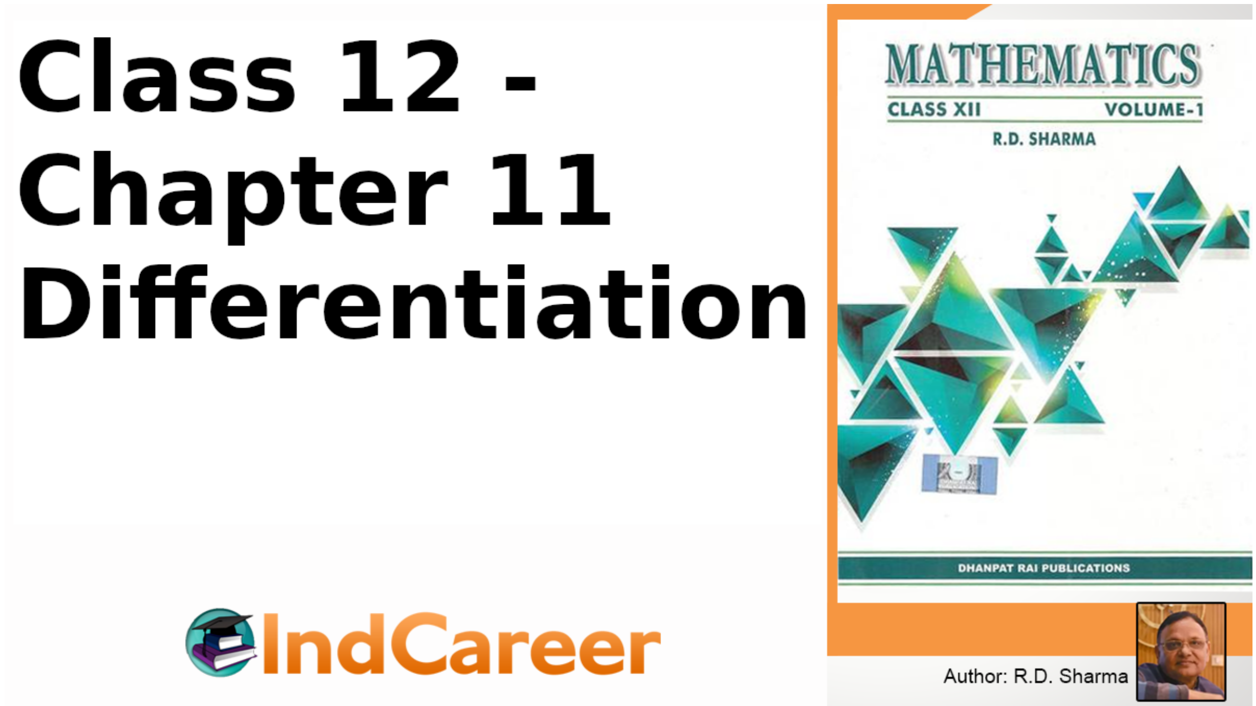

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Exercise 11.1 Page No: 11.17

Differentiate the following functions from the first principles:

1. e^{-x}

Solution:

We have to find the derivative of e^{-x} with the first principle method,

So let $f(x) = e^{-x}$

By using the first principle formula, we get,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^{-(x+h)}-e^{-x}}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^{-x}(e^{-h}-1)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^{-x}(e^{-h}-1)(-1)}{h(-1)}$$

[By using $\lim_{x\to 0}\frac{e^{x}-1}{x} = 1$]

$$f'(x) = -e^{-x}$$

2. e^{3x}

Solution:

We have to find the derivative of e^{3x} with the first principle method,

So, let f (x) = e^{3x}

By using the first principle formula, we get,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{e^{3(x+h)}-e^{3x}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{e^{3x}(e^{3h}-1)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{e^{3x}(e^{3h}-1)3}{3h}$$

[By using $\lim_{x\to 0} \frac{e^{x}-1}{x}$ = 1]

f ' (x) = $3e^{3x}$

3. $e^{ax + b}$

Solution:

We have to find the derivative of e^{ax+b} with the first principle method,

So, let f (x) = e^{ax+b}

By using the first principle formula, we get,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{e^{a(x+h)+b}-e^{ax+b}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{e^{ax+b}(e^{ah}-1)a}{ah}$$

[By using $\lim_{x\to 0} \frac{e^{x}-1}{x}$ = 1]

f ' (x) = a e^{ax+b}

4. $e^{\cos x}$

Solution:

We have to find the derivative of $e^{\cos x}$ with the first principle method,

So, let f (x) = $e^{\cos x}$

By using the first principle formula, we get, f ' (x) = $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

f ' (x) = $\lim_{h\to 0} \frac{e^{\cos(x+h)}-e^{\cos x}}{h}$

f ' (x) = $\lim_{h\to 0} \frac{e^{\cos x}(e^{\cos(x+h)-\cos x}-1)}{h}$

f ' (x) = $\lim_{h\to 0} \frac{e^{\cos x}(e^{\cos(x+h)-\cos x}-1)}{\cos(x+h)-\cos x} \frac{\cos(x+h)-\cos x}{h}$

[By using $\lim_{x\to 0} \frac{e^x-1}{x}$ = 1]

f ' (x) = $\lim_{h\to 0} e^{\cos x} \frac{\cos(x+h)-\cos x}{h}$

Now by using cos (x + h) = cos x cos h – sin x sin h

f ' (x) = $\lim_{h\to 0} e^{\cos x} \frac{\cos x \cos h-\sin x \sin h-\cos x}{h}$

f ' (x) = $\lim_{h\to 0} e^{\cos x} [\frac{\cos x(\cos h-1)}{h} - \frac{\sin x \sin h}{h}]$

[By using $\lim_{x\to 0} \frac{\sin x}{x}$ = 1 and cos 2x = 1–2sin² x]

f ' (x) = $\lim_{h\to 0} e^{\cos x} [\frac{\cos x\left(-2\sin^2\frac{h}{2}\right)(\frac{h}{4})}{h(\frac{h}{4})} - \sin x]$

f ' (x) = $\lim_{h\to 0} e^{\cos x} [\frac{\cos x\left(-2\sin^2\frac{h}{2}\right)(\frac{h}{4})}{\frac{h^2}{2^2}} - \sin x]$

f ' (x) = $-e^{\cos x}$ sin x

5. $e^{\sqrt{2x}}$

Solution:

We have to find the derivative of $e^{\sqrt{2x}}$ with the first principle method,

So, let f(x) = $e^{\sqrt{2x}}$

By using the first principle formula, we get,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^{\sqrt{2(x+h)}}-e^{\sqrt{2x}}}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^{\sqrt{2x}}(e^{\sqrt{2(x+h)}-\sqrt{2x}}-1)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^{\sqrt{2x}}(e^{\sqrt{2(x+h)}-\sqrt{2x}}-1)}{h} \times \frac{\sqrt{2(x+h)}-\sqrt{2x}}{\sqrt{2(x+h)}-\sqrt{2x}}$$

[By using $\lim_{x\to 0}\frac{e^{x}-1}{x}$ = 1]

$$f'(x) = \lim_{h\to 0}\frac{e^{\sqrt{2x}}}{h} \times (\sqrt{2(x+h)} - \sqrt{2x}\,) \times \frac{\sqrt{2(x+h)}+\sqrt{2x}}{\sqrt{2(x+h)}+\sqrt{2x}}$$

[By rationalizing]

$$f'(x) = \lim_{h\to 0}\frac{e^{\sqrt{2x}}}{h} \times \frac{(2(x+h)-2x)}{\sqrt{2(x+h)}+\sqrt{2x}}$$

$$f'(x) = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

Exercise 11.2 Page No: 11.37

Differentiate the following functions with respect to x:

1. Sin (3x + 5)

Solution:

Given Sin (3x + 5)

Let y = sin (3x + 5)

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[\sin(3x+5)]$$

We know $\frac{d}{dx}(\sin x) = \cos x$

$\Rightarrow \frac{dy}{dx} = \cos(3x+5)\frac{d}{dx}(3x+5)$ [Using chain rule]

$$\Rightarrow \frac{dy}{dx} = \cos(3x+5)\left[\frac{d}{dx}(3x) + \frac{d}{dx}(5)\right]$$

$$\Rightarrow \frac{dy}{dx} = \cos(3x+5)\left[3\frac{d}{dx}(x) + \frac{d}{dx}(5)\right]$$

However, $\frac{d}{dx}(x) = 1$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \cos(3x+5)[3 \times 1 + 0]$$

$$\therefore \frac{dy}{dx} = 3\cos(3x+5)$$

Thus, $\frac{d}{dx}[\sin(3x+5)] = 3\cos(3x+5)$

2. $\tan^2 x$

Solution:

Given $\tan^2 x$

Let y = $\tan^2 x$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^2 x)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{dy}{dx} = 2\tan^{2-1} x \frac{d}{dx}(\tan x)$ [Using chain rule]

$$\Rightarrow \frac{dy}{dx} = 2\tan x \frac{d}{dx}(\tan x)$$

However, $\frac{d}{dx}(\tan x) = \sec^2 x$

$$\Rightarrow \frac{dy}{dx} = 2\tan x\,(\sec^2 x)$$

$$\therefore \frac{dy}{dx} = 2\tan x \sec^2 x$$

Thus, $\frac{d}{dx}(\tan^2 x) = 2\tan x \sec^2 x$

3. tan (x° + 45°)

Solution:

Let y = tan (x° + 45°)

First, we will convert the angle from degrees to radians.

Let y = tan (x° + 45°)

First, we will convert the angle from degrees to radians.

We have $1^\circ = \left(\frac{\pi}{180}\right)^c \Rightarrow (x+45)^\circ = \left[\frac{(x+45)\pi}{180}\right]^c$

$$\Rightarrow \; y \; = \tan\left[\frac{(x+45)\pi}{180}\right]$$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left\{\tan\left[\frac{(x+45)\pi}{180}\right]\right\}$$

We know $\frac{d}{dx}(\tan x) = \sec^2 x$

$\Rightarrow \frac{dy}{dx} = \sec^2\left[\frac{(x+45)\pi}{180}\right]\frac{d}{dx}\left[\frac{(x+45)\pi}{180}\right]$ [Using chain rule]

$$\Rightarrow \frac{dy}{dx} = \sec^2(x^\circ + 45^\circ)\frac{\pi}{180}\frac{d}{dx}(x+45)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\pi}{180}\sec^2(x^\circ + 45^\circ)\left[\frac{d}{dx}(x) + \frac{d}{dx}(45)\right]$$

However, $\frac{d}{dx}(x) = 1$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \frac{\pi}{180}\sec^2(x^\circ + 45^\circ)\,[1+0]$$

$$\therefore \frac{dy}{dx} = \frac{\pi}{180}\sec^2(x^\circ + 45^\circ)$$

Thus, $\frac{d}{dx}[\tan(x^\circ + 45^\circ)] = \frac{\pi}{180}\sec^2(x^\circ + 45^\circ)$

4. Sin (log x)

Solution:

Given sin (log x)

Let y = sin (log x)

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[\sin(\log x)]$$

We know $\frac{d}{dx}(\sin x) = \cos x$

5. $e^{sin\sqrt{x}}$

Solution:

Let $y = e^{\sin\sqrt{x}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin\sqrt{x}}\right)$$

We know $\frac{d}{dx}(e^x) = e^x$

$\Rightarrow \frac{dy}{dx} = e^{\sin\sqrt{x}}\frac{d}{dx}\left(\sin\sqrt{x}\right)$ [Using chain rule]

We have $\frac{d}{dx}(\sin x) = \cos x$

$\Rightarrow \frac{dy}{dx} = e^{\sin\sqrt{x}}\cos\sqrt{x}\frac{d}{dx}\left(\sqrt{x}\right)$ [Using chain rule]

$$\Rightarrow \frac{dy}{dx} = e^{\sin\sqrt{x}}\cos\sqrt{x}\frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

However, $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dy}{dx} = e^{\sin\sqrt{x}}\cos\sqrt{x}\left[\frac{1}{2}x^{\left(\frac{1}{2}-1\right)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}e^{\sin\sqrt{x}}\cos\sqrt{x}\,x^{-\frac{1}{2}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x}}e^{\sin\sqrt{x}}\cos\sqrt{x}$$

Thus, $\frac{d}{dx}\left(e^{\sin\sqrt{x}}\right) = \frac{1}{2\sqrt{x}}e^{\sin\sqrt{x}}\cos\sqrt{x}$

6. $e^{\tan x}$

Solution:

Let y = $e^{\tan x}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^{\tan x})$$

We know $\frac{d}{dx}(e^x) = e^x$

$\Rightarrow \frac{dy}{dx} = e^{\tan x}\frac{d}{dx}(\tan x)$ [Using chain rule]

We have $\frac{d}{dx}(\tan x) = \sec^2 x$

$$\therefore \frac{dy}{dx} = e^{\tan x}\sec^2 x$$

Thus, $\frac{d}{dx}(e^{\tan x}) = e^{\tan x}\sec^2 x$

7. Sin² (2x + 1)

Solution:

Let y = $\sin^2(2x + 1)$

On differentiating y with respect to x, we get

$$\therefore \frac{dy}{dx} = 2\sin(4x + 2)$$

Thus, $\frac{d}{dx}[\sin^2(2x + 1)] = 2\sin(4x + 2)$

$$\therefore \frac{dy}{dx} = 2\sin(4x + 2)$$

Thus, $\frac{d}{dx}[\sin^2(2x + 1)] = 2\sin(4x + 2)$

8. $\log_7 (2x - 3)$

Solution:

Let y = $\log_7 (2x - 3)$

We know that $\log_a b = \frac{\log b}{\log a}$.

$$\Rightarrow \log_7(2x-3) = \frac{\log(2x-3)}{\log 7}$$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\log(2x-3)}{\log 7}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1}{\log 7}\right)\frac{d}{dx}[\log(2x-3)]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

Now by using chain rule we get

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1}{\log 7}\right)\left(\frac{1}{2x-3}\right)\frac{d}{dx}(2x-3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(2x-3)\log 7}\left[\frac{d}{dx}(2x) - \frac{d}{dx}(3)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(2x-3)\log 7}\left[2\frac{d}{dx}(x) - \frac{d}{dx}(3)\right]$$

However, $\frac{d}{dx}(x) = 1$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(2x-3)\log 7}[2 \times 1 - 0]$$

$$\therefore \frac{dy}{dx} = \frac{2}{(2x-3)\log 7}$$

Thus, $\frac{d}{dx}[\log_7(2x-3)] = \frac{2}{(2x-3)\log 7}$

9. tan 5x°

Solution:

Let y = tan (5x°)

First, we will convert the angle from degrees to radians. We have

$1° = \left(\frac{\pi}{180}\right)^c \Rightarrow 5x° = 5x \times \frac{\pi}{180}^c$

$\Rightarrow\ y\ = \tan\left(5x \times \frac{\pi}{180}\right)$

On differentiating y with respect to x, we get

$\frac{dy}{dx} = \frac{d}{dx}\left[\tan\left(5x \times \frac{\pi}{180}\right)\right]$

We know $\frac{d}{dx}(\tan x) = \sec^2 x$

Now by using chain rule we have

$\Rightarrow \frac{dy}{dx} = \sec^2\left(5x \times \frac{\pi}{180}\right)\frac{d}{dx}\left(5x \times \frac{\pi}{180}\right)$

$\Rightarrow \frac{dy}{dx} = \sec^2(5x°)\frac{\pi}{180}\frac{d}{dx}(5x)$

$\Rightarrow \frac{dy}{dx} = \frac{\pi}{180}\sec^2(5x°)\left[5\frac{d}{dx}(x)\right]$

However, $\frac{d}{dx}(x) = 1$

$\Rightarrow \frac{dy}{dx} = \frac{\pi}{180}\sec^2(5x°)\,[5]$

$\therefore \frac{dy}{dx} = \frac{5\pi}{180}\sec^2(5x°)$

Thus, $\frac{d}{dx}(\tan 5x°) = \frac{5\pi}{180}\sec^2(5x°)$

10. 2^{x^3}

Solution:

Let $y = 2^{x^3}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(2^{x^3}\right)$$

We know $\frac{d}{dx}(a^x) = a^x \log a$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = 2^{x^3} \log 2 \frac{d}{dx}(x^3)$$

We have $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dy}{dx} = 2^{x^3} \log 2 \times 3x^{3-1}$$

$$\Rightarrow \frac{dy}{dx} = 2^{x^3} \log 2 \times 3x^2$$

$$\therefore \frac{dy}{dx} = 2^{x^3} 3x^2 \log 2$$

Thus, $\frac{d}{dx}\left(2^{x^3}\right) = 2^{x^3} 3x^2 \log 2$

11. 3^{e^x}

Solution:

Let $y = 3^{e^x}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{e^x}\right)$$

We know $\frac{d}{dx}(a^x) = a^x \log a$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = 3^{e^x} \log 3 \frac{d}{dx}(e^x)$$

We have $\frac{d}{dx}(e^x) = e^x$

$$\Rightarrow \frac{dy}{dx} = 3^{e^x} \log 3 \times e^x$$

$$\therefore \frac{dy}{dx} = 3^{e^x} e^x \log 3$$

Thus, $\frac{d}{dx}\left(3^{e^x}\right) = 3^{e^x} e^x \log 3$

12. $\log_x 3$

Solution:

Let y = $\log_x 3$

We know that $\log_a b = \frac{\log b}{\log a}$.

$$\Rightarrow \log_x 3 = \frac{\log 3}{\log x}$$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\log 3}{\log x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \log 3 \frac{d}{dx}\left(\frac{1}{\log x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \log 3 \frac{d}{dx}(\log x)^{-1}$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = \log 3\,[-1 \times (\log x)^{-1-1}]\,\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{dy}{dx} = -\log 3\,(\log x)^{-2}\frac{d}{dx}(\log x)$$

We have $\frac{d}{dx}(\log x) = \frac{1}{x}$

$$\Rightarrow \frac{dy}{dx} = -\log 3\,(\log x)^{-2} \times \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x}\frac{\log 3}{(\log x)^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x}\frac{\log 3}{(\log x)^2} \times \frac{\log 3}{\log 3}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x\log 3}\frac{(\log 3)^2}{(\log x)^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x\log 3}\left(\frac{\log 3}{\log x}\right)^2$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x\log 3 \times \left(\frac{\log x}{\log 3}\right)^2}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{x\log 3\,(\log_3 x)^2}$$

Thus, $\frac{d}{dx}(\log_x 3) = -\frac{1}{x\log 3(\log_3 x)^2}$

13. 3^{x^2+2x}

Solution:

Let $y = 3^{x^2+2x}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x^2+2x}\right)$$

We know $\frac{d}{dx}(a^x) = a^x \log a$

Now by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = 3^{x^2+2x}\log 3 \frac{d}{dx}(x^2 + 2x)$$

$$\Rightarrow \frac{dy}{dx} = 3^{x^2+2x}\log 3\left[\frac{d}{dx}(x^2) + \frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{dy}{dx} = 3^{x^2+2x}\log 3\left[\frac{d}{dx}(x^2) + 2\frac{d}{dx}(x)\right]$$

We have $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{dy}{dx} = 3^{x^2+2x}\log 3\,[2x + 2 \times 1]$$

$$\Rightarrow \frac{dy}{dx} = 3^{x^2+2x}\log 3\,(2x + 2)$$

$$\therefore \frac{dy}{dx} = (2x + 2)3^{x^2+2x}\log 3$$

Thus, $\frac{d}{dx}\left(3^{x^2+2x}\right) = (2x + 2)3^{x^2+2x}\log 3$

14. $\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$

Solution:

Let $y=\sqrt{\frac{a^2-x^2}{a^2+x^2}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{\frac{a^2-x^2}{a^2+x^2}}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}\left[\left(\frac{a^2-x^2}{a^2+x^2}\right)^{\frac{1}{2}}\right]$$

We know $\frac{d}{dx}(x^n)=nx^{n-1}$

Using chain rule

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(\frac{a^2-x^2}{a^2+x^2}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{a^2-x^2}{a^2+x^2}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(\frac{a^2-x^2}{a^2+x^2}\right)^{-\frac{1}{2}}\frac{d}{dx}\left(\frac{a^2-x^2}{a^2+x^2}\right)$$

We know that $\left(\frac{u}{v}\right)'=\frac{vu'-uv'}{v^2}$ (quotient rule)

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(\frac{a^2-x^2}{a^2+x^2}\right)^{-\frac{1}{2}}\left[\frac{(a^2+x^2)\frac{d}{dx}(a^2-x^2)-(a^2-x^2)\frac{d}{dx}(a^2+x^2)}{(a^2+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx}$$

$$= \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{-\frac{1}{2}}\left[\frac{(a^2 + x^2)\left(\frac{d}{dx}(a^2) - \frac{d}{dx}(x^2)\right) - (a^2 - x^2)\left(\frac{d}{dx}(a^2) + \frac{d}{dx}(x^2)\right)}{(a^2 + x^2)^2}\right]$$

However, $\frac{d}{dx}(x^2) = 2x$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{-\frac{1}{2}}\left[\frac{(a^2 + x^2)(0 - 2x) - (a^2 - x^2)(0 + 2x)}{(a^2 + x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{-\frac{1}{2}}\left[\frac{-2x(a^2 + x^2) - 2x(a^2 - x^2)}{(a^2 + x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{-\frac{1}{2}}\left[\frac{-2x(a^2 + x^2 + a^2 - x^2)}{(a^2 + x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{-\frac{1}{2}}\left[\frac{-2x(2a^2)}{(a^2 + x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{-\frac{1}{2}}\left[\frac{-2xa^2}{(a^2 + x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(a^2 - x^2)^{-\frac{1}{2}}}{(a^2 + x^2)^{-\frac{1}{2}}}\left[\frac{-2xa^2}{(a^2 + x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2xa^2(a^2 - x^2)^{-\frac{1}{2}}}{(a^2 + x^2)^{-\frac{1}{2}+2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2xa^2(a^2 - x^2)^{-\frac{1}{2}}}{(a^2 + x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2xa^2}{(a^2 + x^2)^{\frac{3}{2}}(a^2 - x^2)^{\frac{1}{2}}}$$

$$\therefore \frac{dy}{dx} = \frac{-2xa^2}{(a^2 + x^2)^{\frac{3}{2}}\sqrt{a^2 - x^2}}$$

15. $3^{x \log x}$

Solution:

Let $y = 3^{x\log x}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x\log x}\right)$$

We know $\frac{d}{dx}(a^x) = a^x \log a$

Now by using chain rule

$$\Rightarrow \frac{dy}{dx} = 3^{x\log x} \log 3 \frac{d}{dx}(x\log x)$$

We know that by product rule (u v)' = vu' + u v'

$$\Rightarrow \frac{dy}{dx} = 3^{x\log x} \log 3 \frac{d}{dx}(x \times \log x)$$

$$\Rightarrow \frac{dy}{dx} = 3^{x\log x} \log 3 \left[\log x \frac{d}{dx}(x) + x \frac{d}{dx}(\log x)\right]$$

We have $\frac{d}{dx}(\log x) = \frac{1}{x}$ and $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{dy}{dx} = 3^{x\log x} \log 3 \left[\log x \times 1 + x \times \frac{1}{x}\right]$$

$$\Rightarrow \frac{dy}{dx} = 3^{x\log x} \log 3 \, [\log x + 1]$$

$$\therefore \frac{dy}{dx} = (1 + \log x) 3^{x\log x} \log 3$$

Thus, $\frac{d}{dx}\left(3^{x\log x}\right) = (1 + \log x) 3^{x\log x} \log 3$

16. $\sqrt{\frac{1 + sinx}{1 - sinx}}$

Solution:

Let $y=\sqrt{\frac{1+\sin x}{1-\sin x}}$

On differentiating y with respect to x, we get

On differentiating y with respect to x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}\left[\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}}\right]$$

We know $\frac{d}{dx}(x^n)=nx^{n-1}$

Using chain rule, we get

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}}\frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)$$

We know that

$$\left(\frac{u}{v}\right)'=\frac{vu'-uv'}{v^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}}\left[\frac{(1-\sin x)\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx}$$

$$= \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}}\left[\frac{(1-\sin x)\left(\frac{d}{dx}(1)+\frac{d}{dx}(\sin x)\right)-(1+\sin x)\left(\frac{d}{dx}(1)-\frac{d}{dx}(\sin x)\right)}{(1-\sin x)^2}\right]$$

We know $\frac{d}{dx}(\sin x) = \cos x$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}}\left[\frac{(1-\sin x)(0+\cos x)-(1+\sin x)(0-\cos x)}{(1-\sin x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}}\left[\frac{(1-\sin x)\cos x + (1+\sin x)\cos x}{(1-\sin x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}}\left[\frac{(1-\sin x + 1 + \sin x)\cos x}{(1-\sin x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}}\left[\frac{2\cos x}{(1-\sin x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}}\left[\frac{\cos x}{(1-\sin x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\sin x)^{-\frac{1}{2}}}{(1-\sin x)^{-\frac{1}{2}}}\left[\frac{\cos x}{(1-\sin x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\sin x)^{-\frac{1}{2}}\cos x}{(1-\sin x)^{-\frac{1}{2}+2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\sin x)^{-\frac{1}{2}}\cos x}{(1-\sin x)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(1-\sin x)^{1+\frac{1}{2}}(1+\sin x)^{\frac{1}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(1-\sin x)(1-\sin x)^{\frac{1}{2}}(1+\sin x)^{\frac{1}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(1-\sin x)\sqrt{(1-\sin x)(1+\sin x)}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(1-\sin x)\sqrt{1-\sin^2 x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(1-\sin x)\sqrt{\cos^2 x}}$$ ($\because \sin^2\theta + \cos^2\theta = 1$)

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(1-\sin x)\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1-\sin x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1+\sin x}{1-\sin^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1+\sin x}{\cos^2 x}$$ ($\because \sin^2\theta + \cos^2\theta = 1$)

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1}{\cos x}\right)^2 + \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 x + \sec x \tan x$$

$$\therefore \frac{dy}{dx} = \sec x(\sec x + \tan x)$$

Thus, $\frac{d}{dx}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right) = \sec x(\sec x + \tan x)$ **17.** $\sqrt{\frac{1+x^2}{1-x^2}}$

Solution:

Let $y = \sqrt{\frac{1-x^2}{1+x^2}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}\right]$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

Now by using chain rule

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}}\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$$

We know that $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$ (quotient rule)

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}}\left[\frac{(1+x^2)\frac{d}{dx}(1-x^2)-(1-x^2)\frac{d}{dx}(1+x^2)}{(1+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx}$$

$$= \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}}\left[\frac{(1+x^2)\left(\frac{d}{dx}(1)-\frac{d}{dx}(x^2)\right)-(1-x^2)\left(\frac{d}{dx}(1)+\frac{d}{dx}(x^2)\right)}{(1+x^2)^2}\right]$$

However, $\frac{d}{dx}(x^2) = 2x$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}}\left[\frac{(1+x^2)(0-2x)-(1-x^2)(0+2x)}{(1+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}}\left[\frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}}\left[\frac{-2x(1+x^2+1-x^2)}{(1+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}}\left[\frac{-2x(2)}{(1+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}}\left[\frac{-2x}{(1+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1-x^2)^{-\frac{1}{2}}}{(1+x^2)^{-\frac{1}{2}}}\left[\frac{-2x}{(1+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x(1-x^2)^{-\frac{1}{2}}}{(1+x^2)^{-\frac{1}{2}+2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x(1-x^2)^{-\frac{1}{2}}}{(1+x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{(1+x^2)^{\frac{3}{2}}(1-x^2)^{\frac{1}{2}}}$$

$$\therefore \frac{dy}{dx} = \frac{-2x}{(1+x^2)^{\frac{3}{2}}\sqrt{1-x^2}}$$

Thus, $\frac{d}{dx}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right) = \frac{-2x}{(1+x^2)^{\frac{3}{2}}\sqrt{1-x^2}}$

18. $(\log \sin x)^2$

Solution:

Let $y = (\log \sin x)^2$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[(\log(\sin x))^2]$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = 2(\log(\sin x))^{2-1}\frac{d}{dx}[\log(\sin x)]$$

$$\Rightarrow \frac{dy}{dx} = 2\log(\sin x)\frac{d}{dx}[\log(\sin x)]$$

We have $\frac{d}{dx}(\log x) = \frac{1}{x}$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = 2\log(\sin x)\left[\frac{1}{\sin x}\frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sin x}\log(\sin x)\frac{d}{dx}(\sin x)$$

However, $\frac{d}{dx}(\sin x) = \cos x$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sin x}\log(\sin x)\cos x$$

$$\Rightarrow \frac{dy}{dx} = 2\left(\frac{\cos x}{\sin x}\right)\log(\sin x)$$

$$\therefore \frac{dy}{dx} = 2\cot x\log(\sin x)$$

Thus, $\frac{d}{dx}[(\log(\sin x))^2] = 2\cot x\log(\sin x)$

19. $\sqrt{\frac{1+x}{1-x}}$

Solution:

https://www.indcareer.com/schools/rd-sharma-solutions-for-class-12-maths-chapter-11-differentiation/

Let $y = \sqrt{\frac{1+x}{1-x}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{1+x}{1-x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\right]$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

We know that $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$ (quotient rule)

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\left[\frac{(1-x)\frac{d}{dx}(1+x) - (1+x)\frac{d}{dx}(1-x)}{(1-x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\left[\frac{(1-x)\left(\frac{d}{dx}(1)+\frac{d}{dx}(x)\right) - (1+x)\left(\frac{d}{dx}(1)-\frac{d}{dx}(x)\right)}{(1-x)^2}\right]$$

However, $\frac{d}{dx}(x) = 1$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\left[\frac{(1-x)(0+1)-(1+x)(0-1)}{(1-x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\left[\frac{(1-x)+(1+x)}{(1-x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\left[\frac{2}{(1-x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\left[\frac{1}{(1-x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+x)^{-\frac{1}{2}}}{(1-x)^{-\frac{1}{2}}}\left[\frac{1}{(1-x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+x)^{-\frac{1}{2}}}{(1-x)^{-\frac{1}{2}+2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+x)^{-\frac{1}{2}}}{(1-x)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{(1-x)^{\frac{3}{2}}\sqrt{1+x}}$$

Thus, $\frac{d}{dx}\left(\sqrt{\frac{1+x}{1-x}}\right) = \frac{1}{(1-x)^{\frac{3}{2}}\sqrt{1+x}}$ 20. $sin\left(\frac{1+x^2}{1-x^2}\right)$

Solution:

Let $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin\left(\frac{1+x^2}{1-x^2}\right)\right]$$

We know $\frac{d}{dx}(\sin x) = \cos x$

Now by using chain rule

$$\Rightarrow \frac{dy}{dx} = \cos\left(\frac{1+x^2}{1-x^2}\right)\frac{d}{dx}\left(\frac{1+x^2}{1-x^2}\right)$$

We know that $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$ (quotient rule)

$$\Rightarrow \frac{dy}{dx} = \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{(1-x^2)\frac{d}{dx}(1+x^2) - (1+x^2)\frac{d}{dx}(1-x^2)}{(1-x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx}$$

$$= \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{(1-x^2)\left(\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right) - (1+x^2)\left(\frac{d}{dx}(1) - \frac{d}{dx}(x^2)\right)}{(1-x^2)^2}\right]$$

However, $\frac{d}{dx}(x^2) = 2x$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{(1-x^2)(0+2x) - (1+x^2)(0-2x)}{(1-x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \cos\left(\frac{1+x^2}{1-x^2}\right)\left[\frac{2x(1-x^2) + 2x(1+x^2)}{(1-x^2)^2}\right]$$

21. $e^{3x}\cos 2x$

Solution:

Let y = $e^{3x}\cos(2x)$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^{3x}\cos 2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(e^{3x} \times \cos 2x)$$

We know that (u v)’ = vu’ + u v’ (product rule)

$$\Rightarrow \frac{dy}{dx} = \cos 2x \frac{d}{dx}(e^{3x}) + e^{3x}\frac{d}{dx}(\cos 2x)$$

We know $\frac{d}{dx}(e^x) = e^x$ and $\frac{d}{dx}(\cos x) = -\sin x$

Now by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = \cos 2x\left[e^{3x}\frac{d}{dx}(3x)\right] + e^{3x}\left[-\sin 2x\frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{dy}{dx} = e^{3x}\cos 2x\left[\frac{d}{dx}(3x)\right] - e^{3x}\sin 2x\left[\frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{dy}{dx} = e^{3x}\cos 2x\left[3\frac{d}{dx}(x)\right] - e^{3x}\sin 2x\left[2\frac{d}{dx}(x)\right]$$

$$\Rightarrow \frac{dy}{dx} = 3e^{3x}\cos 2x\left[\frac{d}{dx}(x)\right] - 2e^{3x}\sin 2x\left[\frac{d}{dx}(x)\right]$$

We have $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{dy}{dx} = 3e^{3x}\cos 2x \times 1 - 2e^{3x}\sin 2x \times 1$$

$$\Rightarrow \frac{dy}{dx} = 3e^{3x}\cos 2x - 2e^{3x}\sin 2x$$

$$\therefore \frac{dy}{dx} = e^{3x}(3\cos 2x - 2\sin 2x)$$

Thus, $\frac{d}{dx}(e^{3x}\cos 2x) = e^{3x}(3\cos 2x - 2\sin 2x)$

22. Sin (log sin x)

Solution:

Let y = sin (log sin x)

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[\sin(\log(\sin x))]$$

We know $\frac{d}{dx}(\sin x) = \cos x$

By using chain rule,

$$\Rightarrow \frac{dy}{dx} = \cos(\log(\sin x))\frac{d}{dx}[\log(\sin x)]$$

We have $\frac{d}{dx}(\log x) = \frac{1}{x}$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = \cos(\log(\sin x))\left[\frac{1}{\sin x}\frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin x}\cos(\log(\sin x))\frac{d}{dx}(\sin x)$$

However, $\frac{d}{dx}(\sin x) = \cos x$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin x}\cos(\log(\sin x))\cos x$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{\cos x}{\sin x}\right)\cos(\log(\sin x))$$

$$\therefore \frac{dy}{dx} = \cot x \cos(\log(\sin x))$$

Thus, $\frac{d}{dx}[\sin(\log(\sin x))] = \cot x \cos(\log(\sin x))$

23. $e^{\tan 3x}$

Solution:

Let y = $e^{\tan 3x}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^{\tan 3x})$$

We know $\frac{d}{dx}(e^x) = e^x$

By using chain rule,

$$\Rightarrow \frac{dy}{dx} = e^{\tan 3x}\frac{d}{dx}(\tan 3x)$$

We have $\frac{d}{dx}(\tan x) = \sec^2 x$

Now by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = e^{\tan 3x}\sec^2 3x\frac{d}{dx}(3x)$$

$$\Rightarrow \frac{dy}{dx} = 3e^{\tan 3x}\sec^2 3x\frac{d}{dx}(x)$$

However, $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{dy}{dx} = 3e^{\tan 3x}\sec^2 3x \times 1$$

$$\therefore \frac{dy}{dx} = 3e^{\tan 3x}\sec^2 3x$$

Thus, $\frac{d}{dx}(e^{\tan 3x}) = 3e^{\tan 3x}\sec^2 3x$

24. $e^{\sqrt{\cot x}}$

Solution:

Let $y = e^{\sqrt{\cot x}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sqrt{\cot x}}\right)$$

We know $\frac{d}{dx}(e^x) = e^x$

By using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = e^{\sqrt{\cot x}} \frac{d}{dx}\left(\sqrt{\cot x}\right)$$

$$\Rightarrow \frac{dy}{dx} = e^{\sqrt{\cot x}} \frac{d}{dx}\left[(\cot x)^{\frac{1}{2}}\right]$$

We have $\frac{d}{dx}(x^n) = nx^{n-1}$

By using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = e^{\sqrt{\cot x}}\left[\frac{1}{2}(\cot x)^{\frac{1}{2}-1}\frac{d}{dx}(\cot x)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}e^{\sqrt{\cot x}}(\cot x)^{-\frac{1}{2}}\frac{d}{dx}(\cot x)$$

However, $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}e^{\sqrt{\cot x}}(\cot x)^{-\frac{1}{2}}\operatorname{cosec}^2 x$$

$$\Rightarrow \frac{dy}{dx} = -\frac{e^{\sqrt{\cot x}}\operatorname{cosec}^2 x}{2(\cot x)^{\frac{1}{2}}}$$

$$\therefore \frac{dy}{dx} = -\frac{e^{\sqrt{\cot x}}\operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

Thus, $\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right) = -\frac{e^{\sqrt{\cot x}}\operatorname{cosec}^2 x}{2\sqrt{\cot x}}$

$25.\ log\left(\frac{sinx}{1+cosx}\right)$

Solution:

Let $y = \log\left(\frac{\sin x}{1+\cos x}\right)$

$$\Rightarrow y = \log\left(\frac{\sin 2 \times \frac{x}{2}}{1 + \cos 2 \times \frac{x}{2}}\right)$$

We have $\sin 2\theta = 2\sin\theta\cos\theta$ and $1 + \cos 2\theta = 2\cos^2\theta$.

$$\Rightarrow y = \log\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$\Rightarrow y = \log\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$$

$$\Rightarrow y = \log\left(\tan\frac{x}{2}\right)$$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\tan\frac{x}{2}\right)\right]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

Now by using chain rule we have,

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1}{\tan\frac{x}{2}}\right)\frac{d}{dx}\left(\tan\frac{x}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \cot\frac{x}{2}\frac{d}{dx}\left(\tan\frac{x}{2}\right)$$

We have $\frac{d}{dx}(\tan x) = \sec^2 x$

$$\Rightarrow \frac{dy}{dx} = \cot\frac{x}{2}\sec^2\frac{x}{2}\frac{d}{dx}\left(\frac{x}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\cot\frac{x}{2}\sec^2\frac{x}{2}\frac{d}{dx}(x)$$

However, $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\cot\frac{x}{2}\sec^2\frac{x}{2} \times 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \times \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \times \frac{1}{\cos^2\frac{x}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin 2\times\frac{x}{2}}$ [∵ sin2θ = 2sinθcosθ]

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin x}$

$\therefore \frac{dy}{dx} = \operatorname{cosec} x$

Thus, $\frac{d}{dx}\left[\log\left(\frac{\sin x}{1+\cos x}\right)\right] = \operatorname{cosec} x$

26. $\log\sqrt{\frac{1-\cos x}{1+\cos x}}$

Solution:

Let $y = \log\sqrt{\frac{1-\cos x}{1+\cos x}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}\right]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}} \frac{d}{dx}\left[\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1-\cos x}{1+\cos x}\right)^{-\frac{1}{2}} \frac{d}{dx}\left[\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}\right]$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

Again by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = \left(\frac{1-\cos x}{1+\cos x}\right)^{-\frac{1}{2}} \frac{1}{2}\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(\frac{1-\cos x}{1+\cos x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-\cos x}{1+\cos x}\right)^{-\frac{1}{2}}\left(\frac{1-\cos x}{1+\cos x}\right)^{-\frac{1}{2}} \frac{d}{dx}\left(\frac{1-\cos x}{1+\cos x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-\cos x}{1+\cos x}\right)^{-1} \frac{d}{dx}\left(\frac{1-\cos x}{1+\cos x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right) \frac{d}{dx}\left(\frac{1-\cos x}{1+\cos x}\right)$$

We know that $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$ (quotient rule)

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)\frac{d}{dx}(1-\cos x)-(1-\cos x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)\left(\frac{d}{dx}(1)-\frac{d}{dx}(\cos x)\right)-(1-\cos x)\left(\frac{d}{dx}(1)+\frac{d}{dx}(\cos x)\right)}{(1+\cos x)^2}\right]$$

We know $\frac{d}{dx}(\cos x) = -\sin x$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)(0+\sin x)-(1-\cos x)(0-\sin x)}{(1+\cos x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)\sin x+(1-\cos x)\sin x}{(1+\cos x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x+1-\cos x)\sin x}{(1+\cos x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{2\sin x}{(1+\cos x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x}{(1-\cos x)(1+\cos x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x}{1-\cos^2 x}$$

$\Rightarrow \frac{dy}{dx} = \frac{\sin x}{\sin^2 x}$ ($\because \sin^2\theta + \cos^2\theta = 1$)

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin x}$$

$$\therefore \frac{dy}{dx} = \operatorname{cosec} x$$

Thus, $\frac{d}{dx}\left(\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \operatorname{cosec} x$

27. tan ($e^{\sin x}$)

Solution:

Let y = tan ($e^{\sin x}$)

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan\left(e^{\sin x}\right)\right]$$

We know $\frac{d}{dx}(\tan x) = \sec^2 x$

Now by using chain rule,

$$\Rightarrow \frac{dy}{dx} = \sec^2\left(e^{\sin x}\right)\frac{d}{dx}\left(e^{\sin x}\right)$$

We have $\frac{d}{dx}(e^x) = e^x$

Again by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = \sec^2\left(e^{\sin x}\right)e^{\sin x}\frac{d}{dx}(\sin x)$$

However, $\frac{d}{dx}(\sin x) = \cos x$

$$\Rightarrow \frac{dy}{dx} = \sec^2\left(e^{\sin x}\right)e^{\sin x}\cos x$$

$$\therefore \frac{dy}{dx} = e^{\sin x}\cos x \sec^2\left(e^{\sin x}\right)$$

Thus, $\frac{d}{dx}\left[\tan\left(e^{\sin x}\right)\right] = e^{\sin x}\cos x \sec^2\left(e^{\sin x}\right)$

28. $log(x + \sqrt{x^2 + 1})$

Solution:

Let $y = \log\left(x + \sqrt{x^2 + 1}\right)$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(x + \sqrt{x^2 + 1}\right)\right]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

Using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\frac{d}{dx}\left(x + \sqrt{x^2 + 1}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + 1}}\left[\frac{d}{dx}(x) + \frac{d}{dx}\left(\sqrt{x^2 + 1}\right)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[\frac{d}{dx}(x) + \frac{d}{dx}(x^2+1)^{\frac{1}{2}}\right]$$

We know $\frac{d}{dx}(x) = 1$ and $\frac{d}{dx}(x^n) = nx^{n-1}$

Again by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2}(x^2+1)^{\frac{1}{2}-1}\frac{d}{dx}(x^2+1)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2}(x^2+1)^{-\frac{1}{2}}\left(\frac{d}{dx}(x^2) + \frac{d}{dx}(1)\right)\right]$$

However, $\frac{d}{dx}(x^2) = 2x$ and derivative of a constant is 0.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x+0)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \times 2x\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1 + x(x^2+1)^{-\frac{1}{2}}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{x}{\sqrt{x^2+1}}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[\frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}$$

Thus, $\frac{d}{dx}\left[\log\left(x+\sqrt{x^2+1}\right)\right] = \frac{1}{\sqrt{x^2+1}}$ 29. $\frac{e^x \log x}{x^2}$

Solution:

https://www.indcareer.com/schools/rd-sharma-solutions-for-class-12-maths-chapter-11-differentiation/

Let $y = \frac{e^x \log x}{x^2}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x \log x}{x^2}\right)$$

We know that $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$ (quotient rule)

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2)\frac{d}{dx}(e^x \log x) - (e^x \log x)\frac{d}{dx}(x^2)}{(x^2)^2}$$

We have (u v)’ = vu’ + u v’ (product rule)

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2)\left[\log x \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(\log x)\right] - (e^x \log x)\frac{d}{dx}(x^2)}{x^4}$$

We know $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(\log x) = \frac{1}{x}$ and $\frac{d}{dx}(x^2) = 2x$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2)\left[\log x \times e^x + e^x \times \frac{1}{x}\right] - (e^x \log x) \times 2x}{x^4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2)\left[e^x \log x + \frac{e^x}{x}\right] - 2xe^x \log x}{x^4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 e^x \log x + xe^x - 2xe^x \log x}{x^4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 e^x \log x}{x^4} + \frac{xe^x}{x^4} - \frac{2xe^x \log x}{x^4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x \log x}{x^2} + \frac{e^x}{x^3} - \frac{2e^x \log x}{x^3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x}{x^2}\left(\log x + \frac{1}{x} - \frac{2\log x}{x}\right)$$

$$\therefore \frac{dy}{dx} = e^x x^{-2}\left(\log x + \frac{1}{x} - \frac{2}{x}\log x\right)$$

Thus, $\frac{d}{dx}\left(\frac{e^x \log x}{x^2}\right) = e^x x^{-2}\left(\log x + \frac{1}{x} - \frac{2}{x}\log x\right)$

30. log (cosec x – cot x)

Solution:

Let y = log (cosec x – cot x)

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[\log(\operatorname{cosec} x - \cot x)]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

Now by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x}\frac{d}{dx}(\operatorname{cosec} x - \cot x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x}\left[\frac{d}{dx}(\operatorname{cosec} x) - \frac{d}{dx}(\cot x)\right]$$

We know $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$ and $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x}[-\operatorname{cosec} x \cot x - (-\operatorname{cosec}^2 x)]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x}[-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x}[\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x}[(\operatorname{cosec} x - \cot x)\operatorname{cosec} x]$$

$$\therefore \frac{dy}{dx} = \operatorname{cosec} x$$

Thus, $\frac{d}{dx}[\log(\operatorname{cosec} x - \cot x)] = \operatorname{cosec} x$

31. $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

Solution:

Let $y = \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\right)$$

We know that $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$ (quotient rule

$$\Rightarrow \frac{dy}{dx} = \frac{(e^{2x}-e^{-2x})\frac{d}{dx}(e^{2x}+e^{-2x}) - (e^{2x}+e^{-2x})\frac{d}{dx}(e^{2x}-e^{-2x})}{(e^{2x}-e^{-2x})^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(e^{2x}-e^{-2x})\left[\frac{d}{dx}(e^{2x}) + \frac{d}{dx}(e^{-2x})\right] - (e^{2x}+e^{-2x})\left[\frac{d}{dx}(e^{2x}) - \frac{d}{dx}(e^{-2x})\right]}{(e^{2x}-e^{-2x})^2}$$

We know $\frac{d}{dx}(e^x) = e^x$

$$\Rightarrow \frac{dy}{dx} = \frac{(e^{2x}-e^{-2x})\left[e^{2x}\frac{d}{dx}(2x) + e^{-2x}\frac{d}{dx}(-2x)\right] - (e^{2x}+e^{-2x})\left[e^{2x}\frac{d}{dx}(2x) - e^{-2x}\frac{d}{dx}(-2x)\right]}{(e^{2x}-e^{-2x})^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(e^{2x}-e^{-2x})\left[2e^{2x}\frac{d}{dx}(x) - 2e^{-2x}\frac{d}{dx}(x)\right] - (e^{2x}+e^{-2x})\left[2e^{2x}\frac{d}{dx}(x) + 2e^{-2x}\frac{d}{dx}(x)\right]}{(e^{2x}-e^{-2x})^2}$$

However, $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{(e^{2x} - e^{-2x})[2e^{2x} \times 1 - 2e^{-2x} \times 1] - (e^{2x} + e^{-2x})[2e^{2x} \times 1 + 2e^{-2x} \times 1]}{(e^{2x} - e^{-2x})^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(e^{2x} - e^{-2x})[2e^{2x} - 2e^{-2x}] - (e^{2x} + e^{-2x})[2e^{2x} + 2e^{-2x}]}{(e^{2x} - e^{-2x})^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(e^{2x} - e^{-2x})(e^{2x} - e^{-2x}) - 2(e^{2x} + e^{-2x})(e^{2x} + e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2[(e^{2x} - e^{-2x})^2 - (e^{2x} + e^{-2x})^2]}{(e^{2x} - e^{-2x})^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(e^{2x} - e^{-2x} + e^{2x} + e^{-2x})(e^{2x} - e^{-2x} - e^{2x} - e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(2e^{2x})(-2e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-8e^{2x+(-2x)}}{(e^{2x} - e^{-2x})^2}$$

$$\therefore \frac{dy}{dx} = \frac{-8}{(e^{2x} - e^{-2x})^2}$$

Thus, $\frac{d}{dx}\left(\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\right) = \frac{-8}{(e^{2x}-e^{-2x})^2}$

$$32.\ log\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right)$$

Solution:

Let $y = \log\left(\frac{x^2+x+1}{x^2-x+1}\right)$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\frac{x^2+x+1}{x^2-x+1}\right)\right]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

By using chain rule, we have

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\left(\frac{x^2+x+1}{x^2-x+1}\right)}\frac{d}{dx}\left(\frac{x^2+x+1}{x^2-x+1}\right)$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2-x+1}{x^2+x+1}\right)\frac{d}{dx}\left(\frac{x^2+x+1}{x^2-x+1}\right)$$

We know that $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$ (quotient rule)

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2-x+1}{x^2+x+1}\right)\left[\frac{(x^2-x+1)\frac{d}{dx}(x^2+x+1)-(x^2+x+1)\frac{d}{dx}(x^2-x+1)}{(x^2-x+1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2-x+1}{x^2+x+1}\right)\left[\frac{(x^2-x+1)\left(\frac{d}{dx}(x^2)+\frac{d}{dx}(x)+\frac{d}{dx}(1)\right)-(x^2+x+1)\left(\frac{d}{dx}(x^2)-\frac{d}{dx}(x)+\frac{d}{dx}(1)\right)}{(x^2-x+1)^2}\right]$$

We know $\frac{d}{dx}(x^2) = 2x$, $\frac{d}{dx}(x) = 1$ and derivative of constant is 0.

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{(x^2 - x + 1)(2x + 1 + 0) - (x^2 + x + 1)(2x - 1 + 0)}{(x^2 - x + 1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{(2x + 1)(x^2 - x + 1) - (2x - 1)(x^2 + x + 1)}{(x^2 - x + 1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{2x(x^2 - x + 1) + (x^2 - x + 1) - 2x(x^2 + x + 1) + (x^2 + x + 1)}{(x^2 - x + 1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{2x(x^2 - x + 1 - x^2 - x - 1) + (x^2 - x + 1 + x^2 + x + 1)}{(x^2 - x + 1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{2x(-2x) + (2x^2 + 2)}{(x^2 - x + 1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{-4x^2 + 2x^2 + 2}{(x^2 - x + 1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{2 - 2x^2}{(x^2 - x + 1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 - 2x^2}{(x^2 + x + 1)(x^2 - x + 1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(1 - x^2)}{(x^2 + 1)^2 - x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(1 - x^2)}{(x^2 + 1)^2 - x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(1 - x^2)}{x^4 + 2x^2 + 1 - x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(1 - x^2)}{(x^2 + 1)^2 - x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(1 - x^2)}{x^4 + 2x^2 + 1 - x^2}$$

33. $\tan^{-1}(e^x)$

Solution:

Let $y = \tan^{-1}(e^x)$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1} e^x)$$

We know $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

Now by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+(e^x)^2}\frac{d}{dx}(e^x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+e^{2x}}\frac{d}{dx}(e^x)$$

However, $\frac{d}{dx}(e^x) = e^x$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+e^{2x}} \times e^x$$

$$\therefore \frac{dy}{dx} = \frac{e^x}{1+e^{2x}}$$

Thus, $\frac{d}{dx}(\tan^{-1} e^x) = \frac{e^x}{1+e^{2x}}$

34. $e^{\sin^{-1} 2x}$

Solution:

Let $y = e^{\sin^{-1} 2x}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1} 2x}\right)$$

We know $\frac{d}{dx}(e^x) = e^x$

Using chain rule, we can write as

$$\Rightarrow \frac{dy}{dx} = e^{\sin^{-1} 2x} \frac{d}{dx}(\sin^{-1} 2x)$$

We have $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

Using chain rule we get

$$\Rightarrow \frac{dy}{dx} = e^{\sin^{-1} 2x} \frac{1}{\sqrt{1-(2x)^2}} \frac{d}{dx}(2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sin^{-1} 2x}}{\sqrt{1-4x^2}} \times 2 \frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e^{\sin^{-1} 2x}}{\sqrt{1-4x^2}} \times \frac{d}{dx}(x)$$

However, $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{2e^{\sin^{-1} 2x}}{\sqrt{1-4x^2}} \times 1$$

$$\therefore \frac{dy}{dx} = \frac{2e^{\sin^{-1} 2x}}{\sqrt{1-4x^2}}$$

Thus, $\frac{d}{dx}\left(e^{\sin^{-1} 2x}\right) = \frac{2e^{\sin^{-1} 2x}}{\sqrt{1-4x^2}}$

35. sin (2 sin^{-1} x)

Solution:

Let y = sin (2sin^{-1}x)

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[\sin(2\sin^{-1}x)]$$

We know $\frac{d}{dx}(\sin x) = \cos x$

By using chain rule we get,

$$\Rightarrow \frac{dy}{dx} = \cos(2\sin^{-1}x)\frac{d}{dx}(2\sin^{-1}x)$$

$$\Rightarrow \frac{dy}{dx} = \cos(2\sin^{-1}x) \times 2\frac{d}{dx}(\sin^{-1}x)$$

$$\Rightarrow \frac{dy}{dx} = 2\cos(2\sin^{-1}x)\frac{d}{dx}(\sin^{-1}x)$$

We have $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$$\Rightarrow \frac{dy}{dx} = 2\cos(2\sin^{-1}x) \times \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{2\cos(2\sin^{-1}x)}{\sqrt{1-x^2}}$$

Thus, $\frac{d}{dx}[\sin(2\sin^{-1}x)] = \frac{2\cos(2\sin^{-1}x)}{\sqrt{1-x^2}}$

36. $e^{\tan^{-1}\sqrt{x}}$

Solution:

Let $y = e^{\tan^{-1}\sqrt{x}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan^{-1}\sqrt{x}}\right)$$

We know $\frac{d}{dx}(e^x) = e^x$

Now by using chain rule, we can write as

$$\Rightarrow \frac{dy}{dx} = e^{\tan^{-1}\sqrt{x}} \frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right)$$

We have $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

Again by using chain rule we get,

$$\Rightarrow \frac{dy}{dx} = e^{\tan^{-1}\sqrt{x}} \frac{1}{1+(\sqrt{x})^2} \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\tan^{-1}\sqrt{x}}}{1+x} \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

However, $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\tan^{-1}\sqrt{x}}}{1+x}\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\tan^{-1}\sqrt{x}}}{1+x}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\tan^{-1}\sqrt{x}}}{1+x}\left(\frac{1}{2\sqrt{x}}\right)$$

$$\therefore \frac{dy}{dx} = \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}(1+x)}$$

Thus, $\frac{d}{dx}\left(e^{\tan^{-1}\sqrt{x}}\right) = \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}(1+x)}$

37. $\sqrt{\tan^{-1}\left(\frac{x}{2}\right)}$

Solution:

Let $y = \sqrt{\tan^{-1}\frac{x}{2}}$

On differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\tan^{-1}\frac{x}{2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\left(\tan^{-1}\frac{x}{2}\right)^{\frac{1}{2}}\right]$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

Now by using chain rule, we get

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\tan^{-1}\frac{x}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{-\frac{1}{2}}\frac{d}{dx}\left(\tan^{-1}\frac{x}{2}\right)$$

We have $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

Again by using chain rule, we can write as

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{-\frac{1}{2}}\frac{1}{1+\left(\frac{x}{2}\right)^2}\frac{d}{dx}\left(\frac{x}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{-\frac{1}{2}}\frac{1}{1+\frac{x^2}{4}} \times \frac{1}{2}\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)^{-\frac{1}{2}}\frac{4}{4+x^2} \times \frac{1}{2}\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = \left(\tan^{-1}\frac{x}{2}\right)^{-\frac{1}{2}}\frac{1}{4+x^2} \times \frac{d}{dx}(x)$$

However, $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{dy}{dx} = \left(\tan^{-1}\frac{x}{2}\right)^{-\frac{1}{2}}\frac{1}{4+x^2} \times 1$$

$$\Rightarrow \frac{dy}{dx} = \left(\tan^{-1}\frac{x}{2}\right)^{-\frac{1}{2}}\frac{1}{4+x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(4+x^2)\left(\tan^{-1}\frac{x}{2}\right)^{\frac{1}{2}}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{(4+x^2)\sqrt{\tan^{-1}\frac{x}{2}}}$$

Exercise 11.3 Page No: 11.62

Differentiate the following functions with respect to x:

1. $\cos^{-1}\left\{2x\sqrt{1-x^2}\right\},\ \frac{1}{\sqrt{2}}<x<1$

Solution:

Let $y = \cos^{-1}\{2x\sqrt{1-x^2}\}$

let $x = \cos\theta$

Now

$y = \cos^{-1}\{2\cos\theta\sqrt{1-\cos^2\theta}\}$

$= \cos^{-1}\{2\cos\theta\sqrt{\sin^2\theta}\}$

Using $\sin^2\theta + \cos^2\theta = 1$ and $2 \sin \theta \cos \theta = \sin 2\theta$

$= \cos^{-1}(2 \cos \theta \sin \theta)$

$= \cos^{-1}(\sin 2\theta)$

$y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

Now by considering the limits,

$\frac{1}{\sqrt{2}} < x < 1$

$\Rightarrow \frac{1}{\sqrt{2}} < \cos\theta < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{4}$

$\Rightarrow 0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow 0 > -2\theta > -\frac{\pi}{2}$

$$\Rightarrow \frac{\pi}{2} > \frac{\pi}{2} - 2\theta > \frac{\pi}{2} - \frac{\pi}{2}$$

$$\Rightarrow 0 < \frac{\pi}{2} - 2\theta < \frac{\pi}{2}$$

Therefore,

$$y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$$

$$y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$$

$$y = \left(\frac{\pi}{2} - 2\theta\right)$$

$$y = \frac{\pi}{2} - 2\cos^{-1}x$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 2\cos^{-1}x\right)$$

$$\Rightarrow \frac{dy}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1 - x^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1 - x^2}}$$

2. $\cos^{-1}\left\{\sqrt{\frac{1+x}{2}}\right\}, \ -1 < x < 1$

Solution:

$$y = \cos^{-1}\left\{\sqrt{\frac{1+\cos 2\theta}{2}}\right\}$$

$$y = \cos^{-1}\left\{\sqrt{\frac{2\cos^2\theta}{2}}\right\}$$

Now by using $\cos 2\theta = 2\cos^2\theta - 1$

$y = \cos^{-1}(\cos\theta)$

Considering the limits,

$-1 < x < 1$

$-1 < \cos 2\theta < 1$

$0 < 2\theta < \pi$

$$0 < \theta < \frac{\pi}{2}$$

Now, $y = \cos^{-1}(\cos\theta)$

Let

$y = \theta$

$$y = \cos^{-1}\left\{\sqrt{\frac{1+x}{2}}\right\}$$

$$y = \frac{1}{2}\cos^{-1}x$$

Differentiating with respect to x, we get

let $x = \cos 2\theta$

$$\frac{dy}{dx} = \frac{1}{2}\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

Now

3. $$\sin^{-1}\left\{\sqrt{\frac{1-x}{2}}\right\},\ 0 < x < 1$$

Solution:

Let,

let $x = \cos 2\theta$

Now

$$y = \sin^{-1}\left\{\sqrt{\frac{1-\cos 2\theta}{2}}\right\}$$

$$y = \sin^{-1}\left\{\sqrt{\frac{2\sin^2\theta}{2}}\right\}$$

Using $\cos 2\theta = 1 - 2\sin^2\theta$

$y = \sin^{-1}(\sin\theta)$

Considering the limits,

$0 < x < 1$

$0 < \cos 2\theta < 1$

$$0 < 2\theta < \frac{\pi}{2}$$

$$0 < \theta < \frac{\pi}{4}$$

Now, $y = \sin^{-1}(\sin\theta)$

$y = \theta$

$$y = \sin^{-1}\left\{\sqrt{\frac{1-x}{2}}\right\} \qquad y = \frac{1}{2}\cos^{-1} x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

4. $\sin^{-1}\left\{\sqrt{1-x^2}\right\},\ 0 < x < 1$

Solution:

Let,

$$y = \sin^{-1}\left\{\sqrt{1-x^2}\right\}$$

let $x = \cos\theta$

Now

$$y = \sin^{-1}\left\{\sqrt{1-\cos^2\theta}\right\}$$

Using $\sin^2\theta + \cos^2\theta = 1$

$y = \sin^{-1}(\sin\theta)$

Considering the limits,

$0 < x < 1$

$0 < \cos\theta < 1$

$$0 < \theta < \frac{\pi}{2}$$

Now, $y = \sin^{-1}(\sin\theta)$

$y = \theta$

$y = \cos^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

5. $\tan^{-1}\left\{\frac{x}{\sqrt{a^2-x^2}}\right\}, \ -a < x < a$

Solution:

$$y = \tan^{-1}\left\{\frac{x}{\sqrt{a^2 - x^2}}\right\}$$

Let x = a sin θ

Now

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right\}$$

Using $\sin^2\theta + \cos^2\theta = 1$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a\sqrt{1 - \sin^2\theta}}\right\}$$

$$y = \tan^{-1}\left\{\frac{\sin\theta}{\cos\theta}\right\}$$

$y = \tan^{-1}(\tan\theta)$

Considering the limits,

$-a < x < a$

$-a < a\sin\theta < a$

$-1 < \sin\theta < 1$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Now, $y = \tan^{-1}(\tan\theta)$

$y = \theta$

$$y = \sin^{-1}\left(\frac{x}{a}\right)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)\right)$$

$$\frac{dy}{dx} = \frac{a}{\sqrt{a^2 - x^2}} \times \frac{1}{a}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}$$

6. $\sin^{-1}\left\{\dfrac{x}{\sqrt{x^2 + a^2}}\right\}$

Solution:

Let,

$$y = \sin^{-1}\left\{\frac{x}{\sqrt{x^2 + a^2}}\right\}$$

Let x = a tan θ

Now

$$y = \sin^{-1}\left\{\frac{a\tan\theta}{\sqrt{a^2\tan^2\theta + a^2}}\right\}$$

Using $1 + \tan^2\theta = \sec^2\theta$

$$y = \sin^{-1}\left\{\frac{a\tan\theta}{a\sqrt{\tan^2\theta + 1}}\right\}$$

$$y = \sin^{-1}\left\{\frac{a\tan\theta}{a\sqrt{\sec^2\theta}}\right\}$$

$$y = \sin^{-1}\left\{\frac{\tan\theta}{\sec\theta}\right\}$$

$y = \sin^{-1}(\sin\theta)$

$y = \theta$

$$y = \tan^{-1}\left(\frac{x}{a}\right)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right)$$

$$\frac{dy}{dx} = \frac{a^2}{a^2 + x^2} \times \frac{1}{a}$$

$$\frac{dy}{dx} = \frac{a}{a^2 + x^2}$$

7. $Sin^{-1}(2x^2 - 1)$, 0 < x < 1

Solution:

Let,

$y = \sin^{-1}\{2x^2 - 1\}$

let $x = \cos\theta$

Now

$y = \sin^{-1}\left\{\sqrt{2\cos^2\theta - 1}\right\}$

Using $2\cos^2\theta - 1 = \cos2\theta$

$y = \sin^{-1}(\cos2\theta)$

$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\}$

Considering the limits,

$0 < x < 1$

$0 < \cos\theta < 1$

$0 < \theta < \frac{\pi}{2}$

$0 < 2\theta < \pi$

$0 > -2\theta > -\pi$

$\frac{\pi}{2} > \frac{\pi}{2} - 2\theta > -\frac{\pi}{2}$

Now,

$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\}$

$y = \frac{\pi}{2} - 2\theta$

$y = \frac{\pi}{2} - 2\cos^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 2\cos^{-1}x\right)$$

$$\frac{dy}{dx} = 0 - 2\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

8. $\text{Sin}^{-1}(1 - 2x^2)$, $0 < x < 1$

Solution:

Let,

$y = \sin^{-1}\{1 - 2x^2\}$

let $x = \sin\theta$

Now

$y = \sin^{-1}\left\{\sqrt{1 - 2\sin^2\theta}\right\}$

Using $1 - 2\sin^2\theta = \cos 2\theta$

$y = \sin^{-1}(\cos 2\theta)$

$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\}$

Considering the limits,

$0 < x < 1$

$0 < \sin\theta < 1$

$0 < \theta < \frac{\pi}{2}$

$0 < 2\theta < \pi$

$0 > -2\theta > -\pi$

$\frac{\pi}{2} > \frac{\pi}{2} - 2\theta > -\frac{\pi}{2}$

Now,

$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\}$

$y = \frac{\pi}{2} - 2\theta$

$y = \frac{\pi}{2} - 2\sin^{-1}x$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 2\cos^{-1}x\right)$

$\frac{dy}{dx} = 0 - 2\left(\frac{1}{\sqrt{1 - x^2}}\right)$

$\frac{dy}{dx} = \frac{-2}{\sqrt{1 - x^2}}$

9. $\cos^{-1}\left\{\frac{x}{\sqrt{x^2 + a^2}}\right\}$

Solution:

Let,

$$y = \cos^{-1}\left\{\frac{x}{\sqrt{x^2 + a^2}}\right\}$$

Let x = a cot θ

Now

$$y = \cos^{-1}\left\{\frac{a\cot\theta}{\sqrt{a^2\cot^2\theta + a^2}}\right\}$$

Using $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

$$y = \cos^{-1}\left\{\frac{a\cot\theta}{a\sqrt{\cot^2\theta + 1}}\right\}$$

$$y = \cos^{-1}\left\{\frac{a\cot\theta}{a\sqrt{\mathrm{cosec}^2\theta}}\right\}$$

$$y = \cos^{-1}\left\{\frac{\cot\theta}{\mathrm{cosec}\,\theta}\right\}$$

$y = \cos^{-1}(\cos\theta)$

$y = \theta$

$$y = \cot^{-1}\left(\frac{x}{a}\right)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cot^{-1}\left(\frac{x}{a}\right)\right)$$

$$\frac{dy}{dx} = \frac{-a^2}{a^2 + x^2} \times \frac{1}{a}$$

$$\frac{dy}{dx} = \frac{-a}{a^2 + x^2}$$

10. $\sin^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}, -\frac{3\pi}{4} < x < \frac{\pi}{4}$

Solution:

Let,

$$y = \sin^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}$$

Now

$$y = \sin^{-1}\left\{\sin x \frac{1}{\sqrt{2}} + \cos x \frac{1}{\sqrt{2}}\right\}$$

$$y = \sin^{-1}\left\{\sin x \cos\left(\frac{\pi}{4}\right) + \cos x \sin\left(\frac{\pi}{4}\right)\right\}$$

Using sin (A + B) = sin A cos B + cos A sin B

$$y = \sin^{-1}\left\{\sin\left(x + \frac{\pi}{4}\right)\right\}$$

Considering the limits,

$$-\frac{3\pi}{4} < x < \frac{\pi}{4}$$

Differentiating it with respect to x,

$$y = x + \frac{\pi}{4}$$

$$\frac{dy}{dx} = 1$$

11. $\cos^{-1}\left\{\frac{\cos x + \sin x}{\sqrt{2}}\right\}, -\frac{\pi}{4} < x < \frac{\pi}{4}$

Solution:

Let,

$$y = \cos^{-1}\left\{\frac{\cos x + \sin x}{\sqrt{2}}\right\}$$

Now

$$y = \cos^{-1}\left\{\cos x\frac{1}{\sqrt{2}} + \sin x\frac{1}{\sqrt{2}}\right\}$$

$$y = \cos^{-1}\left\{\cos x \cos\left(\frac{\pi}{4}\right) + \sin x \sin\left(\frac{\pi}{4}\right)\right\}$$

Using cos (A – B) = cos A cos B + sin A sin B

$$y = \cos^{-1}\left\{\cos\left(x - \frac{\pi}{4}\right)\right\}$$

Considering the limits,

$$-\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$-\frac{\pi}{2} < x - \frac{\pi}{4} < 0$$

Now,

$$y = -x + \frac{\pi}{4}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -1$$

12. $\tan^{-1}\left\{\frac{x}{1+\sqrt{1-x^2}}\right\},\ -1 < x < 1$

Solution:

Let,

$$y = \tan^{-1}\left\{\frac{x}{1+\sqrt{1-x^2}}\right\}$$

Let x = sin θ

Now

$$y = \tan^{-1}\left\{\frac{\sin\theta}{1+\sqrt{1-\sin^2\theta}}\right\}$$

Using $\sin^2\theta + \cos^2\theta = 1$

$$y = \tan^{-1}\left\{\frac{\sin\theta}{1+\sqrt{\cos^2\theta}}\right\}$$

$$y = \tan^{-1}\left\{\frac{\sin\theta}{1+\cos\theta}\right\}$$

Using $2\cos^2\theta = 1 + \cos 2\theta$ and $2\sin\theta\cos\theta = \sin 2\theta$

$$y = \tan^{-1}\left\{\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right\}$$

$$y = \tan^{-1}\left\{\tan\frac{\theta}{2}\right\}$$

Considering the limits,

$-1 < x < 1$

$-1 < \sin\theta < 1$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$-\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$$

Now,

$$y = \tan^{-1}\left\{\tan\frac{\theta}{2}\right\}$$

$$y = \frac{\theta}{2}$$

$$y = \frac{1}{2}\sin^{-1}x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}\sin^{-1}x\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$$

13. $\tan^{-1}\left\{\frac{x}{a+\sqrt{a^2-x^2}}\right\},\ -a < x < a$

Solution:

Let,

$$y = \tan^{-1}\left\{\frac{x}{a + \sqrt{a^2 - x^2}}\right\}$$

Let x = a sin θ

Now

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a + \sqrt{a^2 - a^2\sin^2\theta}}\right\}$$

Using $\sin^2\theta + \cos^2\theta = 1$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a + a\sqrt{\cos^2\theta}}\right\}$$

$$y = \tan^{-1}\left\{\frac{\sin\theta}{1 + \cos\theta}\right\}$$

Using $2\cos^2\theta = 1 + \cos\theta$ and $2\sin\theta\cos\theta = \sin 2\theta$

$$y = \tan^{-1}\left\{\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right\}$$

$$y = \tan^{-1}\left\{\tan\frac{\theta}{2}\right\}$$

Considering the limits,

$-a < x < a$

$-1 < \sin\theta < 1$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$-\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$$

Now,

$$y = \tan^{-1}\left\{\tan\frac{\theta}{2}\right\}$$

$$y = \frac{\theta}{2}$$

$$y = \frac{1}{2}\sin^{-1}\frac{x}{a}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}\sin^{-1}\frac{x}{a}\right)$$

$$\frac{dy}{dx} = \frac{a}{2\sqrt{a^2 - x^2}} \times \frac{1}{a}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}}$$

14. $\sin^{-1}\left\{\frac{x + \sqrt{1 - x^2}}{\sqrt{2}}\right\}, \ -1 < x < 1$

Solution:

Let,

$$y = \sin^{-1}\left\{\frac{x + \sqrt{1 - x^2}}{\sqrt{2}}\right\}$$

Let x = sin θ

Now

$$y = \sin^{-1}\left\{\frac{\sin\theta + \sqrt{1 - \sin^2\theta}}{\sqrt{2}}\right\}$$

Using $\sin^2\theta + \cos^2\theta = 1$

$$y = \sin^{-1}\left\{\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right\}$$

Now

$$y = \sin^{-1}\left\{\sin\theta\frac{1}{\sqrt{2}} + \cos\theta\frac{1}{\sqrt{2}}\right\}$$

$$y = \sin^{-1}\left\{\sin\theta\cos\left(\frac{\pi}{4}\right) + \cos\theta\sin\left(\frac{\pi}{4}\right)\right\}$$

Using sin (A + B) = sin A cos B + cos A sin B

$$y = \sin^{-1}\left\{\sin\left(\theta + \frac{\pi}{4}\right)\right\}$$

Considering the limits,

$-1 < x < 1$

$-1 < \sin\theta < 1$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$-\frac{\pi}{2} + \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{\pi}{2} + \frac{\pi}{4}$$

$$-\frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$$

Now,

$$y = \sin^{-1}\left\{\sin\left(\theta + \frac{\pi}{4}\right)\right\}$$

$$y = \theta + \frac{\pi}{4}$$

$$y = \sin^{-1}x + \frac{\pi}{4}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1}x + \frac{\pi}{4}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

15. $\cos^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}, \ -1 < x < 1$

Solution:

Let,

$$y = \cos^{-1}\left\{\frac{x + \sqrt{1 - x^2}}{\sqrt{2}}\right\}$$

Let x = sin θ

Now

$$y = \cos^{-1}\left\{\frac{\sin\theta + \sqrt{1 - \sin^2\theta}}{\sqrt{2}}\right\}$$

Using $\sin^2\theta + \cos^2\theta = 1$

$$y = \cos^{-1}\left\{\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right\}$$

Now

$$y = \cos^{-1}\left\{\sin\theta\frac{1}{\sqrt{2}} + \cos\theta\frac{1}{\sqrt{2}}\right\}$$

$$y = \cos^{-1}\left\{\sin\theta\sin\left(\frac{\pi}{4}\right) + \cos\theta\cos\left(\frac{\pi}{4}\right)\right\}$$

Using cos (A – B) = cos A cos B + sin A sin B

$$y = \cos^{-1}\left\{\cos\left(\theta - \frac{\pi}{4}\right)\right\}$$

Considering the limits,

$-1 < x < 1$

$-1 < \sin\theta < 1$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$-\frac{\pi}{2} - \frac{\pi}{4} < \theta - \frac{\pi}{4} < \frac{\pi}{2} - \frac{\pi}{4}$$

$$-\frac{3\pi}{4} < \theta - \frac{\pi}{4} < \frac{\pi}{4}$$

Now,

$$y = \cos^{-1}\left\{\cos\left(\theta - \frac{\pi}{4}\right)\right\}$$

$$y = -\left(\theta - \frac{\pi}{4}\right)$$

$$y = -\sin^{-1} x + \frac{\pi}{4}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(-\sin^{-1} x + \frac{\pi}{4}\right)$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1 - x^2}}$$

16. $\tan^{-1}\left\{\frac{4x}{1 - 4x^2}\right\}, \; -\frac{1}{2} < x < \frac{1}{2}$

Solution:

Let,

$$y = \tan^{-1}\left\{\frac{4x}{1-4x^2}\right\}$$

Let 2x = tan θ

$$y = \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$y = \tan^{-1}(\tan 2\theta)$$

Considering the limits,

$$-\frac{1}{2} < x < \frac{1}{2}$$

$-1 < 2x < 1$

$-1 < \tan\theta < 1$

$$-\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

$$-\frac{\pi}{2} < 2\theta < \frac{\pi}{2}$$

Now,

$y = \tan^{-1}(\tan 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}(2x)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1} 2x)$$

$$\frac{dy}{dx} = 2 \times \frac{2}{1+(2x)^2}$$

$$\frac{dy}{dx} = \frac{4}{1+4x^2}$$

17. $\tan^{-1}\left\{\frac{2^{x+1}}{1-4^x}\right\}, \quad -\infty < x < 0$

Solution:

Let,

$$y = \tan^{-1}\left\{\frac{2^{x+1}}{1-4^x}\right\}$$

Let $2^x = \tan\theta$

$$y = \tan^{-1}\left\{\frac{2\times 2^x}{1-(2^x)^2}\right\}$$

$$y = \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

Using $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$y = \tan^{-1}(\tan 2\theta)$$

Considering the limits,

$$-\infty < x < 0$$

$$2^{-\infty} < 2^x < 2^0$$

$$0 < \tan\theta < 1$$

$$0 < \theta < \frac{\pi}{4}$$

$$0 < 2\theta < \frac{\pi}{2}$$

Now,

$$y = \tan^{-1}(\tan 2\theta)$$

$$y = 2\theta$$

$$y = 2\tan^{-1}(2^x)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1}2^x)$$

$$\frac{dy}{dx} = 2\times\frac{2^x\log 2}{1+(2^x)^2}$$

$$\frac{dy}{dx} = \frac{2^{x+1}\log 2}{1+4^x}$$

18. $\tan^{-1}\left\{\dfrac{2a^x}{1-a^{2x}}\right\}, \ a > 1, -\infty < x < 0$

Solution:

Let,

$$y = \tan^{-1}\left\{\frac{2a^x}{1 - a^{2x}}\right\}$$

Let $a^x = \tan\theta$

$$y = \tan^{-1}\left\{\frac{2\tan\theta}{1 - \tan^2\theta}\right\}$$

Using $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$y = \tan^{-1}(\tan 2\theta)$

Considering the limits,

$-\infty < x < 0$

$a^{-\infty} < a^x < a^0$

$0 < \tan\theta < 1$

$0 < \theta < \frac{\pi}{4}$

$0 < 2\theta < \frac{\pi}{2}$

Now, $y = \tan^{-1}(\tan 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}(a^x)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1} a^x)$$

$$\frac{dy}{dx} = 2 \times \frac{a^x \log a}{1 + (a^x)^2}$$

$$\frac{dy}{dx} = \frac{2a^x \log a}{1 + a^{2x}}$$

19. $\sin^{-1}\left\{\frac{\sqrt{1+x} + \sqrt{1-x}}{2}\right\}, \ 0 < x < 1$

Solution:

Let,

Let,

$$y = \sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$$

Let x = cos2θ

Now

$$y = \sin^{-1}\left\{\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{2}\right\}$$

Using 1 – 2sin²θ = cos 2θ and 2 cos²θ – 1 = cos 2θ

$$y = \sin^{-1}\left\{\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{2}\right\}$$

Now

$$y = \sin^{-1}\left\{\sin\theta\frac{1}{\sqrt{2}}+\cos\theta\frac{1}{\sqrt{2}}\right\}$$

$$y = \sin^{-1}\left\{\sin\theta\cos\left(\frac{\pi}{4}\right)+\cos\theta\sin\left(\frac{\pi}{4}\right)\right\}$$

Using sin (A + B) = sin A cos B + cos A sin B

$$y = \sin^{-1}\left\{\sin\left(\theta+\frac{\pi}{4}\right)\right\}$$

Considering the limits,

0 < x < 1

0 < cos 2θ < 1

$$0 < 2\theta < \frac{\pi}{2}$$

$0 < \theta < \frac{\pi}{4}$

Now,

$y = \sin^{-1}\left\{\sin\left(\theta + \frac{\pi}{4}\right)\right\}$

$y = \ \theta + \frac{\pi}{4}$

$y = \frac{1}{2}\cos^{-1}x + \frac{\pi}{4}$

Differentiating with respect to x, we get

$y = \frac{1}{2}\cos^{-1}x + \frac{\pi}{4}$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}\cos^{-1}x + \frac{\pi}{4}\right)$

$\frac{dy}{dx} = \frac{1}{2} \times \frac{-1}{\sqrt{1 - x^2}}$

$\frac{dy}{dx} = \frac{-1}{2\sqrt{1 - x^2}}$

$20.\ \tan^{-1}\left\{\frac{\sqrt{1 + a^2x^2} - 1}{ax}\right\},\ x \neq 0$

Solution:

Let,

$$y = \tan^{-1}\left\{\frac{\sqrt{1+a^2x^2}-1}{ax}\right\}$$

Let ax = tan θ

Now

$$y = \tan^{-1}\left\{\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right\}$$

Using $\sec^2\theta = 1 + \tan^2\theta$

$$y = \tan^{-1}\left\{\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right\}$$

$$y = \tan^{-1}\left\{\frac{\sec\theta - 1}{\tan\theta}\right\}$$

$$y = \tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$$

Using 2 $\sin^2\theta = 1 - \cos 2\theta$ and $2 \sin \theta \cos \theta = \sin 2\theta$

$$y = \tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\}$$

$$y = \tan^{-1}\left\{\tan\frac{\theta}{2}\right\}$$

$$y = \frac{\theta}{2}$$

$$y = \frac{1}{2}\tan^{-1} ax$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}\tan^{-1} ax\right)$$

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{a}{1 + (ax)^2}$$

$$\frac{dy}{dx} = \frac{a}{2(1 + a^2x^2)}$$

21. $\tan^{-1}\left\{\frac{\sin x}{1 + \cos x}\right\}, \ -\pi < x < \pi$

Solution:

Let,

$$y = \tan^{-1}\left\{\frac{\sin x}{1 + \cos x}\right\}$$

Function y is defined for all real numbers where cos x ≠ −1

Using 2 $\cos^2\theta = 1 + \cos 2\theta$ and $2 \sin \theta \cos \theta = \sin 2\theta$

$$y = \tan^{-1}\left\{\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right\}$$

$$y = \tan^{-1}\left\{\tan\frac{x}{2}\right\}$$

$$y = \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}$$

22. $\sin^{-1}\left\{\frac{1}{\sqrt{1+x^2}}\right\}$

Solution:

Let,

$$y = \sin^{-1}\left\{\frac{1}{\sqrt{1+x^2}}\right\}$$

Let x = cot θ

Now

$$y = \sin^{-1}\left\{\frac{1}{\sqrt{1+\cot^2\theta}}\right\}$$

Using, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Now

$$y = \sin^{-1}\left\{\frac{1}{\sqrt{\operatorname{cosec}^2\theta}}\right\}$$

$$y = \sin^{-1}\left\{\frac{1}{\operatorname{cosec}\theta}\right\}$$

$y = \sin^{-1}(\sin\theta)$

$y = \theta$

$y = \cot^{-1}x$

Differentiating with respect to x we get

$$\frac{dy}{dx} = \frac{d}{dx}(\cot^{-1}x)$$

$$\frac{dy}{dx} = -\frac{1}{1+x^2}$$

23. $\cos^{-1}\left\{\frac{1-x^{2n}}{1+x^{2n}}\right\},\ 0 < x < \infty$

Solution:

Let,

$$y = \cos^{-1}\left\{\frac{1 - x^{2n}}{1 + x^{2n}}\right\}$$

Let $x^n = \tan\theta$

Now

$$y = \cos^{-1}\left\{\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right\}$$

Using $\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \cos 2\theta$

$$y = \cos^{-1}\{\cos 2\theta\}$$

Considering the limits,

$0 < x < \infty$

$0 < x^n < \infty$

$0 < \theta < \frac{\pi}{2}$

Now, $y = \cos^{-1}(\cos 2\theta)$

$y = 2\theta$

$y = \tan^{-1}(x^n)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}(x^n))$$

$$\frac{dy}{dx} = \frac{2nx^{n-1}}{1 + (x^n)^2}$$

$$\frac{dy}{dx} = \frac{2nx^{n-1}}{1 + x^{2n}}$$

Exercise 11.4 Page No: 11.74

Find dy/dx in each of the following:

1. $xy = c^2$

Solution:

Given $xy = c^2$;

Now we have to find $\frac{dy}{dx}$ of given equation, so by differentiating the equation on both sides with respect to x, we get,

By using the product rule on the left hand side,

$$\frac{d(xy)}{dx} = \frac{dc^2}{dx}$$

x (dy/dx) + y (1) = 0

$$\frac{dy}{dx} = \frac{-y}{x}$$

We can further solve it by putting the value of y,

$$\frac{dy}{dx} = \frac{-c^2}{x^2}$$

2. $y^3 – 3xy^2 = x^3 + 3x^2y$

Solution:

Given $y^3 – 3xy^2 = x^3 + 3x^2y$,

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\Rightarrow \frac{d}{dx}\left(y^{3}\right)-\frac{d}{dx}\left(3xy^{2}\right)=\frac{d}{dx}\left(x^{3}\right)+\frac{d}{dx}\left(3x^{2}y\right)$$

Now by using product rule we get,

$$\Rightarrow 3y^{2}\frac{dy}{dx}-3\left[x\frac{d}{dx}\left(y^{2}\right)+y^{2}\frac{d}{dx}(x)\right]=3x^{2}+3\left[x^{2}\frac{d}{dx}(y)+y\frac{d}{dx}\left(x^{2}\right)\right]$$

$$\Rightarrow 3y^{2}\frac{dy}{dx}-3\left[x(2y)\frac{dy}{dx}+y^{2}\right]=3x^{2}+3\left[x^{2}\frac{dy}{dx}+y(2x)\right]$$

$$\Rightarrow 3y^{2}\frac{dy}{dx}-6xy\frac{dy}{dx}-3y^{2}=3x^{2}+3x^{2}\frac{dy}{dx}+6xy$$

$$\Rightarrow 3y^{2}\frac{dy}{dx}-6xy\frac{dy}{dx}-3x^{2}\frac{dy}{dx}=3x^{2}+6xy+3y^{2}$$

$$\Rightarrow 3\frac{dy}{dx}\left(y^{2}-2xy-x^{2}\right)=3\left(x^{2}+2xy+y^{2}\right)$$

Now by taking 3 as common we get,

$$\Rightarrow \frac{dy}{dx}=\frac{3(x+y)^{2}}{3\left(y^{2}-2xy-x^{2}\right)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(x+y)^{2}}{y^{2}-2xy-x^{2}}$$

3. $x^{2/3} + y^{2/3} = a^{2/3}$

Solution:

Given $x^{2/3} + y^{2/3} = a^{2/3}$,

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{2}{3}\frac{1}{x^{1/3}} + \frac{2}{3}\frac{1}{y^{1/3}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y^{1/3}}{x^{1/3}}$$

Now by substituting the value, we get

$$\frac{dy}{dx} = \frac{-\sqrt{a^{2/3} - x^{2/3}}}{x^{1/3}}$$

4. 4x + 3y = log (4x – 3y)

Solution:

Given 4x + 3y = log (4x – 3y),

Now we have to find dy/dx of it, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{d}{dx}(4x)+\frac{d}{dx}(3y)=\frac{d}{dx}\{\log(4x-3y)\}$$

$$\Rightarrow 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\frac{d}{dx}(4x-3y)$$

$$\Rightarrow 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\left(4-3\frac{dy}{dx}\right)$$

$$\Rightarrow 3\frac{dy}{dx}+\frac{3}{(4x-3y)}\frac{dy}{dx}=\frac{4}{(4x-3y)}-4$$

$$\Rightarrow 3\frac{dy}{dx}\left\{1+\frac{1}{(4x-3y)}\right\}=4\left\{\frac{1}{(4x-3y)}-1\right\}$$

$$\Rightarrow 3\frac{dy}{dx}\left\{\frac{4x-3y+1}{(4x-3y)}\right\}=4\left\{\frac{1-4x+3y}{(4x-3y)}\right\}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4}{3}\left\{\frac{1-4x+3y}{(4x-3y)}\right\}\left(\frac{4x-3y}{4x-3y+1}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{4}{3}\left(\frac{1-4x+3y}{4x-3y+1}\right)$$

5. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Solution:

Given $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-xb^2}{ya^2}$$

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-xb^2}{ya^2}$$

6. $x^5 + y^5 = 5xy$

Solution:

Given $x^5 + y^5 = 5xy$

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{d}{dx}\left(x^5\right)+\frac{d}{dx}\left(y^5\right)=\frac{d}{dx}(5xy)$$

Now by using product rule, we get

$$\Rightarrow 5x^4+5y^4\frac{dy}{dx}=5\left[x\frac{dy}{dx}+y\frac{d}{dx}(x)\right]$$

$$\Rightarrow 5x^4+5y^4\frac{dy}{dx}=5\left[x\frac{dy}{dx}+y(1)\right]$$

$$\Rightarrow 5x^4+5y^4\frac{dy}{dx}=5x\frac{dy}{dx}+5y$$

$$\Rightarrow 5y^4\frac{dy}{dx}-5x\frac{dy}{dx}=5y-5x^4$$

$$\Rightarrow 5\frac{dy}{dx}\left(y^4-x\right)=5\left(y-x^4\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{5\left(y-x^4\right)}{5\left(y^4-x\right)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y-x^4}{y^4-x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y-x^4}{y^4-x}$$

7. $(x + y)^2 = 2axy$

Solution:

Given $(x + y)^2 = 2axy$

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\Rightarrow \frac{d}{dx}(x+y)^2 = \frac{d}{dx}(2axy)$$

Now by using product rule, we get

$$\Rightarrow 2(x+y)\frac{d}{dx}(x+y) = 2a\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right]$$

$$\Rightarrow 2(x+y)\left[1+\frac{dy}{dx}\right] = 2a\left[x\frac{dy}{dx} + y(1)\right]$$

$$\Rightarrow 2(x+y) + 2(x+y)\frac{dy}{dx} = 2ax\frac{dy}{dx} + 2ay$$

$$\Rightarrow \frac{dy}{dx}[2(x+y) - 2ax] = 2ay - 2(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2[ay-x-y]}{2[x+y-ax]}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{ay-x-y}{x+y-ax}\right)$$

8. $(x^2 + y^2)^2 = xy$

Solution:

Given $(x + y)^2 = 2axy$

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\Rightarrow \frac{d}{dx}\left[\left(x^2+y^2\right)^2\right]=\frac{d}{dx}(xy)$$

Now by applying product rule we get,

$$\Rightarrow 2\left(x^2+y^2\right)\frac{d}{dx}\left(x^2+y^2\right)=x\frac{dy}{dx}+y\frac{d}{dx}(x)$$

$$\Rightarrow 2\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right)=x\frac{dy}{dx}+y(1)$$

$$\Rightarrow 4x\left(x^2+y^2\right)+4y\left(x^2+y^2\right)\frac{dy}{dx}=x\frac{dy}{dx}+y$$

$$\Rightarrow 4y\left(x^2+y^2\right)\frac{dy}{dx}-x\frac{dy}{dx}=y-4x\left(x^2+y^2\right)$$

$$\Rightarrow \frac{dy}{dx}\left[4y\left(x^2+y^2\right)-x\right]=y-4x\left(x^2+y^2\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{y-4x\left(x^2+y^2\right)}{4y\left(x^2+y^2\right)-x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4x\left(x^2+y^2\right)-y}{x-4y\left(x^2+y^2\right)}$$

9. Tan^{-1} (x^2 + y^2)

Solution:

Given tan $^{-1}$(x^2 + y^2) = a,

Now we have to find dy/dx of given function, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{1}{x^2 + y^2}\left(2x + 2y\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

10. $e^{x-y} = \log\left(\frac{x}{y}\right)$

Solution:

Given $e^{x-y} = log\left(\frac{x}{y}\right)$

Now we have to find dy/dx of given function, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{d}{dx}\left(e^{x-y}\right) = \frac{d}{dx}\left\{\log\left(\frac{x}{y}\right)\right\}$$

$$\Rightarrow e^{(x-y)}\frac{d}{dx}(x-y) = \frac{1}{\left(\frac{x}{y}\right)} \times \frac{d}{dx}\left(\frac{x}{y}\right)$$

Now by applying quotient rule we get

$$\Rightarrow e^{(x-y)}\left(1-\frac{dy}{dx}\right) = \frac{y}{x}\left[\frac{y\frac{d}{dx}(x) - x\frac{dy}{dx}}{y^2}\right]$$

$$\Rightarrow e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{xy}\left[y\,(1) - x\frac{dy}{dx}\right]$$

$$\Rightarrow e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - e^{(x-y)}$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1}{y}-\frac{e^{(x-y)}}{1}\right]=\frac{1}{x}-\frac{e^{(x-y)}}{1}$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1-ye^{(x-y)}}{y}\right]=\frac{1-xe^{(x-y)}}{x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}\left[\frac{1-xe^{(x-y)}}{1-ye^{(x-y)}}\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{-y}{-x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

11. Sin xy + cos (x + y) = 1

Solution:

Given Sin x y + cos (x + y) = 1

Now we have to find dy/dx of given function, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{d}{dx}(\sin xy)+\frac{d}{dx}\cos(x+y)=\frac{d}{dx}(1)$$

$$\Rightarrow \cos xy\frac{d}{dx}(xy)-\sin(x+y)\frac{d}{dx}(x+y)=0$$

$$\Rightarrow \cos xy\left[x\frac{dy}{dx}+y\frac{d}{dx}(x)\right]-\sin(x+y)\left[1+\frac{dy}{dx}\right]=0$$

$$\Rightarrow \cos xy\left[x\frac{dy}{dx}+y(1)\right]-\sin(x+y)-\sin(x+y)\frac{dy}{dx}=0$$

$$\Rightarrow x\cos xy\frac{dy}{dx}+y\cos xy-\sin(x+y)-\sin(x+y)\frac{dy}{dx}=0$$

$$\Rightarrow [x\cos xy-\sin(x+y)]\frac{dy}{dx}=[\sin(x+y)-y\cos xy]$$

$$\Rightarrow \frac{dy}{dx}=\left[\frac{\sin(x+y)-y\cos xy}{x\cos xy-\sin(x+y)}\right]$$

12. $If\ \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y),\ prove\ that\ \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}.$

Solution:

Given $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Let x = sin A and y = sin B

Then given equation becomes,

$$\Rightarrow \sqrt{1-\sin^2 A}+\sqrt{1-\sin^2 B}=a(\sin A-\sin B)$$

$$\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$$

$$\Rightarrow a=\frac{\cos A+\cos B}{\sin A-\sin B}$$

Now by applying the formula we get,

$$\Rightarrow a=\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$\Rightarrow a=\cot\left(\frac{A-B}{2}\right)$$

$$\Rightarrow \cot^{-1} a=\frac{A-B}{2}$$

$$\Rightarrow 2\cot^{-1} a=A-B$$

$$\Rightarrow 2\cot^{-1} a=\sin^{-1} x-\sin^{-1} y.$$

Now by differentiating with respect to x we get,

$$\Rightarrow 0=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(2cot^{-1}a\right)=\frac{d}{dx}\left(\sin^{-1}x\right)-\frac{d}{dx}\left(\sin^{-1}y\right) \quad \Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

13. If $y=\sqrt{1-x^2}+x\sqrt{1-y^2}=1$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

Solution:

Given, $y = \sqrt{1-x^2} + x\sqrt{1-y^2} = 1$

Let x = sin A and y = sin B

Then given equation becomes,

$$\Rightarrow \sin B\sqrt{1-\sin^2 A} + \sin A\sqrt{1-\sin^2 B} = 1$$

Now by applying the identity, we get

$$\Rightarrow \sin B\cos A + \sin A\cos B = 1$$

$$\Rightarrow \sin(A+B) = 1$$

$$\Rightarrow A + B = \sin^{-1}(1)$$

Now by substituting the values of A and B, we get

$$\Rightarrow \sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$$

Now by differentiating with respect to x, we get

Now by differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx}(\sin^{-1}x) + \frac{d}{dx}(\sin^{-1}y) = \frac{d}{dx}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$14.\ If\ xy = 1,\ prove\ that\ \frac{dy}{dx} + y^2 = 0.$$

Solution:

Given x y = 1

Differentiating with respect to x, we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

By using product rule,

$$\Rightarrow x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0.$$

$$\Rightarrow x\frac{dy}{dx} + y(1) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

We have x y = 1, therefore x = 1/y

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{\frac{1}{y}}$$

$$\Rightarrow \frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} + y^2 = 0$$

$$15.\ If\ xy^2 = 1,\ prove\ that\ 2\frac{dy}{dx} + y^3 = 0.$$

Solution:

Given $xy^2 = 1$

Now differentiating given equation with respect to x, we get

$$\frac{d}{dx}\left(xy^2\right) = \frac{d}{dx}(1)$$

$$\Rightarrow x\frac{d}{dx}\left(y^2\right) + y^2\frac{d}{dx}(x) = 0$$

$$\Rightarrow x\left(2y\right)\frac{dy}{dx} + y^2\left(1\right) = 0$$

$$\Rightarrow 2xy\frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2x}$$

Now by substituting $x = 1/y^2$ in above equation we get

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow 2\frac{dy}{dx} = -y^3$$

$$\Rightarrow 2\frac{dy}{dx} + y^3 = 0$$

Exercise 11.5 Page No: 11.88

Differentiate the following functions with respect to x:

1. $x^{1/x}$

Solution:

Let $y = x^{\frac{1}{x}}$

Taking log both the sides:

$$\Rightarrow \log y = \log x^{\frac{1}{x}}$$

We know that log x^a = a log x, substituting this in above equation we get

$$\Rightarrow \log y = \frac{1}{x}\log x$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d\left(\frac{1}{x}\log x\right)}{dx}$$

Now by using the product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{1}{x} \times \frac{d(\log x)}{dx} + \log x \times \frac{d(x^{-1})}{dx}$$

We have $\left\{\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}; \frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}\right\}$, by using this we get,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \times \frac{1}{x}\frac{dx}{dx} + \log x\left(\frac{-1}{x^2}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{1}{x^2}\log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1 - \log x}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{1 - \log x}{x^2}\right)$$

Put the value of $y = x^{\frac{1}{x}}$

Put the value of $y = x^{\frac{1}{x}}$

$$\Rightarrow \frac{dy}{dx} = x^{\frac{1}{x}}\left(\frac{1 - \log x}{x^2}\right)$$

2. $x^{\sin x}$

Solution:

Let $y = x^{\sin x}$

Taking log both the sides

$\log y = \log (x^{\sin x})$

$\log y = \sin x \log x$ $\{\log x^a = a \log x\}$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\sin x \log x)}{dx}$$

Now by using product rule, we can write as

$$\Rightarrow \frac{d(\log y)}{dx} = \sin x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\sin x)}{dx}$$

Again we have, $\left\{\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx} \text{ \& } \frac{d(\sin x)}{dx} = \cos x\right\}$, by using this we can write as

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x \times \frac{1}{x}\frac{dx}{dx} + \log x(\cos x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \log x \cos x$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{\sin x}{x} + \log x \cos x\right)$$

Put the value of $y = x^{\sin x}$

$$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right)$$

3. $(1 + \cos x)^x$

Solution:

Let y = $(1 + \cos x)^x$

Taking log on both the sides

$\Rightarrow \log y = \log (1 + \cos x)^x$

$\Rightarrow \log y = x \log (1+ \cos x)$ $\{\log x^a = a \log x\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d[x \log (1 + \cos x)]}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = x \times \frac{d[\log(1 + \cos x)]}{dx} + \log(1 + \cos x) \times \frac{dx}{dx}$$

Again we have, $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{(1 + \cos x)}\frac{d(1 + \cos x)}{dx} + \log(1 + \cos x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{(1 + \cos x)}(-\sin x) + \log(1 + \cos x)$$

$$\left\{\frac{d(1 + \cos x)}{dx} = \frac{d(1)}{dx} + \frac{d(\cos x)}{dx} = 0 + (-\sin x)\frac{dx}{dx} = -\sin x\right\}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{-x\sin x}{1 + \cos x} + \log(1 + \cos x)$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{-x\sin x}{1 + \cos x} + \log(1 + \cos x)\right\}$$

Put the value of y = $(1 + \cos x)^x$

$$\Rightarrow \frac{dy}{dx} = (1 + \cos x)^x\left\{\frac{-x\sin x}{1 + \cos x} + \log(1 + \cos x)\right\}$$

4. $x^{\cos^{-1} x}$

Solution:

Let $y = x^{\cos^{-1} x}$

Taking log both the sides

$$\Rightarrow \log y = \log x^{\cos^{-1} x}$$

$$\Rightarrow \log y = \cos^{-1} x \log x$$ {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\cos^{-1} x \log x)}{dx}$$

By using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \cos^{-1} x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\cos^{-1} x)}{dx}$$

Again we have, $\left\{\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx} \;\&\; \frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}\right\}$, from this we can write as

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1} x}{x} + \log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right\}$$

Put the value of $y = x^{\cos^{-1} x}$

$$\Rightarrow \frac{dy}{dx} = x^{\cos^{-1} x}\left\{\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right\}$$

5. (log x)^x

Solution:

Let y = (log x)^x

Taking log both the sides

$\Rightarrow$ log y = log (log x)^x

$\Rightarrow$ log y = x log (log x) {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(x \log \log x)}{dx}$$

By product rule, we have

$$\Rightarrow \frac{d(\log y)}{dx} = x \times \frac{d(\log \log x)}{dx} + \log \log x \times \frac{dx}{dx}$$

We know that $\left\{\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}\right\}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{\log x}\frac{d(\log x)}{dx} + \log \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\log x} \times \frac{1}{x} + \log \log x$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{1}{\log x} + \log \log x\right\}$$

Put the value of y = (log x)^x

$$\Rightarrow \frac{dy}{dx} = (\log x)^x\left\{\frac{1}{\log x} + \log \log x\right\}$$

6. (log x)$^{\cos x}$

Solution:

Let y = $(\log x)^{\cos x}$

Taking log both the sides, we get

$\Rightarrow$ Log y = log $(\log x)^{\cos x}$

$\Rightarrow$ Log y = cos x log (log x) {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\cos x \log \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \cos x \times \frac{d(\log \log x)}{dx} + \log \log x \times \frac{d(\cos x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$ & $\frac{d(\cos x)}{dx} = -\sin x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x \times \frac{1}{\log x}\frac{d(\log x)}{dx} + \log \log x\ (-\sin x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{\log x} \times \frac{1}{x} - \sin x \log \log x$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\cos x}{x \log x} - \sin x \log \log x\right\}$$

Put the value of y = $(\log x)^{\cos x}$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x}\left\{\frac{\cos x}{x \log x} - \sin x \log \log x\right\}$$

7. $(\text{Sin } x)^{\cos x}$

Solution:

Let y = $(\sin x)^{\cos x}$

Taking log both the sides

$\Rightarrow$ Log y = log $(\sin x)^{\cos x}$

$\Rightarrow$ Log y = cos x log sin x {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\cos x \log \sin x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \cos x \times \frac{d(\log \sin x)}{dx} + \log \sin x \times \frac{d(\cos x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\cos x)}{dx} = -\sin x$; $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x \times \frac{1}{\sin x}\frac{d(\sin x)}{dx} + \log \sin x\,(-\sin x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cot x\,(\cos x) - \sin x \log \sin x$$

$$\Rightarrow \frac{dy}{dx} = y\{\cos x \cot x - \sin x \log \sin x\}$$

Put the value of y = $(\sin x)^{\cos x}$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^{\cos x}\{\cos x \cot x - \sin x \log \sin x\}$$

8. $e^{x \log x}$

Solution:

Let $y = e^{x \log x}$

Taking log both the sides, we get

⇒ $\text{Log } y = \log (e)^{x \log x}$

⇒ Log y = x log x log e $\{\log x^a = a \log x\}$

⇒ Log y = x log x {log e = 1}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(x \log x)}{dx}$$

Now by using product rule, we get

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = x \times \frac{d(\log x)}{dx} + \log x \times \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \log x$$

$$\Rightarrow \frac{dy}{dx} = y\{1 + \log x\}$$

Put the value of $y = e^{x \log x}$

$$\Rightarrow \frac{dy}{dx} = e^{x \log x}\{1 + \log x\}$$

$$\Rightarrow \frac{dy}{dx} = e^{\log x^x}\{1 + \log x\}$$ $\{e^{\log a} = a;\ a \log x = x^a\}$

$$\Rightarrow \frac{dy}{dx} = x^x\{1 + \log x\}$$

9. $(\text{Sin } x)^{\log x}$

Solution:

Let y = $(\sin x)^{\log x}$

Taking log both the sides

⇒ Log y = log $(\sin x)^{\log x}$

⇒ Log y = log x log sin x {log x^a = a log x}

Differentiating with respect to x, then we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\log x \log \sin x)}{dx}$$

Now by using product rule, we get

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \log x \times \frac{d(\log \sin x)}{dx} + \log \sin x \times \frac{d(\log x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log x \times \frac{1}{\sin x}\frac{d(\sin x)}{dx} + \log \sin x \left(\frac{1}{x}\frac{dx}{dx}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\log x}{\sin x}(\cos x) + \frac{\log \sin x}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\log x \cot x + \frac{\log \sin x}{x}\right\}$$

Put the value of y = $(\sin x)^{\log x}$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^{\log x}\left\{\log x \cot x + \frac{\log \sin x}{x}\right\}$$

10. $10^{\log \sin x}$

Solution:

Let y = $10^{\log \sin x}$

Taking log both the sides

⇒ Log y = log $10^{\log \sin x}$

⇒ Log y = log sin x log 10 {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\log 10 \log \sin x)}{dx}$$

Now by using chain rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \log 10 \times \frac{d(\log \sin x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}; \frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log 10 \times \frac{1}{\sin x}\frac{d(\sin x)}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\log 10}{\sin x}(\cos x)$$

$$\Rightarrow \frac{dy}{dx} = y\{\log 10 \cot x\}$$

Put the value of y = $10^{\log \sin x}$

$$\Rightarrow \frac{dy}{dx} = 10^{\log \sin x}\{\log 10 \cot x\}$$

11. (log x)$^{\log x}$

Solution:

Let y = $(\log x)^{\log x}$

Taking log both the sides

⇒ Log y = log $(\log x)^{\log x}$

⇒ Log y = log x log (log x) {log x^a = a log x}

Differentiating with respect to x, then we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\log x \log(\log x))}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \log x \times \frac{d(\log(\log x))}{dx} + \log(\log x) \times \frac{d(\log x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log x \times \frac{1}{\log x}\frac{d(\log x)}{dx} + \log\log x\left(\frac{1}{x}\frac{dx}{dx}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log x \times \frac{1}{\log x}\frac{d(\log x)}{dx} + \log\log x\left(\frac{1}{x}\frac{dx}{dx}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\log x}{\log x}\left(\frac{1}{x}\frac{dx}{dx}\right) + \frac{\log(\log x)}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{1}{x} + \frac{\log(\log x)}{x}\right\}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{1 + \log(\log x)}{x}\right\}$$

Put the value of y = $(\log x)^{\log x}$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\log x}\left\{\frac{1 + \log(\log x)}{x}\right\}$$

12. $10^{(10^x)}$

Solution:

Let y = $10^{(10^x)}$

Taking log both the sides

⇒ Log y = log $10^{(10^x)}$

⇒ Log y = 10 x log 10 {log x^a = a log x}

⇒ Log y = (10 log 10) x

Differentiating with respect to x,

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d\{(10 \log 10)x\}}{dx}$$

Here 10 log 10 is a constant term, therefore by using chain rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = 10 \times \log(10) \times \frac{d(x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}; \frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 10\log(10)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 10\log(10)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 10\log(10)$$

$$\Rightarrow \frac{dy}{dx} = y\{10\log(10)\}$$

Put the value of y = $10^{(10^x)}$

$$\Rightarrow \frac{dy}{dx} = 10^{10x}\{10\log(10)\}$$

13. Sin (x^x)

Solution:

Let y = sin (x^x)

Take sin inverse both sides

⇒ $\sin^{-1} y = \sin^{-1} (\sin x^x)$

⇒ $\sin^{-1} y = x^x$

Taking log both the sides

⇒ $\text{Log} (\sin^{-1} y) = \log x^x$

⇒ $\text{Log} (\sin^{-1} y) = x \log x$ {$\log x^a = a \log x$}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log(\sin^{-1} y))}{dx} = \frac{d(x \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log(\sin^{-1} y))}{dx} = x \times \frac{d(\log x)}{dx} + \log x \times \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{\sin^{-1} y}\frac{d(\sin^{-1} y)}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$$

$$\Rightarrow \frac{1}{\sin^{-1} y} \frac{d(\sin^{-1} y)}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$$

Again we have, $\frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$ by using this result we get

$$\Rightarrow \frac{1}{\sin^{-1} y} \times \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{x}{x} + \log x$$

$$\Rightarrow \frac{1}{\sin^{-1} y(\sqrt{1-y^2})} \frac{dy}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dy}{dx} = \sin^{-1} y\left(\sqrt{1-y^2}\right)(1 + \log x)$$

Put the value of y = sin (x^x)

$$\Rightarrow \frac{dy}{dx} = \sin^{-1} (\sin x^x)\left(\sqrt{1-\sin^2(x^x)}\right)(1 + \log x)$$

From $\sin^2 x + \cos^2 x = 1$, we can write as

$$\Rightarrow \frac{dy}{dx} = x^x\left(\sqrt{\cos^2(x^x)}\right)(1 + \log x)$$

$$\Rightarrow \frac{dy}{dx} = x^x \cos x^x (1 + \log x)$$

14. (Sin^{-1} x)^x

Solution:

Let $y = (\sin^{-1} x)^x$

Taking log both the sides

$\Rightarrow$ Log $y = \log (\sin^{-1} x)^x$

$\Rightarrow$ Log $y = x \log (\sin^{-1} x)$ $\{\log x^a = a \log x\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(x \log (\sin^{-1}x))}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = x \times \frac{d(\log (\sin^{-1}x))}{dx} + \log(\sin^{-1}x) \times \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{\sin^{-1}x}\frac{d(\sin^{-1}x)}{dx} + \log(\sin^{-1}x)$$

Again we have, $\frac{d(\sin^{-1}u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ by using this result we get

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1}x} \times \frac{1}{\sqrt{1-x^2}}\frac{dx}{dx} + \log(\sin^{-1}x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1}x\sqrt{1-x^2}} + \log(\sin^{-1}x)$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{x}{\sin^{-1}x\sqrt{1-x^2}} + \log(\sin^{-1}x)\right\}$$

Put the value of $y = (\sin^{-1} x)^x$

$$\Rightarrow \frac{dy}{dx} = (\sin^{-1}x)^x\left\{\frac{x}{\sin^{-1}x\sqrt{1-x^2}} + \log(\sin^{-1}x)\right\}$$

15. $x^{\sin^{-1}x}$

Solution:

Let $y = x^{\sin^{-1}x}$

Taking log both the sides

$\Rightarrow \log y = \log x^{\sin^{-1}x}$

$\Rightarrow$ Log y = $\sin^{-1}$ x log x {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\sin^{-1}x \log x)}{dx}$$

By using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \sin^{-1}x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\sin^{-1}x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\sin^{-1}u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin^{-1}x \times \frac{1}{x}\frac{dx}{dx} + \log x \times \frac{1}{\sqrt{1-x^2}}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right\}$$

Put the value of $y = x^{\sin^{-1}x}$:

$$\Rightarrow \frac{dy}{dx} = x^{\sin^{-1}x}\left\{\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right\}$$

16. $(\tan x)^{1/x}$

Solution:

Let $y = (\tan x)^{\frac{1}{x}}$

Taking log both the sides, we get

$$\Rightarrow \log y = \log(\tan x)^{\frac{1}{x}}$$

$$\Rightarrow \log y = \frac{1}{x}\log\tan x$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d\left(\frac{1}{x}\log\tan x\right)}{dx}$$

By using product rule, we can write as

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{1}{x} \times \frac{d(\log\tan x)}{dx} + \log\tan x \times \frac{d(x^{-1})}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}; \frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \times \frac{1}{\tan x}\frac{d(\tan x)}{dx} + \log\tan x\,(-x^{-2})$$

Again we have $\frac{d(\tan x)}{dx} = \sec^2 x$ by using this result in the above expression

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x\tan x}(\sec^2 x) - \frac{\log\tan x}{x^2}$$

$$\frac{dy}{dx} = y\left\{\frac{\sec^2 x}{x\tan x} - \frac{\log\tan x}{x^2}\right\}$$

Put the value of $y = (\tan x)^{\frac{1}{x}}$

$$\frac{dy}{dx} = (\tan x)^{\frac{1}{x}}\left\{\frac{\sec^2 x}{x\tan x} - \frac{\log\tan x}{x^2}\right\}$$

$17.\ x^{\tan^{-1} x}$

Solution:

Let $y = x^{\tan^{-1}x}$

Taking log both the sides

$$\Rightarrow \log y = \log x^{\tan^{-1}x}$$

$\Rightarrow$ Log y = $\tan^{-1}$ x log x {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\tan^{-1}x \ \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\tan^{-1}x \ \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \tan^{-1}x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\tan^{-1}x)}{dx}$$

Again we know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\tan^{-1}u)}{dx} = \frac{1}{u^2+1}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \tan^{-1}x \times \frac{1}{x}\frac{dx}{dx} + \log x \times \frac{1}{x^2+1}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\tan^{-1}x}{x} + \frac{\log x}{x^2+1}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\tan^{-1}x}{x} + \frac{\log x}{x^2+1}\right\}$$

Put the value of $y = x^{\tan^{-1}x}$

$$\Rightarrow \frac{dy}{dx} = x^{\tan^{-1}x}\left\{\frac{\tan^{-1}x}{x} + \frac{\log x}{x^2+1}\right\}$$

18. (i) $(x^x)\sqrt{x}$

Solution:

Let $y = (x)^x \sqrt{x}$

Taking log both the sides

$\Rightarrow \log y = \log(x)^x \sqrt{x}$

$\Rightarrow \log y = \log(x)^x + \log\sqrt{x}$ {Log (ab) = log a +log b}

$\Rightarrow \log y = \log(x)^x + \log x^{\frac{1}{2}}$

$\Rightarrow \log y = x\log x + \frac{1}{2}\log x$ {Log x^a = a log x}

$\Rightarrow \log y = \left(x + \frac{1}{2}\right)\log x$

$$\Rightarrow \log y = \left(x+\frac{1}{2}\right)\log x$$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d\left(\left(x+\frac{1}{2}\right)\log x\right)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \left(x+\frac{1}{2}\right)\times\frac{d(\log x)}{dx} + \log x \times \frac{d\left(x+\frac{1}{2}\right)}{dx}$$

Again we have to use chain rule for the above expression,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left(x+\frac{1}{2}\right)\times\frac{1}{x}\frac{dx}{dx} + \log x\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{(2x+1)}{2}\times\frac{1}{x} + \log x$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{(2x+1)}{2x} + \log x\right\}$$

Put the value of $y = (x)^x\sqrt{x}$

$$\Rightarrow \frac{dy}{dx} = (x)^x\sqrt{x}\left\{\frac{(2x+1)}{2x} + \log x\right\}$$

$$\Rightarrow \frac{dy}{dx} = (x)^x\sqrt{x}\left\{\frac{2x}{2x} + \frac{1}{2x} + \log x\right\}$$

$$\Rightarrow \frac{dy}{dx} = (x)^x\sqrt{x}\left\{1 + \frac{1}{2x} + \log x\right\}$$

18. (ii) $x^{(\sin x - \cos x)} + \frac{x^2-1}{x^2+1}$

Solution:

$$\text{Let } y = x^{(\sin x - \cos x)} + \frac{x^2 - 1}{x^2 + 1}$$

⇒ y = a + b

$$\text{where } a = x^{(\sin x - \cos x)}; b = \frac{x^2 - 1}{x^2 + 1}$$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$a = x^{(\sin x - \cos x)}$$

Taking log both the sides to the above expressions we get

$$\Rightarrow \log a = \log x^{(\sin x - \cos x)}$$

$\Rightarrow \log a = (\sin x - \cos x) \log x$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d((\sin x - \cos x)\log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log a)}{dx} = (\sin x - \cos x) \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\sin x - \cos x)}{dx}$$

To the above expression we have to use chain rule,

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = (\sin x - \cos x) \times \frac{1}{x}\frac{dx}{dx} + \log x(\frac{d(\sin x)}{dx} - \frac{d(\cos x)}{dx})$$

We know that $\frac{d(\cos x)}{dx} = -\sin x ; \frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{(\sin x - \cos x)}{x} + \log x(\cos x - (-\sin x))$$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{(\sin x - \cos x)}{x} + \log x(\cos x + \sin x)$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{\sin x - \cos x}{x} + \log x(\cos x + \sin x)\right\}$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{\sin x - \cos x}{x} + \log x\,(\cos x + \sin x)\right\}$$

Put the value of a = $x^{(\sin x - \cos x)}$

$$\Rightarrow \frac{da}{dx} = x^{(\sin x - \cos x)}\left\{\frac{\sin x - \cos x}{x} + \log x\,(\cos x + \sin x)\right\}$$

$$b = \frac{x^2 - 1}{x^2 + 1}$$

To differentiate above expression with respect to x we have to use quotient rule,

$$\Rightarrow \frac{db}{dx} = \frac{(x^2 + 1)\frac{d(x^2 - 1)}{dx} - (x^2 - 1)\frac{d(x^2 + 1)}{dx}}{(x^2 + 1)^2}$$

Now by using chain rule, we get

$$\Rightarrow \frac{db}{dx} = \frac{(x^2 + 1)(2x) - (x^2 - 1)\,(2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{(2x^3 + 2x) - (2x^3 - 2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{(2x^3 + 2x - 2x^3 + 2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{4x}{(x^2 + 1)^2}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

Now by substituting all the values in above expressions we get

$$\Rightarrow \frac{dy}{dx} = x^{(\sin x - \cos x)}\left\{\frac{\sin x - \cos x}{x} + \log x\,(\cos x + \sin x)\right\} + \frac{4x}{(x^2+1)^2}$$

$$18.(iii)\ x^{x\cos x} + \frac{x^2+1}{x^2-1}$$

Solution:

Let $y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$

⇒ y = a + b

where $a = x^{x\cos x}; b = \frac{x^2+1}{x^2-1}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$a = x^{x\cos x}$

Taking log both the sides to the above equation we get

$$\Rightarrow \log a = \log x^{x\cos x}$$

$$\Rightarrow \log a = x\cos x\log x$$

{Log x^a = a log x}

Differentiating with respect to x,

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(x\cos x\log x)}{dx}$$

Now by using product rule, we can write as

$$\Rightarrow \frac{d(\log a)}{dx} = x\cos x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(x\cos x)}{dx}$$

$$\Rightarrow \frac{d(\log a)}{dx} = x\cos x \times \frac{d(\log x)}{dx} + \log x\left\{x\frac{d(\cos x)}{dx} + \cos x\right\}$$

Again we have, $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$ by using this result in the above expressions we get

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = x\cos x \times \frac{1}{x}\frac{dx}{dx} + \log x\{x(-\sin x) + \cos x\}$$

We know that $\frac{d(\cos x)}{dx} = -\sin x\,;\ \frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{a}\,\frac{da}{dx} = \frac{x\cos x}{x} + \log x(\cos x - x\sin x)$$

$$\Rightarrow \frac{da}{dx} = a\{\cos x + \log x(\cos x - x\sin x)\}$$

Put the value of $a = x^{x\cos x}$:

$$\Rightarrow \frac{da}{dx} = x^{x\cos x}\{\cos x + \log x(\cos x - x\sin x)\}$$

$$\Rightarrow \frac{da}{dx} = x^{x\cos x}\{\cos x + \log x\cos x - x\sin x\log x\}$$

$$\Rightarrow \frac{da}{dx} = x^{x\cos x}\{\cos x\,(1 + \log x) - x\sin x\log x\}$$

$$b = \frac{x^2 + 1}{x^2 - 1}$$

Now we have to differentiate above expression using quotient rule, then we get

$$\Rightarrow \frac{db}{dx} = \frac{(x^2 - 1)\frac{d(x^2 + 1)}{dx} - (x^2 + 1)\frac{d(x^2 - 1)}{dx}}{(x^2 - 1)^2}$$

Now apply chain rule for the above equation,

$$\Rightarrow \frac{db}{dx} = \frac{(x^2 - 1)(2x) - (x^2 + 1)\,(2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{(2x^3 - 2x) - (2x^3 + 2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{(2x^3 - 2x - 2x^3 - 2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{-4x}{(x^2 + 1)^2}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

By substituting all values in the above expression we get

$$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\{\cos x\,(1 + \log x) - x\sin x \log x\} - \frac{4x}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{-4x}{(x^2 + 1)^2}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

By substituting all values in the above expression we get

$$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\{\cos x\,(1 + \log x) - x\sin x \log x\} - \frac{4x}{(x^2 + 1)^2}$$

18. (iv) $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Solution:

Let $y = (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

⇒ y = a + b

where $a = (x\cos x)^x; b = (x\sin x)^{\frac{1}{x}}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$a = (x\cos x)^x$

Taking log both the sides, we get

$\Rightarrow \log a = \log(x\cos x)^x$

$\Rightarrow \log a = x\log(x\cos x)$

$\{\text{Log } x^a = a \log x\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(x\log(x\cos x))}{dx}$$

By using product rule, we get

$$\Rightarrow \frac{d(\log a)}{dx} = x \times \frac{d(\log(x\cos x))}{dx} + \log(x\cos x) \times \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = x \times \frac{1}{x\cos x}\frac{d(x\cos x)}{dx} + \log(x\cos x)$$

Again by using product rule, we can write as

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{x}{x\cos x}\left\{x\frac{d(\cos x)}{dx} + \cos x\right\} + \log(x\cos x)$$

We have $\frac{d(\cos x)}{dx} = -\sin x$ using this result we can write as

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{1}{\cos x}\{x(-\sin x) + \cos x\} + \log(x\cos x)$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{\cos x - x\sin x}{\cos x} + \log(x\cos x)\right\}$$

Put the value of $a = (x\cos x)^x$:

$$\Rightarrow \frac{da}{dx} = (x\cos x)^x\left\{\frac{\cos x - x\sin x}{\cos x} + \log(x\cos x)\right\}$$

$$\Rightarrow \frac{da}{dx} = (x\cos x)^x\{1 - x\tan x + \log(x\cos x)\}$$

$$b = (x\sin x)^{\frac{1}{x}}$$

Taking log both the sides

$$\Rightarrow \log b = \log(x\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log b = \frac{1}{x}\log(x\sin x)$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d\left(\frac{1}{x}\log(x\sin x)\right)}{dx}$$

Now by using product rule, we get

Now by using product rule, we get

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{1}{x} \times \frac{d(\log(x \sin x))}{dx} + \log(x \sin x) \times \frac{d(x^{-1})}{dx}$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{1}{x} \times \frac{1}{x \sin x}\frac{d(x \sin x)}{dx} + \log(x \sin x)\ (-x^{-2})$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{1}{x^2 \sin x}\left(x\frac{d(\sin x)}{dx} + \sin x\frac{dx}{dx}\right) - \frac{\log(x \sin x)}{x^2}$$

We know that $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x \cos x + \sin x}{x^2 \sin x} - \frac{\log(x \sin x)}{x^2}\right\}$$

Put the value of $b = (x \sin x)^{\frac{1}{x}}$:

$$\Rightarrow \frac{db}{dx} = (x \sin x)^{\frac{1}{x}}\left\{\frac{x \cos x + \sin x}{x^2 \sin x} - \frac{\log(x \sin x)}{x^2}\right\}$$

$$\Rightarrow \frac{db}{dx} = (x \sin x)^{\frac{1}{x}}\left\{\frac{x \cot x + 1}{x^2} - \frac{\log(x \sin x)}{x^2}\right\}$$

$$\Rightarrow \frac{db}{dx} = (x \sin x)^{\frac{1}{x}}\left\{\frac{x \cot x + 1 - \log(x \sin x)}{x^2}\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

Now by substituting all the values in above expression we get

$$\Rightarrow \frac{dy}{dx} = (x \cos x)^x\{1 - x \tan x + \log(x \cos x)\} + (x \sin x)^{\frac{1}{x}}\left\{\frac{x \cot x + 1 - \log(x \sin x)}{x^2}\right\}$$

18. (v) $\left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Solution:

Let $y = \left(x + \frac{1}{x}\right)^{x} + x^{\left(1+\frac{1}{x}\right)}$

$\Rightarrow$ y = a + b

where $a = \left(x + \frac{1}{x}\right)^{x}; b = x^{\left(1+\frac{1}{x}\right)}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$a = \left(x + \frac{1}{x}\right)^{x}$$

Taking log both the sides, we get

$$\Rightarrow \log a = \log\left(x + \frac{1}{x}\right)^{x}$$

$$\Rightarrow \log a = x\log\left(x + \frac{1}{x}\right)$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d\left(x\log\left(x + \frac{1}{x}\right)\right)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log a)}{dx} = x \times \frac{d\left(\log\left(x+\frac{1}{x}\right)\right)}{dx} + \log\left(x+\frac{1}{x}\right) \times \frac{dx}{dx}$$

Again we know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$.

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = x \times \frac{1}{x+\frac{1}{x}} \frac{d\left(x+\frac{1}{x}\right)}{dx} + \log\left(x+\frac{1}{x}\right)$$

Again by using chain rule in the above expression we get

Again by using chain rule in the above expression we get

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{\frac{x}{x^2+1}}{x}\left\{\frac{dx}{dx} + \frac{d\left(\frac{1}{x}\right)}{dx}\right\} + \log\left(x + \frac{1}{x}\right)$$

By using $\frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$,

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{x^2}{x^2+1}\left\{1 + \left(-\frac{1}{x^2}\right)\right\} + \log\left(x + \frac{1}{x}\right)$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{x^2}{x^2+1}\left\{1 - \frac{1}{x^2}\right\} + \log\left(x + \frac{1}{x}\right)\right\}$$

Put the value of $a = \left(x + \frac{1}{x}\right)^x$:

$$\Rightarrow \frac{da}{dx} = \left(x + \frac{1}{x}\right)^x\left\{\frac{x^2}{x^2+1}\left\{1 - \frac{1}{x^2}\right\} + \log\left(x + \frac{1}{x}\right)\right\}$$

$$\Rightarrow \frac{da}{dx} = \left(x + \frac{1}{x}\right)^x\left\{\frac{x^2}{x^2+1} - \frac{1}{x^2+1} + \log\left(x + \frac{1}{x}\right)\right\}$$

$$\Rightarrow \frac{da}{dx} = \left(x + \frac{1}{x}\right)^x\left\{\frac{x^2-1}{x^2+1} + \log\left(x + \frac{1}{x}\right)\right\}$$

$b = x^{\left(1+\frac{1}{x}\right)}$

Taking log both the sides

$$\Rightarrow \log b = \log x^{\left(1+\frac{1}{x}\right)}$$

$$\Rightarrow \log b = \left(1+\frac{1}{x}\right)\log x$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d\left(\left(1+\frac{1}{x}\right)\log x\right)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log b)}{dx} = \left(1+\frac{1}{x}\right) \times \frac{d(\log x)}{dx} + \log x \times \frac{d\left(1+\frac{1}{x}\right)}{dx}$$

$$\Rightarrow \frac{d(\log b)}{dx} = \left(1+\frac{1}{x}\right) \times \frac{d(\log x)}{dx} + \log x \times \frac{d\left(1+\frac{1}{x}\right)}{dx}$$

Again for the above expression we have to apply chain rule,

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x+1}{x} \times \frac{1}{x}\frac{dx}{dx} + \log x \left(\frac{d(1)}{dx} + \frac{d\left(\frac{1}{x}\right)}{dx}\right)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x+1}{x^2} + \log x \left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x+1}{x^2} - \frac{\log x}{x^2}\right\}$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x+1-\log x}{x^2}\right\}$$

Put the value of $b = x^{\left(1+\frac{1}{x}\right)}$:

$$\Rightarrow \frac{db}{dx} = x^{\left(1+\frac{1}{x}\right)}\left\{\frac{x+1-\log x}{x^2}\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

Now by substituting the all the values in above expression we get

$$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left\{\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right\} + x^{\left(1+\frac{1}{x}\right)}\left\{\frac{x+1-\log x}{x^2}\right\}$$

18. (vi) $e^{\sin x} + (\tan x)^x$

Solution:

Let $y = e^{\sin x} + (\tan x)^x$

$\Rightarrow y = a + b$

Where $a = e^{\sin x}$; $b = (\tan x)^x$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

a= $e^{\sin x}$

Taking log both the sides, we get

⇒ Log a= log $e^{\sin x}$

⇒ Log a= sin x log e {Log x^a = a log x}

⇒ Log a= sin x {log e =1}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(\sin x)}{dx}$$

Again we have $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \cos x$$

$$\Rightarrow \frac{da}{dx} = a\,(\cos x)$$

Put the value of a = $e^{\sin x}$

$$\Rightarrow \frac{da}{dx} = e^{\sin x}\cos x$$

b = $(\tan x)^x$

Taking log both the sides:

⇒ Log b= log $(\tan x)^x$

⇒ Log b= x log (tan x) {Log x^a = a log x}

Differentiating with respect to x:

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d(x \log(\tan x))}{dx}$$

Again by using product rule,

$$\Rightarrow \frac{d(\log b)}{dx} = x \times \frac{d(\log(\tan x))}{dx} + \log(\tan x) \times \frac{dx}{dx}$$

We know that $\frac{d(\tan x)}{dx} = \sec^2 x$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = x \times \frac{1}{\tan x}\frac{d(\tan x)}{dx} + \log(\tan x)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x}{\tan x}(\sec^2 x) + \log(\tan x)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x\cos x}{\sin x}\left(\frac{1}{\cos^2 x}\right) + \log(\tan x)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x}{\sin x}\left(\frac{1}{\cos x}\right) + \log(\tan x)$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x}{\sin x\cos x} + \log(\tan x)\right\}$$

Put the value of b = $(\tan x)^x$

$$\Rightarrow \frac{db}{dx} = (\tan x)^x\left\{\frac{x}{\sin x\cos x} + \log(\tan x)\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$\Rightarrow \frac{dy}{dx} = e^{\sin x}\cos x + (\tan x)^x\left\{\frac{x}{\sin x\cos x} + \log(\tan x)\right\}$$

18. (vii) $(\cos x)^x + (\sin x)^{1/x}$

Solution:

Let $y = (\cos x)^{x} + (\sin x)^{\frac{1}{x}}$

$\Rightarrow y = a + b$

where $a = (\cos x)^{x}; b = (\sin x)^{\frac{1}{x}}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$a = (\cos x)^x$$

Taking log both the sides

$$\Rightarrow \log a = \log(\cos x)^x$$

$$\Rightarrow \log a = x \log(\cos x)$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(x \log(\cos x))}{dx}$$

Now by using product rule, we have

$$\Rightarrow \frac{d(\log a)}{dx} = x \times \frac{d(\log(\cos x))}{dx} + \log(\cos x) \times \frac{dx}{dx}$$

Again we have $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = x \times \frac{1}{\cos x}\frac{d(\cos x)}{dx} + \log(\cos x)$$

We know that $\frac{d(\cos x)}{dx} = -\sin x$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{x}{\cos x}(-\sin x) + \log(\cos x)$$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{-x\sin x}{\cos x} + \log(\cos x)$$

$$\Rightarrow \frac{da}{dx} = a\{-x\tan x + \log(\cos x)\}$$

Put the value of $a = (\cos x)^x$:

$$\Rightarrow \frac{da}{dx} = (\cos x)^x\{-x\tan x + \log(\cos x)\}$$

$b = (\sin x)^{\frac{1}{x}}$

Taking log both the sides

$$\Rightarrow \log b = \log(\sin x)^{\frac{1}{x}}$$

$\Rightarrow \log b = \frac{1}{x}\log(\sin x)$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d\left(\frac{1}{x}\log(\sin x)\right)}{dx}$$

Again by product rule we have

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{1}{x} \times \frac{d(\log(\sin x))}{dx} + \log(\sin x) \times \frac{d(x^{-1})}{dx}$$

We know that $\frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{1}{x} \times \frac{1}{\sin x}\frac{d(\sin x)}{dx} + \log(\sin x)\,(-x^{-2})$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{1}{x\sin x}(\cos x) - \frac{\log(\sin x)}{x^2}$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{\cos x}{x\sin x} - \frac{\log(\sin x)}{x^2}$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right\}$$

Put the value of $b = (\sin x)^{\frac{1}{x}}$:

$$\Rightarrow \frac{db}{dx} = (\sin x)^{\frac{1}{x}}\left\{\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right\}$$

$$\Rightarrow \frac{db}{dx} = (\sin x)^{\frac{1}{x}}\left\{\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$\Rightarrow \frac{dy}{dx} = (\cos x)^x\{-x\tan x + \log(\cos x)\} + (\sin x)^{\frac{1}{x}}\left\{\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right\}$$

$18.(viii)\ x^{x^2-3} + (x-3)^{x^2}$

Solution:

Let $y = x^{x^2-3} + (x-3)^{x^2}$

$\Rightarrow$ y = a + b

where $a = x^{x^2-3}; b = (x-3)^{x^2}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$a = x^{x^2-3}$

Taking log both the sides

$\Rightarrow \log a = \log x^{x^2-3}$

$\Rightarrow \log a = (x^2-3)\log x$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d((x^2-3)\log x)}{dx}$$

Now by using product rule,

$$\Rightarrow \frac{d(\log a)}{dx} = (x^2-3) \times \frac{d(\log x)}{dx} + \log x \times \frac{d(x^2-3)}{dx}$$

Again by using chain rule we get

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = (x^2-3) \times \frac{1}{x}\frac{dx}{dx} + \log x \times (2x)$$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{(x^2-3)}{x} + 2x\log x$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{(x^2-3)}{x} + 2x\log x\right\}$$

Put the value of $a = x^{x^2-3}$:

$$\Rightarrow \frac{da}{dx} = x^{x^2-3}\left\{\frac{(x^2-3)}{x} + 2x\log x\right\}$$

$b = (x-3)^{x^2}$

Taking log both the sides:

$\Rightarrow \log b = (x-3)^{x^2}$

$\Rightarrow \log b = x^2 \log(x-3)$ {Log x^a = a log x}

Differentiating with respect to x:

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d(x^2\log(x-3))}{dx}$$

Again by using product rule, we get

$$\Rightarrow \frac{d(\log b)}{dx} = x^2 \times \frac{d(\log\,(x-3))}{dx} + \log(x-3) \times \frac{d(x^2)}{dx}$$

For the above expression now we have to use chain rule,

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = x^2 \times \frac{1}{(x-3)}\frac{d(x-3)}{dx} + \log(x-3) \times (2x)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x^2}{(x-3)}\left(\frac{dx}{dx} - \frac{d(3)}{dx}\right) + 2x\log(x-3)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x^2}{(x-3)}(1) + 2x\log(x-3)$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x^2}{(x-3)} + 2x\log(x-3)\right\}$$

Put the value of $b = (x-3)^{x^2}$:

$$\Rightarrow \frac{db}{dx} = (x-3)^{x^2}\left\{\frac{x^2}{(x-3)} + 2x\log(x-3)\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$\Rightarrow \frac{dy}{dx} = x^{x^2-3}\left\{\frac{(x^2-3)}{x} + 2x\log x\right\} + (x-3)^{x^2}\left\{\frac{x^2}{(x-3)} + 2x\log(x-3)\right\}$$

19. $y = e^x + 10^x + x^x$

Solution:

Let $y = e^x + 10^x + x^x$

$\Rightarrow y = a + b + c$

Where $a = e^x$; $b = 10^x$; $c = x^x$

Now we have to differentiate $y = a + b + c$ with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx} + \frac{dc}{dx}$$

$a = e^x$

Taking log both the sides

$\Rightarrow \text{Log } a = \text{Log } e^x$

$\Rightarrow \text{Log } a = x \log e$

$\{\text{Log } x^a = a \log x\}$

$\Rightarrow \text{Log } a = x \{\log e = 1\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = 1$$

$$\Rightarrow \frac{da}{dx} = a$$

Put the value of a = e^x

$$\Rightarrow \frac{da}{dx} = e^x$$

b = 10^x

Taking log both the sides:

⇒ Log b= log 10^x

⇒ Log b= x log 10

{Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d(x \log 10)}{dx}$$

Now by using chain rule,

$$\Rightarrow \frac{d(\log b)}{dx} = \log 10 \times \frac{dx}{dx}$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = b(\log 10)$$

$$\Rightarrow \frac{db}{dx} = b(\log 10)$$

Put the value of b = 10^x

$\Rightarrow \frac{db}{dx} = 10^x(\log 10)$

c = x^x

Taking log both the sides

⇒ Log c= log x^x

⇒ Log c= x log x

{Log x^a = a log x}

Differentiating with respect to x

$\Rightarrow \frac{d(\log c)}{dx} = \frac{d(x \log x)}{dx}$

By using product rule, we get

$\Rightarrow \frac{d(\log c)}{dx} = x \times \frac{d(\log x)}{dx} + \log x \times \frac{dx}{dx}$

$\Rightarrow \frac{1}{c}\frac{dc}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$

$\Rightarrow \frac{1}{c}\frac{dc}{dx} = 1 + \log x$

$\Rightarrow \frac{dc}{dx} = c\{1 + \log x\}$

Put the value of c = x^x

$\Rightarrow \frac{dc}{dx} = x^x\{1 + \log x\}$

$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx} + \frac{dc}{dx}$

$\Rightarrow \frac{dy}{dx} = e^x + 10^x(\log 10) + x^x\{1 + \log x\}$

20. $y = x^n + n^x + x^x + n^n$

Solution:

Let $y = x^n + n^x + x^x + n^n$

$\Rightarrow y = a + b + c + m$

Where $a = x^n$; $b = n^x$; $c = x^x$; $m = n^n$

Now we have to differentiate y = a + b + c + m with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx} + \frac{dc}{dx} + \frac{dm}{dx}$$

$a = x^n$

Taking log both the sides

$\Rightarrow$ Log a= log x^n

$\Rightarrow$ Log a= n log x

{Log x^a = a log x}

$\Rightarrow$ Log a= n log x {log e =1}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(n \log x)}{dx}$$

Again by chain rule, we can write as

$$\Rightarrow \frac{d(\log a)}{dx} = n\frac{d(\log x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$\Rightarrow \frac{1}{a}\frac{da}{dx} = n \times \frac{1}{x}\frac{dx}{dx}$

$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{n}{x}$

$\Rightarrow \frac{da}{dx} = \frac{an}{x}$

Put the value of a = x^n

$$\frac{da}{dx} = \frac{nx^n}{x}$$

$$\frac{da}{dx} = nx^{n-1}$$

b = n^x

Taking log both the sides

⇒ Log b= log n^x

⇒ Log b= x log n {Log x^a = a log x}

Differentiating with respect to x using chain rule, we get

$$\Rightarrow \frac{d(\log b)}{dx} = \log n \times \frac{dx}{dx}$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = b(\log n)$$

$$\Rightarrow \frac{db}{dx} = b(\log n)$$

Put the value of b = n^x

$$\Rightarrow \frac{db}{dx} = n^x(\log n)$$

c = x^x

Taking log both the sides

$\Rightarrow$ Log c= log x^x

$\Rightarrow$ Log c= x log x

{Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log c)}{dx} = \frac{d(x \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log c)}{dx} = x \times \frac{d(\log x)}{dx} + \log x \times \frac{dx}{dx}$$

$$\Rightarrow \frac{1}{c}\frac{dc}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$$

$$\Rightarrow \frac{1}{c}\frac{dc}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dc}{dx} = c\{1 + \log x\}$$

Put the value of $c = x^x$

$$\Rightarrow \frac{dc}{dx} = x^x\{1 + \log x\}$$

$m = n^n$

$$\Rightarrow \frac{dm}{dx} = \frac{d(n^n)}{dx}$$

$$\Rightarrow \frac{dm}{dx} = 0$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx} + \frac{dc}{dx} + \frac{dm}{dx}$$

$$\Rightarrow \frac{dy}{dx} = nx^{n-1} + n^x(\log n) + x^x\{1 + \log x\} + 0$$

$$\Rightarrow \frac{dy}{dx} = nx^{n-1} + n^x(\log n) + x^x\{1 + \log x\}$$

Exercise 11.6 Page No: 11.98

1. $\boldsymbol{If\ y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots\ to\ \infty}}},\ prove\ that\ \frac{dy}{dx} = \frac{1}{2y - 1}.}$

Solution:

Given,

$y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots \text{to } \infty}}}$

$y = \sqrt{x + y}$

Where $y = \sqrt{x + \sqrt{x + \cdots \text{to } \infty}}$

On squaring both sides,

$y^2 = x + y$

Differentiating both sides with respect to x,

$2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\frac{dy}{dx}(2y - 1) = 1$

$\frac{dy}{dx} = \frac{1}{2y - 1}$

Hence proved.

$2. \, If \; y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots.. \, to \, \infty}}}, \; prove \; that \; \frac{dy}{dx} = \frac{\sin x}{1 - 2y}.$

Solution:

Given,

$$y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \cdots \text{to } \infty}}}$$

$$y = \sqrt{\cos x + y}$$

Where y = $\sqrt{\cos x + \sqrt{\cos x + \cdots \text{to } \infty}}$

$$y = \sqrt{\cos x + y}$$

Where y = $\sqrt{\cos x + \sqrt{\cos x + \cdots \text{to } \infty}}$

Squaring on both sides,

$y^2 = \cos x + y$

Differentiating both sides with respect to x,

$$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y-1) = -\sin x$$

$$\frac{dy}{dx} = -\frac{\sin x}{2y-1}$$

$$\frac{dy}{dx} = \frac{\sin x}{1-2y}$$

Hence proved.

3. $If\ y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots\ldots to\ \infty}}},\ prove\ that\ (2y-1)\frac{dy}{dx} = \frac{1}{x}.$

Solution:

Given

$$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots \text{to } \infty}}}$$

$$y = \sqrt{\log x + y}$$

Where $y = \sqrt{\log x + \sqrt{\log x + \cdots \text{to } \infty}}$

Squaring on both sides,

$$y^2 = \text{Log } x + y$$

Differentiating both sides with respect to x,

$$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x(2y - 1)}$$

Hence proved.

4. $If\ y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots.. \ to\ \infty}}},\ prove\ that\ \frac{dy}{dx} = \frac{sec^2 x}{2y - 1}.$

Solution:

Given,

$$y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \cdots \text{ to } \infty}}}$$

$$y = \sqrt{\tan x + y}$$

On squaring both sides,

$$y^2 = \tan x + y$$

Differentiating both sides with respect to x,

$$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{(2y - 1)}$$

Hence proved.

Exercise 11.7 Page No: 11.103

Find dy/dx, when

1. $x = at^2$ and y = 2 at

Solution:

Given that $x = at^2$, $y = 2at$

Now by differentiating $x = at^2$ with respect to t we get

$$\frac{dx}{dt} = \frac{d(at^2)}{dt} = 2at$$

Again by differentiating y = 2at with respect to t we get

$$\frac{dy}{dt} = \frac{d(2at)}{dt} = 2a$$

Therefore,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

2. x = a (θ + sin θ) and y = a (1 – cos θ)

Solution:

Given that x = at^2, y = 2at

Now by differentiating x = at^2 with respect to t we get

$$\frac{dx}{dt} = \frac{d(at^2)}{dt} = 2at$$

Again by differentiating y = 2at with respect to t we get

$$\frac{dy}{dt} = \frac{d(2at)}{dt} = 2a$$

Therefore,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

x = a (θ + sin θ)

Differentiating it with respect to θ,

x = a (θ + sin θ)

Differentiating it with respect to θ,

$\frac{dx}{d\theta} = a(1 + \cos\theta)$ ...... (1)

And,

y = a (1- cos θ)

Differentiating it with respect to θ,

$$\frac{dy}{d\theta} = a(0 + \sin\theta)$$

$\frac{dy}{d\theta} = a\sin\theta$ ...... (2)

Using equation (1) and (2),

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{a\sin\theta}{a(1 - \cos\theta)}$$

$$= \frac{\frac{2\sin\theta}{2}\frac{(\cos\theta)}{2}}{\frac{2\sin^2\theta}{2}},$$

{Since, $1 - \cos\theta = \frac{2\sin^2\theta}{2}$ }

$$= \frac{dy}{dx} = \frac{\tan\theta}{2}$$

3. x = a cos θ and y = b sin θ

Solution:

Given x = a cos θ and y = b sin θ

Given x = a cos θ and y = b sin θ

Now by differentiating x with respect to θ we get,

$$\frac{dx}{d\theta} = \frac{d(a\cos\theta)}{d\theta} = -a\sin\theta$$

Again by differentiating y with respect to θ we get,

$$\frac{dy}{d\theta} = \frac{d(b\sin\theta)}{d\theta} = b\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b}{a}\cot\theta$$

4. x = a e^{θ} (sin θ – cos θ), y = a e^{θ} (sin θ + cos θ)

Solution:

Given that x = a e^{θ} (sin θ – cos θ)

Differentiating it with respect to θ

$$\frac{dx}{d\theta} = a[e^{\theta}\frac{d(\sin\theta - \cos\theta)}{d\theta} + (\sin\theta - \cos\theta)\frac{d(e^{\theta})}{d\theta}]$$

= a [e^{θ} (cos θ + sin θ) + (sin θ-cos θ) e^{θ}]

$\frac{dx}{d\theta} = a[2e^{\theta}\sin\theta]$ …… (1)

And also given that, y = a e^{θ} (sin θ + cos θ)

Differentiating it with respect to θ,

$$\frac{dy}{d\theta} = a[e^{\theta}\frac{d(\sin\theta + \cos\theta)}{d\theta} + (\sin\theta + \cos\theta)\frac{d(e^{\theta})}{d\theta}]$$

= a [e^{θ} (cos θ - sin θ) + (sin θ + cos θ) e^{θ}]

$\frac{dy}{d\theta} = a[2e^{\theta}\cos\theta]$ …… (2)

Dividing equation (2) by equation (1),

$$\frac{dy}{dx} = \frac{a(2e^{\theta}\cos\theta)}{a(2e^{\theta}\sin\theta)}$$

$$\frac{dy}{dx} = \frac{a(2e^{\theta}\cos\theta)}{a(2e^{\theta}\sin\theta)}$$

$$\frac{dy}{dx} = \cot\theta$$

5. x = b $\sin^2$ θ and y = a $\cos^2$ θ

Solution:

Given that $x = b \sin^2 \theta$

Now by differentiating above equation with respect to θ, we get

$$\frac{dx}{d\theta} = \frac{d(b\sin^2\theta)}{d\theta} = 2b\sin\theta\cos\theta$$

And also given that $y = a \cos^2 \theta$

Now by differentiating above equation with respect to θ, we get

$$\frac{dy}{d\theta} = d(a\cos^2\theta) = -2a\cos\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{2a\cos\theta\sin\theta}{2b\sin\theta\cos\theta} = -\frac{a}{b}$$

6. x = a (1 – cos θ) and y = a (θ + sin θ) at θ = π/2

Solution:

Given x = a (1 – cos θ)

Differentiate x with respect to θ, we get

$$\frac{dx}{d\theta} = \frac{d[a(1-\cos\theta)]}{d\theta} = a(\sin\theta)$$

And also given that y = a (θ + sin θ)

Differentiate x with respect to θ, we get

$$\frac{dy}{d\theta} = \frac{d(\theta+\sin\theta)}{d\theta} = a(1+\cos\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1+\cos\theta)}{a(\sin\theta)}\Bigg|\left(\theta = \frac{\pi}{2}\right)$$

$$= \frac{a(1+0)}{a} = 1$$

$$\frac{dy}{d\theta} = \frac{d(\theta+\sin\theta)}{d\theta} = a(1+\cos\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1+\cos\theta)}{a(\sin\theta)}\Bigg|\left(\theta = \frac{\pi}{2}\right)$$

$$= \frac{a(1+0)}{a} = 1$$

$$7.\ x = \frac{e^t + e^{-t}}{2}\ and\ y = \frac{e^t - e^{-t}}{2}$$

Solution:

Given $x=\frac{e^{t}+e^{-t}}{2}$

Differentiating above equation with respect to t

$$\frac{dx}{dt}=\frac{1}{2}\left[\frac{d(e^{t})}{dt}+\frac{d(e^{-t})}{dt}\right]$$

$$=\frac{1}{2}\left[e^{t}+e^{-t}\frac{d(-t)}{dt}\right]$$

$\frac{dx}{dt}=\frac{1}{2}(e^{t}-e^{-t})=y$ ...... (1)

And also given that $y=\frac{e^{t}-e^{-t}}{2}$

Differentiating above equation with respect to t,

$$\frac{dy}{dt}=\frac{1}{2}\left[\frac{d(e^{t})}{dt}-\frac{d(e^{-t})}{dt}\right]$$

$$=\frac{1}{2}\left[e^{t-}-e^{-t}\frac{d(-t)}{dt}\right]$$

$$=\frac{1}{2}\left(e^{t}-e^{-t}(-1)\right)$$

$$= \frac{1}{2}\left(e^{t} - e^{-t}(-1)\right)$$

$$\frac{dy}{dt} = \frac{e^{\theta}+e^{\theta}}{2} = x \text{ ...... } (2)$$

Dividing equation (2) by (1),

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$8.\ x = \frac{3at}{1+t^2} \ and\ y = \frac{3at^2}{1+t^2}$$

Solution:

Given $x = \frac{3at}{1+t^2}$

Differentiating above equation with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{\left((1+t^2)\frac{d(3at)}{dt} - 3at\frac{d(1+t^2)}{dt}\right)}{(1+t^2)^2}\right]$$

$$= \left[\frac{(1+t^2)(3a) - 3at(2t)}{(1+t^2)^2}\right]$$

$$= \left[\frac{(3a) + 3at^2 - 6at^2}{(1+t^2)^2}\right]$$

$$= \left[\frac{3a - 3at^2}{(1+t^2)^2}\right]$$

$\frac{dx}{dt} = \frac{3a(1-t^2)}{(1+t^2)^2}$ ...... (1)

And also given that $y = \frac{3at^2}{1+t^2}$

And also given that $y = \frac{3at^2}{1+t^2}$

Differentiating above equation with respect to t using quotient rule

$$\frac{dy}{dx} = \left[\frac{(1+t^2)\frac{d(3at^2)}{dt} - 3at^2\frac{d(1+t^2)}{dt}}{(1+t^2)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{(1+t^2)(6at) - (3at^2)(2t)}{(1+t^2)^2}\right]$$

$$= \left[\frac{6at + 6at^3 - 6at^3}{(1+t^2)^2}\right]$$

$\frac{dy}{dt} = \frac{6at}{(1+t^2)^2}$ ..... (2)

Dividing equation (2) by (1),

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at}{(1+t^2)^2} \times \frac{3a(1-t^2)}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{2t}{1-t^2}$$

9. x = a (cos θ + θ sin θ) and y = a (sin θ – θ cos θ)

Solution:

Given x = a (cos θ + θ sin θ)

Now differentiating x with respect to θ

$$\frac{dx}{d\theta} = a\left[\frac{d}{d\theta}\cos\theta + \frac{d}{d\theta}(\theta\sin\theta)\right]$$

$$= a\left[-\sin\theta + \frac{\theta d}{d\theta}(\sin\theta) + \sin\theta\frac{d}{d\theta}(\theta)\right]$$

$$= a[-\sin\theta + \theta\cos\theta + \sin\theta] = a\theta\cos\theta$$

And also given y = a (sin θ – cos θ),

Now differentiating x with respect to θ

Now differentiating x with respect to θ

$$\frac{dy}{d\theta} = a\left[\frac{d}{d\theta}(\sin\theta) - \frac{d}{d\theta}(\theta\cos\theta)\right]$$

$$= a\left[\cos\theta - \left\{\frac{\theta d}{d\theta}(\cos\theta) + \cos\theta\frac{d}{d\theta}(\theta)\right\}\right]$$

$$= a[\cos\theta + \theta\sin\theta - \cos\theta]$$

= a θ sin θ

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

$$10.\ x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)\ and\ y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$$

Solution:

Given $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$

Differentiating x with respect to θ using the product rule,

$$\frac{dx}{d\theta} = e^{\theta}\frac{d}{d\theta}\left(\theta + \frac{1}{\theta}\right) + \left(\theta + \frac{1}{\theta}\right)\frac{d}{d\theta}(e^{\theta})$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2}\right) + \frac{\theta^2 + 1}{\theta}(e^{\theta})$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta}\right)$$

$$= e^{\theta}\left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2}\right)$$

$$\frac{dx}{d\theta} = e^{\theta}\left(\frac{\theta^3+\theta^2+\theta-1}{\theta^2}\right) \text{ ...... (1)}$$

And also given that, $y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$

Differentiating y with respect to θ using the product rule,

$$\frac{dy}{d\theta} = e^{-\theta}\frac{d}{d\theta}\left(\theta - \frac{1}{\theta}\right) + \left(\theta - \frac{1}{\theta}\right)\frac{d}{d\theta}(e^{-\theta})$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}\frac{d}{d\theta}(-\theta)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}(-1)$$

$$\frac{dy}{d\theta} = e^{-\theta}\left(1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta}\right)$$

$$= e^{-\theta}\left(\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2}\right)$$

$$\frac{dy}{d\theta} = e^{-\theta}\left(\frac{-\theta^3+\theta^2+\theta+1}{\theta^2}\right) \text{ ...... (2)}$$

Divide equation (2) by (1)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = e^{-\theta}\left(\frac{-\theta^3 + \theta^2 + \theta + 1}{\theta^2}\right) \times \frac{1}{e^{\theta}\left(\frac{\theta^3 + \theta^2 + \theta - 1}{\theta^2}\right)}$$

$$= e^{-2\theta}\left(\frac{-\theta^3 + \theta^2 + \theta + 1}{\theta^3 + \theta^2 + \theta - 1}\right)$$

$$11.\ x = \frac{2t}{1+t^2}\ and\ y = \frac{1-t^2}{1+t^2}$$

Solution:

Given, $x = \frac{2t}{1+t^2}$

Differentiating x with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{(1+t^2)\frac{d}{dt}(2t) - 2t\frac{d}{dt}(1+t^2)}{(1+t^2)^2}\right]$$

$$= \left[\frac{(1+t^2)(2) - 2t(2t)}{(1+t^2)^2}\right]$$

$$= \left[\frac{2+2t^2-4t^2}{(1+t^2)^2}\right]$$

$$= \left[\frac{2-2t^2}{(1+t^2)^2}\right]$$

$\frac{dx}{dt} = \left[\frac{2-2t^2}{(1+t^2)^2}\right]$ ...... (1)

And also given that, $y = \frac{1-t^2}{1+t^2}$

Differentiating y with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{(1+t^2)\frac{d}{dt}(1-t^2) - (1-t^2)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}\right]$$

$$= \left[\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}\right]$$

$$= \left[\frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}\right]$$

$\frac{dy}{dt} = \left[\frac{-4t}{(1+t^2)^2}\right]$ ...... (2)

Dividing equation (2) by (1),

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \left[\frac{-4t}{(1+t^2)^2}\right] \times \frac{1}{\left[\frac{2-2t^2}{(1+t^2)^2}\right]}$$

$$= -\frac{2t}{1-t^2}$$

$$\frac{dy}{dx} = -\frac{x}{y} \text{ [since, } \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2} \text{]}$$

$$\frac{dy}{dx} = -\frac{x}{y} \text{ [since, } \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2} \text{]}$$

12. $x = cos^{-1}\frac{1}{\sqrt{1+t^2}}$ *and* $y = sin^{-1}\frac{1}{\sqrt{1+t^2}},\ t\epsilon R$

Solution:

Given $x=\cos^{-1}\frac{1}{\sqrt{1+t^2}}$

Differentiating x with respect to t using chain rule,

$$\frac{dx}{dt}=-\frac{1}{\sqrt{1-\left(\frac{1}{\sqrt{1+t^2}}\right)^2}}\frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$=-\frac{1}{\sqrt{1-\frac{1}{1+t^2}}}\left\{-\frac{1}{2(1+t^2)^{\frac{3}{2}}}\right\}\frac{d}{dt}(1+t^2)$$

$$=-\frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{(1+t^2-1)}}\left\{-\frac{1}{2(1+t^2)^{\frac{3}{2}}}\right\}(2t)$$

$$=-\frac{t}{\sqrt{t^2}\times(1+t^2)}$$

$\frac{dx}{dt}=-\frac{1}{1+t^2}$ ...... (1)

Also given that, $y=\sin^{-1}\frac{1}{\sqrt{1+t^2}}$

Differentiating y with respect to t using chain rule,

$$\frac{dy}{dt}=\frac{1}{\sqrt{1-\left(\frac{1}{\sqrt{1+t^2}}\right)^2}}\frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{1}{\sqrt{1 - \frac{1}{1+t^2}}} \left\{ -\frac{1}{2(1+t^2)^{\frac{3}{2}}} \right\} \frac{d}{dt}(1+t^2)$$

$$= \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{(1+t^2-1)}} \left\{ -\frac{1}{2(1+t^2)^{\frac{3}{2}}} \right\} (2t)$$

$$= \frac{t}{\sqrt{t^2} \times (1+t^2)}$$

$\frac{dy}{dt} = -\frac{1}{1+t^2}$ ...... (2)

Dividing equation (2) by (1),

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{1}{1+t^2} \times -\frac{1+t^2}{1}$$

$$\frac{dy}{dx} = 1$$

$$13.\ x = \frac{1-t^2}{1+t^2}\ and\ y = \frac{2t}{1+t^2}$$

Solution:

Given $x = \frac{1-t^2}{1+t^2}$

Differentiating x with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{(1+t^2)\frac{d}{dt}(1-t^2) - (1-t^2)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}\right]$$

$$= \left[\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}\right]$$

$$= \left[\frac{-2t - 2t^3 - 2t + 2t^3}{(1+t^2)^2}\right]$$

$\frac{dx}{dt} = \left[\frac{-4t}{(1+t^2)^2}\right]$ ...... (1)

And also given that, $y = \frac{2t}{1+t^2}$

Differentiating y with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{(1+t^2)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}\right]$$

$$= \left[\frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}\right]$$

$$= \left[\frac{2 + 2t^2 - 4t^2}{(1+t^2)^2}\right]$$

$\frac{dy}{dt} = \frac{2(1-t^2)}{(1+t^2)^2}$ ...... (2)

Divide equation (2) by (1) so,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(1-t^2)}{(1+t^2)^2} \times \frac{1}{\frac{-4t}{(1+t^2)^2}}$$

$$\frac{dy}{dx} = \frac{2(1-t^2)}{-4t}$$

$\mathbf{14.}\ \boldsymbol{If\ x = 2\cos\theta - \cos 2\theta\ and\ y = 2\sin\theta - sin2\theta,\ prove\ that\ \frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right).}$

Solution:

Given x = 2cos θ – cos 2θ

Differentiating x with respect to θ using chain rule,

$$\frac{dx}{d\theta} = 2(-\sin\theta) - (-\sin 2\theta)\frac{d}{d\theta}(2\theta)$$

$$= -2\sin\theta + 2\sin 2\theta$$

$$\frac{dx}{d\theta} = 2(-\sin\theta) - (-\sin 2\theta)\frac{d}{d\theta}(2\theta)$$

$$= -2\sin\theta + 2\sin 2\theta$$

$\frac{dx}{d\theta} = 2(\sin 2\theta - \sin\theta)$ ...... (1)

And also given that, y = 2sin θ – sin 2θ

Differentiating y with respect to θ using chain rule,

$$\frac{dy}{d\theta} = 2\cos\theta - \cos 2\theta\frac{d}{d\theta}(2\theta)$$

$$= 2\cos\theta - \cos 2\theta(2)$$

$$= 2\cos\theta - 2\cos 2\theta$$

$\frac{dy}{d\theta} = 2(\cos\theta - \cos 2\theta)$ ...... (2)

Dividing equation (2) by equation (1),

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)}$$

$$= \frac{(\cos\theta - \cos 2\theta)}{(\sin 2\theta - \sin\theta)}$$

$$\frac{dy}{dx} = \frac{-2\sin\left(\frac{\theta + 2\theta}{2}\right)\sin\left(\frac{\theta - 2\theta}{2}\right)}{2\cos\left(\frac{\theta + 2\theta}{2}\right)\sin\left(\frac{2\theta - \theta}{2}\right)}$$

$$\left[\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)\right]$$

$$= -\frac{\sin\left(\frac{3\theta}{2}\right)\left(\sin\left(-\frac{\theta}{2}\right)\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$= -\frac{\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$= -\frac{\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$= \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)}$$

$$= \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)}$$

$$\frac{dy}{dx} = \tan(\frac{3\theta}{2})$$

$$\frac{dy}{dx} = \tan(\frac{3\theta}{2})$$

Exercise 11.8 Page No: 11.112

1. Differentiate x^2 with respect to x^3.

Solution:

Let u = x^2 and v = x^3.

We have to differentiate u with respect to v that is find $\frac{du}{dv}$.

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{du}{dx} = 2x^{2-1}$$

$$\therefore \frac{du}{dx} = 2x$$

Now, on differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(x^3)$$

$$\Rightarrow \frac{dv}{dx} = 3x^{3-1}$$

$$\therefore \frac{dv}{dx} = 3x^2$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{2x}{3x^2}$$

$$\therefore \frac{du}{dv} = \frac{2}{3x}$$

Thus, $\frac{du}{dv} = \frac{2}{3x}$

Thus, $\frac{du}{dv} = \frac{2}{3x}$

2. Differentiate log (1 +x^2) with respect to $\tan^{-1}$ x.

Solution:

Let u = log (1 + x^2) and v = $\tan^{-1}x$.

We have to differentiate u with respect to v that is find $\frac{du}{dv}$.

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}[\log(1 + x^2)]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

$$\Rightarrow \frac{du}{dx} = \frac{1}{1+x^2}\frac{d}{dx}(1 + x^2)$$

Now by using chain rule, we get

$$\Rightarrow \frac{du}{dx} = \frac{1}{1 + x^2}\left[\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right]$$

However, $\frac{d}{dx}(x^n) = nx^{n-1}$ and derivative of a constant is 0.

$$\Rightarrow \frac{du}{dx} = \frac{1}{1 + x^2}[0 + 2x^{2-1}]$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{1 + x^2}[2x]$$

$$\therefore \frac{du}{dx} = \frac{2x}{1 + x^2}$$

Now, on differentiating v with respect to x, we get

$$\therefore \frac{dv}{dx} = \frac{1}{1 + x^2}$$

We have $\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv}=\frac{\frac{2x}{1+x^2}}{\frac{1}{1+x^2}}$$

$$\Rightarrow \frac{du}{dv}=\frac{2x}{1+x^2}\times(1+x^2)$$

$$\therefore \frac{du}{dv}=2x$$

Thus, $\frac{du}{dv}=2x$

3. Differentiate (log x)^x with respect to log x.

Solution:

Let u = $(\log x)^x$ and v = log x.

We need to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have u = $(\log x)^x$

Taking log on both sides, we get

Log u = log $(\log x)^x$

⇒ Log u = x × log (log x) [∵ log a^m = m × log a]

On differentiating both the sides with respect to x, we get

$$\frac{d}{du}(\log u) \times \frac{du}{dx} = \frac{d}{dx}[x \times \log(\log x)]$$

We know that (u v)' = vu' + uv'

$$\Rightarrow \frac{d}{du}(\log u) \times \frac{du}{dx} = \log(\log x)\frac{d}{dx}(x) + x\frac{d}{dx}[\log(\log x)]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$ and $\frac{d}{dx}(x) = 1$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$ and $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{1}{u} \times \frac{du}{dx} = \log(\log x) \times 1 + x\left[\frac{1}{\log x}\frac{d}{dx}(\log x)\right]$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \log(\log x) + \frac{x}{\log x}\frac{d}{dx}(\log x)$$

But, u = (log x)^x and $\frac{d}{dx}(\log x) = \frac{1}{x}$

$$\Rightarrow \frac{1}{(\log x)^x}\frac{du}{dx} = \log(\log x) + \frac{x}{\log x} \times \frac{1}{x}$$

$$\Rightarrow \frac{1}{(\log x)^x}\frac{du}{dx} = \log(\log x) + \frac{1}{\log x}$$

$$\therefore \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$$

Now, on differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(\log x)$$

$$\therefore \frac{dv}{dx} = \frac{1}{x}$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{(\log x)^x \left[\log(\log x) + \frac{1}{\log x}\right]}{\frac{1}{x}}$$

$$\Rightarrow \frac{du}{dv} = x(\log x)^x \left[\log(\log x) + \frac{1}{\log x}\right]$$

$$\Rightarrow \frac{du}{dv} = x(\log x)^x \left[\frac{\log(\log x)\log x + 1}{\log x}\right]$$

$$\Rightarrow \frac{du}{dv} = \frac{x(\log x)^x}{\log x}[\log(\log x)\log x + 1]$$

$$\therefore \frac{du}{dv} = x(\log x)^{x-1}[1 + \log x \log(\log x)]$$

Thus, $\frac{du}{dv} = x(\log x)^{x-1}[1 + \log x \log(\log x)]$

4. Differentiate $\sin^{-1}\sqrt{(1-x^2)}$ with respect to $\cos^{-1}x$, if

(i) $x \in (0, 1)$

(ii) $x \in (-1, 0)$

Solution:

(i) Given $\sin^{-1}\sqrt{(1-x^2)}$

Let $u = \sin^{-1}\sqrt{1-x^2}$ and v = $\cos^{-1}$x.

We need to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \sin^{-1}\sqrt{1-x^2}$

By substituting x = cos θ, we have

$u = \sin^{-1}\sqrt{1-(\cos\theta)^2}$

$\Rightarrow u = \sin^{-1}\sqrt{1-\cos^2\theta}$

$\Rightarrow u = \sin^{-1}\sqrt{\sin^2\theta}$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

⇒ u = $\sin^{-1}$(sin θ)

Given x ϵ (0, 1)

However, x = cos θ.

⇒ Cos θ ϵ (0, 1)

$\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

Hence, u = $\sin^{-1}$(sin θ) = θ.

$\Rightarrow$ u = $\cos^{-1}x$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}(\cos^{-1}x)$$

We know $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

$$\therefore \frac{du}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

Now, on differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(\cos^{-1}x)$$

$$\therefore \frac{dv}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

We have, $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{-\frac{1}{\sqrt{1-x^2}}}{-\frac{1}{\sqrt{1-x^2}}}$$

$$\Rightarrow \frac{du}{dv} = -\frac{1}{\sqrt{1-x^2}} \times \left(-\sqrt{1-x^2}\right)$$

$$\therefore \frac{du}{dv} = 1$$

Thus, $\frac{du}{dv} = 1$

(ii) Given $\sin^{-1}\sqrt{(1-x^2)}$

Let $u = \sin^{-1}\sqrt{1-x^2}$ and v = $\cos^{-1}x$.

Now we have to differentiate u with respect to v that is find $\frac{du}{dv}$.

Now we have to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \sin^{-1}\sqrt{1-x^2}$

By substituting x = cos θ, we get

$u = \sin^{-1}\sqrt{1-(\cos\theta)^2}$

$\Rightarrow u = \sin^{-1}\sqrt{1-\cos^2\theta}$

$\Rightarrow u = \sin^{-1}\sqrt{\sin^2\theta}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow u = \sin^{-1}(\sin\theta)$

Given x ϵ (–1, 0)

However, x = cos θ.

$\Rightarrow$ Cos θ ϵ (–1, 0)

$\Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right)$

Hence, $u = \sin^{-1}(\sin\theta) = \pi - \theta$.

$\Rightarrow u = \pi - \cos^{-1}x$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}(\pi - \cos^{-1}x)$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\pi) - \frac{d}{dx}(\cos^{-1}x)$$

We know $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$ and derivative of a constant is 0.

$$\Rightarrow \frac{du}{dx} = 0 - \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\therefore \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Now, on differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(\cos^{-1}x)$$

Now, on differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(\cos^{-1} x)$$

$$\therefore \frac{dv}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{\frac{1}{\sqrt{1-x^2}}}{-\frac{1}{\sqrt{1-x^2}}}$$

$$\Rightarrow \frac{du}{dv} = \frac{1}{\sqrt{1-x^2}} \times \left(-\sqrt{1-x^2}\right)$$

$$\therefore \frac{du}{dv} = -1$$

Thus, $\frac{du}{dv} = -1$

5. $Differentiate\ \sin^{-1}\left(4x\sqrt{1-4x^2}\right)\ with\ respect\ to\ \sqrt{1-4x^2}\ if,$

$(i)\ x\epsilon\left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)\ (ii)\ x\epsilon\left(\frac{1}{2\sqrt{2}}, \frac{1}{2}\right)\ (iii)\ x\epsilon\left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

Solution:

(i) Let

$u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$ And $v = \sqrt{1-4x^2}$.

We need to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

$\Rightarrow u = \sin^{-1}\left(4x\sqrt{1-(2x)^2}\right)$

By substituting 2x = cos θ, we have

$u = \sin^{-1}\left(2\cos\theta\sqrt{1-(\cos\theta)^2}\right)$

$\Rightarrow u = \sin^{-1}\left(2\cos\theta\sqrt{1-(\cos\theta)^2}\right)$

$\Rightarrow u = \sin^{-1}\left(2\cos\theta\sqrt{\sin^2\theta}\right)$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow u = \sin^{-1}(2\cos\theta\sin\theta)$

$\Rightarrow u = \sin^{-1}(\sin 2\theta)$

Given $x \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$

However, 2x = cos θ ⇒ $x = \frac{\cos\theta}{2}$

$\Rightarrow \frac{\cos\theta}{2} \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$

$\Rightarrow \cos\theta \in \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\Rightarrow 2\theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, $u = \sin^{-1}(\sin 2\theta) = \pi - 2\theta$.

$\Rightarrow u = \pi - 2\cos^{-1}(2x)$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}[\pi - 2\cos^{-1}(2x)]$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\pi) - \frac{d}{dx}[2\cos^{-1}(2x)]$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\pi) - \frac{d}{dx}[2\cos^{-1}(2x)]$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\pi) - 2\frac{d}{dx}[\cos^{-1}(2x)]$$

We know $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$ and derivative of a constant is 0.

$$\Rightarrow \frac{du}{dx} = 0 - 2\left[-\frac{1}{\sqrt{1-(2x)^2}}\frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-4x^2}}\left[\frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-4x^2}}\left[2\frac{d}{dx}(x)\right]$$

$$\Rightarrow \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}}\frac{d}{dx}(x)$$

However, $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \times 1$$

$$\therefore \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}}$$

Now, we have $v = \sqrt{1-4x^2}$

On differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}\left(\sqrt{1-4x^2}\right)$$

$$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(1-4x^2)^{\frac{1}{2}}$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2}(1-4x^2)^{\frac{1}{2}-1}\frac{d}{dx}(1-4x^2)$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2}(1-4x^2)^{\frac{1}{2}-1}\frac{d}{dx}(1-4x^2)$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2}(1-4x^2)^{-\frac{1}{2}}\left[\frac{d}{dx}(1) - \frac{d}{dx}(4x^2)\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}}\left[\frac{d}{dx}(1) - 4\frac{d}{dx}(x^2)\right]$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$ and derivative of a constant is 0.

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}}[0 - 4(2x^{2-1})]$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}}[-8x]$$

$$\therefore \frac{dv}{dx} = -\frac{4x}{\sqrt{1-4x^2}}$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{\frac{4}{\sqrt{1-4x^2}}}{-\frac{4x}{\sqrt{1-4x^2}}}$$

$$\Rightarrow \frac{du}{dv} = \frac{4}{\sqrt{1-4x^2}} \times \left(-\frac{\sqrt{1-x^2}}{4x}\right)$$

$$\therefore \frac{du}{dv} = -\frac{1}{x}$$

Thus, $\frac{du}{dv} = -\frac{1}{x}$

(ii) Let

Let $u = \sin^{-1}(4x\sqrt{1-4x^2})$ and $v = \sqrt{1-4x^2}$.

We need to differentiate u with respect to v that is find $\frac{du}{dv}$.

We need to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \sin^{-1}(4x\sqrt{1-4x^2})$

$\Rightarrow u = \sin^{-1}\left(4x\sqrt{1-(2x)^2}\right)$

By substituting 2x = cos θ, we have

$u = \sin^{-1}\left(2\cos\theta\sqrt{1-(\cos\theta)^2}\right)$

$\Rightarrow u = \sin^{-1}\left(2\cos\theta\sqrt{1-(\cos\theta)^2}\right)$

$\Rightarrow u = \sin^{-1}(2\cos\theta\sqrt{\sin^2\theta})$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow u = \sin^{-1}(2\cos\theta\sin\theta)$

$\Rightarrow u = \sin^{-1}(\sin 2\theta)$

Given $x \in \left(\frac{1}{2\sqrt{2}}, \frac{1}{2}\right)$

However, 2x = cos θ ⇒ $x = \frac{\cos\theta}{2}$

$\Rightarrow \frac{\cos\theta}{2} \in \left(\frac{1}{2\sqrt{2}}, \frac{1}{2}\right)$

$\Rightarrow \cos\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right)$

Hence, $u = \sin^{-1}(\sin 2\theta) = 2\theta$.

$\Rightarrow u = 2\cos^{-1}(2x)$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}[2\cos^{-1}(2x)]$$

$$\Rightarrow \frac{du}{dx} = 2\frac{d}{dx}[\cos^{-1}(2x)]$$

We know $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$ and derivative of a constant is 0.

$$\Rightarrow \frac{du}{dx} = 2\left[-\frac{1}{\sqrt{1-(2x)^2}}\frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{du}{dx} = -\frac{2}{\sqrt{1-4x^2}}\left[\frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{du}{dx} = -\frac{2}{\sqrt{1-4x^2}}\left[2\frac{d}{dx}(x)\right]$$

$$\Rightarrow \frac{du}{dx} = -\frac{4}{\sqrt{1-4x^2}}\frac{d}{dx}(x)$$

However, $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{du}{dx} = -\frac{4}{\sqrt{1-4x^2}} \times 1$$

$$\therefore \frac{du}{dx} = -\frac{4}{\sqrt{1-4x^2}}$$

We have $\frac{dv}{dx} = -\frac{4x}{\sqrt{1-4x^2}}$

We know that $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{-\frac{4}{\sqrt{1-4x^2}}}{-\frac{4x}{\sqrt{1-4x^2}}}$$

$$\Rightarrow \frac{du}{dv} = -\frac{4}{\sqrt{1-4x^2}} \times \left(-\frac{\sqrt{1-x^2}}{4x}\right)$$

$$\therefore \frac{du}{dv} = \frac{1}{x}$$

Thus, $\frac{du}{dv} = \frac{1}{x}$

$$\Rightarrow \frac{du}{dv} = -\frac{4}{\sqrt{1-4x^2}} \times \left(-\frac{\sqrt{1-x^2}}{4x}\right)$$

$$\therefore \frac{du}{dv} = \frac{1}{x}$$

Thus, $\frac{du}{dv} = \frac{1}{x}$

(iii) Let

$u = \sin^{-1}(4x\sqrt{1-4x^2})$ And $v = \sqrt{1-4x^2}$.

We need to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \sin^{-1}(4x\sqrt{1-4x^2})$

$\Rightarrow u = \sin^{-1}\left(4x\sqrt{1-(2x)^2}\right)$

By substituting 2x = cos θ, we have

$u = \sin^{-1}\left(2\cos\theta\sqrt{1-(\cos\theta)^2}\right)$

$\Rightarrow u = \sin^{-1}\left(2\cos\theta\sqrt{1-(\cos\theta)^2}\right)$

$\Rightarrow u = \sin^{-1}(2\cos\theta\sqrt{\sin^2\theta})$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow u = \sin^{-1}(2\cos\theta\sin\theta)$

$\Rightarrow u = \sin^{-1}(\sin 2\theta)$

Given $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

However, 2x = cos θ ⇒ $x = \frac{\cos\theta}{2}$

$\Rightarrow \frac{\cos\theta}{2} \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$\Rightarrow \cos\theta \in \left(-1, -\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \cos\theta \in \left(-1, -\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \theta \in \left(\frac{3\pi}{4}, \pi\right)$

$\Rightarrow 2\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

Hence, $u = \sin^{-1}(\sin 2\theta) = 2\pi - 2\theta$.

$\Rightarrow u = 2\pi - 2\cos^{-1}(2x)$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}[2\pi - 2\cos^{-1}(2x)]$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(2\pi) - \frac{d}{dx}[2\cos^{-1}(2x)]$$

$$\Rightarrow \frac{du}{dx} = 2\frac{d}{dx}(\pi) - 2\frac{d}{dx}[\cos^{-1}(2x)]$$

We know $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$ and derivative of a constant is 0.

$$\Rightarrow \frac{du}{dx} = 0 - 2\left[-\frac{1}{\sqrt{1-(2x)^2}}\frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-4x^2}}\left[\frac{d}{dx}(2x)\right]$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-4x^2}}\left[2\frac{d}{dx}(x)\right]$$

$$\Rightarrow \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}}\frac{d}{dx}(x)$$

However, $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \times 1$$

$$\therefore \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}}$$

$$\therefore \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}}$$

We have $\frac{dv}{dx} = -\frac{4x}{\sqrt{1-4x^2}}$

We know that $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{\frac{4}{\sqrt{1-4x^2}}}{-\frac{4x}{\sqrt{1-4x^2}}}$$

$$\Rightarrow \frac{du}{dv} = \frac{4}{\sqrt{1-4x^2}} \times \left(-\frac{\sqrt{1-x^2}}{4x}\right)$$

$$\therefore \frac{du}{dv} = -\frac{1}{x}$$

Thus, $\frac{du}{dv} = -\frac{1}{x}$

6. *Differentiate* $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ *with respect to* $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, *if* $-1 < x < 1,\ x \neq 0.$

Solution:

Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

We need to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

By substituting x = tan θ, we have

$$u = \tan^{-1}\left(\frac{\sqrt{1+(\tan\theta)^2}-1}{\tan\theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{\sqrt{\sec^2\theta-1}}{\tan\theta}\right)$$ [∵ $\sec^2\theta - \tan^2\theta = 1$]

$$\Rightarrow u = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{1-\cos\left(2\times\frac{\theta}{2}\right)}{\sin\left(2\times\frac{\theta}{2}\right)}\right)$$

But, $\cos 2\theta = 1 - 2\sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$.

$$\Rightarrow u = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$$

$\Rightarrow u = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

Given −1 < x < 1 ⇒ x ϵ (−1, 1)

However, x = tan θ

⇒ Tan θ ϵ (−1, 1)

$\Rightarrow \theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\Rightarrow \frac{\theta}{2} \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right)$

Hence, $u = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$

$\Rightarrow \frac{\theta}{2} \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right)$

Hence, $u = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$

$\Rightarrow u = \frac{1}{2}\tan^{-1} x$

On differentiating u with respect to x, we get

$\frac{du}{dx} = \frac{d}{dx}\left(\frac{1}{2}\tan^{-1} x\right)$

$\Rightarrow \frac{du}{dx} = \frac{1}{2}\frac{d}{dx}(\tan^{-1} x)$

We know $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\Rightarrow \frac{du}{dx} = \frac{1}{2} \times \frac{1}{1 + x^2}$

$\therefore \frac{du}{dx} = \frac{1}{2(1 + x^2)}$

Now, we have $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

By substituting x = tan θ, we have

$v = \sin^{-1}\left(\frac{2\tan\theta}{1 + (\tan\theta)^2}\right)$

$$\Rightarrow v = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$\Rightarrow v = \sin^{-1}\left(\frac{2\tan\theta}{\sec^2\theta}\right)$$ [∵ $\sec^2\theta - \tan^2\theta = 1$]

$$\Rightarrow v = \sin^{-1}\left(\frac{2\times\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}\right)$$

$$\Rightarrow v = \sin^{-1}\left(2\times\frac{\sin\theta}{\cos\theta}\times\cos^2\theta\right)$$

$$\Rightarrow v = \sin^{-1}\left(2 \times \frac{\sin\theta}{\cos\theta} \times \cos^2\theta\right)$$

⇒ v = $\sin^{-1}(2\sin\theta\cos\theta)$

But, $\sin 2\theta = 2\sin\theta\cos\theta$

⇒ v = $\sin^{-1}(\sin 2\theta)$

However, $\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \Rightarrow 2\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Hence, v = $\sin^{-1}(\sin 2\theta) = 2\theta$

⇒ v = $2\tan^{-1}x$

On differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(2\tan^{-1}x)$$

$$\Rightarrow \frac{dv}{dx} = 2\frac{d}{dx}(\tan^{-1}x)$$

We know $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$$\Rightarrow \frac{dv}{dx} = 2 \times \frac{1}{1+x^2}$$

$$\therefore \frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\therefore \frac{du}{dv} = \frac{1}{4}$$

Thus, $\frac{du}{dv} = \frac{1}{4}$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{\frac{1}{2(1+x^2)}}{\frac{2}{1+x^2}}$$

$$\Rightarrow \frac{du}{dv} = \frac{1}{2(1+x^2)} \times \frac{1+x^2}{2}$$

$$\therefore \frac{du}{dv} = \frac{1}{4}$$

Thus, $\frac{du}{dv} = \frac{1}{4}$

7. $\boldsymbol{Differentiate \sin^{-1}\left(2x\sqrt{1-x^2}\right)\ with\ respect\ to \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right),\ if,}$

(i) x ∈ (0, 1/ √2)

(ii) x ∈ (1/√2, 1)

Solution:

(i) Let

$u = \sin^{-1}(2x\sqrt{1-x^2})$ And $v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$.

We have to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \sin^{-1}(2x\sqrt{1-x^2})$

By substituting x = sin θ, we have

$u = \sin^{-1}\left(2\sin\theta\sqrt{1-(\sin\theta)^2}\right)$

$\Rightarrow u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$\Rightarrow u = \sin^{-1}(2\sin\theta\sqrt{\cos^2\theta})$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow u = \sin^{-1}(2\sin\theta\cos\theta)$

$\Rightarrow u = \sin^{-1}(\sin 2\theta)$

Now, we have $v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

By substituting x = sin θ, we have

$v = \sec^{-1}\left(\frac{1}{\sqrt{1-(\sin\theta)^2}}\right)$

$\Rightarrow v = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$

$$\Rightarrow v = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$

$$\Rightarrow v = \sec^{-1}\left(\frac{1}{\sqrt{\cos^2\theta}}\right)$$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

$$\Rightarrow v = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$

$$\Rightarrow v = \sec^{-1}(\sec\theta)$$

Given $x \in \left(0, \frac{1}{\sqrt{2}}\right)$

However, x = sin θ

$$\Rightarrow \sin\theta \in \left(0, \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right)$$

$$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right)$$

Hence, $u = \sin^{-1}(\sin 2\theta) = 2\theta$.

$\Rightarrow u = 2\sin^{-1}(x)$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}(2\sin^{-1}x)$$

$$\Rightarrow \frac{du}{dx} = 2\frac{d}{dx}(\sin^{-1}x)$$

We know $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$$\Rightarrow \frac{du}{dx} = 2 \times \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{du}{dx} = \frac{2}{\sqrt{1-x^2}}$$

We have $\theta \in \left(0, \frac{\pi}{4}\right)$

Hence, $v = \sec^{-1}(\sec\theta) = \theta$

We have $\theta \in \left(0, \frac{\pi}{4}\right)$

Hence, v = $\sec^{-1}(\sec\theta) = \theta$

$\Rightarrow v = \sin^{-1}x$

On differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(\sin^{-1}x)$$

We know $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$$\therefore \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{\frac{2}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}}$$

$$\Rightarrow \frac{du}{dv} = \frac{2}{\sqrt{1-x^2}} \times \sqrt{1-x^2}$$

$$\therefore \frac{du}{dv} = 2$$

Thus, $\frac{du}{dv} = 2$

(ii) Let

$u = \sin^{-1}(2x\sqrt{1-x^2})$ And $v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$.

We have to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \sin^{-1}(2x\sqrt{1-x^2})$

By substituting x = sin θ, we have

By substituting x = sin θ, we have

$u = \sin^{-1}\left(2\sin\theta\sqrt{1-(\sin\theta)^2}\right)$

$\Rightarrow u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$\Rightarrow u = \sin^{-1}(2\sin\theta\sqrt{\cos^2\theta})$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow u = \sin^{-1}(2\sin\theta\cos\theta)$

$\Rightarrow u = \sin^{-1}(\sin 2\theta)$

Now, we have $v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

By substituting x = sin θ, we have

$v = \sec^{-1}\left(\frac{1}{\sqrt{1-(\sin\theta)^2}}\right)$

$\Rightarrow v = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$

$\Rightarrow v = \sec^{-1}\left(\frac{1}{\sqrt{\cos^2\theta}}\right)$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow v = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$

$\Rightarrow v = \sec^{-1}(\sec\theta)$

Given $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$

However, x = sin θ

$\Rightarrow \sin\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$

$\Rightarrow \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\Rightarrow 2\theta \in \left(\frac{\pi}{2}, \pi\right)$

Hence, u = $\sin^{-1}(\sin 2\theta) = \pi - 2\theta$.

$\Rightarrow u = \pi - 2\sin^{-1}(x)$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}(\pi - 2\sin^{-1}x)$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\pi) - \frac{d}{dx}(2\sin^{-1}x)$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(\pi) - 2\frac{d}{dx}(\sin^{-1}x)$$

We know $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$ and derivative of a constant is 0.

$$\Rightarrow \frac{du}{dx} = 0 - 2 \times \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{du}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

We have $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Hence, v = $\sec^{-1}(\sec \theta) = \theta$

$\Rightarrow v = \sin^{-1}x$

On differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(\sin^{-1} x)$$

We know $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$$\therefore \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{-\frac{2}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}}$$

$$\Rightarrow \frac{du}{dv} = -\frac{2}{\sqrt{1-x^2}} \times \sqrt{1-x^2}$$

$$\therefore \frac{du}{dv} = -2$$

Thus, $\frac{du}{dv} = -2$

8. Differentiate $(\cos x)^{\sin x}$ with respect to $(\sin x)^{\cos x}$.

Solution:

Let u = $(\cos x)^{\sin x}$ and v = $(\sin x)^{\cos x}$.

We have to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have u = $(\cos x)^{\sin x}$

Taking log on both sides, we get

Log u = log $(\cos x)^{\sin x}$

⇒ Log u = (sin x) × log (cos x) [∵ log a^m = m × log a]

On differentiating both the sides with respect to x, we get

$$\frac{d}{du}(\log u) \times \frac{du}{dx} = \frac{d}{dx}[\sin x \times \log(\cos x)]$$

We know that (uv)' = vu' + uv'

$$\Rightarrow \frac{d}{du}(\log u) \times \frac{du}{dx} = \log(\cos x)\frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}[\log(\cos x)]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$ and $\frac{d}{dx}(\sin x) = \cos x$

$$\Rightarrow \frac{1}{u} \times \frac{du}{dx} = \log(\cos x) \times \cos x + \sin x\left[\frac{1}{\cos x}\frac{d}{dx}(\cos x)\right]$$

$$\Rightarrow \frac{1}{u} \times \frac{du}{dx} = \log(\cos x) \times \cos x + \sin x \left[\frac{1}{\cos x} \frac{d}{dx}(\cos x)\right]$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \log(\cos x) + \frac{\sin x}{\cos x}\frac{d}{dx}(\cos x)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \log(\cos x) + \tan x \frac{d}{dx}(\cos x)$$

We know $\frac{d}{dx}(\cos x) = -\sin x$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \log(\cos x) + \tan x\,(-\sin x)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \log(\cos x) - \tan x \sin x$$

But, u = $(\cos x)^{\sin x}$

$$\Rightarrow \frac{1}{(\cos x)^{\sin x}}\frac{du}{dx} = \cos x \log(\cos x) - \tan x \sin x$$

$$\therefore \frac{du}{dx} = (\cos x)^{\sin x}[\cos x \log(\cos x) - \tan x \sin x]$$

Now, we have v = $(\sin x)^{\cos x}$

Taking log on both sides, we get

Log v = log $(\sin x)^{\cos x}$

⇒ Log v = (cos x) × log (sin x) [∵ log a^m = m × log a]

On differentiating both the sides with respect to x, we get

$$\frac{d}{dv}(\log v) \times \frac{dv}{dx} = \frac{d}{dx}[\cos x \times \log(\sin x)]$$

We know that (uv)' = vu' + uv' (product rule)

$$\Rightarrow \frac{d}{du}(\log u) \times \frac{dv}{dx} = \log(\sin x)\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}[\log(\sin x)]$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$ and $\frac{d}{dx}(\cos x) = -\sin x$

$$\Rightarrow \frac{1}{v} \times \frac{dv}{dx} = \log(\sin x) \times (-\sin x) + \cos x\left[\frac{1}{\sin x}\frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{1}{v} \times \frac{dv}{dx} = \log(\sin x) \times (-\sin x) + \cos x \left[\frac{1}{\sin x} \frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x \log(\sin x) + \frac{\cos x}{\sin x}\frac{d}{dx}(\sin x)$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x \log(\sin x) + \cot x \frac{d}{dx}(\sin x)$$

We know $\frac{d}{dx}(\sin x) = \cos x$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x \log(\sin x) + \cot x \times (\cos x)$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x \log(\sin x) + \cot x \cos x$$

But, $v = (\sin x)^{\cos x}$

$$\Rightarrow \frac{1}{(\sin x)^{\cos x}}\frac{dv}{dx} = -\sin x \log(\sin x) + \cot x \cos x$$

$$\therefore \frac{dv}{dx} = (\sin x)^{\cos x}[-\sin x \log(\sin x) + \cot x \cos x]$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{(\cos x)^{\sin x}[\cos x \log(\cos x) - \tan x \sin x]}{(\sin x)^{\cos x}[-\sin x \log(\sin x) + \cot x \cos x]}$$

$$\therefore \frac{du}{dv} = \frac{(\cos x)^{\sin x}[\cos x \log(\cos x) - \tan x \sin x]}{(\sin x)^{\cos x}[\cot x \cos x - \sin x \log(\sin x)]}$$

Thus, $\frac{du}{dv} = \frac{(\cos x)^{\sin x}[\cos x \log(\cos x) - \tan x \sin x]}{(\sin x)^{\cos x}[\cot x \cos x - \sin x \log(\sin x)]}$

9. $\textit{Differentiate}\ \sin^{-1}\left(\frac{2x}{1+x^2}\right)\ \textit{with respect to}\ \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right),\ \textit{if}\ 0 < x < 1.$

Solution:

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

We need to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

By substituting x = tan θ, we have

$$u = \sin^{-1}\left(\frac{2\tan\theta}{1+(\tan\theta)^2}\right)$$

$$\Rightarrow u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$\Rightarrow u = \sin^{-1}\left(\frac{2\tan\theta}{\sec^2\theta}\right)$ [∵ $\sec^2\theta - \tan^2\theta = 1$]

$$\Rightarrow u = \sin^{-1}\left(\frac{2\times\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}\right)$$

$$\Rightarrow u = \sin^{-1}\left(2\times\frac{\sin\theta}{\cos\theta}\times\cos^2\theta\right)$$

$\Rightarrow u = \sin^{-1}(2\sin\theta\cos\theta)$

But, $\sin 2\theta = 2\sin\theta\cos\theta$

$\Rightarrow u = \sin^{-1}(\sin 2\theta)$

Given $0 < x < 1 \Rightarrow x \in (0, 1)$

However, $x = \tan\theta$

$\Rightarrow \tan\theta \in (0, 1)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right)$

Hence, $u = \sin^{-1}(\sin 2\theta) = 2\theta$

$\Rightarrow$ u = $2\tan^{-1}x$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}(2\tan^{-1}x)$$

$$\Rightarrow \frac{du}{dx} = 2\frac{d}{dx}(\tan^{-1}x)$$

We know $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$$\Rightarrow \frac{du}{dx} = 2 \times \frac{1}{1+x^2}$$

$$\therefore \frac{du}{dx} = \frac{2}{1+x^2}$$

Now, we have $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

By substituting x = tan θ, we have

$$v = \cos^{-1}\left(\frac{1-(\tan\theta)^2}{1+(\tan\theta)^2}\right)$$

$$\Rightarrow v = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$\Rightarrow v = \cos^{-1}\left(\frac{1-\tan^2\theta}{\sec^2\theta}\right)$$ [∵ $\sec^2\theta - \tan^2\theta = 1$]

$$\Rightarrow v = \cos^{-1}\left(\frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}\right)$$

$$\Rightarrow v = \cos^{-1}\left(\frac{1}{\frac{1}{\cos^2\theta}} - \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}\right)$$

$\Rightarrow v = \cos^{-1}(\cos^2\theta - \sin^2\theta)$

But, $\cos 2\theta = \cos^2\theta - \sin^2\theta$

⇒ v = $\cos^{-1}(\cos 2\theta)$

However, $\theta \in \left(0, \frac{\pi}{4}\right) \Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right)$

Hence, v = $\cos^{-1}(\cos 2\theta) = 2\theta$

⇒ v = $2\tan^{-1}x$

On differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}(2\tan^{-1}x)$$

$$\Rightarrow \frac{dv}{dx} = 2\frac{d}{dx}(\tan^{-1}x)$$

We know $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$$\Rightarrow \frac{dv}{dx} = 2 \times \frac{1}{1+x^2}$$

$$\therefore \frac{dv}{dx} = \frac{2}{1+x^2}$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}}$$

$$\Rightarrow \frac{du}{dv} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\therefore \frac{du}{dv} = 1$$

Thus, $\frac{du}{dv} = 1$

Solution:

Let $u = \tan^{-1}\left(\frac{1+ax}{1-ax}\right)$ and $v = \sqrt{1 + a^2x^2}$.

We have to differentiate u with respect to v that is find $\frac{du}{dv}$.

We have $u = \tan^{-1}\left(\frac{1+ax}{1-ax}\right)$

By substituting ax = tan θ, we have

$$u = \tan^{-1}\left(\frac{1 + \tan\theta}{1 - \tan\theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \theta\right)\right) \left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow u = \frac{\pi}{4} + \theta$$

$$\Rightarrow u = \frac{\pi}{4} + \tan^{-1}(ax)$$

On differentiating u with respect to x, we get

$$\frac{du}{dx} = \frac{d}{dx}\left[\frac{\pi}{4} + \tan^{-1}(ax)\right]$$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) + \frac{d}{dx}[\tan^{-1}(ax)]$$

We know $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$ and derivative of a constant is 0.

$$\Rightarrow \frac{du}{dx} = 0 + \frac{1}{1 + (ax)^2}\frac{d}{dx}(ax)$$

$$\Rightarrow \frac{du}{dx} = 0 + \frac{1}{1 + (ax)^2} \frac{d}{dx}(ax)$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{1 + a^2x^2}\left[a\frac{d}{dx}(x)\right]$$

$$\Rightarrow \frac{du}{dx} = \frac{a}{1 + a^2x^2} \frac{d}{dx}(x)$$

We know $\frac{d}{dx}(x) = 1$

$$\Rightarrow \frac{du}{dx} = \frac{a}{1 + a^2x^2} \times 1$$

$$\therefore \frac{du}{dx} = \frac{a}{1 + a^2x^2}$$

Now, we have $v = \sqrt{1 + a^2x^2}$

On differentiating v with respect to x, we get

$$\frac{dv}{dx} = \frac{d}{dx}\left(\sqrt{1 + a^2x^2}\right)$$

$$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}(1 + a^2x^2)^{\frac{1}{2}}$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2}(1 + a^2x^2)^{\frac{1}{2}-1} \frac{d}{dx}(1 + a^2x^2)$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2}(1 + a^2x^2)^{-\frac{1}{2}}\left[\frac{d}{dx}(1) + \frac{d}{dx}(a^2x^2)\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1 + a^2x^2}}\left[\frac{d}{dx}(1) + a^2\frac{d}{dx}(x^2)\right]$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$ and derivative of a constant is 0.

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1 + a^2x^2}}[0 + a^2(2x^{2-1})]$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1 + a^2x^2}}[2a^2x]$$

$$\therefore \frac{dv}{dx} = \frac{a^2x}{\sqrt{1 + a^2x^2}}$$

We have $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$

$$\Rightarrow \frac{du}{dv} = \frac{\frac{a}{1 + a^2x^2}}{\frac{a^2x}{\sqrt{1 + a^2x^2}}}$$

$$\Rightarrow \frac{du}{dv} = \frac{a}{1 + a^2x^2} \times \frac{\sqrt{1 + a^2x^2}}{a^2x}$$

$$\therefore \frac{du}{dv} = \frac{1}{ax\sqrt{1 + a^2x^2}}$$

Thus, $\frac{du}{dv} = \frac{1}{ax\sqrt{1+a^2x^2}}$

Chapterwise RD Sharma Solutions for Class 12 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.