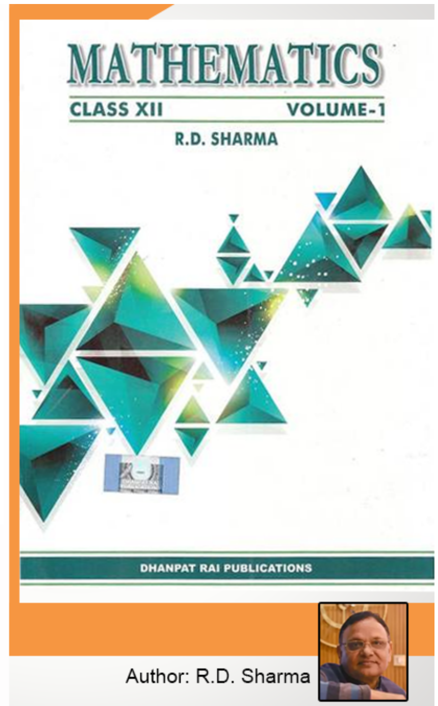
Class 12 - Chapter 10 Differentiability



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Exercise 10.1 Page No: 10.10

1. Show that f (x) = |x – 3| is continuous but not differentiable at x = 3.

Solution:

Given f (x) = |x – 3|

Therefore we can write given function as,

$$f(x) = \begin{cases} -(x-3), x < 3 \\ x-3, x \geq 3 \end{cases}$$

But f (3) = 3 - 3 = 0

LHL = $\lim_{x \to 3} f(x)$

= $\lim_{h \to 0} f(3-h)$

= $\lim_{h \to 0} 3-(3-h)$

= $\lim_{h \to 0} 0$

Now consider,

RHL = $\lim_{x \to 3} f(x)$

= $\lim_{h \to 0} f(3 + h)$

= $\lim_{h \to 0} 3 + h - 3$

= 0

LHL = RHL = f (3)

Since, f(x) is continuous at x = 3

LHD at x = 3 = $\lim_{x-3^-} \frac{f(x)-f(3)}{x-3}$

= $\lim_{h-0^-} \frac{f(3-h)-f(3)}{3-h-3}$

= $\lim_{h-0^-} \frac{3-(3-h)-0}{-h}$

= $\lim_{h-0^-} \frac{h}{-h}$

= - 1

RHD at x = 3 = $\lim_{x-3^+} \frac{f(x)-f(3)}{x-3}$

= $\lim_{h-0^+} \frac{f(3+h)-f(3)}{3+h-3}$

= $\lim_{h-0^+} \frac{3+h-3-0}{h}$

= $\lim_{h-0^+} \frac{h}{h}$

= 1

LHD at x = 3 ≠ RHD at x = 3

Hence, f(x) is continuous but not differentiable at x = 3.

2. Show that f (x) = $x^{1/3}$ is not differentiable at x = 0.

Solution:

For differentiability,

LHD (at x = 0) = RHD (at x = 0)

(LHD at x = 0) = $\lim\limits_{x \to 0^-} \frac{f(x)-f(0)}{x-0}$

$= \lim\limits_{h \to 0^-} \frac{f(0-h)-f(0)}{0-h-0}$

$= \lim\limits_{h \to 0^-} \frac{(-h)^{\frac{1}{3}}-0}{-h}$

$= \lim\limits_{h \to 0^-} \frac{(-h)^{\frac{1}{3}}}{-h}$

$= \lim\limits_{h \to 0^-} \frac{(-1)^{\frac{1}{3}}(h)^{\frac{1}{3}}}{(-1)h}$

$= \lim\limits_{h \to 0} (-1)^{\frac{-2}{3}}\, h^{\frac{-2}{3}}$

= Not defined

(RHD at x = 3) = $\lim\limits_{x \to 0^+} \frac{f(x)-f(0)}{x-0}$

$= \lim\limits_{h \to 0^-} \frac{f(0+h)-f(0)}{0+h-0}$

$= \lim\limits_{h \to 0^+} \frac{(h)^{\frac{1}{3}}-0}{+h}$

$= \lim\limits_{h \to 0^+} \frac{(h)^{\frac{1}{3}}}{+h}$

$= \lim\limits_{h \to 0} h^{\frac{-2}{3}}$

= Not defined

Since, LHD and RHD does not exist at x = 0

Hence, f(x) is not differentiable at x = 0

$3.\ Show\ that\ f(x) = \begin{cases} 12x - 13, & if\ x \leq 3 \\ 2x^2 + 5, & if\ x > 3 \end{cases}\ is\ differentiable\ at\ x = 3.\ Also,\ find\ f'(3)$

Solution:

Now we have to check differentiability of given function at x = 3

That is LHD (at x = 3) = RHD (at x = 3)

(LHD at x = 3) = $\lim_{x-3^-} \frac{f(x)-f(3)}{x-3}$

$= \lim_{h-0^-} \frac{f(3-h)-f(3)}{3-h-3}$

$= \lim_{h-0^-} \frac{[12(3-h)-13]-[12(3)-13]}{-h}$

$= \lim_{h-0^-} \frac{36-12h-13-36+13}{-h}$

$= \lim_{h-0^-} \frac{-12h}{-h}$

= 12

(RHD at x = 3) = $\lim_{x-3^+} \frac{f(x)-f(3)}{x-3}$

$= \lim_{h\to 0^+} \frac{f(3+h)-f(3)}{3+h-3}$

$= \lim_{h\to 0^+} \frac{[2(3+h^2)+5]-[12(3)-13]}{3+h-3}$

$= \lim_{h\to 0^+} \frac{18+12h+2h^2+5-36+13}{h}$

$= \lim_{h\to 0^+} \frac{2h^2+12h}{h}$

$= \lim_{h\to 0^+} \frac{h(2h+12)}{h}$

= 12

Since, (LHD at x = 3) = (RHD at x = 3)

Hence, f(x) is differentiable at x = 3.

4. Show that the function f is defined as follows

$$f(x)=\begin{cases}3x-2, & 0<x\leq 1\\ 2x^{2}-x, & 1<x\leq 2\\ 5x-4, & x>2\end{cases}$$

Is continuous at x = 2, but not differentiable thereat.

Solution:

Given

$$f(x) = \begin{cases} 3x-2, & 0 < x \le 1 \\ 2x^2 - x, & 1 < x \le 2 \\ 5x-4, & x > 2 \end{cases}$$

Now we have to check continuity at x = 2

For continuity,

LHL (at x = 2) = RHL (at x = 2)

f (2) = 2(2)^2 - 2

= 8 - 2 = 6

LHL = $\lim_{x\to 2^-} f(x)$

= $\lim_{h\to 0^-} f(2-h)$

= $\lim_{h\to 0^-} [2(2-h)^2 - (2-h)]$

= 8 - 2

= 6

RHL = $\lim_{x\to 2^+} f(x)$

= $\lim_{h\to 0^+} f(2 + h)$

= $\lim_{h\to 0} 5(2 + h) - 4$

= 6

Since, LHL = RHL = f (2)

Hence, F(x) is continuous at x = 2

Now we have to differentiability at x = 2

(LHD at x = 2) = $\lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{h \to 0} \frac{f(2-h)-f(2)}{2-h-2}$

$= \lim_{h \to 0} \frac{[2(2-h)^2-(2-h)]-[8-2]}{-h}$

$= \lim_{h \to 0} \frac{8-8h+2h^2-h-6}{-h}$

$= \lim_{h \to 0} \frac{2h^2-6h}{-h}$

$= \lim_{h \to 0} \frac{h(2h-6)}{-h}$

$= \lim_{h \to 0} (6-2h)$

= 6

Now consider,

(RHD at x = 2) = $\lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$

$= \lim_{h \to 0} \frac{f(2+h)-f(2)}{2+h-2}$

$= \lim_{h \to 0} \frac{[5(2+h)-4]-[8-2]}{h}$

$= \lim_{h \to 0} \frac{10+5h-4-6}{h}$

= 5

Since, (RHD at x = 2) ≠ (LHD at x = 2)

Hence, f (2) is not differentiable at x = 2.

5. Discuss the continuity and differentiability of the function f (x) = |x| + |x -1| in the interval of (-1, 2).

Solution:

The given function f (x) can be defined as

f (x) = $\begin{cases} x + x + 1, -1 < x < 0 \\ 1, 0 \leq x \leq 1 \\ -x - x + 1, 1 < x < 2 \end{cases}$

f (x) = $\begin{cases} 2x + 1, -1 < x < 0 \\ 1, 0 \leq x \leq 1 \\ -2x + 1, 1 < x < 2 \end{cases}$

We know that a polynomial and a constant function is continuous and differentiable everywhere. So, f(x) is continuous and differentiable for x ∈

(-1, 0) and x ∈ (0, 1) and (1, 2).

We need to check continuity and differentiability at x = 0 and x = 1.

Continuity at x = 0

$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} 2x + 1$ = 1

$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} 1$ = 1

F (0) = 1

$\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = f(0)$

Since, f(x) is continuous at x = 0

Continuity at x = 1

$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} 1$ = 1

$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} 1$ = 1

F (1) = 1

$\log_{x\to0^+} f(x) = \log_{x\to0^-} f(x)$ = 1

Since, f(x) is continuous at x = 1

Now we have to check differentiability at x = 0

For differentiability, LHD (at x = 0) = RHD (at x = 0)

Differentiability at x = 0

(LHD at x = 0) = $\lim_{x-0^-} \frac{f(x)-f(0)}{x-0}$

= $\lim_{x-0^-} \frac{2x+1-1}{x-0}$

= $\lim_{x-0^-} \frac{2x}{x}$

= 2

(RHD at x = 0) = $\lim_{x-0^+} \frac{f(x)-f(0)}{x-0}$

= $\lim_{x-0^+} \frac{1-1}{x}$

= $\lim_{x-0^+} \frac{0}{x}$

= 0

Since, (LHD at x = 0) ≠ (RHD at x = 0)

So, f(x) is differentiable at x = 0.

Now we have to check differentiability at x = 1

For differentiability, LHD (at x = 1) = RHD (at x = 1)

Differentiability at x = 1

(LHD at x = 1) = $\lim_{x-1^-} \frac{f(x)-f(1)}{x-1}$

= $\lim_{x-1^-} \frac{1-1}{x-1}$

= 0

(RHD at x = 1) = $\lim_{x-1^+} \frac{f(x)-f(1)}{x-1}$

= $\lim_{x-1^+} \frac{-2x+1-1}{x-1}$

= ∞

Since, f(x) is not differentiable at x = 1.

So, f(x) is continuous on (- 1, 2) but not differentiable at x = 0, 1

Exercise 10.2 Page No: 10.16

1. If f is defined by f (x) = x^2, find f' (2).

Solution:

We have a polynomial function $f(x) = x^2$, and we have to find whether it is derivable at x = 2 or not, so by using the formula, $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$,

We get, $f'(2) = \lim_{x \to 2} \frac{f(x)-f(2)}{x-2}$

$f'(2) = \lim_{x \to 2} \frac{x^2-2^2}{x-2}$

$f'(2) = \lim_{x \to 2} \frac{(x+2)(x-2)}{x-2}$

[Using $a^2 - b^2 = (a + b)(a - b)$]

$f'(2) = \lim_{x \to 2} x + 2 = 4$

Hence, the function is differentiable at x = 2 and its derivative equals to 4.

2. If f is defined by $f(x) = x^2 - 4x + 7$, show that $f'(5) = 2 f'(7/2)$

Solution:

We have a polynomial function $f(x) = x^2 - 4x + 7$, and we have to f '(x) its value at x = 5 and x = 7/2, so by using the formula, $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$,

We get, $f'(5) = \lim_{x \to 5} \frac{f(x)-f(5)}{x-5}$

$f'(5) = \lim_{x \to 5} \frac{x^2-4x+7-(5^2-4\times5+7)}{x-5}$

$f'(5) = \lim_{x \to 5} \frac{x^2-4x-5}{x-5}$

Hence the proof.

3. Show that the derivative of the function f is given by $f(x) = 2x^3 - 9x^2 + 12x + 9$, at x = 1 and x = 2 are equal.

Solution:

We are given with a polynomial function $f(x) = 2x^3 – 9x^2 + 12x + 9$, and we have

to find f '(x) at x = 1 and x = 2, so by using the formula, $f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$, we get,

$$f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$$

$$f'(1) = \lim_{x\to 1} \frac{2x^3-9x^2+12x+9-[2(1)^3-9(1)^2+12(1)+9]}{x-1}$$

$$f'(1) = \lim_{x\to 1} \frac{2x^3-9x^2+12x-5}{x-1}$$

$$f'(1) = \lim_{x\to 1} \frac{(x-1)(2x^2-7x+5)}{x-1}$$

$$f'(1) = \lim_{x\to 1} 2x^2 - 7x + 5 = 0$$

For x = 2, we get,

$$f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$$

$$f'(2) = \lim_{x\to 2} \frac{2x^3-9x^2+12x+9-[2(2)^3-9(2)^2+12(2)+9]}{x-2}$$

$$f'(2) = \lim_{x\to 2} \frac{2x^3-9x^2+12x-4}{x-2}$$

$$f'(2) = \lim_{x\to 2} \frac{(x-2)(2x^2-5x+2)}{x-2}$$

$$f'(2) = \lim_{x\to 2} 2x^2 - 5x + 2 = 0$$

Hence they are equal at x = 1 and x = 2.

4. If for the function Ø (x) = λ x^2 + 7x – 4, Ø' (5) = 97, find λ.

Solution:

We have to find the value of λ given in the real function and we are given with the differentiability of the function $f(x) = \lambda x^2 + 7x - 4$ at $x = 5$ which is $f'(5) = 97$, so we will adopt the same process but with a little variation.

So by using the formula, $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$, we get,

$$f'(5) = \lim_{x \to 5} \frac{f(x)-f(5)}{x-5}$$

$$f'(5) = \lim_{x \to 5} \frac{\lambda x^2 + 7x - 4 - [\lambda(5)^2 + 7(5) - 4]}{x-5}$$

$$f'(5) = \lim_{x \to 5} \frac{\lambda x^2 + 7x - 4 - [\lambda(5)^2 + 7(5) - 4]}{x-5}$$

$$f'(5) = \lim_{x \to 5} \frac{\lambda x^2 + 7x - 35 - 25\lambda}{x-5}$$

As the limit has some finite value, then there must be the formation of some indeterminate form like $\frac{0}{0}, \frac{\infty}{\infty}$, so if we put the limit value, then the numerator will also be zero as the denominator, but there must be a factor (x - 5) in the numerator, so that this form disappears.

$$f'(5) = \lim_{x \to 5} \frac{(x-5)(\lambda x + 5\lambda + 7)}{x-5}$$

$f'(5) = \lim_{x \to 5} \lambda x + 5\lambda + 7 = 97$

$f'(5) = 10\lambda + 7 = 97$

$10\lambda = 90$

$\lambda = 9$

5. If $f(x) = x^3 + 7x^2 + 8x - 9$, find $f'(4)$.

Solution:

We are given with a polynomial function $f(x) = x^3 + 7x^2 + 8x - 9$, and we have to find whether it is derivable at $x = 4$ or not,

So by using the formula, $f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$,

We get, $f'(4) = \lim_{x\to 4} \frac{f(x)-f(4)}{x-4}$

$f'(4) = \lim_{x\to 4} \frac{x^3 + 7x^2 + 8x - 9 - [4^3 + 7(4)^2 + 8(4) - 9]}{x-4}$

$f'(4) = \lim_{x\to 4} \frac{(x-4)(x^2 + 11x + 52)}{x-4}$

$f'(4) = \lim_{x\to 4} x^2 + 11x + 52$

$f'(4) = 112.$

6. Find the derivative of the function f defined by f (x) = mx + c at x = 0.

Solution:

We are given with a polynomial function $f(x) = mx + c$, and we have to find whether it is derivable at $x = 0$ or not,

So by using the formula, $f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$,

We get, $f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$

$f'(0) = \lim_{x\to 0} \frac{mx + c - [m(0) + c]}{x-0}$

$f'(0) = \lim_{x\to 0} \frac{mx + c - c}{x-0}$

$f'(0) = \lim_{x\to 0} m = m$

This is the derivative of a function at x = 0, and also this is the derivative of this function at every value of x.

Chapterwise RD Sharma Solutions for Class 12 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.