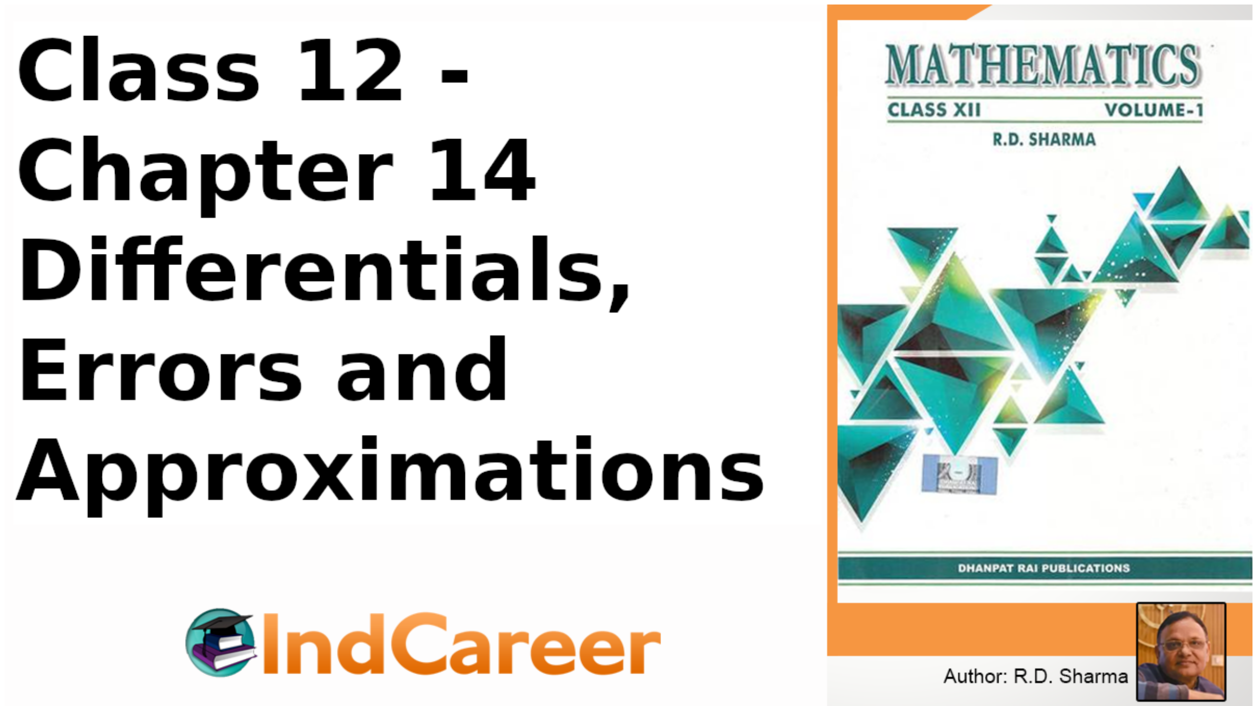

RD Sharma Solutions for Class 12 Maths Chapter 14–Differentials, Errors and Approximations

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RD Sharma Solutions for Class 12 Maths Chapter 14–Differentials, Errors and Approximations

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RD Sharma 12th Maths Chapter 14, Class 12 Maths Chapter 14 solutions

Exercise 14.1 Page No: 14.9

1. If y = sin x and x changes from π/2 to 22/14, what is the approximate change in y?

Solution:

Given y = sin x and x changes from $\frac{\pi}{2}$ to $\frac{22}{14}$.

Let $x = \frac{\pi}{2}$ so that $x + \Delta x = \frac{22}{14}$

$$\Rightarrow \frac{\pi}{2} + \Delta x = \frac{22}{14}$$

$$\therefore \Delta x = \frac{22}{14} - \frac{\pi}{2}$$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)$$

We know that $\frac{d}{dx}(\sin x) = \cos x$

$$\therefore \frac{dy}{dx} = \cos x$$

When $x = \frac{\pi}{2}$, we have $\frac{dy}{dx} = \cos\left(\frac{\pi}{2}\right)$.

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 0$$

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, Δ y = f(x + Δ x) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dy}{dx} = 0$ and $\Delta x = \frac{22}{14} - \frac{\pi}{2}$

$$\Rightarrow \Delta y = (0)\left(\frac{22}{14} - \frac{\pi}{2}\right)$$

∴ Δ y = 0

Thus, there is approximately no change in y.

2. The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.

Solution:

Given the radius of a sphere changes from 10 cm to 9.8 cm.

Let x be the radius of the sphere and Δx be the change in the value of x.

Hence, we have $x = 10$ and $x + \Delta x = 9.8$

$\Rightarrow 10 + \Delta x = 9.8$

$\Rightarrow \Delta x = 9.8 - 10$

$\therefore \Delta x = -0.2$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3}\pi x^3$$

On differentiating V with respect to x, we get

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}\frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}(3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

When x = 10, we have $\frac{dV}{dx} = 4\pi(10)^2$.

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=10} = 4\pi \times 100$$

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=10} = 400\pi$$

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, Δ y = f(x + Δ x) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=10} = 4\pi \times 100$$

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=10} = 400\pi$$

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, Δ y = f(x + Δ x) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dV}{dx} = 400\pi$ and Δ x = –0.2

⇒ ΔV = (400π) (–0.2)

∴ ΔV = –80π

Thus, the approximate decrease in the volume of the sphere is 80π cm^3.

3. A circular metal plate expands under heating so that its radius increases by k%. Find the approximate increase in the area of the plate, if the radius of the plate before heating is 10 cm.

Solution:

Given the radius of a circular plate initially is 10 cm and it increases by k%.

Let x be the radius of the circular plate, and Δx is the change in the value of x.

Hence, we have x = 10 and $\Delta x = \frac{k}{100} \times 10$

$\therefore \Delta x = 0.1k$

The area of a circular plate of radius x is given by

$A = \pi x^2$

On differentiating A with respect to x, we get

$$\frac{dA}{dx} = \frac{d}{dx}(\pi x^2)$$

$$\Rightarrow \frac{dA}{dx} = \pi \frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dA}{dx} = \pi(2x)$$

$$\therefore \frac{dA}{dx} = 2\pi x$$

When x = 10, we have $\frac{dA}{dx} = 2\pi(10)$.

$$\Rightarrow \left(\frac{dA}{dx}\right)_{x=10} = 20\pi$$

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, Δ y = f(x + Δ x) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dA}{dx} = 20\pi$ and Δ x = 0.1k

⇒ ΔA = (20π) (0.1k)

∴ ΔA = 2kπ

Thus, the approximate increase in the area of the circular plate is 2kπ cm^2.

4. Find the percentage error in calculating the surface area of a cubical box if an error of 1% is made in measuring the lengths of the edges of the cube.

Solution:

$\therefore \Delta x = 0.01x$

The surface area of a cubical box of radius x is given by

$S = 6x^2$

On differentiating A with respect to x, we get

$$\frac{dS}{dx} = \frac{d}{dx}(6x^2)$$

$$\Rightarrow \frac{dS}{dx} = 6\frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = 6(2x)$$

$$\therefore \frac{dS}{dx} = 12x$$

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, Δ y = f(x + Δ x) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dS}{dx} = 12x$ and Δ x = 0.01x

$\Rightarrow \Delta S = (12x)(0.01x)$

$\therefore \Delta S = 0.12x^2$

The percentage error is,

$$\text{Error} = \frac{0.12x^2}{6x^2} \times 100\%$$

$\Rightarrow$ Error = 0.02 × 100%

$\therefore$ Error = 2%

Thus, the error in calculating the surface area of the cubical box is 2%.

5. If there is an error of 0.1% in the measurement of the radius of a sphere, find approximately the percentage error in the calculation of the volume of the sphere.

Solution:

Given the error in the measurement of the radius of a sphere is 0.1%. Let x be the radius of the sphere and Δ x be the error in the value of x.

Hence, we have $\Delta x = \frac{0.1}{100} \times x$

$\therefore \Delta x = 0.001x$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3}\pi x^3$$

On differentiating V with respect to x, we get

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}\frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}(3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, Δ y = f(x + Δ x) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dV}{dx} = 4\pi x^2$ and $\Delta x = 0.001x$

$\therefore \Delta V = 0.004\pi x^3$

The percentage error is,

$$\text{Error} = \frac{0.004\pi x^3}{\frac{4}{3}\pi x^3} \times 100\%$$

$$\Rightarrow \text{Error} = \frac{0.004 \times 3}{4} \times 100\%$$

$\Rightarrow$ Error = $0.003 \times 100\%$

$\therefore$ Error = 0.3%

Thus, the error in calculating the volume of the sphere is 0.3%.

6. The pressure p and the volume v of a gas are connected by the relation $pv^{1.4}$ = const. Find the percentage error in p corresponding to a decrease of ½ % in v.

Solution:

Given $pv^{1.4}$ = constant and the decrease in v is ½ %.

Hence, we have $\Delta v = -\frac{\frac{1}{2}}{100} \times v$

∴ $\Delta v = -0.005v$

We have $pv^{1.4}$ = constant

Taking log on both sides, we get

$\log(pv^{1.4}) = \log(\text{constant})$

⇒ $\log p + \log v^{1.4} = 0$ [∵ log (ab) = log a + log b]

⇒ $\log p + 1.4 \log v = 0$ [∵ $\log(a^m) = m \log a$]

On differentiating both sides with respect to v, we get

$$\frac{d}{dp}(\log p) \times \frac{dp}{dv} + \frac{d}{dv}(1.4 \log v) = 0$$

$$\Rightarrow \frac{d}{dp}(\log p) \times \frac{dp}{dv} + 1.4\frac{d}{dv}(\log v) = 0$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

The percentage error is,

$$\text{Error} = \frac{0.007p}{p} \times 100\%$$

⇒ Error = 0.007 × 100%

∴ Error = 0.7%

Thus, the error in p corresponding to the decrease in v is 0.7%.

7. The height of a cone increases by k%, its semi-vertical angle remaining the same. What is the approximate percentage increase in (i) in total surface area, and (ii) in the volume, assuming that k is small.

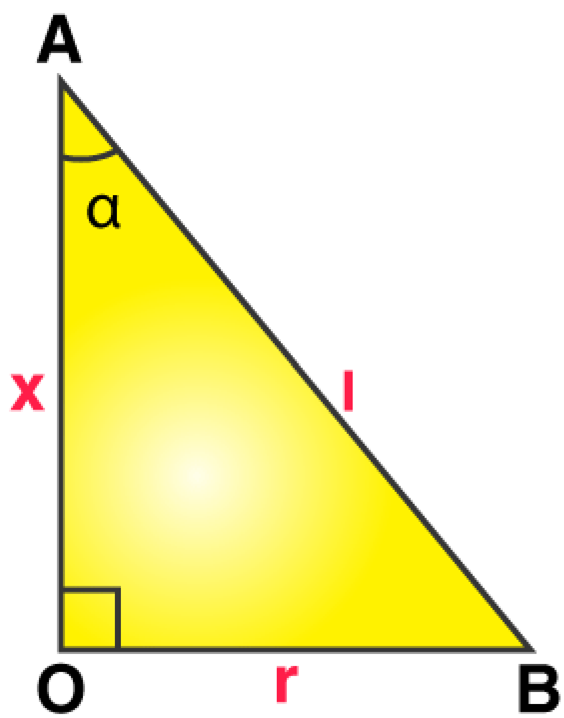
Solution:

Given the height of a cone increases by k%. Let x be the height of the cone and Δx be the change in the value of x.

Hence, we have $\Delta x = \frac{k}{100} \times x$

$\therefore \Delta x = 0.01kx$

Let us assume the radius, the slant height and the semi-vertical angle of the cone to be r, l and α respectively as shown in the figure below.



From the above figure, using trigonometry, we have

$$\tan\alpha = \frac{OB}{OA}$$

$$\Rightarrow \tan\alpha = \frac{r}{x}$$

$\therefore$ r = x tan (α)

We also have

$$\cos\alpha = \frac{OA}{AB}$$

$$\Rightarrow \cos\alpha = \frac{x}{l}$$

$$\Rightarrow l = \frac{x}{\cos\alpha}$$

$\therefore$ l = x sec (α)

$\Rightarrow l = \frac{x}{\cos\alpha}$

∴ l = x sec (α)

(i) The total surface area of the cone is given by

$S = \pi r^2 + \pi r l$

From above, we have r = x tan (α) and l = x sec (α).

$\Rightarrow S = \pi (x \tan(\alpha))^2 + \pi (x \tan(\alpha)) (x \sec(\alpha))$

$\Rightarrow S = \pi x^2 \tan^2\alpha + \pi x^2 \tan(\alpha) \sec(\alpha)$

$\Rightarrow S = \pi x^2 \tan(\alpha) [\tan(\alpha) + \sec(\alpha)]$

On differentiating S with respect to x, we get

$$\frac{dS}{dx} = \frac{d}{dx}[\pi x^2 \tan\alpha (\tan\alpha + \sec\alpha)]$$

$$\Rightarrow \frac{dS}{dx} = \pi \tan\alpha (\tan\alpha + \sec\alpha) \frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = \pi \tan\alpha (\tan\alpha + \sec\alpha)(2x)$$

$$\therefore \frac{dS}{dx} = 2\pi x \tan\alpha (\tan\alpha + \sec\alpha)$$

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dS}{dx} = 2\pi x \tan\alpha\,(\tan\alpha + \sec\alpha)$ and $\Delta x = 0.01kx$

$\Rightarrow \Delta S = (2\pi\, x \tan(\alpha)\,[\tan(\alpha) + \sec(\alpha)])\,(0.01kx)$

$\therefore \Delta S = 0.02\, k\, \pi\, x^2 \tan(\alpha)\,[\tan(\alpha) + \sec(\alpha)]$

The percentage increase in S is,

$$\text{Increase} = \frac{\Delta S}{S} \times 100\%$$

The percentage increase in S is,

$$\text{Increase} = \frac{\Delta S}{S} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.02k\pi x^2 \tan\alpha\,(\tan\alpha + \sec\alpha)}{\pi x^2 \tan\alpha\,(\tan\alpha + \sec\alpha)} \times 100\%$$

⇒ Increase = 0.02k × 100%

∴ Increase = 2k%

Thus, the approximate increase in the total surface area of the cone is 2k%.

(ii) The volume of the cone is given by

$$V = \frac{1}{3}\pi r^2 x$$

From above, we have r = x tan (α).

$$\Rightarrow V = \frac{1}{3}\pi(x\tan\alpha)^2 x$$

$$\Rightarrow V = \frac{1}{3}\pi(x^2\tan^2\alpha)x$$

$$\Rightarrow V = \frac{1}{3}\pi x^3\tan^2\alpha$$

On differentiating V with respect to x, we get

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{1}{3}\pi x^3\tan^2\alpha\right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{1}{3}\pi \tan^2 \alpha \frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{1}{3}\pi \tan^2 \alpha (3x^2)$$

$$\therefore \frac{dV}{dx} = \pi x^2 \tan^2 \alpha$$

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, Δ y = f(x + Δ x) – f(x), is approximately given as

We know that if y = f(x) and Δ x is a small increment in x, then the corresponding increment in y, Δ y = f(x + Δ x) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dV}{dx} = \pi x^2 \tan^2 \alpha$ and Δ x = 0.01kx

$\Rightarrow \Delta V = (\pi x^2 \tan^2 \alpha)(0.01kx)$

$\therefore \Delta V = 0.01k\pi x^3 \tan^2 \alpha$

The percentage increase in V is,

$$\text{Increase} = \frac{\Delta V}{V} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.01k\pi x^3 \tan^2 \alpha}{\frac{1}{3}\pi x^3 \tan^2 \alpha} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.01k}{\frac{1}{3}} \times 100\%$$

$\Rightarrow$ Increase = 0.03k × 100%

$\therefore$ Increase = 3k%

Thus, the approximate increase in the volume of the cone is 3k%.

8. Show that the relative error in computing the volume of a sphere, due to an error in measuring the radius, is approximately equal to three times the relative error in the radius.

Solution:

Let the error in measuring the radius of a sphere be k%.

Let x be the radius of the sphere and Δx be the error in the value of x.

Hence, we have $\Delta x = \frac{k}{100} \times x$

$\therefore \Delta x = 0.01kx$

The volume of a sphere of radius x is given by

$\therefore \Delta x = 0.01kx$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3}\pi x^3$$

On differentiating V with respect to x, we get

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}\frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}(3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

We know that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dV}{dx} = 4\pi x^2$ and $\Delta x = 0.01kx$

$\Rightarrow \Delta V = (4\pi x^2)(0.01kx)$

$\therefore \Delta V = 0.04k\pi x^3$

The percentage error is,

$$\text{Error} = \frac{0.04k\pi x^3}{\frac{4}{3}\pi x^3} \times 100\%$$

$$\Rightarrow \text{Error} = \frac{0.04k \times 3}{4} \times 100\%$$

$\Rightarrow$ Error = 0.03k × 100%

$\therefore$ Error = 3k%

$\Rightarrow$ Error = 0.03k × 100%

$\therefore$ Error = 3k%

Chapterwise RD Sharma Solutions for Class 12 Maths :

- Chapter 1–Relation
- Chapter 2–Functions
- Chapter 3–Binary Operations
- Chapter 4–Inverse Trigonometric Functions
- Chapter 5–Algebra of Matrices
- Chapter 6–Determinants
- Chapter 7–Adjoint and Inverse of a Matrix
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- Chapter 19–Indefinite Integrals

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.