NCERT Solutions for 12th Class Physics: Chapter 2-Electrostatic Potential and Capacitance

Class 12: Physics Chapter 2 solutions. Complete Class 12 Physics Chapter 2 Notes.

NCERT Solutions for 12th Class Physics: Chapter 2-Electrostatic Potential and Capacitance

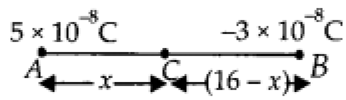
NCERT 12th Physics Chapter 2, Class 12 Physics Chapter 2 solutions

Question 1.

Two charges $5 \times 10^{-8}8$ C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. .

Solution:

(i) Let C be the point on the line joining the two charges, where electric potential is zero, then



$$V_C = 0$$

$$\text{or} \quad V_{CA} + V_{CB} = 0$$

$$\text{or} \quad V_{CA} = -V_{CB}$$

$$\text{or} \quad \frac{1}{4\pi\varepsilon_0}\frac{q_A}{r_{CA}} = -\frac{1}{4\pi\varepsilon_0}\frac{q_B}{r_{CB}}$$

$$\text{or} \quad \frac{5\times10^{-8}\text{C}}{x\times10^{-2}\text{m}} = -\frac{(-3\times10^{-8}\text{C})}{[(16-x)\times10^{-2}\text{m}]}$$

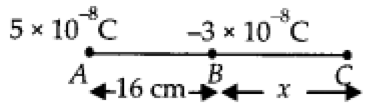
$$\text{or} \quad \frac{5}{x} = \frac{3}{16-x}$$

$$\text{or} \quad 80 - 5x = 3x \quad \text{or} \quad 80 = 8x$$

$$\text{or} \quad x = \frac{80}{8} \quad \text{or} \quad x = 10 \text{ cm}$$

So, electric potential is zero at distance of 10 cm from charge of 5×10^{-8} C on line joining the two charges between them. If point C is not between the two charges, then

$V_{CA} + V_{CB} = 0$ or $V_{CA} = -V_{CB}$



or $\frac{1}{4\pi\varepsilon_0}\frac{q_A}{r_{CA}} = \frac{-1}{4\pi\varepsilon_0}\frac{q_B}{r_{CB}}$

or $\frac{5\times10^{-8}\text{ C}}{\left[(16+x)\times10^{-2}\text{m}\right]} = \frac{-(-3\times10^{-8}\text{C})}{[x\times10^{-2}\text{ m}]}$

$\frac{5}{16+x} = \frac{3}{x}$

or $5x = 48 + 3x$ or $2x = 48$ or $x = 24$ cm

So, electric potential is also equal to zero a*a distance of 24 cm from charge of -3×10^{-8} C and at a distance of (24 +16) = 40 cm from charge of 5×10^{-8} C, on the side of charge of -3×10^{-8} C.

Question 2.

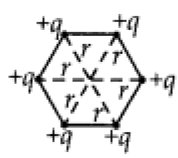
A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the center of the hexagon.

Solution:

$V = 6 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

or $V = 6 \times 9 \times 10^9 \times \frac{5\times10^{-6}}{10\times10^{-2}}$

or $V = 2.7 \times 10^6$ volts.



Question 3.

Two charges 2 μC and -2 μC are placed at points A and B 6 cm apart.

1. Identify an equipotential surface of the system.
2. What is the direction of the electric field at every point on this surface?

Solution:

1. Since it is an electric dipole, so a plane normal to AB and passing through its mid-point has zero potential everywhere.
2. Normal to the plane in the direction AB.

Question 4.

A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field

(a) inside the sphere

(b) just outside the sphere

(c) at a point 18 cm from the centre of the sphere?

Solution:

$R = 12 \text{ cm}, q = 1.6 \times 10^{-7} \text{C}$

(a) $E_{in} = 0$

(b) $E_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 9 \times 10^9 \times \frac{1.6 \times 10^{-7}}{(12 \times 10^{-2})^2}$

or $E_{out} = 1.0 \times 10^5 \text{ N C}^{-1}$

(c) $E_{on} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = 9 \times 10^9 \times \frac{1.6 \times 10^{-7}}{(18 \times 10^{-2})^2}$

or $E_{on} = 4.44 \times 10^4 \text{ N C}^{-1}$

Question 5.

A parallel plate capacitor with air between the plates has a capacitance of 8 μF. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Solution:

$$C_0 = \frac{\varepsilon_0 A}{d} = 8 \text{ pF}$$

$$C = \frac{K\varepsilon_0 A}{d/2} = \frac{2K\varepsilon_0 A}{d} = 2\,KC_0 = 2 \times 6 \times 8 \text{ pF}$$

or $C = 96$ pF

Question 6.

Three capacitors each of capacitance 9 μF are connected in series.

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitors if the combination is connected to a 120 V supply?

Solution:

(a) $C = \frac{C_1}{3} = \frac{9}{3}$ pF

or $C = 3$ pF

(b) Net charge stored in combination of capacitors is

$Q = CV = 3 \times 10^{-12} \times 120 = 360$ pC

So, potential difference across each capacitor is

$V_1 = \frac{Q}{C_1} = \frac{360 \text{ pC}}{9 \text{ pF}}$ or $V_1 = 40$ volts.

Question 7.

Three capacitors of capacitances 2 μF, 3 μF and 4 μF are connected in parallel.

1. What is the total capacitance of the combination.
2. Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Solution:

1. $C = C_1 + C_2 + C_3 = 2 + 3 + 4$ or $C = 9$ μF

2. Since the capacitors are in parallel, so potential difference across each of them is same i.e.

$V_1 = V_2 = V_3 = 100$ V

So, charges stored on capacitors are

$Q_1 = C_1V_1 = 2 \times 100 = 200$ pC

$Q_2 = C_2V_2 = 3 \times 100 = 300$ pC

$Q_3 = C_3V_3 = 4 \times 100 = 400$ pC

Question 8.

In a parallel plate capacitor with air between the plates, each plate has an area of 6 × 10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this, capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Solution:

$A = 6 \times 10^{-3}$ m^2, $d = 3$ mm $= 3 \times 10^{-3}$ m,

$V = 100$ V

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$= 17.7 \times 10^{-12}$ F or $C_0 = 17.7$ pF

On connecting the capacitor across 100 V supply, charge on each plate of the capacitor is

$Q_0 = C_0V = 17.7 \times 10^{-12} \times 100$

or $Q_0 = 1.77 \times 10^{-9}$ C.

Question 9.

Explain what would happen if in the capacitor given in above question, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates.

(a) While the voltage supply remained connected.

(b) After the supply was disconnected.

Solution:

$C = KC_0 = 6 \times 17.7 = 106.2$ pF

(a) If the voltage supply remained connected, then the potential difference across the capacitor will remain the same *i.e.* $V = 100$ V and hence charge on the capacitor becomes

$Q = CV = 106.2 \times 10^{-12} \times 100$

or $Q = 1.062 \times 10^{-8}$ C

(b) If the voltage supply was disconnected, then charge on the capacitor remains the same

i.e. $Q = 1.77 \times 10^{-9}$ C

and hence potential difference across the capacitor becomes

$$V = \frac{Q}{C} = \frac{1.77 \times 10^{-9}\,\text{C}}{106.2 \times 10^{-12}\,\text{F}} \quad \text{or} \quad V = 16.7\ \text{V}$$

Question 10.

A 12 μF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Solution:

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times 50^2$$

$$= 15000 \times 10^{-12} \quad \text{or} \quad U = 1.5 \times 10^{-8}\ \text{J}.$$

Question 11.

A 600 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 μF capacitor. How much electrostatic energy is lost in the process?

Solution:

$C_1 = 600$ pF, $V_1 = 200$ V, $C_2 = 600$ pF, $V_2 = 0$

On connecting charged capacitor to uncharged capacitor, the common potential V across the capacitors is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{600 \times 10^{-12} \times 200 + 0}{(600 + 600) \times 10^{-12}}$$

or $V = 100$ V

Energy stored in capacitors before connection is

$$U_i = \frac{1}{2}C_1V_1^2 + 0 = \frac{1}{2} \times 600 \times 10^{-12} \times 200^2$$

or $U_i = 12\ \mu J$

and energy stored in capacitors after connection is

$$U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(600 + 600) \times 10^{-12} \times 100^2$$

or $U_f = 6\ \mu J$

Hence the energy lost in the process is

$\Delta = U_f - U_i = (6 - 12)\ \mu J$ or $\Delta U = -6\ \mu J$.

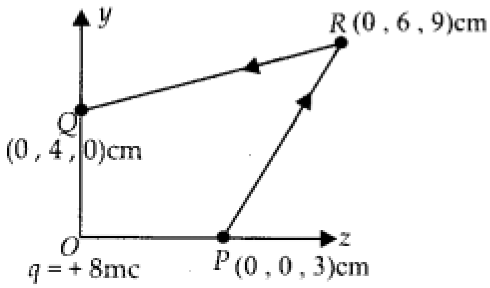
C, = 600 μF, V_1 = 200 V, C_2 = 600 μF, V_2 = 0

Question 12.

Acharge of 8 mC is located at theorigin. Calculate the work done in taking a small charge of – 2 × 10^{-9} C from a point P(0,0,3 cm) to a point Q (0,4 cm, 0), via a point R (0,6 cm, 9 cm).

Solution:

As electric field is conservative field, so work done in moving a charge in electric field is independent of path chosen to move the charge in electric field and depends only on the electric potential difference between the two end points. So



$$W_{PQ} = q_0[V_Q - V_P] = q_0\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{r_{OQ}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{OP}}\right]$$

or $$W_{PQ} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_{OQ}} - \frac{1}{r_{OP}}\right]$$

$$= 9 \times 10^9 \times 8 \times 10^{-3}\,C \times (-2 \times 10^{-9}\,C) \times \left(\frac{1}{4} - \frac{1}{3}\right) \times \frac{1}{10^{-2}}$$

$$= -144 \times 10^{-1} \times \left(\frac{3-4}{12}\right) = 12 \times 10^{-1}$$

$W_{PQ} = 1.2$ J.

Question 13.

A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Solution:

The length of diagonal of the cube of each side b is

$$\sqrt{3b^2} = b\sqrt{3}$$

Distance of any of the vertices from the centre of cube,

$$= \frac{\sqrt{3}}{2} b$$

$$V = 8 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{r} = 8 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{\sqrt{3}\frac{b}{2}}$$

or $$V = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

E = 0, as electric field at centre due to a charge at any corner of cube is just equal and opposite to that of another charge at diagonally opposite corner of cube.

Question 14.

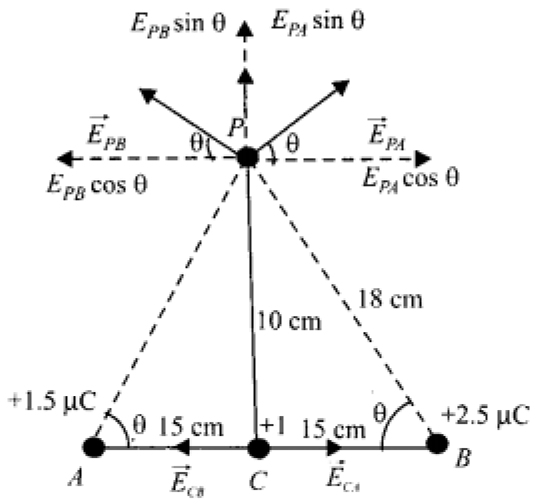
Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Solution:

(a) At mid point C of line joining two charges, electric potential is $V_c = V_{CA} + V_{CB}$



or $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_{CA}} + \frac{q_B}{r_{CB}}\right]$

$= 9 \times 10^9\left[\frac{1.5}{15} + \frac{2.5}{15}\right] \times \frac{10^{-6}}{10^{-2}}$ or $V_C = 2.4 \times 10^5$ V

and electric field at point C is

$E_C = E_{CB} - E_{CA}$ [As $E_{CB} > E_{CA}$ and they are directed opposite to each other]

or $E_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_B}{r_{CB}^2} - \frac{q_A}{r_{CA}^2}\right]$

$= 9 \times 10^9\left[\frac{2.5}{15^2} - \frac{1.5}{15^2}\right] \times \frac{10^{-6}\,\text{C}}{10^{-4}\,\text{m}^2}$

or $E_C = 4 \times 10^5$ V m^{-1}

directed in direction of $\vec{E}_{CB}$ *i.e.* from C to A.

(b) At given point P on perpendicular bisector of line joining two charges, electric potential is

$V_P = V_{PA} + V_{PB}$ or $V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_{PA}} + \frac{q_B}{r_{PB}}\right]$

$= 9 \times 10^9\left[\frac{1.5}{18} + \frac{2.5}{18}\right] \times \frac{10^{-6}\,\text{C}}{10^{-2}\,\text{m}}$

or $V_P = 2.0 \times 10^5$ V

and horizontal component of net electric field at point P is

$E_x = E_{PA} \cos\theta - E_{PB} \cos\theta = (E_{PA} - E_{PB}) \cos\theta$

or $E_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_{PA}^2} + \frac{q_B}{r_{PB}^2}\right]\cos\theta$

$= 9 \times 10^9 \left[\frac{1.5}{18^2} - \frac{2.5}{18^2}\right] \times \frac{10^{-6}}{10^{-4}} \times \frac{15}{18}$

or $E_x = -2.3 \times 10^5$ V m^{-1}

whereas vertical component of net electric field at point p is

$E_y = E_{PA} \sin\theta + E_{PB} \sin\theta = [E_{PA} + E_{PB}] \sin\theta$

or $E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_{PA}^2} + \frac{q_B}{r_{PB}^2}\right]\sin\theta$

$= 9 \times 10^9 \left[\frac{1.5}{18^2} + \frac{2.5}{18^2}\right] \times \frac{10^{-6}}{10^{-4}} \times \frac{10}{18}$

or $E_y = 6.2 \times 10^5$ V m^{-1}

So, magnitude of net electric field at point P is

$E_P = \sqrt{E_x^2 + E_y^2} = \sqrt{2.3^2 + 6.2^2} \times 10^5$

or $E_P = 6.6 \times 10^5$ V m^{-1} directed at an angle

$\tan\theta = \frac{E_y}{E_x} = \frac{6.2 \times 10^5}{-2.3 \times 10^5} = -2.6956$

or $\theta = -69.6°$

with the horizontal in –ve x-direction *i.e.* at 69.6° with BA.

Question 15.

A spherical conducting shell of inner radius r, and outer radius r_2 has a charge Q.

1. A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?
2. Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

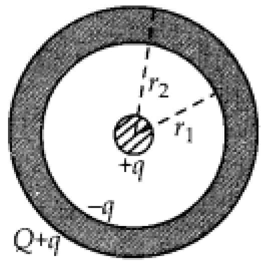
Solution:

1. Surface charge density on the inner surface of shell is

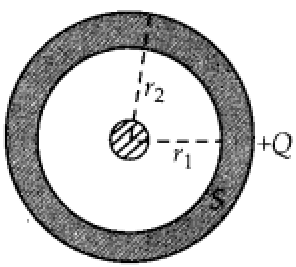
$$\sigma_{in} = \frac{-q}{4\pi r_1^2}$$

and on the outer surface of shell is

$$\sigma_{out} = \frac{Q+q}{4\pi r_2^2}$$



1. The electric flux linked with any closed surface S inside the conductor is zero as electric field inside conductor is zero.
   So, by Gauss's theorem net charge enclosed by closed surface S is also zero i.e., $q_{net} = 0$
   So, if there is no charge inside the cavity, then there cannot be any charge on the inner surface of the shell and hence electric field inside the cavity will be zero, even though the shell may not be spherical.



Question 16.

Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$\left(\vec{E}_2 - \vec{E}_1\right).\hat{n} = \frac{\sigma}{\varepsilon_0}$$

where n is a unit vector normal eo to the surface at a point and o is the surface charge density at that point. (The direction of n is from side 1 to side 2). Hence show that just outside a conductor, the electric field is

$$\sigma\frac{\hat{n}}{\varepsilon_0}$$

.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

Solution:

Normal component of Electric field intensity due to a thin infinite plane sheet of

charge, on left side (side 1) $\vec{E}_1 = -\frac{\sigma}{2\varepsilon_0}\hat{n}$

and on right side (side 2), $\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{n}$

Discontinuity in the normal component from one side to the other is

$$\vec{E}_2 - \vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{n} + \frac{\sigma}{2\varepsilon_0}\hat{n} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

or $(\vec{E}_2 - \vec{E}_1).\hat{n} = \frac{\sigma}{\varepsilon_0}\hat{n}.\hat{n} = \frac{\sigma}{\varepsilon_0}$

Inside a closed conductor, $\vec{E}_1 = 0$

$\therefore \quad E = \vec{E}_2 = \frac{\sigma}{\varepsilon_0}\hat{n}.$

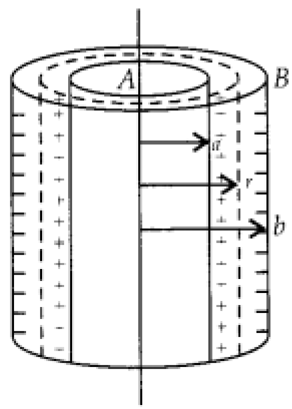
(b) To show that the tangential component of electrostatic field is continuous from one side of a charged surface to another, we use the fact that work done by electrostatic field on a closed loop is zero.

Question 17.

A long charged cylinder of linear charge density k is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Solution:

In Figure, A is a long charged cylinder of linear charge density k, length l and radius a. A hollow co-axial conducting cylinder B of of length l and radius b surrounds A. The charge q = kl spreads uniformly on the outer surface of A. It induces – q charge on the cylinder B, which spreads on the inner surface of B. An electric field E is produced in the space between the two cylinders, which is directed radially outwards. Let us consider a co-axial cylindrical Gaussian surface of radius r. The electric flux through the cylindrical Gaussian surface is



$$\phi_E = \int \vec{E}\cdot\vec{ds} = \int E ds \cos 0^\circ = E\int ds = E(2\pi r l)$$

The electric flux through the end faces of the cylindrical Gaussian surface is zero, as E is parallel to them. According to Gauss's

theorem, $\phi_E = E(2\pi r l) = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Question 18.

In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 A:

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 A separation?

Solution:

(a) $U = \frac{1}{4\pi\varepsilon_0}\frac{+e \times -e}{r}$

$= -9 \times 10^9 \times \frac{(1.6\times10^{-19})^2}{0.53\times10^{-10}}$ J

or $U = \frac{-9\times10^9\times(1.6\times10^{-19})^2}{0.53\times10^{-10}\times1.6\times10^{-19}}$ eV

or $U = -27.2$ eV

(b) K.E. $= +\frac{|U|}{2} = 13.6$ eV

$\therefore$ Total energy E = P.E + K.E = –27.2 + 13.6

= –13.6 eV

Now $W = \Delta U = 0 - (-13.6)$

or $W = +13.6$ eV

(c) P.E. at 1.06×10^{-10} m separation,

$U' = \frac{9\times10^9\times(-1.6\times10^{-19})\times(1.6\times10^{-19})}{1.06\times10^{-10}}$

$= -21.74 \times 10^{-19}$ J

or $U' = \frac{-21.74\times10^{-19}}{1.06\times10^{-19}} = -13.585$ eV

For (a): Taking –13.585 eV as zero of P.E., then

P.E. of the system = –27.17 – (–13.585)

= –13.585 eV = –13.6 eV.

For (b): ΔU = – 13.6 eV – (–13.6 eV) = 0

$\therefore$ $W = 0$

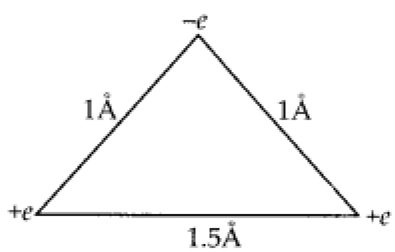
Question 19.

If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an

, the two protons are separated by roughly 1.5 A, and the electron is roughly 1 A from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Solution:

It is a system of three point charges and the potential energy stored is this system of charges is



$$U' = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

$$U = 9\times10^9\left[\frac{e\times -e}{1} + \frac{e\times e}{1.5} + \frac{e\times -e}{1}\right]\times\frac{1}{10^{-10}\,\text{m}}$$

$$\text{or}\quad U = 9\times10^9\times(1.6\times10^{-19})^2 \times 10^{10}\left[-1+\frac{1}{1.5}-1\right]\text{J}$$

$$\text{or}\quad U = \frac{9\times10^{19}\times(1.6\times10^{-19})^2}{1.6\times10^{-19}}\times\left(\frac{-2}{1.5}\right)\text{eV}$$

$$\text{or}\quad U = -19.2\ \text{eV}$$

with zero potential energy at infinity.

Question 20.

TWO charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Solution:

As the two spheres are connected to each other by wire, so they have same electric potential i.e.,$V_a = V_b$

if b > a, then $E_a > E_b$

i.e. sphere with smaller radius produces more electric field on its surface. Hence, the charge density on the sharp and pointed ends of conductor is higher than on its flatter portions.

Question 21.

Two charges -q and +q are located at points (0,0, -a) and (0,0, a), respectively.

(a) What is the electrostatic potential at the points (0,0, z) and (x, y, 0)?

(b) Obtain the dependence of potential on the distance r of a point from the origin when r/a »1.

(c) How much work is done in moving a small test charge from the point (5, 0, 0) to (-7, 0, 0) along the x-axis? Does the

answer change if the path of the test charge between the some points is not along the x-axis?

Solution:

(a) The two point charges form an electric dipole of moment p = q 2_a directed along + z-axis. Point A (0, 0, z) lies on the axis of electric dipole, so electric potential at point A is

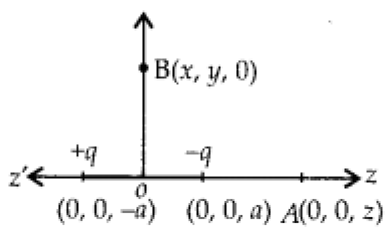
$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{z+a} + \frac{q}{z-a}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{-z+a+z+a}{z^2-a^2}\right]$$

$$\text{or} \quad V_A = \frac{1}{4\pi\varepsilon_0}\frac{p}{(z^2-a^2)}$$

Point B (x, y, 0) lies on the equatorial plane of electric dipole, so electric potential at point B is zero i.e. $V_B = 0$

(b) Electric potential at any point on the axis of electric dipole at distance r from its centre is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2-a^2)}$$



when $r >>> a$, then electric potential becomes

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

so in that case $V \propto \frac{1}{r^2}$

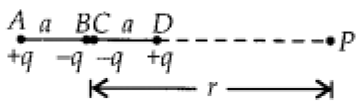
(c) As both the points C (5, 0, 0) and D (-7, 0, 0) lie on the perpendicular bisector of electric dipole, so electric potential at both the points is zero. Hence work done in moving the charge from C to D is

$W_{CD} = q_0 [V_D – V_C] \times O$ or $W_{CD} = O$

This work done will remain equal to zero even if the path of the test charge between the same points is changed, as electric field is conservative field and work done in moving a charge between the two points in electric field is independent of the path chosen to move the charge.

Question 22.

Figure shows a charge array known as an electric quadrupole. For a point on the axis of quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



Solution:

$$V_P = V_{PA} + V_{PB} + V_{PC} + V_{PD}$$

or $$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{+q}{r+a} - \frac{q}{r} - \frac{q}{r} + \frac{q}{r-a}\right]$$

or $$V_P = \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r-a) - 2(r^2-a^2) + r(r+a)}{r(r^2-a^2)}\right]$$

or $$V_P = \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2-a^2)}\right]$$

or $$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q\cdot 2a^2}{r(r^2-a^2)} = \frac{1}{4\pi\varepsilon_0}\frac{p\cdot a}{r(r^2-a^2)}$$

For $\frac{r}{a} >>$ or $r >> a$

$$V_P \approx \frac{1}{4\pi\varepsilon_0}\frac{pa}{r^3} \quad \text{or} \quad V_P \propto \frac{1}{r^3}.$$

However, electric potential at any point on axis of electric dipole is $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$ or $V \propto \frac{1}{r^2}$

and due to point charge is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad \text{or} \quad V \propto \frac{1}{r}$$

Question 23.

An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Solution:

Minimum number of capacitors that must be connected in series in a row are

$$n = \frac{1000\,\text{V}}{400\,\text{V}} = 2.5 \approx 3$$

capacitance of 3 capacitors in series in a row is

$$C' = \frac{1}{3}\,\mu\text{F}$$

Minimum number of rows of 3 capacitors each to be connected in parallel to obtain net capacitance of 2 μF are

$$m = \frac{2\,\mu\text{F}}{\frac{1}{3}\,\mu\text{F}} = 6$$

So minimum number of capacitors required are $m \times n = 6 \times 3 = 18$

Question 24.

What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm?

Solution:

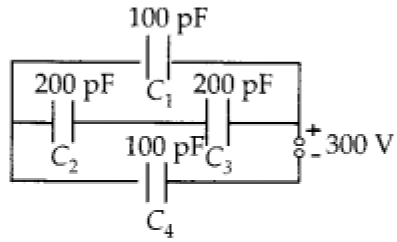
$$C = \frac{\varepsilon_0 A}{d} \quad \text{or} \quad A = \frac{C \cdot d}{\varepsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.85 \times 10^{-12}}$$

or $A = 1.13 \times 10^9\ \text{m}^2 = 1130\ \text{km}^2$

This is why ordinary capacitors are in the range of μF or less.

Question 25.

Obtain the equivalent capacitance of the network in figure. For a 300 V supply, determine the charge and voltage across each capacitor.



Solution:

$\because$ C_2 and C_3 are in series, so, $C' = 100$ pF

$\because$ C_1 and C' are parallel,

so $C'' = C_1 + C' = 100 + 100$ or $C'' = 200$ pF

$\because$ C_4 and C'' are in series, so net capacitance of the network is

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C_4} = \frac{1}{200} + \frac{1}{100} = \frac{1+2}{200}$$

or $C_2 = \frac{200}{3}$ pF $= 66.7$ pF

Net charge stored on the combination is

$$Q = CV = \frac{200}{3} \times 10^{-12} \times 300 = 2 \times 10^{-8}\,\text{C}$$

As C'' and C_4 are in series, so

$Q'' = Q_4 = Q$

or $Q'' = Q_4 = 2 \times 10^{-8}$ C

and hence $V''' = \frac{Q''}{C''} = \frac{2 \times 10^{-8}\,\text{C}}{200 \times 10^{-12}\,\text{F}} = 100$ V

and $V_4 = \frac{Q_4}{C_4} = \frac{2 \times 10^{-8}\,\text{C}}{100 \times 10^{-12}\,\text{F}} = 200$ V

$\because$ C_1 and C' are in parallel, so

$V_1 = V' = V''$ or $V_1 = V' = 100$ V

and hence $Q_1 = C_1V_1 = 100 \times 10^{-12} \times 100$

$= 1 \times 10^{-8}$ C

and $Q' = C'V' = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8}$ C

$\because$ C_2 and C_3 are in parallel, so $Q_2 = Q_3 = Q'$

or $Q_2 = Q_3 = 1 \times 10^{-8}$ C

and hence $V_2 = \frac{Q_2}{C_2} = \frac{1 \times 10^{-8}\,\text{C}}{200 \times 10^{-12}\,\text{F}} = 50$ V

and $V_3 = \frac{Q_3}{C_3} = \frac{1 \times 10^{-8}\,\text{C}}{200 \times 10^{-12}\,\text{F}} = 50$ V.

Question 26.

The plates of a parallel plate capacitor have an area of 90 cm^2 eacn and are separated by 2.5 mm.The capacitor is charged by connecting it to a 400 V supply

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electro-static field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Solution:

$A = 90\ cm^2 = 90 \times 10^{-4}\ m^2,\ d = 2.5\ mm$
$= 2.5 \times 10^{-3}\ m$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 9 \times 10^{-3}}{2.5 \times 10^{-3}}$$

or $C = 32\ pF$

(a) $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 32 \times 10^{-12} \times 400^2$

or $U = 2.56\ \mu J$

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{\varepsilon_0 A}{d} \times (E.d)^2$

$= \frac{1}{2}\varepsilon_0 A E^2 d$

or $\frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$

or Energy per unit volume $u = \frac{1}{2}\varepsilon_0 E^2$

Question 27.

A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Solution:

$C_1 = 4\ \mu F,\ V_1 = 200\ V,\ C_2 = 2\ \mu F,\ V_2 = 0$

So, common potential difference across the two capacitors after connection is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{4\times10^{-6}\times200+0}{(4+2)\times10^{-6}} = 133.33\ V$$

Initially, total energy stored in capacitors before connection is

$$U_i = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times4\times10^{-6}\times200^2 = 0.08\ J$$

and total energy stored in capacitors after connection is

$$U_f = \frac{1}{2}(C_1+C_2)V^2 = \frac{1}{2}(4+2)\times10^{-6}\times133.33^2$$

or $U_f = 0.053\ J$

So, energy lost due to connection is

$\Delta U = U_f - U_i = 0.053 - 0.08$

or $\Delta U = -0.027\ J.$

Question 28.

Show that the force on each plate of a parallel plate capacitor has magnitude equal to

$\frac{1}{2}$

QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor

$\frac{1}{2}$

.

Solution:

Magnitude of electric field between the

plates of charged capacitor is

$E\frac{\sigma}{\varepsilon_0}$

However, magnitude of electric field of one plate or! the other plate of charged capacitor is

$E_1\frac{\sigma}{2\varepsilon_0}$

So, force on the one plate of charged capacitor due to the other is

$$F = QE_1 = Q\cdot\frac{\sigma}{2\varepsilon_0} = \frac{1}{2}\times Q\times\frac{\sigma}{\varepsilon_0} \quad \text{or} \quad F = \frac{1}{2}QE$$

The factor

$\frac{1}{2}$

is because the electric field of one 2 plate on the other plate of charged capacitor is

$\frac{1}{2}$

of the resultant electric field E between the 2 plates of charged capacitor.

Question 29.

A spherical capacitor consists of two concentric spherical conductors held in position by suitable insulating supports. Show that the capacitance of a spherical capacitor is given by

$$C = \frac{4\Pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

Where r_1 and r_2 are the radii of outer and inner spheres, respectively.

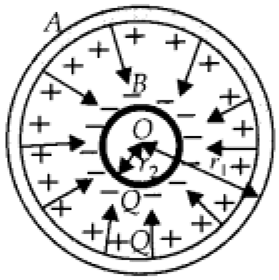
Solution:

Let + Q be the charge on outer spherical shell A of radius r, and + Q be the charge on inner spherical shell B of radius r2. Then electric potential on shell A is

$$V_A = V_{AA} + V_{AB} = \frac{1}{4\pi\varepsilon_0}\left[\frac{+Q}{r_1} - \frac{Q}{r_1}\right]$$

or $V_A = 0$

and electric potential on shell B is



$$V_B = V_{BA} + V_{BB} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r_1} - \frac{Q}{r_2}\right]$$

or $V_B = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$

So, the potential difference between the two spherical shells A and B is

$$V = V_A - V_B = 0 - \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

or $V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$

or $\frac{Q}{V} = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$ or $C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$

This gives the capacitance of the spherical capacitor.

Question 30.

A spherical capacitor has an inner sphere of radius 12 cm and outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 µC. The space between the concentric spheres is filled with liquid of dielectric constant 32.

(a) Determine the capacitance of the capacitor.

(b) What is the potential of the inner sphere.

(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Solution:

$r_1 = 13$ cm, $r_2 = 12$ cm, $K = 32$, $Q = 2.5\ \mu C$

(a) Capacitance of capacitor is

$$C = \frac{4\pi\varepsilon_0 k r_1 r_2}{r_1 - r_2} = \frac{1 \times 32}{9 \times 10^9} \times \frac{13 \times 10^{-2} \times 12 \times 10^{-2}}{(13-12) \times 10^{-2}}$$

or $C = 5.5 \times 10^{-9}$ F

(b) Electric potential of inner sphere is

$$V_B = V_{BB} + V_{BA}$$

$$= \frac{1}{4\pi\varepsilon_0 k}\left[+\frac{Q}{r_2} - \frac{Q}{r_1}\right] = \frac{Q}{4\pi\varepsilon_0 k}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

$$= \frac{9 \times 10^9}{32} 2.5 \times 10^{-6}\left[\frac{13-12}{13 \times 12}\right] \times \frac{10^{-2}}{10^{-4}}$$

$= 4.5 \times 10^2$ V.

(c) Capacitance of isolated sphere of radius 12 cm is

$$C_0 = 4\pi\varepsilon_0 r_2 = \frac{1}{9 \times 10^9} \times 12 \times 10^{-2}$$

or $C_0 = 1.3 \times 10^{-11}$ F

Here $C > C_0$, because a single conductor A can be charged to a electric potential till it reaches the breakdown value of surroundings. But when another earthed metallic conductor B is ; brought near it, negative charge induced on it decreases the electric potential on A, hence more charge can not be stored on A.

Question 31.

Answer carefully:

1. Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other, is the magnitude of electrostatic force between them exactly given by r is the distance between their centres?
2. If coulomb's law involved 1 /r^3 dependence (instead of 1/r^2), would Gauss's law be still true?
3. A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?
4. What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?
5. We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?
6. What meaning would you give to the capacitance of a single conductor?
7. Guess a possible reason why water has a much greater dielectric constant {- 80) then say, mica (= 6).

Solution:

1. No, because coulomb's law holds good only for point charges. '
2. No, because in that case electric flux linked with the closed surface will also become dependent on V other than charge enclosed by it.
3. No, it will travel along the field line only if it is a straight line.
4. Zero, whatever may be the shape of orbit may be. It is because work done in moving a charge in closed path in electric field is zero, as electric field is a conservative field.
5. No, electric potential is continuous there. As E = 0, so = 0 or V = constant.
6. It means that a single conductor is a capacitor whose other plate can be considered to be at infinity.
7. A water molecule is a polar molecule with non-zero electric dipole moment, however mica does not have polar molecules. So, dielectric constant of water is high.

Question 32.

A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e. bending of the field lines at the ends).

Solution:

Capacitance of cylindrical capacitor is

$$C = \frac{2\pi\varepsilon_0 L}{2.303\log_{10}\frac{b}{a}} = \frac{2\times15\times10^{-2}}{9\times10^{9}\times2.303\times\log_{10}\frac{1.5}{1.4}}$$

or $C = 1.21 \times 10^{-10}$ F

Electric potential of the inner cylinder is

$$V = \frac{Q}{C} = \frac{3.5\times10^{-6}\text{C}}{1.21\times10^{-10}\text{F}} \quad \text{or} \quad V = 2.89 \times 10^{4}\text{ V.}$$

Question 33.

A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 V m^{-1}. For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 μF?

Solution:

$V = 1 \times 10^3$ V, $K = 3$, $C = 50 \times 10^{-12}$ F

Given E = 10% of dielectric strength

or $E = \frac{10}{100} \times 10^7 \text{ V m}^{-1} = 10^6 \text{ V m}^{-1}$

As $V = E.d$, so $d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}$ m.

and $C = \frac{k\varepsilon_0 A}{d}$

or $A = \frac{C \cdot d}{k\varepsilon_0} = \frac{50 \times 10^{-12} \times 10^{-3}}{3 \times 8.85 \times 10^{-12}}$

or $A = 1.9 \times 10^{-3} \text{ m}^2 = 19 \text{ cm}^2$.

Question 34.

Describe schematically the equipotential surfaces corresponding to

- A constant electric field in the z-direction.
- a field that uniformly increases in magnitude but remains in a constant (say, z) direction.
- a single positive charge at the origin, and
- a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Solution:

- Planes parallel to x-y plane or normal to the electric field in z-direction.
- Planes parallel to x-y plane or normal to the electric field in z-direction, but the planes having different fixed potentials will become closer with increase in electric field intensity.
- Concentric spherical surfaces with their centres at origin.
- A time dependent changing shape nearer to grid, and at far off distances from the grid, it slowly becomes planar and parallel to the grid.

Question 35.

In a Van-de-Graff type generator, a spherical metal shell is to be a 15 × 106 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 V m^1. What is the minimum radius of the spherical shell required?

Solution:

$$V = 15 \times 10^6 \text{ V}, E = 5 \times 10^7 \text{ V m}^{-1}$$

As $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

$$R = \frac{V}{E} = \frac{15\times10^6}{5\times10^7} = 3 \times 10^{-1} \text{ m}$$

or $R = 0.3 \text{ m} = 30 \text{ cm}.$

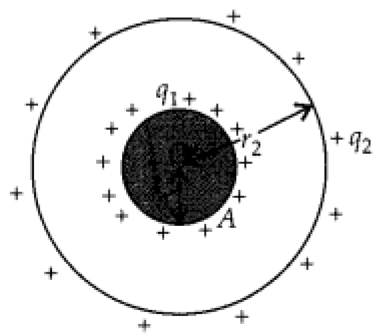
Question 36.

A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q: is positive, charge will necessary flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q^2 on the shell is.

Solution:

The potential on inner small sphere is VA

$V_A = V_{AA} + V_{AB}$



or $V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

whereas the potential on the outer shell B is

$V_B = V_{BA} + V_{BB} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

So, $V_A - V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} - \frac{q_1}{r_2}\right] = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$

As $r_2 > r_1$, so $V_A > V_B$ *i.e.* inner sphere A is at

higher potential than outer conducting shell B, for any value of charge qv So, when inner sphere A is connected to outer shell B, then charge will flow from inner sphere A to outer shell B, until electric potentials on them is same i.e.

$V_A – V_B = 0$ or $q_1 = 0$ [As r_1 # r_2]

So, charge ql given to sphere A will flow on the shell B, no matter what the charge on the shell B is.

Question 37.

Answer the following:

1. The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 V m^{-1}. Why then do we not get an

electric shock as we step out of our house into the open? (Assume the house to be a steel cage, so there is no field inside).

2. A man fixes outside house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m_2. Will he get an electric shock if he touches the metal sheet next morning?
3. The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?
4. What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?

Solution:

1. Our body and the ground are at same electric potential. As we step out into the open, the original equipotential surfaces of open air change, keeping our head and the ground at the same potential.
2. Yes, it is because the aluminium sheet gets charged due to discharging current and raises to the extent depending on the capacitor formed by the sheet and the ground.
3. The atmosphere of earth gets continuously charged due to lightning, thunderstorms but simultaneously it gets discharged through normal weather zones. This keeps the system balanced.
4. Light, sound and heat energy. Light energy in lightning and heat and sound energy in the accompanying thunder.

Chapterwise NCERT Solutions for Class 12 Physics :

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