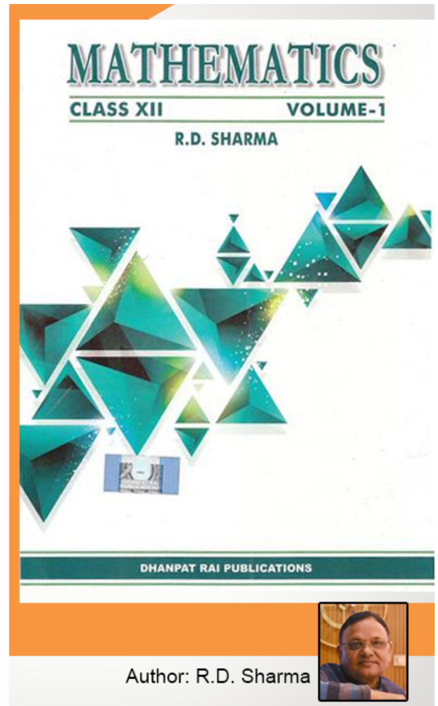
Class 12 - Chapter 17 Increasing and Decreasing Functions



RD Sharma Solutions for Class 12 Maths Chapter 17–Increasing and Decreasing Functions

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Exercise 17.1 Page No: 17.10

1. Prove that the function f(x) = $\log_e x$ is increasing on (0, ∞).

Solution:

Let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, f(x) is increasing in (0, ∞)

2. Prove that the function f(x) = $\log_a x$ is increasing on (0, ∞) if a > 1 and decreasing on (0, ∞), if 0 < a < 1.

Solution:

Case I

When $a > 1$

Let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, f(x) is increasing in $(0, \infty)$

Case II

When $0 < a < 1$

$f(x) = \log_a x = \frac{\log x}{\log a}$

When $a < 1 \Rightarrow \log a < 0$

Let $x_1 < x_2$

$\Rightarrow \log x_1 < \log x_2$

$\Rightarrow \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a}$ $[\because \log a < 0]$

$\Rightarrow f(x_1) > f(x_2)$

So, f(x) is decreasing in $(0, \infty)$

$\Rightarrow \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a}$ $[\because \log a < 0]$

$\Rightarrow f(x_1) > f(x_2)$

So, f(x) is decreasing in $(0, \infty)$

3. Prove that f(x) = ax + b, where a, b are constants and a > 0 is an increasing function on R.

Solution:

Given,

$f(x) = ax + b, a > 0$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow ax_1 > ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b > ax_2 + b$ for some b

$\Rightarrow f(x_1) > f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

So, f(x) is increasing function of R

4. Prove that f(x) = ax + b, where a, b are constants and a < 0 is a decreasing function on R.

Solution:

Given,

$f(x) = ax + b, a < 0$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow ax_1 < ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b < ax_2 + b$ for some b

$\Rightarrow f(x_1) < f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, f(x) is decreasing function of R

Exercise 17.2 Page No: 17.33

1. Find the intervals in which the following functions are increasing or decreasing.

(i) $f(x) = 10 - 6x - 2x^2$

Solution:

Given f (x) = 10 – 6x – 2x^2

By differentiating above equation we get,

$\Rightarrow f'(x) = \frac{d}{dx}(10 - 6x - 2x^2)$

$\Rightarrow f'(x) = -6 - 4x$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow -6 - 4x > 0$

$\Rightarrow -4x > 6$

$\Rightarrow x < -\frac{6}{4}$

$\Rightarrow x < -\frac{3}{2}$

$\Rightarrow x \in (-\infty, -\frac{3}{2})$

Thus f(x) is increasing on the interval $\left(-\infty, -\frac{3}{2}\right)$

Again, for f(x) to be increasing, we must have

$f'(x) < 0$

$\Rightarrow -6 - 4x < 0$

$\Rightarrow -4x < 6$

$\Rightarrow x > -\frac{6}{4}$

$\Rightarrow x > -\frac{3}{2}$

$\Rightarrow x \in (-\frac{3}{2}, \infty)$

Thus f(x) is decreasing on interval $x \in (-\frac{3}{2}, \infty)$

$\Rightarrow x > -\frac{3}{2}$

$\Rightarrow x \in (-\frac{3}{2}, \infty)$

Thus f(x) is decreasing on interval $x \in (-\frac{3}{2}, \infty)$

(ii) f (x) = x^2 + 2x – 5

Solution:

Given f (x) = $x^2 + 2x - 5$

Now by differentiating the given equation we get,

$\Rightarrow f'(x) = \frac{d}{dx}(x^2 + 2x - 5)$

$\Rightarrow f'(x) = 2x + 2$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow 2x + 2 > 0$

$\Rightarrow 2x < -2$

$\Rightarrow x < -\frac{2}{2}$

$\Rightarrow x < -1$

$\Rightarrow x \in (-\infty, -1)$

Thus f(x) is increasing on interval $(-\infty, -1)$

Again, for f(x) to be increasing, we must have

$f'(x) < 0$

$\Rightarrow 2x + 2 < 0$

$\Rightarrow 2x > -2$

$\Rightarrow x > -\frac{2}{2}$

$\Rightarrow x > -\frac{2}{2}$

$\Rightarrow x > -1$

$\Rightarrow x \in (-1, \infty)$

Thus f(x) is decreasing on interval $x \in (-1, \infty)$

(iii) f (x) = 6 – 9x – x^2

Solution:

Given f (x) = $6 - 9x - x^2$

$\Rightarrow f'(x) = \frac{d}{dx}(6 - 9x - x^2)$

$\Rightarrow f'(x) = -9 - 2x$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow -9 - 2x > 0$

$\Rightarrow -2x > 9$

$\Rightarrow x < -\frac{9}{2}$

$\Rightarrow x < -\frac{9}{2}$

$\Rightarrow x \in (-\infty, -\frac{9}{2})$

Thus f(x) is increasing on interval $\left(-\infty, -\frac{9}{2}\right)$

Again, for f(x) to be decreasing, we must have

$f'(x) < 0$

$\Rightarrow -9 - 2x < 0$

$\Rightarrow -2x < 9$

$\Rightarrow x > -\frac{9}{2}$

$\Rightarrow x > -\frac{9}{2}$

$\Rightarrow x \in (-\frac{9}{2}, \infty)$

Thus f(x) is decreasing on interval $x \in (-\frac{9}{2}, \infty)$

(iv) f(x) = $2x^3 - 12x^2 + 18x + 15$

Solution:

Given f (x) = $2x^3 - 12x^2 + 18x + 15$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 12x^2 + 18x + 15)$

$\Rightarrow f'(x) = 6x^2 - 24x + 18$

For f(x) we have to find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 24x + 18 = 0$

$\Rightarrow 6(x^2 - 4x + 3) = 0$

$\Rightarrow 6(x^2 - 3x - x + 3) = 0$

$\Rightarrow 6(x - 3)(x - 1) = 0$

$\Rightarrow (x - 3)(x - 1) = 0$

$\Rightarrow x = 3, 1$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$ and $f'(x) < 0$ if $1 < x < 3$

Thus, f(x) increases on $(-\infty, 1) \cup (3, \infty)$ and f(x) is decreasing on interval $x \in (1, 3)$

(v) f (x) = $5 + 36x + 3x^2 - 2x^3$

Solution:

Given f (x) = $5 + 36x + 3x^2 - 2x^3$

$\Rightarrow$

$f'(x) = \frac{d}{dx}(5 + 36x + 3x^2 - 2x^3)$

⇒ $f'(x) = 36 + 6x - 6x^2$

For f(x) now we have to find critical point, we must have

⇒ $f'(x) = 0$

⇒ $36 + 6x - 6x^2 = 0$

⇒ $6(-x^2 + x + 6) = 0$

⇒ $6(-x^2 + 3x - 2x + 6) = 0$

⇒ $-x^2 + 3x - 2x + 6 = 0$

⇒ $x^2 - 3x + 2x - 6 = 0$

⇒ $(x - 3)(x + 2) = 0$

⇒ $x = 3, -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$ and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, f(x) increases on $x \in (-2, 3)$ and f(x) is decreasing on interval $(-\infty, -2) \cup (3, \infty)$

(vi) $f(x) = 8 + 36x + 3x^2 - 2x^3$

Solution:

Given $f(x) = 8 + 36x + 3x^2 - 2x^3$

Now differentiating with respect to x

⇒

$$f'(x) = \frac{d}{dx}(8 + 36x + 3x^2 - 2x^3)$$

⇒ $f'(x) = 36 + 6x - 6x^2$

For f(x) we have to find critical point, we must have

⇒ $f'(x) = 0$

⇒ $36 + 6x - 6x^2 = 0$

⇒ $6(-x^2 + x + 6) = 0$

⇒ $6(-x^2 + 3x - 2x + 6) = 0$

⇒ $-x^2 + 3x - 2x + 6 = 0$

⇒ $x^2 - 3x + 2x - 6 = 0$

⇒ $(x - 3)(x + 2) = 0$

⇒ $x = 3, -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$ and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, f(x) increases on $x \in (-2, 3)$ and f(x) is decreasing on interval $(-\infty, 2) \cup (3, \infty)$

(vii) $f(x) = 5x^3 - 15x^2 - 120x + 3$

Solution:

Given $f(x) = 5x^3 - 15x^2 - 120x + 3$

Now by differentiating above equation with respect x, we get

⇒

$$f'(x) = \frac{d}{dx}(5x^3 - 15x^2 - 120x + 3)$$

⇒ $f'(x) = 15x^2 - 30x - 120$

For f(x) we have to find critical point, we must have

⇒ $f'(x) = 0$

⇒ $15x^2 - 30x - 120 = 0$

⇒ $15(x^2 - 2x - 8) = 0$

⇒ $15(x^2 - 4x + 2x - 8) = 0$

⇒ $x^2 - 4x + 2x - 8 = 0$

⇒ $(x - 4)(x + 2) = 0$

⇒ $x = 4, -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 4$ and $f'(x) < 0$ if $-2 < x < 4$

Thus, f(x) increases on $(-\infty,-2) \cup (4, \infty)$ and f(x) is decreasing on interval $x \in (-2, 4)$

(viii) $f(x) = x^3 - 6x^2 - 36x + 2$

Solution:

Given $f(x) = x^3 - 6x^2 - 36x + 2$

⇒

$$f'(x) = \frac{d}{dx}(x^3 - 6x^2 - 36x + 2)$$

⇒ $f'(x) = 3x^2 - 12x - 36$

For f(x) we have to find critical point, we must have

⇒ $f'(x) = 0$

⇒ $3x^2 - 12x - 36 = 0$

⇒ $3(x^2 - 4x - 12) = 0$

⇒ $3(x^2 - 6x + 2x - 12) = 0$

⇒ $x^2 - 6x + 2x - 12 = 0$

⇒ $(x - 6)(x + 2) = 0$

⇒ $x = 6, -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 6$ and $f'(x) < 0$ if $-2< x < 6$

Thus, f(x) increases on $(-\infty,-2) \cup (6, \infty)$ and f(x) is decreasing on interval $x \in (-2, 6)$

(ix) $f(x) = 2x^3 - 15x^2 + 36x + 1$

Solution:

Given $f(x) = 2x^3 - 15x^2 + 36x + 1$

Now by differentiating above equation with respect x, we get

⇒

$$f'(x) = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 1)$$

⇒ $f'(x) = 6x^2 - 30x + 36$

For f(x) we have to find critical point, we must have

⇒ $f'(x) = 0$

⇒ $6x^2 - 30x + 36 = 0$

⇒ $6(x^2 - 5x + 6) = 0$

⇒ $6(x^2 - 3x - 2x + 6) = 0$

⇒ $x^2 - 3x - 2x + 6 = 0$

⇒ $(x - 3)(x - 2) = 0$

⇒ $x = 3, 2$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 3$ and $f'(x) < 0$ if $2 < x < 3$

Thus, f(x) increases on $(-\infty, 2) \cup (3, \infty)$ and f(x) is decreasing on interval $x \in (2, 3)$

(x) $f(x) = 2x^3 + 9x^2 + 12x + 20$

Solution:

Given $f(x) = 2x^3 + 9x^2 + 12x + 20$

Differentiating above equation we get

⇒

$$f'(x) = \frac{d}{dx}(2x^3 + 9x^2 + 12x + 20)$$

⇒ $f'(x) = 6x^2 + 18x + 12$

For f(x) we have to find critical point, we must have

⇒ $f'(x) = 0$

⇒ $6x^2 + 18x + 12 = 0$

⇒ $6(x^2 + 3x + 2) = 0$

⇒ $6(x^2 + 2x + x + 2) = 0$

⇒ $x^2 + 2x + x + 2 = 0$

⇒ $(x + 2)(x + 1) = 0$

⇒ $x = -1, -2$

Clearly, $f'(x) > 0$ if $-2 < x < -1$ and $f'(x) < 0$ if $x < -1$ and $x > -2$

Thus, f(x) increases on $x \in (-2,-1)$ and f(x) is decreasing on interval $(-\infty, -2) \cup (-2, \infty)$

2. Determine the values of x for which the function $f(x) = x^2 - 6x + 9$ is increasing or decreasing. Also, find the coordinates of the point on the curve $y = x^2 - 6x + 9$ where the normal is parallel to the line $y = x + 5$.

Solution:

Given $f(x) = x^2 - 6x + 9$

⇒

$$f'(x) = \frac{d}{dx}(x^2 - 6x + 9)$$

⇒ $f'(x) = 2x - 6$

⇒ $f'(x) = 2(x - 3)$

For f(x) let us find critical point, we must have

⇒ $f'(x) = 0$

⇒ $2(x - 3) = 0$

⇒ $(x - 3) = 0$

$\Rightarrow x = 3$

Clearly, $f'(x) > 0$ if $x > 3$ and $f'(x) < 0$ if $x < 3$

Thus, f(x) increases on $(3, \infty)$ and f(x) is decreasing on interval $x \in (-\infty, 3)$

Now, let us find coordinates of point

Equation of curve is $f(x) = x^2 - 6x + 9$

Slope of this curve is given by

$\Rightarrow m_1 = \frac{dy}{dx}$

$\Rightarrow m_1 = \frac{d}{dx}(x^2 - 6x + 9)$

$\Rightarrow m_1 = 2x - 6$

Equation of line is $y = x + 5$

Slope of this curve is given by

$\Rightarrow m_2 = \frac{dy}{dx}$

$\Rightarrow m_2 = \frac{d}{dx}(x + 5)$

$\Rightarrow m_2 = 1$

Since slope of curve is parallel to line

Therefore, they follow the relation

$\Rightarrow \frac{-1}{m_1} = m_2$

$\Rightarrow \frac{-1}{2x-6} = 1$

$\Rightarrow 2x - 6 = -1$

$\Rightarrow x = \frac{5}{2}$

Thus putting the value of x in equation of curve, we get

$\Rightarrow y = x^2 - 6x + 9$

$\Rightarrow 2x - 6 = -1$

$\Rightarrow x = \frac{5}{2}$

Thus putting the value of x in equation of curve, we get

$\Rightarrow y = x^2 - 6x + 9$

$\Rightarrow y = (\frac{5}{2})^2 - 6\left(\frac{5}{2}\right) + 9$

$\Rightarrow y = \frac{25}{4} - 15 + 9$

$\Rightarrow y = \frac{25}{4} - 6$

$\Rightarrow y = \frac{1}{4}$

Thus the required coordinates is $(\frac{5}{2}, \frac{1}{4})$

3. Find the intervals in which f(x) = sin x – cos x, where 0 < x < 2π is increasing or decreasing.

Solution:

Given f (x) = sin x – cos x

$\Rightarrow f'(x) = \frac{d}{dx}(\sin x - \cos x)$

⇒ f′(x) = cos x + sin x

For f(x) let us find critical point, we must have

⇒ f′(x) = 0

⇒ Cos x + sin x = 0

⇒ Tan (x) = –1

$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Here these points divide the angle range from 0 to 2 π since we have x as angle

Clearly, f′(x) > 0 if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$ and f′(x) < 0 if $\frac{3\pi}{4} < x < \frac{7\pi}{4}$

Thus, f(x) increases on $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$ and f(x) is decreasing on interval $(\frac{3\pi}{4}, \frac{7\pi}{4})$

Clearly, f′(x) > 0 if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$ and f′(x) < 0 if $\frac{3\pi}{4} < x < \frac{7\pi}{4}$

Thus, f(x) increases on $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$ and f(x) is decreasing on interval $(\frac{3\pi}{4}, \frac{7\pi}{4})$

4. Show that f(x) = e^{2x} is increasing on R.

Solution:

Given f (x) = e^{2x}

⇒

$$f'(x) = \frac{d}{dx}(e^{2x})$$

⇒ $f'(x) = 2e^{2x}$

For f(x) to be increasing, we must have

⇒ $f'(x) > 0$

⇒ $2e^{2x} > 0$

⇒ $e^{2x} > 0$

Since, the value of e lies between 2 and 3

So, whatever be the power of e (that is x in domain R) will be greater than zero.

Thus f(x) is increasing on interval R

5. Show that f (x) = $e^{1/x}$, x ≠ 0 is a decreasing function for all x ≠ 0.

Solution:

Given $f(x) = e^{\frac{1}{x}}$

$\Rightarrow f'(x) = \frac{d}{dx}\left(e^{\frac{1}{x}}\right)$

$\Rightarrow f'(x) = e^{\frac{1}{x}}.\left(\frac{-1}{x^2}\right)$

$\Rightarrow f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$

As given $x \in R$, $x \neq 0$

$\Rightarrow \frac{1}{x^2} > 0$ and $e^{\frac{1}{x}} > 0$

Their ratio is also greater than 0

$\Rightarrow \frac{e^{\frac{1}{x}}}{x^2} > 0$

Their ratio is also greater than 0

$\Rightarrow \frac{e^{\frac{1}{x}}}{x^2} > 0$

$\Rightarrow -\frac{e^{\frac{1}{x}}}{x^2} < 0$; as by applying negative sign change in comparison sign

$\Rightarrow f'(x) < 0$

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing for all $x \neq 0$

6. Show that $f(x) = \log_a x$, $0 < a < 1$ is a decreasing function for all $x > 0$.

Solution:

Given f (x) = $\log_a x$, 0 < a < 1

$\Rightarrow f'(x) = \frac{d}{dx}(\log_a x)$

$\Rightarrow f'(x) = \frac{1}{x\log a}$

As given 0 < a < 1

⇒ log (a) < 0 and for x > 0

$\Rightarrow \frac{1}{x} > 0$

Therefore f'(x) is

$\Rightarrow \frac{1}{x\log a} < 0$

⇒ f'(x) < 0

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing for all x > 0

7. Show that f(x) = sin x is increasing on (0, π/2) and decreasing on (π/2, π) and neither increasing nor decreasing in (0, π).

Solution:

Given f(x) = sin x

$\Rightarrow f'(x) = \frac{d}{dx}(\sin x)$

$\Rightarrow f'(x) = \cos x$

Taking different region from 0 to 2π

Let $x \in (0, \frac{\pi}{2})$

$\Rightarrow \cos(x) > 0$

$\Rightarrow f'(x) > 0$

Thus f(x) is increasing in $(0, \frac{\pi}{2})$

Let $x \in (\frac{\pi}{2}, \pi)$

$\Rightarrow \cos(x) < 0$

$\Rightarrow f'(x) < 0$

Thus f(x) is decreasing in $(\frac{\pi}{2}, \pi)$

Therefore, from above condition we find that

$\Rightarrow$ f (x) is increasing in $(0, \frac{\pi}{2})$ and decreasing in $(\frac{\pi}{2}, \pi)$

Hence, condition for f(x) neither increasing nor decreasing in (0, π)

8. Show that f(x) = log sin x is increasing on (0, π/2) and decreasing on (π/2, π).

Solution:

Given f(x) = log sin x

$\Rightarrow f'(x) = \frac{d}{dx}(\log \sin x)$

$\Rightarrow f'(x) = \frac{1}{\sin x} \times \cos x$

$\Rightarrow f'(x) = \frac{1}{\sin x} \times \cos x$

$\Rightarrow f'(x) = \cot(x)$

Taking different region from 0 to π

Let $x \in (0, \frac{\pi}{2})$

$\Rightarrow \cot(x) > 0$

$\Rightarrow f'(x) > 0$

Thus f(x) is increasing in $(0, \frac{\pi}{2})$

Let $x \in (\frac{\pi}{2}, \pi)$

$\Rightarrow \cot(x) < 0$

$\Rightarrow f'(x) < 0$

Thus f(x) is decreasing in $(\frac{\pi}{2}, \pi)$

Hence proved

9. Show that f(x) = x – sin x is increasing for all x ϵ R.

Solution:

Given f (x) = x – sin x

⇒

$f'(x) = \frac{d}{dx}(x - \sin x)$

⇒ f′(x) = 1 – cos x

Now, as given x ϵ R

⇒ –1 < cos x < 1

⇒ $-1 > \cos x > 0$

⇒ $f'(x) > 0$

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval $x \in R$

10. Show that $f(x) = x^3 – 15x^2 + 75x – 50$ is an increasing function for all $x \in R$.

Solution:

Given $f(x) = x^3 – 15x^2 + 75x – 50$

⇒

$$f'(x) = \frac{d}{dx}(x^3 - 15x^2 + 75x - 50)$$

⇒ $f'(x) = 3x^2 – 30x + 75$

⇒ $f'(x) = 3(x^2 – 10x + 25)$

⇒ $f'(x) = 3(x – 5)^2$

Now, as given $x \in R$

⇒ $(x – 5)^2 > 0$

⇒ $3(x – 5)^2 > 0$

⇒ $f'(x) > 0$

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval $x \in R$

11. Show that $f(x) = \cos^2 x$ is a decreasing function on $(0, \pi/2)$.

Solution:

Given $f(x) = \cos^2 x$

⇒

$$f'(x) = \frac{d}{dx}(\cos^2 x)$$

⇒ f'(x) = 2 cos x (–sin x)

⇒ f'(x) = –2 sin (x) cos (x)

⇒ f'(x) = –sin2x

Now, as given x belongs to (0, π/2).

⇒ 2x ∈ (0,

π)

⇒ Sin (2x)> 0

⇒ –Sin (2x) < 0

⇒ f'(x) < 0

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing on interval (0, π/2).

Hence proved

12. Show that f(x) = sin x is an increasing function on (–π/2, π/2).

Solution:

Given f (x) = sin x

⇒

$$f'(x) = \frac{d}{dx}(\sin x)$$

⇒ f'(x) = cos x

Now, as given x ∈ (–π/2, π/2).

That is 4^{th} quadrant, where

⇒ Cos x> 0

⇒ f'(x) > 0

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval (–π/2, π/2).

13. Show that f(x) = cos x is a decreasing function on (0, π), increasing in (–π, 0) and neither increasing nor decreasing in (–π, π).

Solution:

Given f(x) = cos x

⇒

$$f'(x) = \frac{d}{dx}(\cos x)$$

⇒ f'(x) = –sin x

Taking different region from 0 to 2π

Let x ∈ (0, π).

⇒ Sin(x) > 0

⇒ –sin x < 0

⇒ f'(x) < 0

Thus f(x) is decreasing in (0, π)

Let x ∈ (–π, o).

⇒ Sin (x) < 0

⇒ –sin x > 0

⇒ f'(x) > 0

Thus f(x) is increasing in (–π, 0).

Therefore, from above condition we find that

⇒ f (x) is decreasing in (0, π) and increasing in (–π, 0).

Hence, condition for f(x) neither increasing nor decreasing in (–π, π)

14. Show that f(x) = tan x is an increasing function on (–π/2, π/2).

Solution:

Given f (x) = tan x

⇒

$$f'(x) = \frac{d}{dx}(\tan x)$$

⇒ $f'(x) = \sec^2 x$

Now, as given

$x \in (-\pi/2, \pi/2)$.

That is 4th quadrant, where

⇒ $\sec^2 x > 0$

⇒ $f'(x) > 0$

Hence, Condition for f(x) to be increasing

Thus f(x) is increasing on interval (–π/2, π/2).

15. Show that f(x) = $\tan^{-1}$ (sin x + cos x) is a decreasing function on the interval (π/4, π /2).

Solution:

Given f (x) = $\tan^{-1}$ (sin x + cos x)

$$\Rightarrow f'(x) = \frac{d}{dx}(\tan^{-1}(\sin x + \cos x))$$

$$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now, as given

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

⇒ Cos x – sin x < 0; as here cosine values are smaller than sine values for same angle

$$\Rightarrow \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0$$

⇒ f′(x) < 0

Hence, Condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

16. Show that the function f (x) = sin (2x + π/4) is decreasing on (3π/8, 5π/8).

Solution:

Given, $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$

$\Rightarrow f'(x) = \frac{d}{dx}\left\{\sin\left(2x + \frac{\pi}{4}\right)\right\}$

$\Rightarrow f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$

$\Rightarrow f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$

Now, as given $x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

$\Rightarrow \frac{3\pi}{8} < x < \frac{5\pi}{8}$

$\Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$

$\Rightarrow \pi < 2x + \frac{\pi}{4} < \frac{3\pi}{2};$

As here $2x + \frac{\pi}{4}$ lies in 3rd quadrant

$\Rightarrow \cos\left(2x + \frac{\pi}{4}\right) < 0$

$\Rightarrow 2\cos\left(2x + \frac{\pi}{4}\right) < 0$

$\Rightarrow f'(x) < 0$

Hence, condition for f(x) to be decreasing

Thus f (x) is decreasing on the interval (3π/8, 5π/8).

17. Show that the function f(x) = $\cot^{-1}$ (sin x + cos x) is decreasing on (0, π/4) and increasing on (π/4, π/2).

Solution:

Given f(x) = $\cot^{-1}$ (sin x + cos x)

$\Rightarrow f'(x) = \frac{d}{dx}\{\cot^{-1}(\sin x + \cos x)\}$

$\Rightarrow f'(x) = \frac{1}{1+(\sin x + \cos x)^2} \times (\cos x - \sin x)$

$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$

$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$

Now, as given $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

⇒ Cos x – sin x < 0; as here cosine values are smaller than sine values for same angle

$\Rightarrow \frac{\cos x - \sin x}{2(1+\sin x \cos x)} < 0$

⇒ f'(x) < 0

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

18. Show that f(x) = (x – 1) e^x + 1 is an increasing function for all x > 0.

Solution:

Given f (x) = (x – 1) e^x + 1

Now differentiating the given equation with respect to x, we get

⇒

$f'(x) = \frac{d}{dx}((x-1)e^x + 1)$

⇒ f'(x) = e^x + (x – 1) e^x

⇒ $f'(x) = e^x(1 + x - 1)$

⇒ $f'(x) = x e^x$

As given $x > 0$

⇒ $e^x > 0$

⇒ $x e^x > 0$

⇒ $f'(x) > 0$

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval $x > 0$

19. Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on (0, 1).

Solution:

Given $f(x) = x^2 - x + 1$

Now by differentiating the given equation with respect to x, we get

⇒

$$f'(x) = \frac{d}{dx}(x^2 - x + 1)$$

⇒ $f'(x) = 2x - 1$

Taking different region from (0, 1)

Let $x \in (0, ½)$

⇒ $2x - 1 < 0$

⇒ $f'(x) < 0$

Thus f(x) is decreasing in (0, ½)

Let $x \in (½, 1)$

⇒ $2x - 1 > 0$

⇒ f'(x) > 0

Thus f(x) is increasing in (½, 1)

Therefore, from above condition we find that

⇒ f (x) is decreasing in (0, ½) and increasing in (½, 1)

Hence, condition for f(x) neither increasing nor decreasing in (0, 1)

20. Show that f(x) = $x^9 + 4x^7 + 11$ is an increasing function for all x ϵ R.

Solution:

Given f (x) = $x^9 + 4x^7 + 11$

Now by differentiating above equation with respect to x, we get

⇒

$$f'(x) = \frac{d}{dx}(x^9 + 4x^7 + 11)$$

⇒ f'(x) = $9x^8 + 28x^6$

⇒ f'(x) = $x^6(9x^2 + 28)$

As given x ϵ R

⇒ $x^6 > 0$ and $9x^2 + 28 > 0$

⇒ $x^6 (9x^2 + 28) > 0$

⇒ f'(x) > 0

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval x ∈ R

Chapterwise RD Sharma Solutions for Class 12 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.