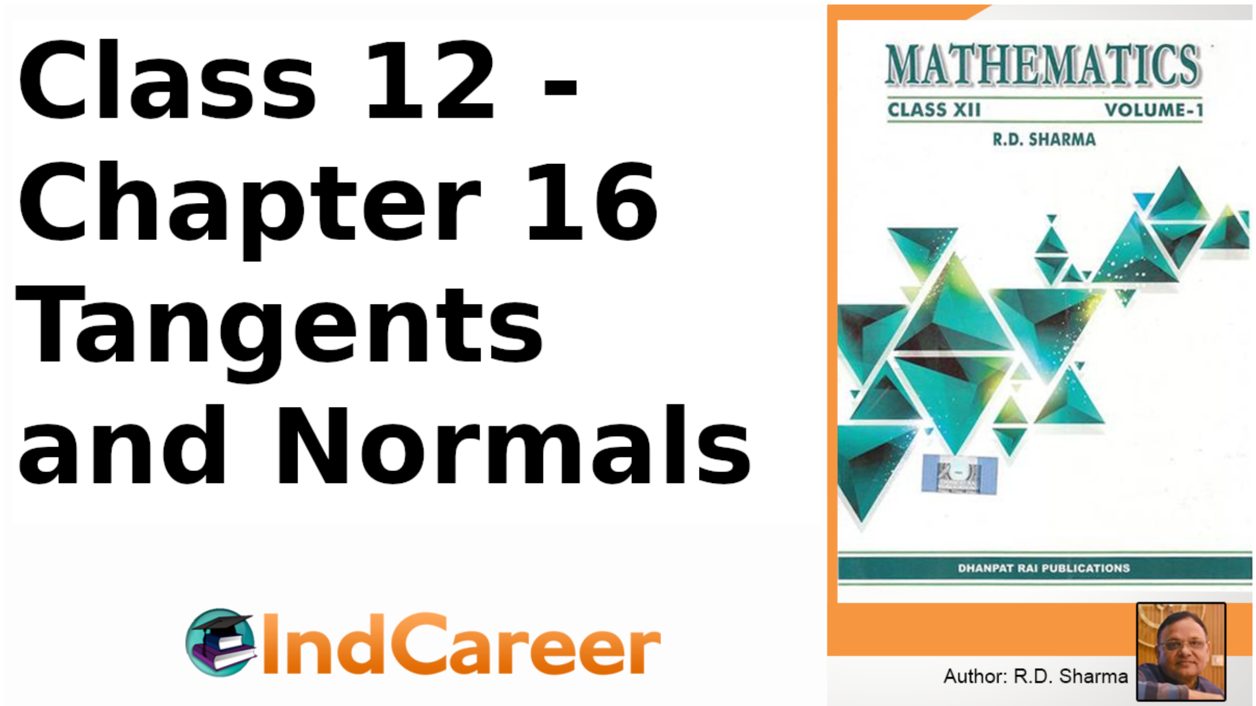


# RD Sharma Solutions for Class 12 Maths Chapter 16–Tangents and Normals

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## RD Sharma Solutions for Class 12 Maths Chapter 16–Tangents and Normals

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Exercise 16.1 Page No: 16.10

**1. Find the Slopes of the tangent and the normal to the following curves at the indicated points:**

(i) $y = \sqrt{x^3}$ at $x = 4$

**Solution:**

Given y = $\sqrt{x^3}$ at x = 4

First, we have to find $\frac{dy}{dx}$ of given function, f(x) that is to find the derivative of f (x)

y = $\sqrt{x^3}$

$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$

$\Rightarrow$ y = $(x^3)^{\frac{1}{2}}$

$\Rightarrow$ y = $(x)^{\frac{3}{2}}$

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

We know that the Slope of the tangent is $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{3}{2}(x)^{\frac{3}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2}(x)^{\frac{1}{2}}$$

Since, x = 4

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2}(4)^{\frac{1}{2}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times \sqrt{4}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times 2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times 2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = 3$$

The Slope of the tangent at x = 4 is 3

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{x=4}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{3}$$

**(ii) y = √x at x = 9**

**Solution:**

Given y = $\sqrt{x}$ at x = 9

First, we have to find $\frac{dy}{dx}$ of given function, f(x) that is to find the derivative of f(x)

$\Rightarrow y = \sqrt{x}$

$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$

$\Rightarrow y = (x)^{\frac{1}{2}}$

$\therefore \frac{dy}{dx}(x^n) = n\, x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = (x)^{\frac{1}{2}}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{-1}{2}}$$

Since, x = 9

$$\left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2}(9)^{\frac{-1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{-1}{2}}$$

Since, x = 9

$$\left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2}(9)^{\frac{-1}{2}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{(9)^{\frac{1}{2}}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{\sqrt{9}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{3}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{6}$$

The Slope of the tangent at x = 9 is $\frac{1}{6}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{x=9}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\frac{1}{6}}$

$\Rightarrow$ The Slope of the normal = – 6

**(iii) $y = x^3 - x$ at $x = 2$**

**Solution:**

First, we have to find $\frac{dy}{dx}$ of given function f(x) that is to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = x^3 - x$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dx}(x^3) + 3 \times \frac{dy}{dx}(x)$

$\Rightarrow \frac{dy}{dx} = 3.x^{3-1} - 1.x^{1-0}$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 1$

Since, x = 2

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 3 \times (2)^2 - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = (3 \times 4) - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 12 - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 11$

$\therefore$The Slope of the tangent at x = 2 is 11

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

⇒ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{x=2}}$

⇒ The Slope of the normal = $\frac{-1}{11}$

**(iv) $y = 2x^2 + 3 \sin x$ at $x = 0$**

**Solution:**

Given $y = 2x^2 + 3\sin x$ at $x = 0$

First, we have to find $\frac{dy}{dx}$ of given function f(x) that is to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = 2x^2 + 3\sin x$

$\Rightarrow \frac{dy}{dx} = 2\ dy/dx\ (x^2) + 3\ dy/dx\ (\sin x)$

$\Rightarrow \frac{dy}{dx} = 2\ (2x^{2-1}) + 3\ (\cos x)$

$\therefore \frac{d}{dx}\ (\text{Sin } x) = \cos x$

$\Rightarrow \frac{dy}{dx} = 4x + 3\cos x$

Since, x = 2

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 4\ (0) + 3\ \cos\ (0)$

We know cos (0) = 1

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 0 + 3\ (1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 3$

∴The Slope of the tangent at x = 0 is 3

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)x = 0}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{3}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{3}$

**(v) x = a (θ – sin θ), y = a (1 + cos θ) at θ = -π /2**

**Solution:**

Given x = a (θ - sin θ), y = a (1 + cos θ) at θ = -π /2

Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get our desired $\frac{dy}{dx}$.

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

$\Rightarrow x = a\,(\theta - \sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a\,\{\frac{dx}{d\theta}(\theta) - \frac{dx}{d\theta}(\sin\theta)\}$

$\Rightarrow \frac{dx}{d\theta} = a\,(1 - \cos\theta)$ ... (1)

$\therefore \frac{d}{dx}(\text{Sin } x) = \cos x$

$\Rightarrow y = a\,(1 + \cos\theta)$

$\Rightarrow \frac{dy}{d\theta} = a\,[\frac{dx}{d\theta}(1) + \frac{dx}{d\theta}(\cos\theta)]$

$\therefore \frac{d}{dx}(\text{Cos } x) = -\sin x$

$\therefore \frac{d}{dx}(\text{Constant}) = 0$

$\Rightarrow \frac{dy}{d\theta} = a\,(0 + (-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = a\,(-\sin\theta)$

$\Rightarrow \frac{dy}{d\theta} = -\,a \sin\theta$ ... (2)

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)}$

$\Rightarrow \frac{dy}{dx} = \frac{-\sin\theta}{(1-\cos\theta)}$

The Slope of the tangent is $\frac{-\sin\theta}{(1-\cos\theta)}$

Since, $\theta = \frac{-\pi}{2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{-\sin\frac{-\pi}{2}}{(1-\cos\frac{-\pi}{2})}$

We know Cos (π /2) = 0 and sin (π /2) = 1

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{-(-1)}{(1-(-0))}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{1}{(1-0)}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = 1$

∴The Slope of the tangent at x = $-\frac{\pi}{2}$ is 1

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{\theta = \frac{-\pi}{2}}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{1}$

$\Rightarrow$ The Slope of the normal = – 1

**(vi) x = a cos$^3$ θ, y = a sin$^3$ θ at θ = π /4**

**Solution:**

Given x = a $\cos^3\theta$, y = a $\sin^3\theta$ at $\theta = \pi/4$

Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get $\frac{dy}{dx}$.

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

$\Rightarrow x = a\cos^3\theta$

$\Rightarrow \frac{dx}{d\theta} = a\left(\frac{dx}{d\theta}(\cos^3\theta)\right)$

$\therefore \frac{d}{dx}(\text{Cos } x) = -\sin x$

$\Rightarrow \frac{dx}{d\theta} = a\,(3\cos^{3-1}\theta \times -\sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a\,(3\cos^2\theta \times -\sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$... (1)

$\Rightarrow y = a\sin^3\theta$

$\Rightarrow \frac{dy}{d\theta} = a\left(\frac{dy}{d\theta}(\sin^3\theta)\right)$

$\therefore \frac{d}{dx}(\text{Sin } x) = \cos x$

$\Rightarrow \frac{dy}{d\theta} = a\,(3\sin^{3-1}\theta\cos\theta)$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

The Slope of the tangent is $-\tan\theta$

Since, $\theta = \pi/4$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -\tan(\pi/4)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -1$$

We know $\tan(\pi/4) = 1$

The Slope of the tangent at $x = \pi/4$ is $-1$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{-1}$$

$$\Rightarrow \text{The Slope of the normal} = 1$$

$$\Rightarrow \frac{dy}{d\theta} = a(3\sin^2\theta\cos\theta)$$

$$\Rightarrow \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta \ldots (2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}$$

**(vii) x = a (θ – sin θ), y = a (1 – cos θ) at θ = π /2**

**Solution:**

Given x = a (θ - sin θ), y = a (1 - cos θ) at θ = π /2

Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get $\frac{dy}{dx}$.

$\therefore \frac{dy}{dx}(x^n) = n\, x^{n-1}$

$\Rightarrow x = a\,(\theta - \sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a\,\{\frac{dx}{d\theta}(\theta) - \frac{dx}{d\theta}(\sin\theta)\}$

$\Rightarrow \frac{dx}{d\theta} = a\,(1 - \cos\theta)$ ... (1)

$\therefore \frac{d}{dx}(\text{Sin } x) = \cos x$

$\Rightarrow y = a\,(1 - \cos\theta)$

$\Rightarrow \frac{dy}{d\theta} = a\,(\frac{dx}{d\theta}(1) - \frac{dx}{d\theta}(\cos\theta))$

$\therefore \frac{d}{dx}(\text{Cos } x) = -\sin x$

$\therefore \frac{d}{dx}(\text{Constant}) = 0$

$\Rightarrow \frac{dy}{d\theta} = a\,(\theta - (-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = a \sin\theta$ ... (2)

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$

$\Rightarrow \frac{dy}{dx} = \frac{-\sin\theta}{(1-\cos\theta)}$

The Slope of the tangent is $\frac{-\sin\theta}{(1-\cos\theta)}$

Since, $\theta = \frac{\pi}{2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{\sin\frac{\pi}{2}}{(1-\cos\frac{\pi}{2})}$

We know cos (π /2) = 0 and sin (π /2) = 1

We know cos (π /2) = 0 and sin (π /2) = 1

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{(1)}{(1-(-0))}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{1}{(1-0)}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 1$

The Slope of the tangent at x = $\frac{\pi}{2}$ is 1

⇒ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

⇒ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}}$

⇒ The Slope of the normal = $\frac{-1}{1}$

⇒ The Slope of the normal = – 1

**(viii) y = (sin 2x + cot x + 2)$^2$ at x = π /2**

**Solution:**

Given y = (sin 2x + cot x + 2)$^2$ at x = π /2

First, we have to find $\frac{dy}{dx}$ of given function f(x) that is to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = (\sin 2x + \cot x + 2)^2$

$\frac{dy}{dx} = 2 \times (\sin 2x + \cot x + 2)^{2-1}\left\{\frac{dy}{dx}(\sin 2x) + \frac{dy}{dx}(\cot x) + \frac{dy}{dx}(2)\right\}$

$\frac{dy}{dx} = 2 \times (\sin 2x + \cot x + 2)^{2-1}\left\{\frac{dy}{dx}(\sin 2x) + \frac{dy}{dx}(\cot x) + \frac{dy}{dx}(2)\right\}$

$\Rightarrow \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)\ \{(\cos 2x) \times 2 + (-\operatorname{cosec}^2 x) + (0)\}$

$\therefore \frac{d}{dx}(\operatorname{Sin} x) = \cos x$

$\therefore \frac{d}{dx}(\operatorname{Cot} x) = -\operatorname{cosec}^2 x$

$\Rightarrow \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)\ (2 \cos 2x - \operatorname{cosec}^2 x)$

Since, $x = \pi/2$

$\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 2 \times (\sin 2\ (\pi/2) + \cot (\pi/2) + 2)\ (2 \cos 2\ (\pi/2) - \operatorname{cosec}^2 (\pi/2))$

$\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 2 \times (\sin (\pi) + \cot (\pi/2) + 2) \times (2\cos (\pi) - \operatorname{cosec}^2 (\pi/2))$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 2 \times (0 + 0 + 2) \times (2(-1) - 1)$

We know $\sin (\pi) = 0$, $\cos (\pi) = -1$

$\operatorname{Cot} (\pi/2) = 0$, $\operatorname{cosec} (\pi/2) = 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 2\ (2) \times (-2 - 1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 4 \times -3$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = -12$

The Slope of the tangent at $x = \frac{\pi}{2}$ is $-12$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{-12}$

$\Rightarrow$ The Slope of the normal $= \frac{1}{12}$

**(ix) $x^2 + 3y + y^2 = 5$ at (1, 1)**

**Solution:**

Given $x^2 + 3y + y^2 = 5$ at (1, 1)

Here we have to differentiate the above equation with respect to x.

$\Rightarrow \frac{d}{dx}(x^2 + 3y + y^2) = \frac{d}{dx}(5)$

$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(3y) + \frac{d}{dx}(y^2) = \frac{d}{dx}(5)$

$\therefore \frac{dy}{dx}(x^n) = n\,x^{n-1}$

$\Rightarrow 2x + 3 \times \frac{dy}{dx} + 2y \times \frac{dy}{dx} = 0$

$\Rightarrow 2x + \frac{dy}{dx}(3 + 2y) = 0$

$\Rightarrow \frac{dy}{dx}(3 + 2y) = -2x$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{(3 + 2y)}$

The Slope of the tangent at (1, 1) is

$$\Rightarrow \frac{dy}{dx} = \frac{-2 \times 1}{(3 + 2 \times 1)}$$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{(3 + 2)}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{5}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{5}$

The Slope of the tangent at (1, 1) is $\frac{-2}{5}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\frac{-2}{5}}$

$\Rightarrow$ The Slope of the normal = $\frac{5}{2}$

**(x) x y = 6 at (1, 6)**

**Solution:**

Given xy = 6 at (1, 6)

Here we have to use the product rule for above equation, then we get

$\frac{d}{dx}(x\,y) = \frac{d}{dx}(6)$

$\Rightarrow x \times \frac{d}{dx}(y) + y\frac{d}{dx}(x) = \frac{d}{dx}(5)$

$\therefore \frac{d}{dx}$ (Constant) = 0

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow x\frac{dy}{dx} = -y$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

The Slope of the tangent at (1, 6) is

$\Rightarrow \frac{dy}{dx} = \frac{-6}{1}$

$\Rightarrow \frac{dy}{dx} = -6$

The Slope of the tangent at (1, 6) is – 6

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{-6}$

$\Rightarrow$ The Slope of the normal = $\frac{1}{6}$

**2. Find the values of a and b if the Slope of the tangent to the curve x y + a x + by = 2 at (1, 1) is 2.**

**Solution:**

Given the Slope of the tangent to the curve xy + ax + by = 2 at (1, 1) is 2

First, we will find The Slope of tangent by using product rule, we get

$\Rightarrow$ x y + ax + by = 2

$\Rightarrow x\frac{d}{dx}(y) + y\frac{d}{dx}(x) + a\frac{d}{dx}(x) + b\frac{d}{dx}(y) + = \frac{d}{dx}(2)$

$\Rightarrow x\frac{dy}{dx} + y + a + b\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(x + b) + y + a = 0$

$\Rightarrow \frac{dy}{dx}(x + b) = -(a + y)$

$\Rightarrow \frac{dy}{dx} = \frac{-(a+y)}{x+b}$

$\Rightarrow \frac{dy}{dx} = \frac{-(a+y)}{x+b}$

Since, the Slope of the tangent to the curve xy + ax + by = 2 at (1, 1) is 2 that is, $\frac{dy}{dx} = 2$

$\Rightarrow \left\{\frac{-(a+y)}{x+b}\right\}_{(x=1,\, y=1)} = 2$

$\Rightarrow \frac{-(a+1)}{1+b} = 2$

$\Rightarrow$ – a – 1 = 2(1 + b)

$\Rightarrow$ – a – 1 = 2 + 2b

⇒ $a + 2b = -3$ … (1)

Also, the point (1, 1) lies on the curve $xy + ax + by = 2$, we have

$1 \times 1 + a \times 1 + b \times 1 = 2$

⇒ $1 + a + b = 2$

⇒ $a + b = 1$ … (2)

From (1) & (2), we get $b = -4$

Substitute $b = -4$ in $a + b = 1$

$a - 4 = 1$

⇒ $a = 5$

So the value of $a = 5$ & $b = -4$

**3. If the tangent to the curve $y = x^3 + a x + b$ at (1, – 6) is parallel to the line $x - y + 5 = 0$, find a and b**

**Solution:**

Given the Slope of the tangent to the curve $y = x^3 + ax + b$ at $(1, -6)$

First, we will find the slope of tangent

$y = x^3 + ax + b$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(ax) + \frac{d}{dx}(b)$$

$$\Rightarrow \frac{dy}{dx} = 3x^{3-1} + a\left(\frac{dx}{dx}\right) + 0$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 + a$$

The Slope of the tangent to the curve $y = x^3 + ax + b$ at $(1, -6)$ is

$$\Rightarrow \frac{dy}{dx}_{(x=1, y=-6)} = 3(1)^2 + a$$

$$\Rightarrow \frac{dy}{dx}_{(x=1, y=-6)} = 3 + a \ldots (1)$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 + a$$

The Slope of the tangent to the curve $y = x^3 + ax + b$ at $(1, -6)$ is

$$\Rightarrow \frac{dy}{dx}_{(x=1, y=-6)} = 3(1)^2 + a$$

$$\Rightarrow \frac{dy}{dx}_{(x=1, y=-6)} = 3 + a \ldots (1)$$

The given line is $x - y + 5 = 0$

$y = x + 5$ is the form of equation of a straight line $y = mx + c$, where m is the Slope of the line.

So the slope of the line is $y = 1 \times x + 5$

So the Slope is 1. … (2)

Also the point (1, – 6) lie on the tangent, so

x = 1 & y = – 6 satisfies the equation, $y = x^3 + ax + b$

$– 6 = 1^3 + a × 1 + b$

⇒ – 6 = 1 + a + b

⇒ a + b = – 7 … (3)

Since, the tangent is parallel to the line, from (1) & (2)

Hence, 3 + a = 1

⇒ a = – 2

From (3)

a + b = – 7

⇒ – 2 + b = – 7

⇒ b = – 5

So the value is a = – 2 & b = – 5

**4. Find a point on the curve $y = x^3 – 3x$ where the tangent is parallel to the chord joining (1, – 2) and (2, 2).**

**Solution:**

Given curve $y = x^3 - 3x$

First, we will find the Slope of the tangent

$y = x^3 - 3x$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(3x)$$

$$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 3\left(\frac{dx}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 3 \ldots (1)$$

$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(3x)$

$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 3(\frac{dx}{dx})$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 3$ ... (1)

The equation of line passing through $(x_0, y_0)$ and The Slope m is $y - y_0 = m(x - x_0)$.

So the Slope, $m = \frac{y - y_0}{x - x_0}$

The Slope of the chord joining (1, – 2) & (2, 2)

$\Rightarrow \frac{dy}{dx} = \frac{2-(-2)}{2-1}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{1}$

$\Rightarrow \frac{dy}{dx} = 4$ ... (2)

From (1) & (2)

$3x^2 - 3 = 4$

$\Rightarrow 3x^2 = 7$

$\Rightarrow x^2 = \frac{7}{3}$

$$\Rightarrow x = \pm\sqrt{\frac{7}{3}}$$

$y = x^3 - 3x$

$$\Rightarrow y = x(x^2 - 3)$$

$$\Rightarrow y = \pm\sqrt{\frac{7}{3}}\left(\left(\pm\sqrt{\frac{7}{3}}\right)^2 - 3\right)$$

$$\Rightarrow y = \pm\sqrt{\frac{7}{3}}\left(\left(\frac{7}{3} - 3\right)\right)$$

$$\Rightarrow y = \pm\sqrt{\frac{7}{3}}\left(\frac{-2}{3}\right)$$

$$\Rightarrow y = \mp\left(\frac{-2}{3}\right)\sqrt{\frac{7}{3}}$$

Thus, the required point is $x = \pm\sqrt{\frac{7}{3}}$ & $y = \mp\left(\frac{-2}{3}\right)\sqrt{\frac{7}{3}}$

**5. Find a point on the curve $y = x^3 – 2x^2 – 2x$ at which the tangent lines are parallel to the line $y = 2x – 3$.**

**Solution:**

Given the curve $y = x^3 – 2x^2 – 2x$ and a line $y = 2x – 3$

First, we will find the slope of tangent

$y = x^3 – 2x^2 – 2x$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(2x^2) - \frac{d}{dx}(2x)$$

$$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 2 \times 2\,(x^{2-1}) - 2 \times x^{1-1}$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 4x - 2 \ldots (1)$$

$y = 2x – 3$ is the form of equation of a straight line $y = mx + c$, where m is the Slope of the line.

So the slope of the line is $y = 2 \times (x) – 3$

Thus, the Slope = 2. … (2)

From (1) & (2)

$\Rightarrow 3x^2 - 4x - 2 = 2$

$\Rightarrow 3x^2 - 4x = 4$

$\Rightarrow 3x^2 - 4x - 4 = 0$

We will use factorization method to solve the above Quadratic equation.

$\Rightarrow 3x^2 - 6x + 2x - 4 = 0$

$\Rightarrow 3\,x\,(x - 2) + 2\,(x - 2) = 0$

$\Rightarrow (x - 2)\,(3x + 2) = 0$

$\Rightarrow (x - 2) = 0\ \&\ (3x + 2) = 0$

$\Rightarrow x = 2$ or

$x = -2/3$

Substitute $x = 2$ & $x = -2/3$ in $y = x^3 - 2x^2 - 2x$

When $x = 2$

$\Rightarrow y = (2)^3 - 2 \times (2)^2 - 2 \times (2)$

$\Rightarrow y = 8 - (2 \times 4) - 4$

$\Rightarrow y = 8 - 8 - 4$

$\Rightarrow y = -4$

When $x = \frac{-2}{3}$

$\Rightarrow y = (\frac{-2}{3})^3 - 2 \times (\frac{-2}{3})^2 - 2 \times (\frac{-2}{3})$

$\Rightarrow y = (\frac{-8}{27}) - 2 \times (\frac{4}{9}) + (\frac{4}{3})$

$\Rightarrow y = (\frac{-8}{27}) - (\frac{8}{9}) + (\frac{4}{3})$

Taking LCM

$\Rightarrow y = \frac{(-8\times1)-(8\times3)+(4\times9)}{27}$

$\Rightarrow y = \frac{-8-24+36}{27}$

$\Rightarrow y = \frac{4}{27}$

Thus, the points are $(2, -4)$ & $(\frac{-2}{3}, \frac{4}{27})$

**6. Find a point on the curve $y^2 = 2x^3$ at which the Slope of the tangent is 3**

**Solution:**

Given the curve $y^2 = 2x^3$ and the Slope of tangent is 3

$y^2 = 2x^3$

Differentiating the above with respect to x

$\Rightarrow 2y^{2-1}\frac{dy}{dx} = 2 \times 3x^{3-1}$

$\Rightarrow y\frac{dy}{dx} = 3x^2$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{y}$

Since, The Slope of tangent is 3

$\frac{3x^2}{y} = 3$

$\Rightarrow \frac{x^2}{y} = 1$

$\Rightarrow x^2 = y$

Substituting $x^2 = y$ in $y^2 = 2x^3$,

$(x^2)^2 = 2x^3$

$x^4 - 2x^3 = 0$

$x^3(x - 2) = 0$

$x^3 = 0$ or $(x - 2) = 0$

$x = 0$ or $x = 2$

If $x = 0$

$\Rightarrow \frac{dy}{dx} = \frac{3(0)^2}{y}$

dy/dx = 0 which is not possible.

So we take x = 2 and substitute it in $y^2 = 2x^3$, we get

$y^2 = 2(2)^3$

$y^2 = 2 \times 8$

$y^2 = 16$

$y = 4$

Thus, the required point is (2, 4)

**7. Find a point on the curve x y + 4 = 0 at which the tangents are inclined at an angle of 45° with the x–axis.**

**Solution:**

Given the curve is xy + 4 = 0

If a tangent line to the curve y = f(x) makes an angle θ with x – axis in the positive direction, then

$\frac{dy}{dx}$ = The Slope of the tangent = tan θ

xy + 4 = 0

Differentiating the above with respect to x

$\Rightarrow x\frac{d}{dx}(y) + y\frac{d}{dx}(x) + \frac{d}{dx}(4) = 0$

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow x\frac{dy}{dx} = -y$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$ ... (1)

Also, $\frac{dy}{dx} = \tan 45° = 1$ ... (2)

From (1) & (2), we get,

$\Rightarrow \frac{-y}{x} = 1$

$\Rightarrow x = -y$

Substitute in xy + 4 = 0, we get

$\Rightarrow x(-x) + 4 = 0$

$\Rightarrow -x^2 + 4 = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x =$

$\pm 2$

So when x = 2, y = – 2

And when x = – 2, y = 2

Thus, the points are (2, – 2) & (– 2, 2)

**8. Find a point on the curve $y = x^2$ where the Slope of the tangent is equal to the x – coordinate of the point.**

**Solution:**

Given the curve is $y = x^2$

$y = x^2$

Differentiating the above with respect to x

$$\Rightarrow \frac{dy}{dx} = 2x^{2-1}$$

$$\Rightarrow \frac{dy}{dx} = 2x \ldots (1)$$

Also given the Slope of the tangent is equal to the x – coordinate,

$$\frac{dy}{dx} = x \ldots (2)$$

From (1) & (2), we get,

2x = x

⇒ x = 0.

Substituting this in $y = x^2$, we get,

$y = 0^2$

⇒ y = 0

Thus, the required point is (0, 0)

**9. At what point on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to x – axis.**

**Solution:**

Given the curve is $x^2 + y^2 - 2x - 4y + 1 = 0$

Differentiating the above with respect to x

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$

$\Rightarrow 2x^{2-1} + 2y^{2-1} \times \frac{dy}{dx} - 2 - 4 \times \frac{dy}{dx} + 0 = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(2y - 4) = -2x + 2$

$\Rightarrow \frac{dy}{dx} = \frac{-2(x-1)}{2(y-2)}$

$\Rightarrow \frac{dy}{dx} = \frac{-(x-1)}{(y-2)}$ ... (1)

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

Since, the tangent is parallel to x – axis

$\Rightarrow \frac{dy}{dx} = \tan(0) = 0$ ... (2)

Because $\tan(0) = 0$

From (1) & (2), we get,

$\Rightarrow \frac{-(x-1)}{(y-2)} = 0$

$\Rightarrow \frac{dy}{dx}(2y - 4) = -2x + 2$

$\Rightarrow \frac{dy}{dx} = \frac{-2(x-1)}{2(y-2)}$

$\Rightarrow \frac{dy}{dx} = \frac{-(x-1)}{(y-2)}$ ... (1)

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = tan θ

Since, the tangent is parallel to x – axis

$\Rightarrow \frac{dy}{dx}$ = tan (0) = 0 ... (2)

Because tan (0) = 0

From (1) & (2), we get,

$\Rightarrow \frac{-(x-1)}{(y-2)} = 0$

⇒ – (x – 1) = 0

⇒ x = 1

Substituting x = 1 in $x^2 + y^2 - 2x - 4y + 1 = 0$, we get,

⇒ $1^2 + y^2 - 2(1) - 4y + 1 = 0$

⇒ $1 - y^2 - 2 - 4y + 1 = 0$

⇒ $y^2 - 4y = 0$

⇒ y (y – 4) = 0

⇒ y = 0 and y = 4

Thus, the required point is (1, 0) and (1, 4)

**10. At what point of the curve $y = x^2$ does the tangent make an angle of 45° with the x–axis?**

**Solution:**

Given the curve is $y = x^2$

Differentiating the above with respect to x

$\Rightarrow y = x^2$

$\Rightarrow \frac{dy}{dx} = 2x^{2-1}$

$\Rightarrow \frac{dy}{dx} = 2x$ ... (1)

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = tan θ

Since, the tangent make an angle of 45° with x – axis

$\Rightarrow \frac{dy}{dx}$ = tan (45°) = 1 ... (2)

Because tan (45°) = 1

From (1) & (2), we get,

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

Substituting $x = \frac{1}{2}$ in $y = x^2$, we get,

$\Rightarrow y = (\frac{1}{2})^2$

$\Rightarrow y = \frac{1}{4}$

Thus, the required point is $(\frac{1}{2}, \frac{1}{4})$

Exercise 16.2 Page No: 15.27

**1. Find the equation of the tangent to the curve √x + √y = a, at the point ($a^2/4$, $a^2/4$).**

**Solution:**

Given √x + √y = a

To find the slope of the tangent of the given curve we have to differentiate the given equation

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{x}}{\sqrt{y}}$$

At $\left(\frac{a^2}{4}, \frac{a^2}{4}\right)$ slope m, is – 1

The equation of the tangent is given by $y - y_1 = m(x - x_1)$

$$y - \frac{a^2}{4} = -1\left(x - \frac{a^2}{4}\right)$$

$$x + y = \frac{a^2}{2}$$

**2. Find the equation of the normal to y = 2x³ – x² + 3 at (1, 4).**

**Solution:**

Given $y = 2x^3 - x^2 + 3$

By differentiating the given curve, we get the slope of the tangent

$$m = \frac{dy}{dx} = 6x^2 - 2x$$

m = 4 at (1, 4)

Normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(\text{normal}) = -\frac{1}{4}$$

Equation of normal is given by $y - y_1 = m\ (\text{normal})\ (x - x_1)$

$$y - 4 = \left(-\frac{1}{4}\right)(x - 1)$$

x + 4y = 17

$$m(\text{normal}) = -\frac{1}{4}$$

Equation of normal is given by $y - y_1 = m\ (\text{normal})\ (x - x_1)$

$$y - 4 = \left(-\frac{1}{4}\right)(x - 1)$$

x + 4y = 17

**3. Find the equation of the tangent and the normal to the following curves at the indicated points:**

**(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5)**

**Solution:**

Given $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5)

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

m (tangent) at (0, 5) = – 10

$$m(\text{normal}) \text{ at } (0,5) = \frac{1}{10}$$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$y - 5 = -10x$

$y + 10x = 5$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y - 5 = \frac{1}{10}x$$

**(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $x = 1$ $y = 3$**

**Solution:**

Given $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $x = 1$ $y = 3$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

m (tangent) at (x = 1) = 2

Normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(\text{normal}) \text{ at } (x = 1) = -\frac{1}{2}$$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$y - 3 = 2(x - 1)$

$y = 2x + 1$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y - 3 = -\frac{1}{2}(x - 1)$$

$$2y = 7 - x$$

**(iii) y = $x^2$ at (0, 0)**

**Solution:**

Given $y = x^2$ at (0, 0)

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 2x$$

m (tangent) at (x = 0) = 0

Normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(\text{normal}) \text{ at } (x = 0) = \frac{1}{0}$$

We can see that the slope of normal is not defined

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$y = 0$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$x = 0$

**(iv) $y = 2x^2 - 3x - 1$ at $(1, -2)$**

**Solution:**

Given $y = 2x^2 - 3x - 1$ at $(1, -2)$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 4x - 3$$

m (tangent) at $(1, -2) = 1$

Normal is perpendicular to tangent so, $m_1m_2 = -1$

m (normal) at $(1, -2) = -1$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$y + 2 = 1(x - 1)$

$y = x - 3$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$y + 2 = -1(x - 1)$

$y + x + 1 = 0$

$$(v)\ y^2 = \frac{x^3}{4-x}$$

**Solution:**

By differentiating the given curve, we get the slope of the tangent

$$2y\frac{dy}{dx} = \frac{(4-x)3x^2 + x^4}{(4-x)^2}$$

$$\frac{dy}{dx} = \frac{(4-x)3x^2 + x^4}{2y(4-x)^2}$$

m (tangent) at (2, – 2) = – 2

$$m(\text{normal}) \text{ at } (2,-2) = \frac{1}{2}$$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

y + 2 = – 2(x – 2)

y + 2x = 2

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y + 2 = \frac{1}{2}(x-2)$$

2y + 4 = x – 2

2y – x + 6 = 0

**4. Find the equation of the tangent to the curve x = θ + sin θ, y = 1 + cos θ at θ = π/4.**

**Solution:**

Given x = θ + sin θ, y = 1 + cos θ at θ = π/4

By differentiating the given equation with respect to θ, we get the slope of the tangent

$$\frac{dx}{d\theta} = 1 + \cos\theta$$

$$\frac{dy}{d\theta} = -\sin\theta$$

Dividing both the above equations

$$\frac{dy}{dx} = -\frac{\sin\theta}{1 + \cos\theta}$$

m at θ = ( $\pi/4$) = $-1 + \frac{1}{\sqrt{2}}$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$$y - 1 - \frac{1}{\sqrt{2}} = \left(-1 + \frac{1}{\sqrt{2}}\right)\left(x - \frac{\pi}{4} - \frac{1}{\sqrt{2}}\right)$$

**5. Find the equation of the tangent and the normal to the following curves at the indicated points:**

**(i) x = θ + sin θ, y = 1 + cos θ at θ = π/2**

**Solution:**

Given x = θ + sin θ, y = 1 + cos θ at θ = π/2

By differentiating the given equation with respect to θ, we get the slope of the tangent

$$\frac{dx}{d\theta} = 1 + \cos\theta$$

$$\frac{dy}{d\theta} = -\sin\theta$$

Dividing both the above equations

$$\frac{dy}{dx} = -\frac{\sin\theta}{1 + \cos\theta}$$

m (tangent) at $\theta = (\pi/2) = -1$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

m (normal) at $\theta = (\pi/2) = 1$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$$y - 1 = -1\left(x - \frac{\pi}{2} - 1\right)$$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y - 1 = 1\left(x - \frac{\pi}{2} - 1\right)$$

$$(ii)\ x = \frac{2at^2}{1+t^2},\ y = \frac{2at^3}{1+t^2}\ at\ t = \frac{1}{2}$$

**Solution:**

By differentiating the given equation with respect to t, we get the slope of the tangent

$$\frac{dx}{dt} = \frac{(1 + t^2)4at - 2at^2(2t)}{(1 + t^2)^2}$$

$$\frac{dx}{dt} = \frac{4at}{(1 + t^2)^2}$$

$$\frac{dy}{dt} = \frac{(1 + t^2)6at^2 - 2at^3(2t)}{(1 + t^2)^2}$$

$$\frac{dy}{dt} = \frac{6at^2 + 2at^4}{(1 + t^2)^2}$$

$$\frac{dy}{dt} = \frac{6at^2 + 2at^4}{(1 + t^2)^2}$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{6at^2 + 2at^4}{4at}$$

m (tangent) at t = $\frac{1}{2}$ is $\frac{13}{16}$

Normal is perpendicular to tangent so, $m_1m_2 = -1$

m (normal) at t = $\frac{1}{2}$ is $-\frac{16}{13}$

Equation of tangent is given by $y - y_1$ = m (tangent) $(x - x_1)$

$$y - \frac{a}{5} = \frac{13}{16}\left(x - \frac{2a}{5}\right)$$

Equation of normal is given by $y - y_1$ = m (normal) $(x - x_1)$

$$y - \frac{a}{5} = -\frac{16}{13}\left(x - \frac{2a}{5}\right)$$

**(iii) $x = at^2$, $y = 2at$ at $t = 1$.**

**Solution:**

Given x = $at^2$, y = 2at at t = 1.

By differentiating the given equation with respect to t, we get the slope of the tangent

$$\frac{dx}{dt} = 2at$$

$$\frac{dy}{dt} = 2a$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{1}{t}$$

m (tangent) at t = 1 is 1

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

m (normal) at t = 1 is – 1

Equation of tangent is given by $y - y_1$ = m (tangent) $(x - x_1)$

y – 2a = 1(x – a)

Equation of normal is given by $y - y_1$ = m (normal) $(x - x_1)$

y – 2a = – 1(x – a)

**(iv) x = a sec t, y = b tan t at t.**

**Solution:**

Given x = a sec t, y = b tan t at t.

By differentiating the given equation with respect to t, we get the slope of the tangent

$$\frac{dx}{dt} = a\sec t \tan t$$

$$\frac{dy}{dt} = b\sec^2 t$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{b\operatorname{cosec} t}{a}$$

m (tangent) at t = $\frac{b\operatorname{cosec} t}{a}$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

m (normal) at t = $-\frac{a}{b}\sin t$

Equation of tangent is given by $y - y_1$ = m (tangent) $(x - x_1)$

$$y - b\tan t = \frac{b\operatorname{cosec} t}{a}(x - a\sec t)$$

Equation of normal is given by $y - y_1$ = m (normal) $(x - x_1)$

$$y - b\tan t = -\frac{a\sin t}{b}(x - a\sec t)$$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$$y - b\tan t = \frac{b\operatorname{cosec} t}{a}(x - a\sec t)$$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y - b\tan t = -\frac{a\sin t}{b}(x - a\sec t)$$

**(v) x = a (θ + sin θ), y = a (1 – cos θ) at θ**

**Solution:**

Given x = a (θ + sin θ), y = a (1 – cos θ) at θ

By differentiating the given equation with respect to θ, we get the slope of the tangent

$$\frac{dx}{d\theta} = a(1 + \cos\theta)$$

$$\frac{dy}{d\theta} = a(\sin\theta)$$

Now dividing $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{\sin\theta}{1 + \cos\theta}$$

m (tangent) at theta is $\frac{\sin\theta}{1 + \cos\theta}$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

m (normal) at theta is $-\frac{\sin\theta}{1 + \cos\theta}$

Equation of tangent is given by $y - y_1 = m\text{ (tangent) } (x - x_1)$

$$y - a(1 - \cos\theta) = \frac{\sin\theta}{1 + \cos\theta}(x - a(\theta + \sin\theta))$$

Equation of normal is given by $y - y_1 = m\text{ (normal) } (x - x_1)$

$$y - a(1 - \cos\theta) = \frac{1 + \cos\theta}{-\sin\theta}(x - a(\theta + \sin\theta))$$

$$y - a(1 - \cos\theta) = \frac{1 + \cos\theta}{-\sin\theta}(x - a(\theta + \sin\theta))$$

**(vi) x = 3 cos θ – cos³ θ, y = 3 sin θ – sin³θ**

**Solution:**

Given x = 3 cos θ – $\cos^3$ θ, y = 3 sin θ – $\sin^3$θ

By differentiating the given equation with respect to θ, we get the slope of the tangent

$$\frac{dx}{d\theta} = -3\sin\theta + 3\cos^2\theta\sin\theta$$

$$\frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta\cos\theta$$

Now dividing $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{3\cos\theta - 3\sin^2\theta\cos\theta}{-3\sin\theta + 3\cos^2\theta\sin\theta} = -\tan^3\theta$$

m (tangent) at theta is $-\tan^3\theta$

Normal is perpendicular to tangent so, $m_1m_2 = -1$

m (normal) at theta is $\cot^3\theta$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$$y - 3\sin\theta + \sin^3\theta = -\tan^3\theta(x - 3\cos\theta + 3\cos^3\theta)$$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y - 3\sin\theta + \sin^3\theta = \cot^3\theta(x - 3\cos\theta + 3\cos^3\theta)$$

**6. Find the equation of the normal to the curve $x^2 + 2y^2 - 4x - 6y + 8 = 0$ at the point whose abscissa is 2.**

**Solution:**

Given $x^2 + 2y^2 - 4x - 6y + 8 = 0$

By differentiating the given curve, we get the slope of the tangent

$$2x + 4y\frac{dy}{dx} - 4 - 6\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{4 - 2x}{4y - 6}$$

Finding y co – ordinate by substituting x in the given curve

$2y^2 - 6y + 4 = 0$

$y^2 - 3y + 2 = 0$

y = 2 or y = 1

m (tangent) at x = 2 is 0

Normal is perpendicular to tangent so, $m_1m_2 = -1$

m (normal) at x = 2 is 1/0, which is undefined

Equation of normal is given by $y - y_1 = m\text{ (normal) } (x - x_1)$

x = 2

**7. Find the equation of the normal to the curve $ay^2 = x^3$ at the point $(am^2, am^3)$.**

**Solution:**

Given $ay^2 = x^3$

By differentiating the given curve, we get the slope of the tangent

$$2ay\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2ay}$$

m (tangent) at $(am^2, am^3)$ is $\frac{3m}{2}$

Normal is perpendicular to tangent so, $m_1m_2 = -1$

m (normal) at $(am^2, am^3)$ is $-\frac{2}{3m}$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y - am^3 = -\frac{2}{3m}(x - am^2)$$

$$y - am^3 = -\frac{2}{3m}(x - am^2)$$

**8. The equation of the tangent at (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.**

**Solution:**

Given $y^2 = ax^3 + b$ is $y = 4x - 5$

By differentiating the given curve, we get the slope of the tangent

$$2y\frac{dy}{dx} = 3ax^2$$

$$\frac{dy}{dx} = \frac{3ax^2}{2y}$$

m (tangent) at (2, 3) = 2a

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

Now comparing the slope of a tangent with the given equation

$2a = 4$

$a = 2$

Now (2, 3) lies on the curve, these points must satisfy

$3^2 = 2 \times 2^3 + b$

$b = -7$

**9. Find the equation of the tangent line to the curve $y = x^2 + 4x - 16$ which is parallel to the line $3x - y + 1 = 0$.**

**Solution:**

Given $y = x^2 + 4x - 16$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 2x + 4$$

m (tangent) = 2x + 4

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

Now comparing the slope of a tangent with the given equation

2x + 4 = 3

$$x = -\frac{1}{2}$$

Now substituting the value of x in the curve to find y

$$y = \frac{1}{4} - 2 - 16 = -\frac{71}{4}$$

Therefore, the equation of tangent parallel to the given line is

$$y + \frac{71}{4} = 3\left(x + \frac{1}{2}\right)$$

2x + 4 = 3

$$x = -\frac{1}{2}$$

Now substituting the value of x in the curve to find y

$$y = \frac{1}{4} - 2 - 16 = -\frac{71}{4}$$

Therefore, the equation of tangent parallel to the given line is

$$y + \frac{71}{4} = 3\left(x + \frac{1}{2}\right)$$

**10. Find the equation of normal line to the curve $y = x^3 + 2x + 6$ which is parallel to the line $x + 14y + 4 = 0$.**

**Solution:**

Given $y = x^3 + 2x + 6$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 3x^2 + 2$$

m (tangent) = $3x^2 + 2$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

m (normal) = $\frac{-1}{3x^2 + 2}$

Equation of normal is given by $y - y_1$ = m (normal) $(x - x_1)$

Now comparing the slope of normal with the given equation

m (normal) = $-\frac{1}{14}$

$$-\frac{1}{14} = -\frac{1}{3x^2 + 2}$$

x = 2 or – 2

Hence the corresponding value of y is 18 or – 6

So, equations of normal are

$y - 18 = -\frac{1}{14}(x - 2)$ Or

$$y + 6 = -\frac{1}{14}(x + 2)$$

Hence the corresponding value of y is 18 or – 6

So, equations of normal are

$$y - 18 = -\frac{1}{14}(x - 2)$$

Or

$$y + 6 = -\frac{1}{14}(x + 2)$$

Exercise 16.3 Page No: 16.40

**1. Find the angle to intersection of the following curves:**

**(i) $y^2 = x$ and $x^2 = y$**

**Solution:**

Given curves $y^2 = x$ ... (1)

And $x^2 = y$ ... (2)

First curve is $y^2 = x$

Differentiating above with respect to x,

$\Rightarrow 2y.\frac{dy}{dx} = 1$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{1}{2x}$ ... (3)

The second curve is $x^2 = y$

$\Rightarrow 2x = \frac{dy}{dx}$

$\Rightarrow m_2 = \frac{dy}{dx} = 2x$ ... (4)

Substituting (1) in (2), we get

$\Rightarrow x^2 = y$

$\Rightarrow (y^2)^2 = y$

$\Rightarrow y^4 - y = 0$

$\Rightarrow y (y^3 - 1) = 0$

$\Rightarrow y = 0$ or $y = 1$

Substituting y = 0 & y = 1 in (1) in (2),

$x = y^2$

When y = 0, x = 0

When y = 1, x = 1

Substituting above values for $m_1$ & $m_{2,}$ we get, When x = 0,

$m_1 = \frac{dy}{dx} = \frac{1}{2\times0} = \infty$

When x = 1,

$m_1 = \frac{dy}{dx} = \frac{1}{2\times1} = \frac{1}{2}$

Values of $m_1$ is $\infty$ & $\frac{1}{2}$

When y = 0,

$m_2 = \frac{dy}{dx} = 2x = 2 \times 0 = 0$

When x = 1,

$m_2 = \frac{dy}{dx} = 3x = 2\times1 = 2$

Values of $m_2$ is 0 & 2

When $m_1 = \infty$ & $m_2 = 0$

$\text{Tan }\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\text{Tan }\theta = \left|\frac{0-\infty}{1 + \infty\times0}\right|$

$\text{Tan }\theta = \infty$

$\theta = \tan^{-1}(\infty)$

$\therefore \text{Tan}^{-1}(\infty) = \frac{\pi}{2}$

$\theta = \frac{\pi}{2}$

When $m_1 = \frac{1}{2}$ & $m_2 = 2$

Angle of intersection of two curves is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\text{Tan}\,\theta = \left|\frac{2-\frac{1}{2}}{1+\frac{1}{2}\times 2}\right|$

$\text{Tan}\,\theta = \left|\frac{2-\frac{1}{2}}{1+\frac{1}{2}\times 2}\right|$

$\text{Tan}\,\theta = \left|\frac{\frac{3}{2}}{2}\right|$

$\text{Tan}\,\theta = \left|\frac{3}{4}\right|$

$\theta = \tan^{-1}(\frac{3}{4})$

$\theta \cong 36.86$

**(ii) $y = x^2$ and $x^2 + y^2 = 20$**

**Solution:**

Given curves $y = x^2$ ... (1) and $x^2 + y^2 = 20$ ... (2)

Now consider first curve $y = x^2$

$\Rightarrow m_1 = \frac{dy}{dx} = 2x$ ... (3)

Consider second curve is $x^2 + y^2 = 20$

Differentiating above with respect to x,

$\Rightarrow 2x + 2y.\frac{dy}{dx} = 0$

$\Rightarrow y.\frac{dy}{dx} = -x$

$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y}$ ... (4)

Substituting (1) in (2), we get

$\Rightarrow y + y^2 = 20$

$\Rightarrow y^2 + y - 20 = 0$

We will use factorization method to solve the above Quadratic equation

$\Rightarrow y^2 + 5y - 4y - 20 = 0$

$\Rightarrow y (y + 5) - 4 (y + 5) = 0$

$\Rightarrow (y + 5) (y - 4) = 0$

$\Rightarrow (y + 5)(y - 4) = 0$

$\Rightarrow y = -5$ & $y = 4$

Substituting $y = -5$ & $y = 4$ in (1) in (2),

$y = x^2$

When $y = -5$,

$\Rightarrow -5 = x^2$

$\Rightarrow x = \sqrt{-5}$

When $y = 4$,

$\Rightarrow 4 = x^2$

$\Rightarrow x = \pm 2$

Substituting above values for $m_1$ & $m_2$, we get,

When $x = 2$,

$m_1 = \frac{dy}{dx} = 2 \times 2$

$= 4$

When $x = 1$,

$m_1 = \frac{dy}{dx} = 2 \times -2$

$= -4$

Values of $m_1$ is $4$ & $-4$

When $y = 4$ & $x = 2$

$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{-2}{4} = \frac{-1}{2}$

When $y = 4$ & $x = -2$

$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{2}{4} = \frac{1}{2}$

Values of $m_2$ is $\frac{-1}{2}$ & $\frac{1}{2}$

When $m_1 = \infty$ & $m_2 = 0$

Values of $m_2$ is $\frac{-1}{2}$ & $\frac{1}{2}$

When $m_1 = \infty$ & $m_2 = 0$

Angle of intersection of two curves is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\text{Tan }\theta = \left|\frac{\frac{-1}{2} - 4}{1 + 2\times4}\right|$

$\text{Tan }\theta = \left|\frac{\frac{-9}{2}}{1-2}\right|$

$\text{Tan }\theta = \left|\frac{9}{2}\right|$

$\theta = \tan^{-1}(\frac{9}{2})$

**(iii) $2y^2 = x^3$ and $y^2 = 32x$**

**Solution:**

Given curves $2y^2 = x^3$ ... (1) and $y^2 = 32x$ ... (2)

First curve is $2y^2 = x^3$

Differentiating above with respect to x,

$\Rightarrow 4y.\frac{dy}{dx} = 3x^2$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{3x^2}{4y}$ ... (3)

Second curve is $y^2 = 32x$

$\Rightarrow 2y.\frac{dy}{dx} = 32$

$\Rightarrow y.\frac{dy}{dx} = 16$

$\Rightarrow m_2 = \frac{dy}{dx} = \frac{16}{y}$ ... (4)

Substituting (2) in (1), we get

$\Rightarrow 2y^2 = x^3$

$\Rightarrow 2(32x) = x^3$

$\Rightarrow 64\ x = x^3$

$\Rightarrow x^3 - 64x = 0$

$\Rightarrow x\ (x^2 - 64) = 0$

$\Rightarrow x = 0\ \&\ (x^2 - 64) = 0$

Substituting (2) in (1), we get

$\Rightarrow 2y^2 = x^3$

$\Rightarrow 2(32x) = x^3$

$\Rightarrow 64\ x = x^3$

⇒ $x^3 - 64x = 0$

⇒ $x (x^2 - 64) = 0$

⇒ $x = 0$ & $(x^2 - 64) = 0$

⇒ $x = 0$ & $\pm 8$

Substituting $x = 0$ & $x = \pm 8$ in (1) in (2),

$y^2 = 32x$

When $x = 0$, $y = 0$

When $x = 8$

⇒ $y^2 = 32 \times 8$

⇒ $y^2 = 256$

⇒ $y = \pm 16$

Substituting above values for $m_1$ & $m_{2,}$ we get,

When $x = 0$, $y = 16$

$m_1 = \frac{dy}{dx}$

$\Rightarrow \frac{3\times 0^2}{4\times 8}$

= 0

When x = 8, y = 16

$m_1 = \frac{dy}{dx}$

$\Rightarrow \frac{3\times 8^2}{4\times 16}$

= 3

Values of $m_1$ is 0 & 3

When x = 0, y = 0,

$m_2 = \frac{dy}{dx}$

$\Rightarrow \frac{16}{y} = \frac{16}{0} = \infty$

When y = 16,

$m_2 = \frac{dy}{dx}$

$\Rightarrow \frac{16}{y} = \frac{16}{16}$

$\Rightarrow \frac{16}{y} = \frac{16}{0} = \infty$

When y = 16,

$m_2 = \frac{dy}{dx}$

$\Rightarrow \frac{16}{y} = \frac{16}{16}$

= 1

Values of $m_2$ is ∞ & 1

When $m_1 = 0$ & $m_2 = \infty$

$\Rightarrow \text{Tan}\ \theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \text{Tan}\ \theta = \left|\frac{\infty - 0}{1 + \infty\times 0}\right|$

$\Rightarrow \text{Tan}\ \theta = \infty$

$\Rightarrow \theta = \tan^{-1}(\infty)$

$\therefore \text{Tan}^{-1}(\infty) = \frac{\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{2}$

When $m_1 = \frac{1}{2}$ & $m_2 = 2$

Angle of intersection of two curves is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow$ Tan $\theta = \left|\frac{3-1}{1+3\times1}\right|$

$\Rightarrow$ Tan $\theta = \left|\frac{2}{4}\right|$

$\Rightarrow$ Tan $\theta = \left|\frac{1}{2}\right|$

$\Rightarrow \theta = \tan^{-1}(\frac{1}{2})$

**(iv) $x^2 + y^2 - 4x - 1 = 0$ and $x^2 + y^2 - 2y - 9 = 0$**

**Solution:**

Given curves $x^2 + y^2 - 4x - 1 = 0$ … (1) and $x^2 + y^2 - 2y - 9 = 0$ … (2)

First curve is $x^2 + y^2 - 4x - 1 = 0$

$\Rightarrow x^2 - 4x + 4 + y^2 - 4 - 1 = 0$

$\Rightarrow (x - 2)^2 + y^2 - 5 = 0$

Now, Subtracting (2) from (1), we get

$\Rightarrow x^2 + y^2 - 4x - 1 - ( x^2 + y^2 - 2y - 9) = 0$

$\Rightarrow x^2 + y^2 - 4x - 1 - x^2 - y^2 + 2y + 9 = 0$

$\Rightarrow -4x - 1 + 2y + 9 = 0$

$\Rightarrow -4x + 2y + 8 = 0$

$\Rightarrow 2y = 4x - 8$

$\Rightarrow y = 2x - 4$

Substituting $y = 2x - 4$ in (3), we get,

$\Rightarrow (x - 2)^2 + (2x - 4)^2 - 5 = 0$

⇒ $(x - 2)^2 + 4(x - 2)^2 - 5 = 0$

⇒ $(x - 2)^2(1 + 4) - 5 = 0$

⇒ $5(x - 2)^2 - 5 = 0$

⇒ $(x - 2)^2 - 1 = 0$

⇒ $(x - 2)^2 = 1$

⇒ $(x - 2) = \pm 1$

⇒ $x = 1 + 2$ or $x = - 1 + 2$

⇒ $x = 3$ or $x = 1$

So, when $x = 3$

$y = 2 \times 3 - 4$

⇒ $y = 6 - 4 = 2$

So, when $x = 3$

$y = 2 \times 1 - 4$

⇒ $y = 2 - 4 = - 2$

The point of intersection of two curves are (3, 2) & (1, – 2)

Now, differentiating curves (1) & (2) with respect to x, we get

$\Rightarrow x^2 + y^2 - 4x - 1 = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 4 - 0 = 0$

$\Rightarrow x + y\frac{dy}{dx} - 2 = 0$

$\Rightarrow y\frac{dy}{dx} = 2 - x$

$\Rightarrow \frac{dy}{dx} = \frac{2-x}{y}$ ... (3)

$\Rightarrow x^2 + y^2 - 2y - 9 = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} - 0 = 0$

$\Rightarrow x + y\frac{dy}{dx} - \frac{dy}{dx} = 0$

$\Rightarrow x + (y - 1)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{y-1}$ ... (4)

At (3, 2) in equation (3), we get

$\Rightarrow \frac{dy}{dx} = \frac{2-3}{2}$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-1}{2}$

$\Rightarrow x + y\frac{dy}{dx} - 2 = 0$

$\Rightarrow y\frac{dy}{dx} = 2 - x$

$\Rightarrow \frac{dy}{dx} = \frac{2-x}{y}$ ... (3)

$\Rightarrow x^2 + y^2 - 2y - 9 = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} - 0 = 0$

$\Rightarrow x + y\frac{dy}{dx} - \frac{dy}{dx} = 0$

$\Rightarrow x + (y-1)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{y-1}$ ... (4)

At (3, 2) in equation (3), we get

$$\Rightarrow \frac{dy}{dx} = \frac{2-3}{2}$$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-1}{2}$

At (3, 2) in equation (4), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-3}{2-1}$$

$\Rightarrow \frac{dy}{dx} = -3$

$\Rightarrow m_2 = \frac{dy}{dx} = -3$

When $m_1 = \frac{-1}{2}$ & $m_2 = 0$

Angle of intersection of two curves is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \text{Tan}\,\theta = \left|\frac{\frac{-1}{2} + 3}{1 + \frac{3}{2}}\right| = 1$

$\Rightarrow \theta = \tan^{-1}(1) = \frac{\pi}{4}$

$\Rightarrow \text{Tan}\,\theta = \left|\frac{\frac{-1}{2} + 3}{1 + \frac{3}{2}}\right| = 1$

$\Rightarrow \theta = \tan^{-1}(1) = \frac{\pi}{4}$

$(v)\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ and\ x^2 + y^2 = ab$

**Solution:**

Given curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ... (1) and $x^2 + y^2 = ab$ ... (2)

Second curve is $x^2 + y^2 = ab$

$y^2 = ab - x^2$

Substituting this in equation (1),

$\Rightarrow \frac{x^2}{a^2}+\frac{ab-x^2}{b^2}=1$

$\Rightarrow \frac{x^2b^2+a^2(ab-x^2)}{a^2b^2}=1$

$\Rightarrow x^2b^2 + a^3b - a^2x^2 = a^2b^2$

$\Rightarrow x^2b^2 - a^2x^2 = a^2b^2 - a^3b$

$\Rightarrow x^2(b^2 - a^2) = a^2b(b - a)$

$\Rightarrow x^2=\frac{a^2b(b-a)}{x^2\,(b^2-a^2)}$

$\Rightarrow x^2=\frac{a^2b(b-a)}{x^2\,(b-a)(b+a)}$

$\Rightarrow x^2=\frac{a^2b}{(b+a)}$

$\therefore a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow x=\pm\sqrt{\frac{a^2b}{(b+a)}}$ ... (3)

$\Rightarrow x = \pm\sqrt{\frac{a^2b}{(b+a)}}$ ... (3)

Since, $y^2 = ab - x^2$

$\Rightarrow y^2 = ab - (\frac{a^2b}{(b+a)})$

$\Rightarrow y^2 = \frac{ab^2 + a^2b - a^2b}{(b+a)}$

$\Rightarrow y^2 = \frac{ab^2}{(b+a)}$

$\Rightarrow y = \pm\sqrt{\frac{ab^2}{(b+a)}}$ ... (4)

Since, curves are $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ & $x^2 + y^2 = ab$

Differentiating above with respect to x

$\Rightarrow \frac{2x}{a^2} + \frac{2y}{b^2}.\frac{dy}{dx} = 0$

$\Rightarrow \frac{y}{b^2}.\frac{dy}{dx} = -\frac{x}{a^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-\frac{x}{a^2}}{\frac{y}{b^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{-b^2x}{a^2y}$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-b^2x}{a^2y}$ ... (5)

Second curve is $x^2 + y^2 = ab$

$\Rightarrow 2x + 2y.\frac{dy}{dx} = 0$

$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y}$ ... (6)

Substituting (3) in (4), above values for $m_1$ & $m_2$, we get, At $\left(\sqrt{\frac{a^2b}{(b+a)}}, \sqrt{\frac{ab^2}{(b+a)}}\right)$ in equation (5), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 \times \sqrt{\frac{a^2b}{(b+a)}}}{a^2 \times \sqrt{\frac{ab^2}{(b+a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 \times a\sqrt{\frac{b}{(b+a)}}}{a^2 \times b\sqrt{\frac{a}{(b+a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 a\sqrt{b}}{a^2 b\sqrt{a}}$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-b\sqrt{b}}{a\sqrt{a}}$$

At $\left(\sqrt{\frac{a^2b}{(b+a)}}, \sqrt{\frac{ab^2}{(b+a)}}\right)$ in equation (6), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{\frac{a^2b}{(b+a)}}}{\sqrt{\frac{ab^2}{(b+a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-a\sqrt{\frac{b}{(b+a)}}}{b\sqrt{\frac{a}{(b+a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-a\sqrt{b}}{b\sqrt{a}}$$

$$\Rightarrow m_2 = \frac{dy}{dx} = -\sqrt{\frac{a}{b}}$$

When $m_1 = \frac{-b\sqrt{b}}{a\sqrt{a}}$ & $m_2 = -\sqrt{\frac{a}{b}}$

Angle of intersection of two curves is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \operatorname{Tan}\theta = \left|\frac{\frac{-b\sqrt{b}}{a\sqrt{a}} - -\sqrt{\frac{a}{b}}}{1 + \frac{-b\sqrt{b}}{a\sqrt{a}} \times -\sqrt{\frac{a}{b}}}\right|$$

$$\Rightarrow \operatorname{Tan}\theta = \left|\frac{\frac{-b\sqrt{b}}{a\sqrt{a}} + \sqrt{\frac{a}{b}}}{1 + \frac{b}{a}}\right|$$

$$\Rightarrow \operatorname{Tan}\theta = \left|\frac{\frac{-b\sqrt{b}\times\sqrt{b} + a\sqrt{a}\times\sqrt{a}}{a\sqrt{a}\times\sqrt{b}}}{1 + \frac{b}{a}}\right|$$

$$\Rightarrow \operatorname{Tan}\theta = \left|\frac{\frac{-b\times b + a\times a}{a\sqrt{ab}}}{1 + \frac{b}{a}}\right|$$

$$\Rightarrow \operatorname{Tan}\theta = \left|\frac{\frac{a^2 - b^2}{a\sqrt{ab}}}{\frac{a + b}{a}}\right|$$

$$\Rightarrow \operatorname{Tan}\theta = \left|\frac{\frac{(a+b)(a-b)}{\sqrt{ab}}}{a + b}\right|$$

$$\Rightarrow \operatorname{Tan}\theta = \left|\frac{(a-b)}{\sqrt{ab}}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{(a-b)}{\sqrt{ab}}\right)$$

**2. Show that the following set of curves intersect orthogonally:**

**(i) $y = x^3$ and $6y = 7 - x^2$**

**Solution:**

Given curves $y = x^3$ … (1) and $6y = 7 - x^2$ … (2)

Solving (1) & (2), we get

⇒ $6y = 7 – x^2$

⇒ $6(x^3) = 7 – x^2$

⇒ $6x^3 + x^2 – 7 = 0$

Since $f(x) = 6x^3 + x^2 – 7$,

We have to find f(x) = 0, so that x is a factor of f(x).

When x = 1

$f(1) = 6(1)^3 + (1)^2 – 7$

f (1) = 6 + 1 – 7

f (1) = 0

Hence, x = 1 is a factor of f(x).

Substituting x = 1 in $y = x^3$, we get

$y = 1^3$

y = 1

The point of intersection of two curves is (1, 1)

First curve $y = x^3$

Differentiating above with respect to x,

$\Rightarrow 6\frac{dy}{dx} = 0 - 2x$

$\Rightarrow m_2 = \frac{-2x}{6}$

$\Rightarrow m_2 = \frac{-x}{3}$ ... (4)

At (1, 1), we have,

$m_1 = 3x^2$

$\Rightarrow 3 \times (1)^2$

$m_1 = 3$

At (1, 1), we have,

$\Rightarrow m_2 = \frac{-x}{3}$

$\Rightarrow \frac{-1}{3}$

$\Rightarrow m_2 = \frac{-1}{3}$

When $m_1 = 3$ & $m_2 = \frac{-1}{3}$

Two curves intersect orthogonally if $m_1m_2 = -1$

$\Rightarrow 3 \times \frac{-1}{3} = -1$

∴ Two curves $y = x^3$ & $6y = 7 - x^2$ intersect orthogonally.

$\Rightarrow 3 \times \frac{-1}{3} = -1$

∴ Two curves $y = x^3$ & $6y = 7 - x^2$ intersect orthogonally.

**(ii) $x^3 - 3xy^2 = -2$ and $3x^2y - y^3 = 2$**

**Solution:**

Given curves $x^3 - 3xy^2 = -2$ … (1) and $3x^2y - y^3 = 2$ … (2)

Adding (1) & (2), we get

$\Rightarrow x^3 - 3xy^2 + 3x^2y - y^3 = -2 + 2$

$\Rightarrow x^3 - 3xy^2 + 3x^2y - y^3 = -0$

$\Rightarrow (x - y)^3 = 0$

$\Rightarrow (x - y) = 0$

$\Rightarrow x = y$

Substituting x = y on $x^3 - 3xy^2 = -2$

$\Rightarrow x^3 - 3 \times x \times x^2 = -2$

$\Rightarrow x^3 - 3x^3 = -2$

$\Rightarrow -2x^3 = -2$

$\Rightarrow x^3 = 1$

$\Rightarrow x = 1$

Since x = y

y = 1

The point of intersection of two curves is (1, 1)

First curve $x^3 - 3xy^2 = -2$

Differentiating above with respect to x,

$\Rightarrow 3x^2 - 3(1\times y^2 + x\times 2y\frac{dy}{dx}) = 0$

$\Rightarrow 3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 3y^2 = 6xy\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2-3y^2}{6xy}$

$\Rightarrow \frac{dy}{dx} = \frac{3(x^2-y^2)}{6xy}$

$\Rightarrow m_1 = \frac{(x^2-y^2)}{2xy}$ ... (3)

Second curve $3x^2y - y^3 = 2$

Differentiating above with respect to x

$\Rightarrow 3(2x \times y + x^2 \times \frac{dy}{dx}) - 3y^2\frac{dy}{dx} = 0$

$\Rightarrow 6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$

$\Rightarrow 6xy + (3x^2 - 3y^2)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-6xy}{3x^2-3y^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{x^2-y^2}$

$\Rightarrow m_2 = \frac{-2xy}{x^2-y^2}$ ... (4)

When $m_1 = \frac{(x^2-y^2)}{2xy}$ & $m_2 = \frac{-2xy}{x^2-y^2}$

Two curves intersect orthogonally if $m_1m_2 = -1$

$\Rightarrow \frac{(x^2-y^2)}{2xy} \times \frac{-2xy}{x^2-y^2} = -1$

∴ Two curves $x^3 - 3xy^2 = -2$ & $3x^2y - y^3 = 2$ intersect orthogonally.

**(iii) $x^2 + 4y^2 = 8$ and $x^2 - 2y^2 = 4$.**

**Solution:**

Given curves $x^2 + 4y^2 = 8$ ... (1) and $x^2 - 2y^2 = 4$ ... (2)

Solving (1) & (2), we get,

From 2nd curve,

$x^2 = 4 + 2y^2$

Substituting on $x^2 + 4y^2 = 8$,

$\Rightarrow 4 + 2y^2 + 4y^2 = 8$

$\Rightarrow 6y^2 = 4$

$\Rightarrow y^2 = \frac{4}{6}$

Substituting on $x^2 + 4y^2 = 8$,

$\Rightarrow 4 + 2y^2 + 4y^2 = 8$

$\Rightarrow 6y^2 = 4$

$\Rightarrow y^2 = \frac{4}{6}$

$\Rightarrow y = \pm\sqrt{\frac{2}{3}}$

Substituting on $y = \pm\sqrt{\frac{2}{3}}$, we get,

$\Rightarrow x^2 = 4 + 2(\pm\sqrt{\frac{2}{3}})^2$

$\Rightarrow x^2 = 4 + 2(\frac{2}{3})$

$\Rightarrow x^2 = 4 + \frac{4}{3}$

$\Rightarrow x^2 = \frac{16}{3}$

$\Rightarrow x = \pm\sqrt{\frac{16}{3}}$

$\Rightarrow x = \pm\frac{4}{\sqrt{3}}$

∴ The point of intersection of two curves $(\frac{4}{\sqrt{3}}, \sqrt{\frac{2}{3}})$ & $(-\frac{4}{\sqrt{3}}, -\sqrt{\frac{2}{3}})$

Now, differentiating curves (1) & (2) with respect to x, we get

$\Rightarrow x^2 + 4y^2 = 8$

$\Rightarrow 2x + 8y.\frac{dy}{dx} = 0$

$\Rightarrow 8y.\frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{4y}$ ... (3)

$\Rightarrow x^2 - 2y^2 = 4$

$\Rightarrow 2x - 4y.\frac{dy}{dx} = 0$

$\Rightarrow x - 2y.\frac{dy}{dx} = 0$

$\Rightarrow 4y\frac{dy}{dx} = x$

$\Rightarrow \frac{dy}{dx} = \frac{x}{2y}$ ... (4)

At ($\frac{4}{\sqrt{3}}$, $\sqrt{\frac{2}{3}}$) in equation (3), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{4}{\sqrt{3}}}{4 \times \sqrt{\frac{2}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{1}{\sqrt{3}}}{\sqrt{\frac{2}{3}}}$$

$\Rightarrow m_1 = \frac{-1}{\sqrt{2}}$

At ($\frac{4}{\sqrt{3}}$, $\sqrt{\frac{2}{3}}$) in equation (4), we get

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{4}{\sqrt{3}}}{2 \times \sqrt{\frac{2}{3}})}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{2}{\sqrt{3}}}{\sqrt{\frac{2}{3}})}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{2}$$

$\Rightarrow m_2 = 1$

When $m_1 = \frac{-1}{\sqrt{2}}$ & $m_2 = \sqrt{2}$

Two curves intersect orthogonally if $m_1 m_2 = -1$

$$\Rightarrow \frac{-1}{\sqrt{2}} \times \sqrt{2} = -1$$

∴ Two curves $x^2 + 4y^2 = 8$ & $x^2 - 2y^2 = 4$ intersect orthogonally.

**3. $x^2 = 4y$ and $4y + x^2 = 8$ at (2, 1)**

**Solution:**

Given curves $x^2 = 4y$ ... (1) and $4y + x^2 = 8$ ... (2)

The point of intersection of two curves (2, 1)

Solving (1) & (2), we get,

First curve is $x^2 = 4y$

Differentiating above with respect to x,

$\Rightarrow 2x = 4.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{4}$

$\Rightarrow m_1 = \frac{x}{2}$ ... (3)

Second curve is $4y + x^2 = 8$

$\Rightarrow 4.\frac{dy}{dx} + 2x = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{4}$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{4}$

$\Rightarrow m_2 = \frac{-x}{2}$ ... (4)

Substituting (2, 1) for $m_1$ & $m_2$, we get,

$m_1 = \frac{x}{2}$

$\Rightarrow \frac{2}{2}$

$m_1 = 1$ ... (5)

$m_2 = \frac{-x}{2}$

$\Rightarrow \frac{-2}{2}$

$m_2 = -1$ ... (6)

When $m_1 = 1$ & $m_2 = -1$

Two curves intersect orthogonally if $m_1m_2 = -1$

$\Rightarrow 1 \times -1 = -1$

∴ Two curves $x^2 = 4y$ & $4y + x^2 = 8$ intersect orthogonally.

**(ii) $x^2 = y$ and $x^3 + 6y = 7$ at (1, 1)**

**Solution:**

Given curves $x^2 = y$ … (1) and $x^3 + 6y = 7$ … (2)

The point of intersection of two curves (1, 1)

Solving (1) & (2), we get,

First curve is $x^2 = y$

Differentiating above with respect to x,

$\Rightarrow 2x = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow m_1 = 2x$ ... (3)

Second curve is $x^3 + 6y = 7$

Differentiating above with respect to x,

$\Rightarrow 3x^2 + 6.\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-3x^2}{6}$

$\Rightarrow \frac{dy}{dx} = \frac{-x^2}{2}$

$\Rightarrow m_2 = \frac{-x^2}{2}$ ... (4)

Substituting (1, 1) for $m_1$ & $m_2$, we get,

$m_1 = 2x$

$\Rightarrow 2 \times 1$

$m_1 = 2$ ... (5)

$m_2 = \frac{-x^2}{2}$

$\Rightarrow \frac{-1^2}{2}$

$m_2 = -\frac{-1}{2}$ ... (6)

When $m_1 = 2$ & $m_2 = -\frac{-1}{2}$

Two curves intersect orthogonally if $m_1 m_2 = -1$

$\Rightarrow 2 \times \frac{-1}{2} = -1$

∴ Two curves $x^2 = y$ & $x^3 + 6y = 7$ intersect orthogonally.

**(iii) $y^2 = 8x$ and $2x^2 + y^2 = 10$ at $(1, 2\sqrt{2})$**

**Solution:**

Given curves $y^2 = 8x$ … (1) and $2x^2 + y^2 = 10$ … (2)

The point of intersection of two curves are (0, 0) & (1, 2√

Now, differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow y^2 = 8x$

$\Rightarrow 2y.\frac{dy}{dx} = 8$

$\Rightarrow \frac{dy}{dx} = \frac{8}{2y}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{y}$ ... (3)

$\Rightarrow 2x^2 + y^2 = 10$

Differentiating above with respect to x,

$\Rightarrow 4x + 2y.\frac{dy}{dx} = 0$

$\Rightarrow 2x + y.\frac{dy}{dx} = 0$

$\Rightarrow y.\frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{y}$ ... (4)

Substituting (1, 2√2) for $m_1$ & $m_2$, we get,

$m_1 = \frac{4}{y}$

$\Rightarrow \frac{4}{2\sqrt{2}}$

$m_1 = \sqrt{2}$ ... (5)

$m_2 = \frac{-2x}{y}$

$\Rightarrow \frac{-2\times 1}{2\sqrt{2}}$

$m_2 = -\frac{-1}{\sqrt{2}}$ ... (6)

When $m_1 = \sqrt{2}$ & $m_2 = \frac{-1}{\sqrt{2}}$

When $m_1 = \sqrt{2}$ & $m_2 = \frac{-1}{\sqrt{2}}$

Two curves intersect orthogonally if $m_1 m_2 = -1$

$\Rightarrow \sqrt{2} x \frac{-1}{\sqrt{2}} = -1$

∴ Two curves $y^2 = 8x$ & $2x^2 + y^2 = 10$ intersect orthogonally.

**4. Show that the curves $4x = y^2$ and $4xy = k$ cut at right angles, if $k^2 = 512$.**

**Solution:**

Given curves $4x = y^2$ ... (1) and $4xy = k$ ... (2)

We have to prove that two curves cut at right angles if $k^2 = 512$

Now, differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow 4x = y^2$

$\Rightarrow 4 = 2y.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$

$m_1 = \frac{2}{y}$ ... (3)

$\Rightarrow 4xy = k$

Differentiating above with respect to x,

$\Rightarrow 4(y + x\frac{dy}{dx}) = 0$

$\Rightarrow y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

$\Rightarrow m_2 = \frac{-y}{x}$ ... (4)

Two curves intersect orthogonally if $m_1 m_2 = -1$

Since $m_1$ and $m_2$ cuts orthogonally, Since $m_1$ and $m_2$ cuts orthogonally,

$\Rightarrow \frac{2}{y} \times \frac{-y}{x} = -1$

$\Rightarrow \frac{-2}{x} = -1$

$\Rightarrow x = 2$

Now, Solving (1) & (2), we get,

$4xy = k$ & $4x = y^2$

$\Rightarrow (y^2)\, y = k$

$\Rightarrow y^3 = k$

$\Rightarrow y = k^{\frac{1}{3}}$

Substituting $y = k^{\frac{1}{3}}$ in $4x = y^2$, we get,

$\Rightarrow 4x = (k^{\frac{1}{3}})^2$

$\Rightarrow 4 \times 2 = k^{\frac{2}{3}}$

$\Rightarrow k^{\frac{2}{3}} = 8$

$\Rightarrow k^2 = 8^3$

$\Rightarrow k^2 = 512$

**5. Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles, if $k^2 = 8$.**

**Solution:**

Given curves $2x = y^2$ … (1) and $2xy = k$ … (2)

We have to prove that two curves cut at right angles if $k^2 = 8$

Now, differentiating curves (1) & (2) with respect to x, we get

⇒ $2x = y^2$

$\Rightarrow 2 = 2y.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{y}$

$m_1 = \frac{1}{y}$ ... (3)

$\Rightarrow 2xy = k$

Differentiating above with respect to x,

$\Rightarrow 2(y + x\frac{dy}{dx}) = 0$

$\Rightarrow y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

$\Rightarrow m_2 = \frac{-y}{x}$ ... (4)

Two curves intersect orthogonally if $m_1 m_2 = -1$

Since $m_1$ and $m_2$ cuts orthogonally,

$\Rightarrow \frac{1}{y} \times \frac{-y}{x} = -1$

$\Rightarrow \frac{-1}{x} = -1$

$\Rightarrow x = 1$

Now, solving (1) & (2), we get,

$2xy = k$ & $2x = y^2$

$\Rightarrow (y^2)\, y = k$

$\Rightarrow y^3 = k$

$\Rightarrow y = k^{\frac{1}{3}}$

Substituting $y = k^{\frac{1}{3}}$ in $2x = y^2$, we get,

Substituting $y = k^{\frac{1}{3}}$ in $2x = y^2$, we get,

$\Rightarrow 2x = (k^{\frac{1}{3}})^2$

$\Rightarrow 2 \times 1 = k^{\frac{2}{3}}$

$\Rightarrow k^{\frac{2}{3}} = 2$

$\Rightarrow k^2 = 2^3$

$\Rightarrow k^2 = 8$

# Chapterwise RD Sharma Solutions for Class 12 Maths :

# About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.