}$ and $x, y \in A$.

$R = \{(1, 1), (2, 3), (3, 5)\}$

Therefore, Domain of R = {1, 2, 3}.

Hence, the domain of the relation on A defined by $R = \{(x, y): y = 2x - 1\}$, is {1, 2, 3}.

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Question 6:

If R s a relation defined on set A ={1, 2, 3} by the rule (a,b) ∈R⇔||a2−b2||≤5,∈R⇔a2-b2≤5, then R^{-1} =_____________________.

ANSWER:

Given: R = {(a, b): ||a2−b2||≤5a2-b2≤5}, where A = {1, 2, 3} and a, b ∈ A.

$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)\}$

Therefore, $R^{-1} = \{(1, 1), (2, 1), (1, 2), (2, 2), (3, 2), (2, 3), (3, 3)\} = R$

Hence, if R is a relation defined on set A = {1, 2, 3} by the rule (a,b) ∈R⇔||a2−b2||≤5,∈R⇔a2-b2≤5, then R^{-1} = $\underline{R}$.

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Question 7:

If R is a relation from A = {11, 12, 13} to B = {8, 10 12} defined by y = x-3, then R^{-1} =________________________.

ANSWER:

Given: $R = \{(x, y): y = x - 3, x \in A$ and $y \in B\}$, where A = {11, 12, 13} and B = {8, 10 12}.

R = {(11, 8), (13, 10)}

Therefore, R^{-1} = {(8, 11), (10, 13)}

Hence, R^{-1} = {(8, 11), (10, 13)}.

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Question 8:

The smallest equivalence relation on the set A = {a, b, c, d} is ________________________.

ANSWER:

Given: A = {a, b, c, d}

Identity relation is the smallest equivalence relation.

Therefore, $R = \{(a, a), (b, b), (c, c)\}$ is the smallest equivalence relation.

Hence, the smallest equivalence relation on the set $A = \{a, b, c, d\}$ is $\{(a, a), (b, b), (c, c)\}$.

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Question 9:

The largest equivalence relation on the set $A = \{1, 2, 3\}$ is ___________________.

ANSWER:

Given: $A = \{1, 2, 3\}$

The largest equivalence relation contains all the possible ordered pairs.

Therefore, $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, 1)\}$ is the largest equivalence relation.

Hence, the largest equivalence relation on the set $A = \{1, 2, 3\}$ is $\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, 1)\}$.

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Question 10:

Let R be the equivalence relation on the set Z of integers given by $R = \{(a, b)$: 3 divides $a-b\}$. Then the equivalence class [0] is equal to ____________________.

ANSWER:

Given: R is the equivalence relation on the set Z of integers given by $R = \{(a, b)$: 3 divides $a - b\}$.

To find the equivalence class [0], we put $b = 0$ in the given relation and find all the possible values of a.

Thus,

$R = \{(a, 0): 3 \text{ divides } a - 0\}$

$\Rightarrow a - 0$ is a multiple of 3

$\Rightarrow a$ is a multiple of 3

$\Rightarrow a = 3n$, where $n \in Z$

$\Rightarrow a = 0, \pm 3, \pm 6, \pm 9, ...$

Therefore, equivalence class $[0] = \{0, \pm 3, \pm 6, \pm 9, ...\}$

Hence, the equivalence class [0] is equal to $\{0, \pm 3, \pm 6, \pm 9, ...\}$.

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Question 11:

Let R be a relation on the set Z of all integers defined as $(x, y) \in R \Leftrightarrow x\text{-}y$ is divisible by 2. Then, the equivalence class [1] is ________________.

ANSWER:

Given: R is the equivalence relation on the set Z of integers defined as $(x, y) \in R \Leftrightarrow x - y$ is divisible by 2.

To find the equivalence class [1], we put $y = 1$ in the given relation and find all the possible values of x.

Thus,

R = {(x, 1): x − 1 is divisible by 2}

⇒ x − 1 is divisible by 2

⇒ x = ±1, ±3, ±6, ±9, ...

Therefore, equivalence class [0] = {±1, ±3, ±6, ±9, ...}

Hence, the equivalence class [1] is {±1, ±3, ±6, ±9, ...}.

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Question 12:

The relation R = {(1, 2,), (1, 3)} on set A = [1, 2, 3] is ________________ only.

ANSWER:

Given: A relation R on the set {1, 2, 3} be defined by R = R = {(1, 2,), (1, 3)}.

R = {(1, 2,), (1, 3)}

Since, (1, 1) ∉ R

Therefore, It is not reflexive.

Since, (1, 2) ∈ R but (2, 1) ∉ R

Therefore, It is not symmetric.

But there is no counter example to disapprove transitive condition.

Therefore, it is transitive.

Hence, The relation R = {(1, 2,), (1, 3)} on set A = {1, 2, 3} is transitive only.

Page No 1.33:

Question 1:

Write the domain of the relation R defined on the set Z of integers as follows:

$(a, b) \in R \Leftrightarrow a^2 + b^2 = 25$

ANSWER:

Domain of R is the set of values satisfying the relation R.

As a should be an integer, we get the given values *of a*:

0, ±3, ±4, ±5Thus,Domain of R={0, ±3, ±4, ±5}0, ±3, ±4, ±5Thus,Domain of R=0, ±3, ±4, ±5

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Question 2:

If $R = \{(x, y) : x^2 + y^2 \leq 4;\ x, y \in Z\}$ is a relation on Z, write the domain of R.

ANSWER:

Domain of R is the set of values of x satisfying the relation R.

As x must be an integer, we get the given values of x:

0, ±1, ±2Thus, Domain of R={0, ±1, ±2}0, ±1, ±2Thus, Domain of R=0, ±1, ±2

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Question 3:

Write the identity relation on set A = {a, b, c}.

ANSWER:

Identity set of A is

I = {(a, a), (b, b), (c, c)}

Every element of this relation is related to itself.

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Question 4:

Write the smallest reflexive relation on set A = {1, 2, 3, 4}.

ANSWER:

Here,

A = {1, 2, 3, 4}

Also, a relation is reflexive iff every element of the set is related to itself.

So, the smallest reflexive relation on the set A is

R = {(1, 1), (2, 2), (3, 3), (4, 4)}

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Question 5:

If R = {(x, y) : x + 2y = 8} is a relation on N by, then write the range of R.

ANSWER:

R = {(x, y) : x + 2y = 8, x, y∈∈N}

Then, the values of y can be 1, 2, 3 only.

Also, y = 4 cannot result in x = 0 because x is a natural number.

Therefore, range of R is {1, 2, 3}.

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Question 6:

If R is a symmetric relation on a set A, then write a relation between R and R^{-1}.

ANSWER:

Here, R is symmetric on the set A.

Let (a, b)∈R⇒(b, a)∈R [Since R is symmetric]⇒(a, b)∈R−1 [By definition of inverse relation]⇒R⊂R−1Let (x, y)∈R−1⇒(y, x)∈R [By definition of inverse relation]⇒(x, y)∈R [Since R is symmetric]⇒ R−1⊂RThus, R=R−1Let a, b∈R⇒b, a∈R Since R is symmetric⇒a, b∈R-1 By definition of inverse relation⇒R⊂R-1Let x, y∈R-1⇒y, x∈R By definition of inverse relation⇒x, y∈R Since R is symmetric⇒ R-1⊂RThus, R=R-1

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Question 7:

Let R = {(x, y) : |x^2 − y^2| <1) be a relation on set A = {1, 2, 3, 4, 5}. Write R as a set of ordered pairs.

ANSWER:

R is the set of ordered pairs satisfying the above relation. Also, no two different elements can satisfy the relation; only the same elements can satisfy the given relation.

So, R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)}

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Question 8:

If $A = \{2, 3, 4\}$, $B = \{1, 3, 7\}$ and $R = \{(x, y) : x \in A, y \in B \text{ and } x < y\}$ is a relation from A to B, then write R^{-1}.

ANSWER:

Since $R = \{(x, y) : x \in\in A, y \in\in A \text{ and } x < y\}$,

$R = \{(2, 3), (2, 7), (3, 7), (4, 7)\}$

So, $R^{-1} = \{(3, 2), (7, 2), (7, 3), (7, 4)\}$

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Question 9:

Let $A = \{3, 5, 7\}$, $B = \{2, 6, 10\}$ and R be a relation from A to B defined by $R = \{(x, y) : x$ and y are relatively prime$\}$. Then, write R and R^{-1}.

ANSWER:

$R = \{(x, y) : x$ and y are relatively prime$\}$

Then,

$R = \{(3, 2), (5, 2), (7, 2), (3, 10), (7, 10), (5, 6), (7, 6)\}$

So, $R^{-1} = \{(2, 3), (2, 5), (2, 7), (10, 3), (10, 7), (6, 5), (6, 7)\}$

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Question 10:

Define a reflexive relation.

ANSWER:

A relation *R* on *A* is said to be reflexive iff every element of *A* is related to itself.

i.e. *R* is reflexive ⇔(a, a)∈R for all a∈A⇔a, a∈R for all a∈A

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Question 11:

Define a symmetric relation.

ANSWER:

A relation *R* on a set *A* is said to be symmetric iff

(a, b)∈R⇒(b, a)∈R for all a, b∈Ai.e. aRb⇒bRa for all a, b∈Aa, b∈R⇒b, a∈R for all a, b∈Ai.e. aRb⇒bRa for all a, b∈A

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Question 12:

Define a transitive relation.

ANSWER:

A relation *R* on a set *A* is said to be transitive iff

(a, b)∈R and (b, c)∈R⇒(a, c)∈R for all a, b, c∈Ri.e. aRb and bRc⇒aRc for all a, b, c∈R a, b∈R and b, c∈R⇒a, c∈R for all a, b, c∈Ri.e. aRb and bRc⇒aRc for all a, b, c∈R

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Question 13:

Define an equivalence relation.

ANSWER:

A relation *R* on set *A* is said to be an equivalence relation iff

(i) it is reflexive,

(ii) it is symmetric and

(iii) it is transitive.

Relation *R* on set *A* satisfying all the above three properties is an equivalence relation.

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Question 14:

If *A* = {3, 5, 7} and *B* = {2, 4, 9} and *R* is a relation given by "is less than", write *R* as a set ordered pairs.

ANSWER:

Since, R={(x, y) : x, y∈N and x<y},R = {(3, 4), (3, 9), (5, 9), (7,9)}Since, R=x, y : x, y∈N and x<y,R = {(3, 4), (3, 9), (5, 9), (7,9)}

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Question 15:

A = {1, 2, 3, 4, 5, 6, 7, 8} and if *R* = {(*x*, *y*) : *y* is one half of *x*; *x*, *y* ∈ *A*} is a relation on *A*, then write *R* as a set of ordered pairs.

ANSWER:

Since *R* = {(*x*, *y*) : *y* is one half of *x*; *x*, *y*∈∈*A*}

So, R = {(2, 1), (4, 2), (6, 3), (8, 4)}

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Question 16:

Let A = {2, 3, 4, 5} and B = {1, 3, 4}. If R is the relation from A to B given by a R b if "a is a divisor of b". Write R as a set of ordered pairs.

ANSWER:

Since R = {(a, b) : a, b∈N : a is a divisor of b}Since R = a, b : a, b∈N : a is a divisor of b

So, R = {(2, 4), (3, 3), (4, 4)}

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Question 17:

State the reason for the relation R on the set {1, 2, 3} given by R = {(1, 2), (2, 1)} to be transitive.

ANSWER:

Since (1, 2)∈R, (2, 1)∈R but (1, 1)∉R, R is not transitive on the set {1, 2, 3}.For R to be in a transitive relation, we must have (1, 1)∈R.Since 1, 2∈R, 2, 1∈R but 1, 1∉R, R is not transitive on the set 1, 2, 3.For R to be in a transitive relation, we must have 1, 1∈R.

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Question 18:

Let R = {(a, a^3) : a is a prime number less than 5} be a relation. Find the range of R. [CBSE 2014]

ANSWER:

We have,

R = {(*a*, a^3) : *a* is a prime number less than 5}

Or,

R = {(2, 8), (3, 27)}

So, the range of *R* is {8, 27}.

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Question 19:

Let *R* be the equivalence relation on the set **Z** of the integers given by *R* = {(*a*, *b*) : 2 divides *a* −- *b*}. Write the equivalence class [0].

[NCERT EXEMPLAR]

ANSWER:

We have,

An equivalence relation, *R* = {(*a*, *b*) : 2 divides *a* −- *b*}

If b=0, then a−b=a−0=aAs, 2 divides a−bAnd, the set of integers which are divided by 2 is {0,±2,±4,±6,...}So, the equivalence class [0]={0,±2,±4,±6,...}If b=0, then a-b=a-0=aAs, 2 divides a-bAnd, the set of integers which are divided by 2 is 0,±2,±4,±6,...So, the equivalence class [0]=0,±2,±4,±6,...

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Question 20:

For the set *A* = {1, 2, 3}, define a relation *R* on the set *A* as follows:

R = {(1, 1), (2, 2), (3, 3), (1, 3)}

Write the ordered pairs to be added to R to make the smallest equivalence relation.

ANSWER:

We have,

R = {(1, 1), (2, 2), (3, 3), (1, 3)}

As, (a, a) ∈∈ R, for all values of a ∈∈ A

So, R is a reflexive relation

R can be a symmetric and transitive relation only when element (3, 1) is added

Hence, the ordered pairs to be added to R to make the smallest equivalence relation is (3, 1).

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Question 21:

Let A = {0, 1, 2, 3} and R be a relation on A defined as

R = {(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)}

Is R reflexive? symmetric? transitive?

ANSWER:

We have,

R = {(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)}

As, (a,a)∈R∀a∈ASo, R is a reflexive relationAlso, (a,b)∈R and (b,a)∈RSo, R is a symmetric relation as wellAnd, (0,1)∈R but (1,2)∉R and (2,3)∉RSo, R is not a transitive relationAs, a,a∈R∀a∈ASo, R is a reflexive relationAlso, a,b∈R and b,a∈RSo, R is a symmetric relation as wellAnd, 0,1∈R but 1,2∉R and 2,3∉RSo, R is not a transitive relation

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Question 22:

Let the relation R be defined on the set A = {1, 2, 3, 4, 5} by R = {(a, b) : |a^2 −- b^2| < 8}. Write R as a set of ordered pairs.

ANSWER:

As, R = {(a, b) : |a^2 −- b^2| < 8}

So, R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)}

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Question 23:

Let the relation R be defined on **N** by aRb iff 2a + 3b = 30. Then write R as a set of ordered pairs.

ANSWER:

As, R = {(a, b) : 2a + 3b = 30; a, b ∈∈ **N**}

So, R = {(3, 8), (6, 6), (9, 4), (12, 2)}

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Question 24:

Write the smallest equivalence relation on the set A = {1, 2, 3}.

ANSWER:

The smallest equivalence relation on the set A = {1, 2, 3} is R = {(1, 1), (2, 2), (3, 3)}

Chapterwise RD Sharma Solutions for Class 12 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.