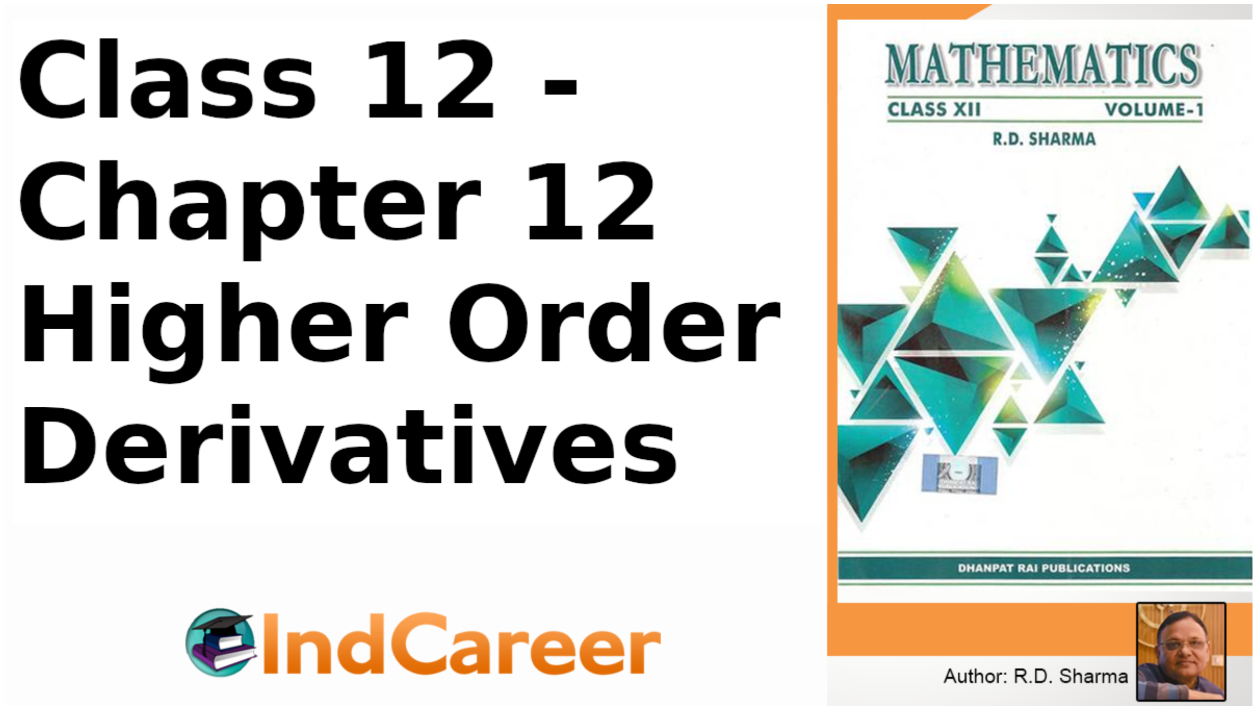

RD Sharma Solutions for Class 12 Maths Chapter 12–Higher Order Derivatives

Class 12: Maths Chapter 12 solutions. Complete Class 12 Maths Chapter 12 Notes.

RD Sharma Solutions for Class 12 Maths Chapter 12–Higher Order Derivatives

RD Sharma 12th Maths Chapter 12, Class 12 Maths Chapter 12 solutions

https://www.indcareer.com/schools/rd-sharma-solutions-for-class-12-maths-chapter-12-higher-order-derivatives/

IndCareer

Exercise 12.1 Page No: 12.17

1. Find the second order derivatives of the each of the following functions:

(i) x^3 + tan x

Solution:

Given, $y = x^3 + \tan x$

We have to find $\frac{d^2y}{dx^2}$

As $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So let's first find $\frac{dy}{dx}$ and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^3 + \tan x) = \frac{d}{dx}(x^3) + \frac{d}{dx}(\tan x)$

$= 3x^2 + \sec^2 x$

$\therefore \frac{dy}{dx} = 3x^2 + \sec^2 x$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(3x^2 + \sec^2 x) = \frac{d}{dx}(3x^2) + \frac{d}{dx}(\sec^2 x)$$

$$\frac{d^2y}{dx^2} = 6x + 2\sec x \sec x \tan x$$

$$\frac{d^2y}{dx^2} = 6x + 2\sec^2 x \tan x$$

(ii) Sin (log x)

Solution:

Let, y = sin (log x)

We have to find $\frac{d^2y}{dx^2}$

We know that $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

We know that $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin(\log x))$

Differentiating $\sin(\log x)$ using the chain rule,

Let, t = log x and y = sin t

$\because \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ [using chain rule]

$$\frac{dy}{dx} = \cos t \times \frac{1}{x}$$

$$\frac{dy}{dx} = \cos(\log x) \times \frac{1}{x}$$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\ (\cos(\log x) \times \frac{1}{x})$$

$$\frac{d^2y}{dx^2} = \ \cos(\log x) \times \frac{-1}{x^2} + \frac{1}{x} \times \frac{1}{x}\ (-\sin(\log x))$$

Now by using product rule for differentiation we get,

$$= \frac{-1}{x^2}\cos(\log x) - \frac{1}{x^2}\sin(\log x)$$

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}\cos(\log x) - \frac{1}{x^2}\sin(\log x)$$

(iii) Log (sin x)

Solution:

Let, y = log (sin x)

We have to find $\frac{d^2y}{dx^2}$

We know $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\log(\sin x))$

Differentiating sin (log x) using chain rule,

Let, t = sin x and y = log t

$\because \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ [using chain rule]

$$\frac{dy}{dx} = \cos x \times \frac{1}{t}$$

$[\because \frac{d}{dx}\log x = \frac{1}{x}$ & $\frac{d}{dx}(\sin x) = \cos x]$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$$

Differentiating again with respect to x:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\cot x)$$

$\frac{d^2y}{dx^2} = -\mathrm{cosec}^2 x$ $[\because \frac{d}{dx}\cot x = -\mathrm{cosec}^2 x]$

$$\frac{d^2y}{dx^2} = -\mathrm{cosec}^2 x$$

(iv) e^x sin 5x

Solution:

Let, y = $e^x \sin 5x$

Now we have to find $\frac{d^2y}{dx^2}$

We know, $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(e^x \sin 5x)$

Let u = e^x and v = sin 5x

As, y = uv

Now by using product rule of differentiation:

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = e^x \frac{d}{dx}(\sin 5x) + \sin 5x \frac{d}{dx} e^x$$

$$\frac{dy}{dx} = 5e^x \cos 5x + e^x \sin 5x$$

$[\because \frac{d}{dx}(\sin ax) = a \cos ax, \text{where a is any constant} \ \& \frac{d}{dx} e^x = e^x]$

Again differentiating with respect to x:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(5e^x \cos 5x + e^x \sin 5x)$$

$$= \frac{d}{dx}(5e^x \cos 5x) + \frac{d}{dx}(e^x \sin 5x)$$

Again using the product rule

$$\frac{d^2y}{dx^2} = e^x \frac{d}{dx}(\sin 5x) + \sin 5x \frac{d}{dx} e^x + 5e^x \frac{d}{dx}(\cos 5x) + \cos 5x \frac{d}{dx}(5e^x)$$

$$\frac{d^2y}{dx^2} = 5e^x \cos 5x - 25e^x \sin 5x + e^x \sin 5x + 5e^x \cos 5x$$

$$\frac{d^2y}{dx^2} = 10e^x \cos 5x - 24e^x \sin 5x$$

(v) e^{6x} cos 3x

Solution:

Let, y = e^{6x} cos 3x

We have to find $\frac{d^2y}{dx^2}$

We know, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(e^{6x} \cos 3x)$

Let $u = e^{6x}$ and $v = \cos 3x$

We have, $y = uv$

Now by using product rule of differentiation

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$\therefore \frac{dy}{dx} = e^{6x}\frac{d}{dx}(\cos 3x) + \cos 3x\frac{d}{dx}e^{6x}$

$\frac{dy}{dx} = -3e^{6x}\sin 3x + 6e^{6x}\cos 3x$

$[\because \frac{d}{dx}(\cos ax) = -a\sin ax, \text{a is any constant} \; \& \frac{d}{dx}e^{ax} = ae^{x}]$

Again differentiating with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-3e^{6x}\sin 3x + 6e^{6x}\cos 3x)$$

$$= \frac{d}{dx}\,(-3e^{6x}\sin 3x) + \frac{d}{dx}(6e^{6x}\cos 3x)$$

Again using the product rule

$$\frac{d^2y}{dx^2} = -3e^{6x}\frac{d}{dx}(\sin 3x) - 3\sin 3x\frac{d}{dx}e^{6x} + 6e^{6x}\frac{d}{dx}(\cos 3x) + \cos 3x\frac{d}{dx}(6e^{6x})$$

$$\frac{d^2y}{dx^2} = -9e^{6x}\cos 3x - 18e^{6x}\sin 3x - 18e^{6x}\sin 3x + 36e^{6x}\cos 3x$$

$$\frac{d^2y}{dx^2} = 27e^{6x}\cos 3x - 36e^{6x}\sin 3x$$

(vi) $x^3 \log x$

Solution:

Let, y = x^3 log x

We have to find $\frac{d^2y}{dx^2}$

We know $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^3 \log x)$

Let u = x^3 and v = log x

We have, y = uv

Now by using product rule of differentiation

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$\therefore \frac{dy}{dx} = x^3\frac{d}{dx}(\log x) + \log x\frac{d}{dx}x^3$

$$\frac{dy}{dx} = 3x^2 \log x + \frac{x^3}{x}$$

$[\because \frac{d}{dx}(\log x) = \frac{1}{x} \text{ and } \frac{d}{dx}(x^n) = nx^{n-1}]$

Again differentiating with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(3x^2 \log x + x^2)$$

$= \frac{d}{dx}(3x^2 \log x) + \frac{d}{dx}(x^2)$

Again using the product rule

$$\frac{d^2y}{dx^2} = 3\log x \frac{d}{dx}x^2 + 3x^2 \frac{d}{dx}\log x + \frac{d}{dx}x^2$$

$$\frac{d^2y}{dx^2} = 3\log x \frac{d}{dx}x^2 + 3x^2 \frac{d}{dx}\log x + \frac{d}{dx}x^2$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$ and $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\frac{d^2y}{dx^2} = 6x\log x + \frac{3x^2}{x} + 2x$$

$$\frac{d^2y}{dx^2} = 6x\log x + 5x$$

(vii) $\tan^{-1}x$

Solution:

Let, y = tan $^{-1}$ x

We have to find $\frac{d^2y}{dx^2}$

As $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x)$

$\therefore \frac{dy}{dx} = \frac{1}{1+x^2}$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{1+x^2}\right)$$

Differentiating $\frac{1}{1+x^2}$ using chain rule,

Let t = 1 +x^2 and z = 1/t

$\because \frac{dz}{dx} = \frac{dz}{dt} \times \frac{dt}{dx}$

$\therefore \frac{dz}{dx} = \frac{-1}{t^2} \times 2x = -\frac{2x}{1+x^2}$ $[\because \frac{d}{dx}(x^n) = nx^{n-1}]$

$\therefore \frac{d^2y}{dx^2} = -\frac{2x}{(1+x^2)^2}$

$\therefore \frac{dz}{dx} = \frac{-1}{t^2} \times 2x = -\frac{2x}{1+x^2}$ $[\because \frac{d}{dx}(x^n) = nx^{n-1}]$

$\therefore \frac{d^2y}{dx^2} = -\frac{2x}{(1+x^2)^2}$

(viii) x cos x

Solution:

Let, y = x cos x

We have to find $\frac{d^2y}{dx^2}$

We know $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(x \cos x)$

Let u = x and v = cos x

As, y = u v

Now by using product rule of differentiation:

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = x\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}x$$

$$\frac{dy}{dx} = -x\sin x + \cos x$$

$[\because \frac{d}{dx}(\cos x) = -\sin x \text{ and } \frac{d}{dx}(x^n) = nx^{n-1}]$

Again differentiating with respect to x:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-x\sin x + \cos x)$$

$$= \frac{d}{dx}(-x\sin x) + \frac{d}{dx}\cos x$$

Again using the product rule

$$\frac{d^2y}{dx^2} = -x\frac{d}{dx}\sin x + \sin x\frac{d}{dx}(-x) + \frac{d}{dx}\cos x$$

$$[\because \frac{d}{dx}(\sin x) = \cos x \text{ and } \frac{d}{dx}(x^n) = nx^{n-1}]$$

$$\frac{d^2y}{dx^2} = -x\cos x - \sin x - \sin x$$

$$\frac{d^2y}{dx^2} = -x\cos x - 2\sin x$$

(ix) Log (log x)

Solution:

Let, y = log (log x)

We have to find $\frac{d^2y}{dx^2}$

We know, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\log \log x)$

Let y = log t and t = log x

Using chain rule of differentiation:

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$\therefore \frac{dy}{dx} = \frac{1}{t} \times \frac{1}{x} = \frac{1}{x \log x}$

Again differentiating with respect to x:

As, $\frac{dy}{dx} = u \times v$

Where u = $\frac{1}{x}$ and v = $\frac{1}{\log x}$

Now by using product rule of differentiation:

$$\frac{d^2y}{dx^2} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{x}\frac{d}{dx}\left(\frac{1}{\log x}\right) + \frac{1}{\log x}\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\frac{d^2y}{dx^2} = -\frac{1}{x^2(\log x)^2} - \frac{1}{x^2 \log x}$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{1}{x^2(\log x)^2} - \frac{1}{x^2 \log x}$$

2. $If\ y = e^{-x}\cos x,\ show\ that\ \frac{d^2y}{dx^2} = 2e^{-x}\sin x.$

Solution:

Let $y=e^{-x}\cos x$

We have to find $\frac{d^2y}{dx^2}$

We have, $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(e^{-x}\cos x)$

Let $u = e^{-x}$ and $v = \cos x$

We have, $y = u\,v$

Differentiate the above by using product rule,

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = e^{-x}\frac{d}{dx}(\cos x) + \cos x\frac{dy}{dx}e^{-x}$$

$$\frac{dy}{dx} = -e^{-x}\sin x - e^{-x}\cos x$$

$$[\because \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}e^{-x} = -e^{-x}]$$

Again differentiating with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-e^{-x}\sin x - e^{-x}\cos x)$$

$$= \frac{d}{dx}(-e^{-x}\sin x) - \frac{d}{dx}(e^{-x}\cos x)$$

Again by using product rule we get

$$\frac{d^2y}{dx^2} = -e^{-x}\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}e^{-x} - e^{-x}\frac{d}{dx}(\cos x) - \cos x\frac{d}{dx}(e^{-x})$$

$$\frac{d^2y}{dx^2} = -e^{-x}\cos x + e^{-x}\sin x + e^{-x}\sin x + e^{-x}\cos x$$

$$[\because \frac{d}{dx}(\cos x) = -\sin x, \frac{d}{dx}e^{-x} = -e^{-x}]$$

$$\frac{d^2y}{dx^2} = 2e^{-x}\sin x$$

Hence proved.

$$\textbf{3. } If\ y = x + \tan x,\ show\ that\ \cos^2 x\frac{d^2y}{dx^2} - 2y + 2x = 0.$$

Solution:

Given y = x + tan x

Let's find $\frac{d^2y}{dx^2}$

As $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(x + \tan x) = \frac{d}{dx}(x) + \frac{d}{dx}(\tan x) = 1 + \sec^2 x$

$\therefore \frac{dy}{dx} = 1 + \sec^2 x$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(1 + \sec^2 x) = \frac{d}{dx}(1) + \frac{d}{dx}(\sec^2 x)$$

By using chain rule, we get

$$\frac{d^2y}{dx^2} = 0 + 2\sec x \sec x \tan x$$

$\frac{d^2y}{dx^2} = 2\sec^2 x \tan x$

As we got an expression for the second order, as we need $\cos^2x$ term with $\frac{d^2y}{dx^2}$

Multiply both sides of equation 1 with $\cos^2x$

We have,

$\cos^2 x \frac{d^2y}{dx^2} = 2\cos^2 x \sec^2 x \tan x$ [∵ cos x × sec x = 1]

$$\cos^2 x \frac{d^2y}{dx^2} = 2\tan x$$

From the given equation tan x = y – x

$$\therefore \cos^2 x \frac{d^2y}{dx^2} = 2(y - x)$$

$$\therefore \cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$$

4. $If\ y = x^3 \log x,\ prove\ that\ \frac{d^4y}{dx^4} = \frac{6}{x}.$

Solution:

Given, $y = x^3 \log x$

Let's find $\frac{d^4y}{dx^4}$

As $\frac{d^4y}{dx^4} = \frac{d}{dx}(\frac{d^3y}{dx^3}) = \frac{d}{dx}\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{dy}{dx}\right)\right)\right)$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^3 \log x)$

Again differentiating by using product rule, we get

$$\frac{dy}{dx} = x^3\frac{d}{dx}\log x + \log x\frac{d}{dx}x^3$$

$$\frac{dy}{dx} = \frac{x^3}{x} + 3x^2\log x$$

$$\frac{dy}{dx} = x^2(1 + 3\log x)$$

Again differentiating using product rule:

$$\frac{d^2y}{dx^2} = x^2\frac{d}{dx}(1 + 3\log x) + (1 + 3\log x)\frac{d}{dx}x^2$$

$$\frac{d^2y}{dx^2} = x^2 \times \frac{3}{x} + (1 + 3\log x) \times 2x$$

$$\frac{d^2y}{dx^2} = x(5 + 6\log\ x)$$

Again differentiating using product rule

$$\frac{d^3y}{dx^3} = x\frac{d}{dx}(5 + 6\log x) + (5 + 6\log x)\frac{d}{dx}x$$

$$\frac{d^3y}{dx^3} = x \times \frac{6}{x} + (5 + 6\log x)$$

$$\frac{d^3y}{dx^3} = 11 + 6\log x$$

Again differentiating with respect to x

$$\frac{d^4y}{dx^4} = \frac{6}{x}$$

Hence proved.

5. $If\ y = \log\ (\sin x),\ prove\ that\ \frac{d^3y}{dx^3} = 2\cos x\ cosec^3 x.$

Solution:

Given, y = log (sin x)

Let's find – $\frac{d^3y}{dx^3}$

As $\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}(\frac{d}{dx}\left(\frac{dy}{dx}\right))$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\log(\sin x))$

Differentiating log (sin x) using the chain rule,

Let, t = sin x and y = log t

$\because \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ [using chain rule]

$$\frac{dy}{dx} = \cos x \times \frac{1}{t}$$

[$\because \frac{d}{dx}\log x = \frac{1}{x}$ & $\frac{d}{dx}(\sin x) = \cos x$]

$$\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\ (\cot x)$$

$$\frac{d^2y}{dx^2} = \ -\mathrm{cosec}^2 x$$

$[\because \frac{d}{dx}\cot x = \ -\mathrm{cosec}^2 x]$

$$\frac{d^2y}{dx^2} = -\mathrm{cosec}^2 x$$

Differentiating again with respect to x:

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}\ (\ -\mathrm{cosec}^2 x)$$

Using the chain rule and $\frac{d}{dx}\mathrm{cosec}\, x = \ -\mathrm{cosec}\, x \cot x$

$$\frac{d^3y}{dx^3} = \ -2\mathrm{cosec}\, x(-\mathrm{cosec}\, x \cot x)$$

$$= 2\mathrm{cosec}^2 x\ \cot x\ = 2\ \mathrm{cosec}^2 x\ \frac{\cos x}{\sin x}$$

$$\therefore \frac{d^3y}{dx^3} = 2\mathrm{cosec}^3 x \cos x$$

Hence proved.

$$\mathbf{6.}\ \boldsymbol{If\ \ y = 2\sin x + 3\cos x,\ \ show\ that\ \frac{d^2y}{dx^2} + y = 0.}$$

Solution:

Given y = 2 sin x + 3 cos x

Let's find $\frac{d^2y}{dx^2}$

We know $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(2\sin x + 3\cos x) = 2\frac{d}{dx}(\sin x) + 3\frac{d}{dx}(\cos x)$

$= 2\cos x - 3\sin x$

$\therefore \frac{dy}{dx} = 2\cos x - 3\sin x$

Differentiating again with respect to x:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(2\cos x - 3\sin x) = \frac{2d}{dx}\cos x - 3\frac{d}{dx}\sin x$$

$$\frac{d^2y}{dx^2} = -2\sin x - 3\cos x$$

We have, y = 2 sin x + 3 cos x

$\therefore \frac{d^2y}{dx^2} = -(2\sin x + 3\cos x) = -y$

$\therefore \frac{d^2y}{dx^2} + y = 0$

Hence proved.

7. $If\ y = \frac{\log x}{x},\ show\ that\ \frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}.$

Solution:

As y is the product of two functions u and v

Let u = log x and v = 1/x

Now by using product rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\,\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{\log x}{x}\right) = \log x\frac{d}{dx}\frac{1}{x} + \frac{1}{x}\frac{d}{dx}\log x$$

$$\frac{dy}{dx} = -\frac{1}{x^2}\log x + \frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2}\,(1 - \log x)$$

Again using the product rule to find $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = (1 - \log x)\frac{d}{dx}\frac{1}{x^2} + \frac{1}{x^2}\frac{d}{dx}(1 - \log x)$$

$$= -2\left(\frac{1 - \log x}{x^3}\right) - \frac{1}{x^3}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$$

Hence proved.

8. $If\ x = a\ sec\ \theta,\ y = b\tan\theta,\ prove\ that\ \frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}.$

Solution:

If y = f (θ) and x = g (θ) that is y is a function of θ and x is also some other function of θ.

Then dy/dθ = f' (θ) and dx/dθ = g' (θ)

We can write: $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

We can write: $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Given,

x = a sec θ ......equation 1

y = b tan θ ......equation 2

We have to prove $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

Let's find $\frac{d^2y}{dx^2}$

As, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So, let's first find dy/dx using parametric form and differentiate it again.

$\frac{dx}{d\theta} = \frac{d}{d\theta} a \sec\theta = a \sec\theta \tan\theta$ .....equation 3

Similarly, $\frac{dy}{d\theta} = b \sec^2\theta$ ...... equation 4

$[\because \frac{d}{dx}\sec x = \sec x \tan x, \frac{d}{dx}\tan x = \sec^2 x]$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b \sec^2\theta}{a \sec\theta \tan\theta} = \frac{b}{a}\operatorname{cosec}\theta$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\frac{b}{a}\operatorname{cosec}\theta)$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a}\operatorname{cosec}\theta\cot\theta\frac{d\theta}{dx}$$ .....equation 5 [using chain rule]

From equation 3:

$$\frac{dx}{d\theta} = a\sec\theta\tan\theta$$

$$\therefore \frac{d\theta}{dx} = \frac{1}{a\sec\theta\tan\theta}$$

Putting the value in equation 5

$$\frac{d^2y}{dx^2} = -\frac{b}{a}\operatorname{cosec}\theta\cot\theta\frac{1}{a\sec\theta\tan\theta}$$

$$\frac{d^2y}{dx^2} = \frac{-b}{a^2\tan^3\theta}$$

From equation 1:

y = b tan θ

$$\therefore \frac{d^2y}{dx^2} = \frac{-b}{\frac{a^2y^3}{b^3}} = -\frac{b^4}{a^2y^3}.$$

$9. If\ x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta),\ prove\ that$ $\frac{d^2x}{d\theta^2} = a(\cos\theta - \theta\sin\theta), \frac{d^2y}{d\theta^2} = a(\sin\theta + \theta\cos\theta)\ and\ \frac{d^2y}{dx^2} = \frac{sec^3\theta}{a\theta}.$

Solution:

If y = f (θ) and x = g (θ), that is y is a function of θ and x is also some other function of θ.

Then dy/dθ = f' (θ) and dx/dθ = g' (θ)

We can write: $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Given,

x =a (cos θ + θ sin θ) ……equation 1

y =a (sin θ – θ cos θ) ……equation 2

Let's find $\frac{d^2y}{dx^2}$

We know $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}a(\cos\theta + \theta\sin\theta)$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta} a(\cos\theta + \theta\sin\theta)$$

$$= a(-\sin\theta + \theta\cos\theta + \sin\theta)$$

= a θ cos θ... Equation 4

Again differentiating with respect to θ using product rule

$$\frac{d^2x}{d\theta^2} = a(-\theta\sin\theta + \cos\theta)$$

$$\therefore \frac{d^2x}{d\theta^2} = a(\cos\theta - \theta\sin\theta)$$

Similarly,

$$\frac{dy}{d\theta} = \frac{d}{d\theta} a(\sin\theta - \theta\cos\theta) = a\frac{d}{d\theta}\sin\theta - a\frac{d}{d\theta}(\theta\cos\theta)$$

$$= a\cos\theta + a\theta\sin\theta - a\cos\theta$$

$\therefore \frac{dy}{d\theta} = a\theta\sin\theta$ .... equation 5

Again differentiating with respect to θ using product rule

$$\frac{d^2x}{d\theta^2} = a(\theta\cos\theta + \sin\theta)$$

$$\therefore \frac{d^2x}{d\theta^2} = a(\sin\theta + \theta\cos\theta)$$

$\because \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Using equation 4 and 5, we have

$$\frac{dy}{dx} = \frac{a\theta \sin\theta}{a\theta \cos\theta} = \tan\theta$$

We have $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

Again differentiating with respect to x

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\tan\theta$$

$$= \sec^2\theta \frac{d\theta}{dx}$$

$$\because \frac{dx}{d\theta} = a\theta\cos\theta \Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$$

Putting a value in the above equation we get

$$\frac{d^2y}{dx^2} = \sec^2\theta \times \frac{1}{a\theta\cos\theta}$$

$$\frac{d^2y}{dx^2} = \frac{\sec^3\theta}{a\theta}$$

10. $If\ y = e^x \cos x,\ prove\ that\ \frac{d^2y}{dx^2} = 2e^x \cos\left(x + \frac{\Pi}{2}\right).$

Solution:

Given, y = e^x cos x

We have to find $\frac{d^2y}{dx^2}$

As $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So let's first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(e^x \cos x)$

Let u = e^x and v = cos x

As, y = u v

Now by using product rule we get

$$\therefore \frac{dy}{dx} = e^x \frac{d}{dx}(\cos x) + \cos x \frac{dy}{dx} e^x$$

$$\frac{dy}{dx} = -e^x \sin x + e^x \cos x \; [\because \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx} e^x = e^x]$$

Again differentiating with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-e^x \sin x + e^x \cos x)$$

$$= \frac{d}{dx}(-e^x \sin x) + \frac{d}{dx}(e^x \cos x)$$

Again using the product rule

$$\frac{d^2y}{dx^2} = -e^x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx} e^x + e^x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(e^x)$$

$$\frac{d^2y}{dx^2} = -e^x \cos x - e^x \sin x - e^x \sin x + e^x \cos x$$

$$[\because \frac{d}{dx}(\cos x) = -\sin x, \frac{d}{dx} e^{-x} = -e^{-x}]$$

$$\frac{d^2y}{dx^2} = -2e^x \sin x \; [\because -\sin x = \cos(x + \pi/2)]$$

$$\frac{d^2y}{dx^2} = -2e^x \cos(x + \frac{\pi}{2})$$

Hence proved.

11. $If\ x = a\cos\theta\ y = b\sin\theta,\ show\ that\ \frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}.$

Solution:

Given,

x = a cos θ ......equation 1

y = b sin θ ......equation 2

If y = f (θ) and x = g (θ) that is y is a function of θ and x is also some other function of θ.

function of θ.

Then dy/dθ = f’ (θ) and dx/dθ = g’ (θ)

We can write $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Now we have to prove $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

Let’s find $\frac{d^2y}{dx^2}$

As $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So, let’s first find dy/dx using parametric form and differentiate it again.

$\frac{dx}{d\theta} = \frac{d}{d\theta} a\cos\theta = -a\sin\theta$ .....equation 3

Similarly, $\frac{dy}{d\theta} = b\cos\theta$ ...... equation 4

[$\because \frac{d}{dx}\cos x = -\sin x \tan x, \frac{d}{dx}\sin x = \cos x$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{b\cos\theta}{a\sin\theta} = -\frac{b}{a}\cot\theta$$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{b}{a}\cot\theta\right)$$

By using chain rule, we get

$$\frac{d^2y}{dx^2} = \frac{b}{a}\operatorname{cosec}^2\theta\frac{d\theta}{dx}$$ .....equation 5

From equation 3

$$\frac{dx}{d\theta} = -a\sin\theta$$

$$\therefore \frac{d\theta}{dx} = \frac{-1}{a\sin\theta}$$

Putting the value in equation 5

$$\frac{d^2y}{dx^2} = -\frac{b}{a}\operatorname{cosec}^2\theta\frac{1}{a\sin\theta}$$

$$\frac{d^2y}{dx^2} = \frac{-b}{a^2\sin^3\theta}$$

From equation 1:

y = b sin θ

$$\therefore \frac{d^2y}{dx^2} = \frac{-b}{\frac{a^2y^3}{b^3}} = -\frac{b^4}{a^2y^3}$$

Hence proved.

12. $If\ x = a(1-\cos^3\theta), y = s\sin^3\theta,\ prove\ that\ \frac{d^2y}{dx^2} = \frac{32}{27a}\ at\ \theta = \frac{\pi}{6}.$

Solution:

Given,

$x = a(1 - \cos^3\theta)$ …… equation 1

$y = a \sin^3 \theta$ …… equation 2

If $y = f(\theta)$ and $x = g(\theta)$ that is y is a function of θ and x is also some other function of θ.

Then $dy/d\theta = f'(\theta)$ and $dx/d\theta = g'(\theta)$

We can write $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Let's find $\frac{d^2y}{dx^2}$

We know $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

Let’s find $\frac{d^2y}{dx^2}$

We know $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So, let’s first find dy/dx using parametric form and differentiate it again.

Now by using chain rule,

$\frac{dx}{d\theta} = \frac{d}{d\theta} a\,(1 - \cos^3\theta) = 3a\cos^2\theta \sin\theta$ .....equation 3

Similarly,

$\frac{dy}{d\theta} = \frac{d}{d\theta} a\sin^3\theta = 3a\sin^2\theta\cos\theta$ ......equation 4

$[\because \frac{d}{dx}\cos x = -\sin x \,\&\, \frac{d}{dx}\cos x = \sin x]$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta} = \tan\theta$$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan\theta)$$

$\frac{d^2y}{dx^2} = \sec^2\theta\frac{d\theta}{dx}$ ..... Equation 5

From equation 3

$$\frac{dx}{d\theta} = 3\,a\cos^2\theta\sin\,\theta$$

$$\therefore \frac{d\theta}{dx} = \frac{1}{3\,a\cos^2\theta\sin\,\theta}$$

Putting the value in equation 5

$$\frac{d^2y}{dx^2} = \sec^2\theta\frac{1}{3\,a\cos^2\theta\sin\,\theta}$$

$$\frac{d^2y}{dx^2} = \frac{1}{3\,a\cos^4\theta\sin\theta}$$

Put θ = π/6

$$\left(\frac{d^2y}{dx^2}\right)\text{at}\left(x = \frac{\pi}{6}\right) = \frac{1}{3\,a\cos^4\frac{\pi}{6}\sin\frac{\pi}{6}} = \frac{1}{3a\left(\frac{\sqrt{3}}{2}\right)^4\frac{1}{2}}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)\text{at}\left(x = \frac{\pi}{6}\right) = \frac{32}{27\,a}$$

Hence proved

13. If $x = a(\theta + \sin\theta), y = a(1 + \cos\theta),$ prove that $\frac{d^2y}{dx^2} = -\frac{a}{y^2}$.

Solution:

Given,

x = a (θ + sin θ) ......equation 1

y = a (1+ cos θ) ......equation 2

If y = f (θ) and x = g (θ) that is y is a function of θ and x is also some other function of θ.

Then dy/dθ = f' (θ) and dx/dθ = g' (θ)

We can write $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Let's find $\frac{d^2y}{dx^2}$

As, $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So, let's first find dy/dx using parametric form and differentiate it again.

$\frac{dx}{d\theta} = \frac{d}{d\theta} a\ (\theta\ +\ \sin\theta) = a(1 + \cos\theta) = y$

Similarly,

$\frac{dy}{d\theta} = \frac{d}{d\theta} a\,(1 + \cos\theta) = -a\sin\theta$ ...... equation 4

$[\because \frac{d}{dx}\cos x = -\sin x, \frac{d}{dx}\sin x = \cos x$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1+\cos\theta)} = \frac{-\sin\theta}{(1+\cos\theta)} = \frac{-a\sin\theta}{y}$

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = -a\frac{d}{dx}\left(\frac{\sin\theta}{y}\right)$$

Using product rule and chain rule together, we get

$$\frac{d^2y}{dx^2} = -a\left(\frac{\sin\theta}{-y^2}\frac{dy}{dx} + \frac{1}{y}\cos\theta\frac{d\theta}{dx}\right)$$

By using equation 3 and 5

$$\frac{d^2y}{dx^2} = -a\left(\frac{\sin\theta}{-y^2}\frac{(-a\sin\theta)}{y} + \frac{1}{y}\cos\theta\frac{1}{y}\right)$$

$$\frac{d^2y}{dx^2} = -a\left(\frac{a\sin^2\theta}{y^3} + \frac{1}{y^2}\cos\theta\right)$$

$$\frac{d^2y}{dx^2} = -\frac{a}{y^2}\left(\frac{a\sin^2\theta}{a\,(1+\cos\theta)} + \cos\theta\right)$$

$$\frac{d^2y}{dx^2} = -\frac{a}{y^2}\left(\frac{1-\cos^2\theta}{(1+\cos\theta)} + \cos\theta\right)$$

$$\frac{d^2y}{dx^2} = -\frac{a}{y^2}\left(\frac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)} + \cos\theta\right)$$

$$\frac{d^2y}{dx^2} = -\frac{a}{y^2}(1-\cos\theta + \cos\theta)$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{a}{y^2}$$

Hence proved.

Hence proved.

$$14. \; If \; x = a(\theta - \sin\theta), y = a(1+\cos\theta), \; find \; \frac{d^2y}{dx^2}.$$

Solution:

Given,

x = a (θ – sin θ) ……equation 1

y = a (1+ cos θ) ……equation 2

If y = f (θ) and x = g (θ) that is y is a function of θ and x is also some other function of θ.

Then dy/dθ = f’ (θ) and dx/dθ = g’ (θ)

We can write $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Now we have to find $\frac{d^2y}{dx^2}$

As, $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So, let’s first find dy/dx using parametric form and differentiate it again.

$\frac{dx}{d\theta} = \frac{d}{d\theta}a\ (\theta - \sin\theta) = a(1 - \cos\theta)$ …..equation 3

Similarly,

$\frac{dy}{d\theta} = \frac{d}{d\theta}a\ (1 + \cos\theta) = -a\sin\theta$ ……equation 4

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-\sin\theta}{(1-\cos\theta)}$ ….. Equation 5

Differentiating again with respect to x, we get

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = -\frac{d}{dx}(\frac{\sin\theta}{1-\cos\theta})$

Using product rule and chain rule together, we get

$$\frac{d^2y}{dx^2} = \{-\frac{1}{1-\cos\theta}\frac{d}{d\theta}\sin\theta - \sin\theta\frac{d}{d\theta}\frac{1}{(1-\cos\theta)}\}\frac{d\theta}{dx}$$

Apply chain rule to determine $\frac{d}{d\theta}\frac{1}{(1-\cos\theta)}$

$$\frac{d^2y}{dx^2} = \left\{\frac{-\cos\theta}{1-\cos\theta} + \frac{\sin^2\theta}{(1-\cos\theta)^2}\right\}\frac{1}{a(1-\cos\theta)}$$

$$\frac{d^2y}{dx^2} = \left\{\frac{-\cos\theta(1-\cos\theta) + \sin^2\theta}{(1-\cos\theta)^2}\right\}\frac{1}{a(1-\cos\theta)}$$

$$\frac{d^2y}{dx^2} = \left\{\frac{-\cos\theta + \cos^2\theta + \sin^2\theta}{(1-\cos\theta)^2}\right\}\frac{1}{a(1-\cos\theta)}$$

$$\frac{d^2y}{dx^2} = \left\{\frac{1-\cos\theta}{(1-\cos\theta)^2}\right\}\frac{1}{a(1-\cos\theta)} \; [\because \cos^2\theta + \sin^2\theta = 1]$$

$$\frac{d^2y}{dx^2} = \frac{1}{a(1-\cos\theta)^2}$$

We know $1 - \cos\theta = 2\sin^2\theta/2$

$$\frac{d^2y}{dx^2} = \frac{1}{a\left(2\sin^2\frac{\theta}{2}\right)^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{4a}\operatorname{cosec}^4\frac{\theta}{2}$$

Solution:

Given,

y = a (θ + sin θ) ……equation 1

x = a (1– cos θ) ……equation 2

If y = f (θ) and x = g (θ) that is y is a function of θ and x is also some other function of θ.

Then dy/dθ = f’ (θ) and dx/dθ = g’ (θ)

Then dy/dθ = f' (θ) and dx/dθ = g' (θ)

We can write $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Now we have to prove $\frac{d^2y}{dx^2} = -\frac{1}{a}$ at $\theta = \frac{\pi}{2}$.

Let's find $\frac{d^2y}{dx^2}$

As $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So, let's first find dy/dx using parametric form and differentiate it again.

$\frac{dy}{d\theta} = \frac{d}{d\theta} a\,(\theta + \sin\theta) = a(1 + \cos\theta)$ .....equation 3

Similarly,

$\frac{dx}{d\theta} = \frac{d}{d\theta} a\,(1 - \cos\theta) = a\sin\theta$ ......equation 4

$[\because \frac{d}{dx}\cos x = -\sin x, \frac{d}{dx}\sin x = \cos x]$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1+\cos\theta)}{a\sin\theta} = \frac{(1+\cos\theta)}{\sin\theta}$ .....equation 5

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{(1+\cos\theta)}{\sin\theta}\right) = \frac{d}{dx}(1 + \cos\theta)\operatorname{cosec}\theta$$

Using product rule and chain rule together we get

$$\frac{d^2y}{dx^2} = \{\operatorname{cosec}\theta\frac{d}{d\theta}(1+\cos\theta) + (1+\cos\theta)\frac{d}{d\theta}\operatorname{cosec}\theta\}\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \{\operatorname{cosec}\theta(-\sin\theta) + (1+\cos\theta)(-\operatorname{cosec}\theta\cot\theta)\}\frac{1}{a\sin\theta}$$

$$\frac{d^2y}{dx^2} = \{-1 - \operatorname{cosec}\theta\cot\theta - \cot^2\theta\}\frac{1}{a\sin\theta}$$

As we have to find $\frac{d^2y}{dx^2} = -\frac{1}{a}$ at **θ=** $\frac{\pi}{2}$

∴ put θ = π/2 in above equation:

$$\frac{d^2y}{dx^2} = \{-1 - \operatorname{cosec}\frac{\pi}{2}\cot\frac{\pi}{2} - \cot^2\frac{\pi}{2}\}\frac{1}{a\sin\frac{\pi}{2}}$$

$$= \frac{\{-1-0-0\}1}{a}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{a}$$

16. $\boldsymbol{If\ x = a(1-\cos\theta), y = a(\theta+\sin\theta),\ prove\ that\ \frac{d^2y}{dx^2} = -\frac{1}{a}\ at\ \theta = \frac{\pi}{2}.}$

Solution:

Given,

y = a (θ + sin θ) ……equation 1

x = a (1+ cos θ) ……equation 2

If y = f (θ) and x = g (θ) that is y is a function of θ and x is also some other function of θ.

Then dy/dθ = f' (θ) and dx/dθ = g' (θ)

We can write: $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Given,

y = a (θ + sin θ) ……equation 1

x = a (1+ cos θ) ……equation 2

Now we have to prove $\frac{d^2y}{dx^2} = -\frac{1}{a}$ at $\theta = \frac{\pi}{2}$.

Let's find $\frac{d^2y}{dx^2}$

We know, $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So, let's first find dy/dx using parametric form and differentiate it again.

$\frac{dy}{d\theta} = \frac{d}{d\theta}a\,(\theta + \sin\theta) = a(1 + \cos\theta)$ .....equation 3

Similarly,

$\frac{dx}{d\theta} = \frac{d}{d\theta}a\,(1 + \cos\theta) = -a\sin\theta$ ......equation 4

$[\because \frac{d}{dx}\cos x = -\sin x, \frac{d}{dx}\sin x = \cos x]$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1+\cos\theta)}{-a\sin\theta} = -\frac{(1+\cos\theta)}{\sin\theta}$ .....equation 5

Differentiating again with respect to x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{(1+\cos\theta)}{\sin\theta}\right) = -\frac{d}{dx}(1+\cos\theta)\text{cosec}\,\theta$$

Using product rule and chain rule together

$$\frac{d^2y}{dx^2} = -\{\text{cosec}\,\theta\frac{d}{d\theta}(1+\cos\theta) + (1+\cos\theta)\frac{d}{d\theta}\text{cosec}\,\theta\}\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = -\{\text{cosec}\,\theta(-\sin\theta) + (1+\cos\theta)(-\text{cosec}\,\theta\cot\theta)\}\frac{1}{(-a\sin\theta)}$$

$$\frac{d^2y}{dx^2} = \{-1 - \text{cosec}\theta\cot\theta - \cot^2\theta\}\frac{1}{a\sin\theta}$$

As we have to find $\frac{d^2y}{dx^2} = -\frac{1}{a}$ at $\theta = \frac{\pi}{2}$

∴ put θ = π/2 in above equation:

$\frac{d^2y}{dx^2} = \{-1 - \text{cosec}\frac{\pi}{2}\ \cot\frac{\pi}{2} - \cot^2\frac{\pi}{2}\}\frac{1}{a\sin\frac{\pi}{2}} = \frac{\{-1-0-0\}1}{a}$

$\frac{d^2y}{dx^2} = -\frac{1}{a}$

17. $If\ x = \cos\theta, y = \sin^3\theta),\ prove\ that\ y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta(5\cos^2\theta - 1).$

Solution:

Given,

$y = \sin^3\theta$ ......equation 1

$x = \cos\theta$ ......equation 2

If y = f (θ) and x = g (θ), that is y is a function of θ and x is also some other function of θ.

Then dy/dθ = f’ (θ) and dx/dθ = g’ (θ)

We can write: $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

To prove: $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\,(5\cos^2\theta - 1)$

Now we have to find $\frac{d^2y}{dx^2}$

We know, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So, let’s first find dy/dx using parametric form and differentiate it again.

$\frac{dx}{d\theta} = -\sin\theta$ ....equation 3

Applying chain rule to differentiate $\sin^3\theta$, then

$\frac{dy}{d\theta} = 3\sin^2\theta\cos\theta$ ....equation 4

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3\sin^2\theta\cos\theta}{-\sin\theta} = -3\sin\theta\cos\theta$ ....equation 5

Again differentiating with respect to x

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-3\sin\theta\cos\theta)$$

Applying product rule and chain rule together, we get

$$\frac{d^2y}{dx^2} = -3\{\sin\theta\frac{d}{d\theta}\cos\theta + \cos\theta\frac{d}{d\theta}\sin\theta\}\frac{d\theta}{dx}$$

Put the value of dθ/dx

$$\frac{d^2y}{dx^2} = 3\{-\sin^2\theta + \cos^2\theta\}\frac{1}{\sin\theta}$$

Multiplying y both sides to approach towards the expression we want to prove

$$y\frac{d^2y}{dx^2} = 3\{-\sin^2\theta + \cos^2\theta\}\frac{y}{\sin\theta}$$

Substitute the value of y

$$y\frac{d^2y}{dx^2} = 3\{-\sin^2\theta + \cos^2\theta\}\sin^2\theta$$

Adding equation 5 and squaring we get

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\{-\sin^2\theta + \cos^2\theta\}\sin^2\theta + 9\sin^2\theta\cos^2\theta$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\{-\sin^2\theta + \cos^2\theta + 3\cos^2\theta\}$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\{5\cos^2\theta - 1\}$$

18. $If\ y = \sin(\sin x),\ prove\ that\ \frac{d^2y}{dx^2} + \tan x.\frac{dy}{dx} + y\cos^2 x = 0.$

Solution:

Given, y = sin (sin x) ......equation 1

To prove: $\frac{d^2y}{dx^2} + \tan x . \frac{dy}{dx} + y\cos^2 x = 0$

To prove: $\frac{d^2y}{dx^2} + \tan x . \frac{dy}{dx} + y\cos^2 x = 0$

Now we have to find $\frac{d^2y}{dx^2}$

We know $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$

So, first we have to find dy/dx

$$\frac{dy}{dx} = \frac{d}{dx}\sin(\sin x)$$

Using chain rule, we will differentiate the above expression

Let t = sin x $\Longrightarrow \frac{dt}{dx} = \cos x$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$$

$\frac{dy}{dx} = \cos t \cos x = \cos(\sin x)\cos x$ ....equation 2

Again differentiating with respect to x applying product rule, we get

$$\frac{d^2y}{dx^2} = \cos x \frac{d}{dx}\cos(\sin x) + \cos(\sin x)\frac{d}{dx}\cos x$$

Using chain rule we get

$$\frac{d^2y}{dx^2} = -\cos x \cos x \sin(\sin x) - \sin x \cos(\sin x)$$

$$\frac{d^2y}{dx^2} = -y\cos^2 x - \tan x \cos x \cos(\sin x)$$

And using equation 2, we have:

$$\frac{d^2y}{dx^2} = -y\cos^2 x - \tan x \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} + y\cos^2 x + \tan x \frac{dy}{dx} = 0$$

Chapterwise RD Sharma Solutions for Class 12 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.