NCERT Solutions for 12th Class Physics: Chapter 13-Nuclei

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Class 12: Physics Chapter 13 solutions. Complete Class 12 Physics Chapter 13 Notes.

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Question 1.

(a) Two stable isotopes of lithium

${}^{6}_{3}Li$

and

${}^{3}_{7}Li$

have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotope.s,

${}^{10}_{5}B$

and

${}^{11}_{5}B$

Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is

10.811 u. Find the abundances of

${}^{10}_{5}B$

and

${}^{11}_{5}B$

.

Solution:

Abundance of

${}^{6}_{3}Li$

is 7.5% and abundance

of ${}^{7}_{3}\text{Li}$ is 92.5%.

Hence atomic mass of lithium,

$$A = \frac{7.5\,(6.01512\text{ u}) + 92.5\,(7.01600\text{ u})}{100}$$

$$A = \frac{451134 + 648.98}{100}\text{ u} = 6.941\text{ u}$$

(b) Let abundance of

${}^{10}_{5}B$

x% than abundance of

${}^{11}_{5}B$

will be (100 – x)%.

Atomic mass of boron

$$= \frac{x[10.01294\text{ u}] + (100 - x)[11.00931\text{ u}]}{100}$$

$$\Rightarrow 100 \times 10.811\text{ u} = 1100.931\text{ u} - 0.99637x\text{ u}$$

Solving we get, $x = \frac{19.831}{0.99637} = 19.9\%$

So, relative abundance of ${}^{10}_{5}\text{B}$ isotope = 19.9%

Relative abundance of ${}^{11}_{5}\text{B}$ isotope = 80.1%

Question 2.

The three stable isotopes of neon :

$^{20}_{10}Ne$

,

$^{21}_{10}Ne$

and have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Solution:

Average atomic mass of neon with the given abundances,

$$A = \frac{90.51\,(19.99\text{ u}) + 0.27\,(20.99\text{ u}) + 9.22\,(21.99\text{ u})}{100}$$

$$A = \frac{2017.7}{100}\text{ u} = 20.18\text{ u}$$

Question 3.

Obtain the binding energy (in MeV) of a nitrogen nucleus

$(^{14}_{7}N)$

, given m

$(^{14}_{7}N)$

= 14.00307 u

Solution:

The nucleus contains 7 protons and

Question 4.

Obtain the binding energy of the nuclei

$^{56}_{26}Fe$

and

$^{209}_{83}Bi$

in units of MeV from the following data:

$m\left(^{56}_{26}Fe\right)$

= 55.934939 u

$m\left(^{209}_{83}Bi\right)$

= 208.980388 u

Solution:

Let us first find the binding energy of

$^{56}_{26}\text{Fe}$.

No. of protons in Fe = $Z = 26$

Mass of protons $= 26 \times 1.007825$ u

$= 26.203450$ u

No. of neutrons in Fe, $n = A - Z = 56 - 26 = 30$

Mass of neutrons $= 30 \times 1.008665$ u

$= 30.259950$ u

Total theoretical mass of nucleus

$= 26.203450$ u $+ 30.259950$ u $= 56.463400$ u

Actual mass of Fe nucleus 55.934939 u

Mass defect Δm = Total mass – Actual mass

$= 0.528461$ u

B.E. of $^{56}_{26}\text{Fe}$ nucleus $E = \Delta mc^2 = \Delta m\ 931.5$ MeV

$= 0.528461\ (931.5)$ MeV $= 492.26$ MeV

$$\frac{\text{B.E}}{\text{nucleon}} \text{ of } ^{56}_{26}\text{Fe} = \frac{492.26}{56} \text{ MeV} = 8.79 \text{ MeV}$$

(b) Now binding energy of $^{209}_{83}\text{Bi}$

No. of protons in Bi = $Z = 83$

No. of neutrons in Bi $\Rightarrow n = A - Z = 209 - 83$

$= 126$

Mass of protons $= 83 \times 1.007825$ u

$= 83.649475$ u

Mass of neutrons = 126 × 1.008665 u

= 127.091790 u

Total theoretical mass of nucleus

= 210.741265 u

Actual mass of Bi nucleus = 208.980388 u

Mass defect, Δm = 210.741260 − 208.980388

= 1.760877 u

B.E. of $^{209}_{83}\text{Bi}$ nucleus $\Rightarrow \Delta mc^2$

$\Rightarrow \quad \Delta m$ (931.5 MeV)

$\Rightarrow \quad 1.760877 \times 931.5$ MeV

$\Rightarrow \quad 1640.3$ MeV

$$\frac{\text{B.E}}{\text{nucleon}} \text{ of } ^{209}_{83}\text{Bi} = \frac{1640.3}{209} \text{ MeV} = 7.85 \text{ MeV}$$

So, $^{56}_{26}\text{Fe}$ is much more stable than $^{209}_{83}\text{Bi}$, due to more binding energy per nucleon.

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Question 5.

A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the

coin is entirely made of

$^{63}_{29}Cu$

atoms (of mass 62.92960 u).

Solution:

Let us first find the B.E. of each copper nucleus and then we can find binding energy of 300 g of $^{63}_{29}$Cu.

Mass of 29 protons = 29 × 1.00783 = 29.22707 u

Mass of 34 neutrons = 34 × 1.00867

= 34.29478 u

Total theoretical mass = 63.52185 u

Actual mass of Cu nucleus = 62.92960 u

Mass of defect = Theoretical mass – Actual mass

= 0.59225 u

B.E. of each Cu nucleus = Δm [931.5 MeV]

= 0.59225 [931.5 MeV] = 551.385 MeV

Number of atoms in 3 g of copper

$$n = \frac{\text{Avogadro number}}{\text{Mass number}} \times 3$$

or $$n = \frac{6.023 \times 10^{23} \times 3}{63} = 2.86 \times 10^{22}$$

Total binding energy in 3 g of copper

$= 2.86 \times 10^{22} \times 551.385$ MeV

$= 1.6 \times 10^{25}$ MeV

So, the energy required to separate all the neutrons and protons from each other in 3 g copper coin will be 1.6×10^{25} MeV.

Question 6.

Write nuclear reaction equations for

(i) a-decay of

$^{226}_{88}Ra$

(ii) a-decay of

$^{242}_{94}Pu$

(iii) p-decay of

$^{32}_{15}P$

(iv) p-decay of

$^{210}_{83}Bi$

(v) p+-decay of

$^{11}_{6}C$

(vi) p+-decay of

$^{97}_{43}Tc$

(vii) Electron capture of

$^{120}_{54}Xe$

Solution:

(i) $^{226}_{83}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$

(ii) $^{242}_{94}\mathrm{Pu} \rightarrow {}^{238}_{92}\mathrm{U} + {}^{4}_{2}\mathrm{He}$

(iii) $^{32}_{15}\mathrm{P} \rightarrow {}^{32}_{16}\mathrm{S} + e^{-} + \overline{v}$

(iv) $^{210}_{83}\mathrm{Bi} \rightarrow {}^{210}_{84}\mathrm{Po} + e^{-} + \overline{v}$

(v) $^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{5}\mathrm{B} + e^{+} + v$

(vi) $^{97}_{43}\mathrm{Tc} \rightarrow {}^{97}_{42}\mathrm{Mo} + e^{+} + v$

(vii) $^{120}_{54}\mathrm{Xe} + e^{+} \rightarrow {}^{120}_{53}\mathrm{I} + v$

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Question 7.

A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to

(a) 3.125%

(b) 1% of its original value?

Required time, as cannot be solved by direct calculation as in part (a).

Solution:

Activity $R = R_0 e^{-\lambda t}$

Also instantaneous activity, $R = -\lambda N$

$$R = -\frac{0.693}{T} N$$

Initial activity, $R_0 = -\lambda N_0$

So, $R_0 = -\frac{0.693}{T} N_0$

(a) $\frac{R}{R_0} = \frac{N}{N_0} = \frac{3.125}{100} = \frac{1}{32}$

or $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^5$ or $n = 5$

$\therefore$ $t = nT = 5T$ years.

(b) $\frac{R}{R_0} = \frac{N}{N_0} = e^{-\lambda t} = \frac{1}{100}$

Required time, as cannot be solved by direct calculation as in part (a)

$$t = \frac{2.303}{\lambda} \log \frac{N_0}{N} = \frac{2.303\,T}{0.693} \log 100$$

$$= \frac{2.303 \times 2 \times T}{0.693} \approx 6.65\,T \text{ years.}$$

Question 8.

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive

$^{14}_{6}C$

present with the stable carbon isotope

$^{12}_{6}C$

. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of

$^{14}_{6}C$

, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of

$^{14}_{6}C$

dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilization.

Solution:

In order to estimate age, let us first find the activity ratio in form of time ‘t’. Given normal activity, R0 = 15 decays min^{-1} Present activity, R = 9 decays min^{-1}, Tin = 5730 years Since activity is proportional to the number of radioactive atoms, therefore,

$$\frac{R}{R_0} = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

$$\text{or} \quad \frac{9}{15} = e^{-\lambda t} \quad \text{or} \quad e^{\lambda t} = \frac{15}{9}$$

Taking natural logarithm,

$$\log_e e^{\lambda t} = \log_e \frac{15}{9}$$

$$\text{or} \quad \lambda t \log_e e = 2.303 \log_{10} \frac{5}{3} = 2.303 \times 0.2218$$

$$\text{or} \quad t = \frac{0.5109}{\lambda} \qquad \ldots\text{(i)} \qquad [\because \log_e e = 1]$$

Now we know, half life, $T_{1/2} = \dfrac{0.693}{\lambda}$

$$\therefore \quad t = \frac{0.5109}{0.693 / T_{1/2}} = \frac{0.5109}{0.693} \times T_{1/2}$$

$$= \frac{0.5109 \times 5730}{0.693} \text{ years} = 4224 \text{ years.}$$

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Question 9.

Obtain the amount of ${}^{16}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of

${}^{16}_{27}Co$

is 5.3 years.

Solution:

Here rate of disintegration required

Half life $T_{1/2} = 5.3 \text{ years} = 5.3 \times 3.16 \times 10^7 \text{ s}$

But $R = \lambda N = \dfrac{0.693}{T_{1/2}} \cdot N$

No. of atoms for given rate required,

$$N = \frac{RT_{1/2}}{0.693}$$

$R = 8.0 \text{ mCi}$

$= 8.0 \times 10^{-3} \times 3.7 \times 10^{10} \text{ diss}^{-1}$

$= 29.6 \times 10^7 \text{ diss}^{-1}$

$$= \frac{29.6 \times 10^7 \times 5.3 \times 3.16 \times 10^7}{0.693}$$

$= 7.15 \times 10^{16}$ atoms

As 1 mole i.e., 60 g of cobalt contains 6.023×10^{23} atoms, so, the mass of cobalt required for given rate of disintegration

$$= \frac{60 \times 7.15 \times 10^{16}}{6.023 \times 10^{23}} = 7.123 \times 10^{-6} \text{ g.}$$

Question 10.

The half-life of

$^{90}_{38}Sr$

is 28 years. What is the disintegration rate of 15 mg of this isotope?

Solution:

Question 11.

Obtain approximately the ratio of the nuclear radii of the gold isotope

$^{197}_{79}Au$

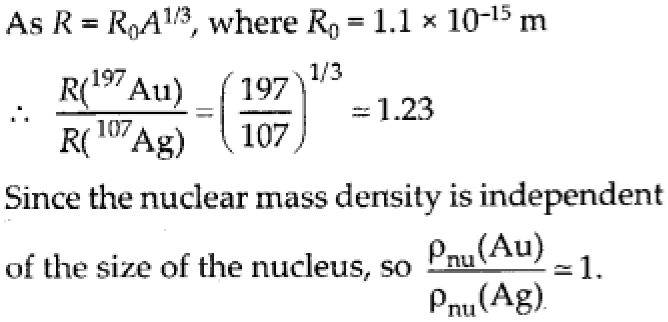
and the silver isotope

$^{107}_{47}Au$

.

Solution:

We know the radius of nucleus depend upon mass number 'A'



As $R = R_0A^{1/3}$, where $R_0 = 1.1 \times 10^{-15}$ m

$$\therefore \quad \frac{R(^{197}\text{Au})}{R(^{107}\text{Ag})} = \left(\frac{197}{107}\right)^{1/3} \simeq 1.23$$

Since the nuclear mass density is independent of the size of the nucleus, so $\frac{\rho_{nu}(\text{Au})}{\rho_{nu}(\text{Ag})} \simeq 1.$

Question 12.

Find the Q-value and the kinetic energy of the emitted a-particle in the a-decay of

(a)

$^{226}_{88}Ra$

and (b)

$^{220}_{86}Rn$

.

Given

$m\left(^{226}_{88}Ra\right)$

= 226.02540 u,

$m\left(^{222}_{86}Rn\right)$

= 222.01750 u,

$m\left(^{220}_{86}Rn\right)$

= 220.01137 u,

$m\left(^{216}_{84}Po\right)$

= 216.00189 u, and

m_x = 4.00260 u.

Solution:

(a) α-decay of $^{226}_{88}\text{Ra}$

$$^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\text{He} + Q$$

so, Q value

$$Q = [m(^{226}_{88}\text{Ra}) - m(^{222}_{86}\text{Rn}) - m(^{4}_{2}\text{He})]c^2$$

$$Q = [226.02540 - 222.01750 - 4.00260] \times 931.5\ \text{MeV}$$

$$Q = 0.0053 \times 931.5\ \text{MeV} = 4.937\ \text{MeV}$$

Kinetic energy of emitted α-particle

$$K_\alpha = \frac{Q}{A}(A - 4)$$

or $$K_\alpha = \frac{4.937}{226} \times (226 - 4)\ \text{MeV}$$

$$K_\alpha = 4.85\ \text{MeV}$$

(b) α-decay of $^{220}_{86}\text{Rn}$

$$^{220}_{86}\text{Rn} \rightarrow ^{216}_{84}\text{Po} + ^{4}_{2}\text{He} + Q$$

So, Q value

$$Q = \left[m(^{220}_{86}\text{Rn}) - m(^{216}_{84}\text{Po}) - m(^{4}_{2}\text{He})\right]c^2$$

$$Q = [220.01137 - 216.00189 - 4.00260]\, 931.5\ \text{MeV} = 6.41\ \text{MeV}$$

Kinetic energy of emitted α particle

Question 13.

The radionuclide “C decays according to

$$^{11}_{6}C \rightarrow ^{11}_{5} B$$

$+ e^+ + v$: $T_{1/2} = 20.3$ min

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

$m\left({}^{11}_{6}C\right)$

= 11.011434 u

$m\left({}^{11}_{5}B\right)$

= 11.009305 u

Calculate Q and compare it with the maximum energy of the positron emitted.

Solution:

The given equation

$${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} + \upsilon + Q$$

The Q value is

$$Q = \left[m({}^{11}_{6}\text{C}) - 6m_e - \underset{\text{(positron)}}{m({}^{11}_{5}\text{B})} + 5m_e - m_e\right]c^2$$

$$Q = [11.011434 - 11.009305 - 2 \times 0.000548]c^2$$
$$931.5 \text{ MeV} \approx 0.96 \text{ MeV}$$

As we know that different positrons comes out with different possible energies shared between daughter nucleus and positron.

So, the Q value of reaction is almost same as the maximum energy of positron emitted.

Question 14.

The nucleus

${}^{23}_{10}Ne$

decays by β^- emission. Write t down the β^- decay equation and determine

r the maximum kinetic energy of the electrons emitted. Given that:

$m\left({}^{23}_{10}Ne\right)$

= 22.994466 amu,

$m\left({}^{23}_{11}Na\right)$

= 22.989770 amu.

Solution:

The β^- decay of

${}^{23}_{10}Ne$

may be explained as

$${}^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{-1}e + \bar{v} + Q$$

The expression or the kinetic energy released may be written as

$$Q = \left[m({}^{23}_{10}\text{Ne}) - m({}^{23}_{11}\text{Na}) - m_e\right]c^2$$

$$\approx \left[m({}^{23}_{10}\text{Ne}) - m({}^{23}_{11}\text{Na})\right]c^2$$

$$\approx [22.994466 - 22.989770] \times 931.5 \text{ MeV}$$

$$\approx 0.004696 \times 931.5 \text{ MeV} = 4.374 \text{ MeV}$$

As

${}^{23}_{11}Na$

is massive, the kinetic energy released is mainly shared by electron-positron pair. When the neutrino carries no energy, the electron has a maximum kinetic energy equal to 4.374 MeV.

Question 15.

The Q value of a nuclear reaction A + b—>C+d is defined by $Q = [m_A + m_b - m_c - m_d]\, c^2$, where the masses refer to the respective nuclei, Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i)

${}^{1}_{1}H + {}^{3}_{1}H \rightarrow {}^{2}_{1}H + {}^{2}_{1}H$

(ii)

$${}^{12}_{6}C + {}^{12}_{6}C \rightarrow {}^{20}_{10}C + {}^{4}_{2}C$$

Atomic masses are given to be

$m({}^{1}_{1}\mathrm{H}) = 1.007825\,\mathrm{u},$

$m({}^{2}_{1}\mathrm{H}) = 2.014102\,\mathrm{u},$

$m({}^{3}_{1}\mathrm{H}) = 3.016049\,\mathrm{u},$

$m({}^{12}_{6}\mathrm{C}) = 12.000000\,\mathrm{u},$

$m({}^{20}_{10}\mathrm{Ne}) = 19.992439\,\mathrm{u}$

Solution:

(i) Let us find the Q value in given first equation,

$$\begin{aligned}
&{}^{1}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \\
&Q = \left[m({}^{1}_{1}\mathrm{H}) + m({}^{3}_{1}\mathrm{H}) - 2m({}^{2}_{1}\mathrm{H})\right]c^2 \\
&\quad = [1.007825 + 3.016049 - 2 \times 2.014102] \\
&\qquad\qquad\qquad\qquad \times (931\ \mathrm{MeV}) \\
&Q = [4.023874 - 4.028204]\ 931.5\ \mathrm{MeV} \\
&\quad = -4.033\ \mathrm{MeV}
\end{aligned}$$

Negative Q value shows that reaction is endothermic.

(ii) Q value in the given second equation

$${}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{20}_{10}\mathrm{Ne} + {}^{4}_{2}\mathrm{He}$$

$$Q = \left[2m({}^{12}_{6}\mathrm{C}) - m({}^{20}_{10}\mathrm{Ne}) - m({}^{4}_{2}\mathrm{He})\right]c^2$$

$$Q = [2 \times 12.0000 - 19.992439 - 4.002603] \times 931.5\ \mathrm{MeV}$$

$$Q = 0.004958 \times 931.5\ \mathrm{MeV} = 4.618\ \mathrm{MeV}$$

Positive Q shows that the reaction is exothermic.

Question 16.

Suppose, we think of fission of a

$${}^{56}_{26}Fe$$

nucleus into two equal fragments,

$${}^{28}_{13}AI$$

. IS the fission energetically possible? Argue by working out Q of the process.

Given,

$$m\left({}^{56}_{26}Fe\right)$$

= 55.93494 u

and

$$m\left({}^{28}_{13}Ai\right)$$

= 27.98191 u.

Solution:

The fission of Fe-56 into two fragments of

$^{28}_{13}\mathrm{Al}$ with energy released Q can be written as

$$^{56}_{26}\mathrm{Fe} \rightarrow {}^{28}_{13}\mathrm{Al} + {}^{28}_{13}\mathrm{Al} + Q$$

$$Q = \left[m({}^{56}_{26}\mathrm{Fe}) - 2m({}^{28}_{13}\mathrm{Al}) \right] c^2$$

$$= [55.93494 - 2 \times 27.98191] \times 931.5 \text{ MeV}$$

$$= -0.02888 \times 931.5 = -26.90 \text{ MeV}$$

As the Q-value is negative, the fission is not possible energycally.

Question 17.

The fission properties of

$^{239}_{94}Pu$

are very similar to those of

$^{235}_{92}U$

. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure

$^{239}_{94}Pu$

undergo fission?

Solution:

Number of atoms present in 1 mole *i.e.,*

239 g of $^{239}_{94}\mathrm{Pu} = 6.023 \times 10^{23}$

$\therefore$ Number of atoms present in 1000 g of $^{239}_{94}\mathrm{Pu}$

$$= \frac{6.023 \times 10^{23} \times 1000}{239} = 2.52 \times 10^{24}$$

Energy released per fission = 180 MeV

Total energy released = $2.52 \times 10^{24} \times 180$ MeV

$= 4.54 \times 10^{26}$ MeV.

Question 18.

A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much

$^{235}_{92}U$

did it contain initially? Assume that the reactor operates 80% of the time and that all the energy generated arises from the fission of

$^{235}_{92}U$

and that this nuclide is consumed by the fission process.

Solution:

In the fission of one nucleus of

$^{235}_{92}U$

, energy generated is 200 MeV.

$\therefore$ Energy generated in fission of 1 kg of $^{235}_{92}\text{U}$

$= 200 \times \dfrac{6.023 \times 10^{23}}{235} \times 1000\ \text{M}$

$= 5.106 \times 10^{26}$ MeV

$= 5.106 \times 10^{26} \times 1.6 \times 10^{-13}\ \text{J} = 8.17 \times 10^{13}\ \text{J}$

Time for which reactor operates

$= \dfrac{80}{100} \times 5\ \text{yr} = 4\ \text{yr}.$

Total energy generated in 5 years

$= 1000 \times 10^{6} \times 60 \times 60 \times 24 \times 365 \times 4\ \text{J}$

$\therefore$ Amount of $^{235}_{92}\text{U}$ consumed in 5 years

$= \dfrac{1000 \times 10^{6} \times 60 \times 60 \times 24 \times 365 \times 4}{8.17 \times 10^{13}}\ \text{kg} = 1544\ \text{kg}$

$\therefore$ Initial amount of $^{235}_{92}\text{U} = 2 \times 1544\ \text{kg} = 3088\ \text{kg}$

Question 19.

How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n + 3.27 MeV$$

Solution:

Number of atoms present in 2 g of deuterium = 6.023 × 10^{23} Total number of atoms present in 2000 g of deuterium

$$= \frac{6.023 \times 10^{23} \times 2000}{2} = 6.023 \times 10^{26}$$

Energy released in the fusion of 2 deuterium atoms = 3.27 MeV

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} = 9.81 \times 10^{26} \text{ MeV}$$

$$= 15.696 \times 10^{13} \text{ J}$$

Energy consumed by the bulb per second

$= 100$ J

Time for which the bulb will glow

$$t = \frac{15.69 \times 10^{13}}{100} \text{ s} \quad \text{or} \quad t = \frac{15.69 \times 10^{11}}{3.15 \times 10^{7}} \text{ years}$$

$$= 4.9 \times 10^{4} \text{ years.}$$

Question 20.

Calculate the height of potential barrier for a head-on collision of two deuterons. The effective radius of deuteron can be taken to be 2fm.

Solution:

For head on collision, distance between centers of two deuterons

$= r = 2 \times \text{radius}$

$r = 4 \text{ fm} = 4 \times 10^{-15} \text{ m}$

charge of each deuteron, $e = 1.6 \times 10^{-19}$ C

Potential energy

$$= \frac{e^2}{4\pi\varepsilon_0 r} = \frac{9\times10^9(1.6\times10^{-19})^2}{4\times10^{-15}} \text{ joule}$$

$$= \frac{9\times1.6\times1.6\times10^{-14}}{4\times1.6\times10^{-16}} \text{keV}$$

P.E. = 360 keV

P.E. = 2 × K.E. of each deuteron = 360 keV

$$\therefore \quad \text{K.E. of each deuteron} = \frac{360}{2} = 180 \text{ keV.}$$

This is a measure of height of coulomb barrier.

Question 21.

From the relation R = R_0 $A^{1/3}$, where R_2 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e., independent of A).

Solution:

$$\text{Density of nuclear matter} = \frac{\text{Mass of nucleus}}{\text{Volume}}$$

$$\rho = \frac{A\times1\,\text{amu}}{\frac{4}{3}\pi R^3}, \quad \text{where } R = R_0\ A^{1/3}$$

$$\text{Density, } \rho = \frac{A\times1\,\text{amu}}{\frac{4}{3}\pi R_0^3 A} = \frac{1\,\text{amu}}{\frac{4}{3}\pi R_0^3} = \frac{3\,\text{amu}}{4\pi R_0^3}$$

As R is constant, p is contact so, nuclear density is constant irrespective of mass number or size.

Question 22.

For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the /(-shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}^A_ZX \rightarrow {}_{Z-1}^{\;\;A}Y + v$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Solution:

Let us first consider positron emission.

$${}^A_ZX \rightarrow {}_{Z-1}^{\;\;A}Y + e^+ + v + Q_1$$

The Q value is

$$Q_1 = \left[m({}^A_ZX) - m({}_{Z-1}^{\;\;A}Y) - m_e\right]c^2 \qquad \ldots(i)$$

Let us now consider electron capture

$$e^- + {}^A_ZX \rightarrow {}_{Z-1}^{\;\;A}Y + v + Q_2$$

The Q value

$$Q_2 = \left[m({}^A_ZX) + m_e - m({}_{Z-1}^{\;\;A}Y)\right]c^2$$

So, $Q_2 > Q_1$

This mean if $Q_1 > 0$ then $Q_2 > 0$ but vice vesa is not necessarily allowed. So, electron capture is not necessary for positron emission.

Question 23.

In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are $^{24}_{12}Mg$ (23.98504 u),

$^{25}_{12}Mg$

(24.98584 u) and

$^{26}_{12}Mg$

(25.98259 u).The natural abundance of

$^{24}_{12}Mg$

78.99% by mass. Calculate the abundances of the other two isotopes.

Solution:

Let the abundance of isotope

$^{26}_{12}Mg$

is

$x\%$, then the abundance of isotope $^{25}_{12}\text{Mg}$ is $[100-(x+78.99)]\%$.

Average atomic mass of Mg 24.312 =

$$\frac{78.99 \times 23.98504 + [100-(x+78.99)]\,24.98584 + x\,[25.98259]}{100}$$

$$24.312 = 1894.5783 + 2498.564 + 0.99675\,x - 1973.6157$$

Question 24.

The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei

$^{41}_{20}Ca$

and

$^{27}_{13}Ai$

from

the following data:

$$m\left({}^{40}_{20}\text{Ca}\right) = 39.962591\,\text{u},$$

$$m\left({}^{41}_{20}\text{Ca}\right) = 40.962278\,\text{u},$$

$$m\left({}^{26}_{13}\text{Al}\right) = 25.986895\,\text{u},$$

$$m\left({}^{27}_{13}\text{Al}\right) = 26.981541\,\text{u},$$

$$m_n = 1.008665\,\text{u}$$

Solution:

Neutron separation of

$^{40}_{20}Ca$

can be obtained as E = Energy equivalent of total mass afterward – Energy equivalent of nucleus before

$$E = \left\{m({}^{40}_{20}\text{Ca}) + m_n - m({}^{41}_{20}\text{Ca})\right\}c^2$$

$$E = \{39.962591 + 1.008665 - 40.962278\}\ 931.5\ \text{MeV}$$

$$E = 0.008978 \times 931.5\ \text{MeV} = 8.363\ \text{MeV}$$

Similarly, neutron separation energy of $^{27}_{13}\text{Al}$ can be calculated as

E = [Energy equivalent of ^{26}Al + Energy equivalent of mass of neutron

– Energy equivalent of nucleus ^{27}Al before]

$$E = [m^{26}(\text{Al}) + m_n - m^{27}(\text{Al})]c^2$$

$$E = [25.986895 + 1.008665 - 26.981541]\ 931.5\ \text{MeV}$$

$$E = 0.014079 \times 931.5\ \text{MeV} = 13.058\ \text{MeV}.$$

Question 25.

A source contains two phosphorus radio -nuclides

${}^{32}_{15}P$

(T$_{1/2}$ = 14.3 days) and

${}^{32}_{15}P$

(Tv2 = 25.3 days). Initially, 10% of the decays come from

${}^{32}_{15}P$

How long one must wait until 90% do so?

Solution:

In the mixture of P-32 and P-33 initially 10% decay came from P-33. Hence initially 90% of the mixture is P-32 and 10% of the mixture is P-33. Let after time't' the mixture is left with 10% of P-32 and 90% of P-33. Half life of both P-32 and P-33 are given as 14.3 days and 25.3 days respectively. Let V be total mass undecayed initially and 'y' be total mass undecayed finally. Let initial number of P-32 nuclides = 0.9 x Final number of P-32 nuclides = 0.1 y Similarly, initial number of P-33 nuclides = 0.I x Final number of P-33 nuclides = 0.9 y For isotope P-32

$$N = N_0\, 2^{-t/T_{1/2}} \text{ or } 0.1\,y = 0.9\,x\,2^{-t/14.3} \qquad \text{...(i)}$$

For isotope P-33

$$N = N_0\, 2^{-t/T_{1/2}} \text{ or } 0.9\,y = 0.1\,x\,2^{-t/25.3} \qquad \text{...(ii)}$$

On dividing, we get

$$\frac{1}{9} = 9\,\frac{2^{-t/14.3}}{2^{-t/25.3}} \text{ or } \frac{1}{81} = 2^{\left(-\frac{t}{14.3}+\frac{t}{25.3}\right)}$$

$$\frac{1}{81} = 2^{-t\left[\frac{11}{14.3\times 25.3}\right]} \text{ or } 81 = 2^{t\left[\frac{11}{14.3\times 25.3}\right]}$$

Taking log

$$\log_e 81 = t\left(\frac{11}{14.3\times 25.3}\right)\log_e 2$$

$$\text{or} \quad 1.9082 = \frac{11\,t}{25.3\times 14.3}\times 0.3010$$

$$t = 208.5 \text{ days} \simeq 209 \text{ days}$$

Question 26.

Under certain circumstances, a nucleus can decay by emitting a particle more massive than an a-partide. Consider the following decay processes:

$${}^{223}_{88}\text{Ra} \rightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C},$$

$${}^{223}_{88}\text{Ra} \rightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Solution:

Let us calculate Q value for the given decay process. For first decay process

$$Q = m\left({}^{223}_{88}\text{Ra}\right) - m\left({}^{209}_{82}\text{Pb}\right) - m\left({}^{14}_{6}\text{C}\right)$$

$$Q = [223.01850 - 208.98107 - 14.00324]\ (c^2)\text{u}$$
$$= [0.034109] \times 931.5\ \text{MeV} = 31.85\ \text{MeV}$$

For the second decay process

$$Q = m\left({}^{223}_{88}\text{Ra}\right) - m\left({}^{219}_{86}\text{Rn}\right) - m\left({}^{4}_{2}\text{He}\right)$$

$$Q = [223.01850 - 219.00948 - 4.00260]\ c^2\ \text{u}$$
$$Q = 0.00642 \times 931.5\ \text{MeV} = 5.98\ \text{MeV}$$

Since, Q value is positive in both the cases, hence decay process in both ways are possible

Question 27.

Consider the fission of

$${}^{238}_{92}U$$

by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments are

$${}^{140}_{58}Ce$$

and

$${}^{32}_{15}Ru$$

. Calculate Q for this fission process. The relevant atomic and particle masses are:

$$m\left({}^{238}_{92}\text{U}\right) = 238.05079\,\text{u},$$
$$m\left({}^{140}_{58}\text{Ce}\right) = 139.90543\,\text{u},$$
$$m\left({}^{99}_{44}\text{Ru}\right) = 98.90594\,\text{u}$$

Solution:

The fission of U-238 by fast neutrons into fragments Ce-140 and Ru-99 with energy released Q can be written as

$$^{238}_{92}\text{U} + ^{1}_{1}n \rightarrow ^{140}_{58}\text{Ce} + ^{99}_{34}\text{Ru} + Q$$

The Q value,

$$Q = [m(\text{U-238}) + m_n - m\ (\text{Ce-140}) - m(\text{Ru-99})c^2$$

$$Q = [238.05079 + 1.00867 - 139.90543 - 98.90594]\ \text{amu} \times c^2$$

$$Q = 0.24809 \times 931.5\ \text{MeV} = 231.09\ \text{MeV}$$

$$= 231.1\ \text{MeV.}$$

Question 28.

Consider the D-T reaction (deuterium – tritium fusion)

$$^{2}_{1}H +^{3}_{1} H \rightarrow^{4}_{2} He + n$$

(a) Calculate the energy released in MeV in this reaction from the data

m

$(^{2}_{1}H)$

= 2.014102 u, m

$(^{3}_{1}H)$

= 3.016049 u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gases be heated to initiate the reaction?

Solution:

(a) For the process ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + n + Q$

$$Q = [m({}^2_1\text{H}) + m({}^3_1\text{H}) - m({}^4_2\text{He}) - m_n] \times 931 \text{ MeV}$$

$$= [2.014102 + 3.016049 - 4.002603 - 1.00867] \times 931 \text{ MeV}$$

$$= 0.018878 \times 931 = 17.58 \text{ MeV}$$

(b) Repulsive potential energy of two nuclei when they almost touch each other is

$$= \frac{q^2}{4\pi\varepsilon_0(2r)}$$

$$= \frac{9\times10^9(1.6\times10^{-19})^2}{2\times2\times10^{-15}} \text{joule}$$

$$= 5.76\times10^{-14} \text{ joule.}$$

Classically, K.E. at least equal to this amount is required to overcome Coulomb repulsion. Using the relation

$$\text{K.E.} = 2\times\frac{3}{2}kT$$

$$T = \frac{(\text{K.E.})}{3k} = \frac{5.76\times10^{-14}}{3\times1.38\times10^{-23}} = 1.39\times10^{-9} \text{ K}$$

Question 29.

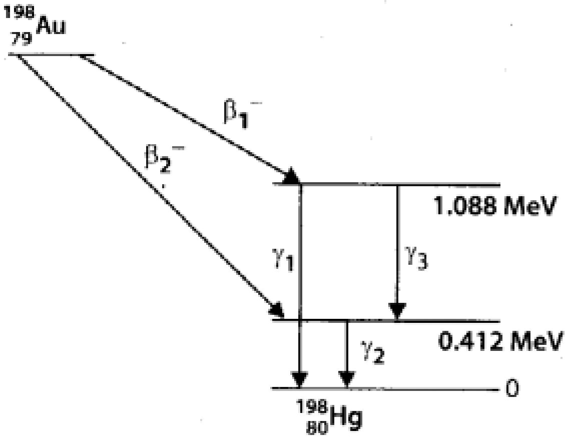
Obtain the maximum kinetic energy of β-particles and the radiation frequencies of y-decays in the decay scheme shown in figure. You are given that

$m\left({}^{198}_{79}Au\right)$

= 197.968233 u,

$m\left({}^{198}_{80}Ag\right)$

= 197.966760 u



Solution:

Energy corresponding to y_1

$$E_1 = 1.088 - 0 = 1.088 \text{ MeV}$$

$$= 1.088 \times 1.6 \times 10^{-13} \text{ joule}$$

$\therefore$ Frequency, $\upsilon(\gamma_1)$

$$= \frac{E_1}{h} = \frac{1.088 \times 1.6 \times 10^{-13}}{6.63 \times 10^{-34}} = 2.626 \times 10^{20} \text{ H}$$

$$\text{Similarly, } \upsilon(\gamma_2) = \frac{E_2}{h} = \frac{0.412 \times 1.6 \times 10^{-13}}{6.63 \times 10^{-34}}$$

$$= 9.95 \times 10^{19} \text{ Hz}$$

$$\text{and } \upsilon(\gamma_3) = \frac{E_3}{h} = \frac{(1.088 - 0.412) \times 1.6 \times 10^{-13}}{6.63 \times 10^{-34}}$$

$$= 1.631 \times 10^{20} \text{ Hz}$$

Maximum K.E. of β_1 particle

$$K_{\max}(\beta_1) = [m(^{198}_{79}\text{Au}) - \text{mass of second excited state of } ^{198}_{80}\text{Hg}] \times 931 \text{ MeV}$$

$$= \left[m(^{198}_{79}\text{Au} - m(^{198}_{80}\text{Hg}) - \frac{1.088}{931} \right] \times 931 \text{ MeV}$$

$$= 931\,[197.968233 - 197.966760] - 1.088 \text{ MeV}$$

$$= 1.371 - 1.088 = 0.283 \text{ MeV}$$

Similarly, $K_{\max}(\beta_2) = 0.957$ MeV.

Question 30.

Calculate and compare the energy released by

(a) fusion of 1.0 kg of hydrogen deep within the Sun and

(b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Solution:

(a) In the fusion reactions taking place within core of sun, 4 hydrogen nuclei combines to form a helium nucleus with the release of 26 MeV of energy.

$$4\,{}_1^1\text{H} \rightarrow {}_2^4\text{He} + 2e^+ + 26\,\text{MeV}$$

Number of atoms in 1 kg of ${}_1^1\text{H}$,

$$n = \frac{1000\text{ g} \times 6 \times 10^{23}}{\text{Atomic mass}} = \frac{1000\text{ g}}{1\text{ g}} \times 6 \times 10^{23}$$

$$= 6 \times 10^{26}\text{ atoms}$$

Energy released in the fusion of 1 kg of ${}_1^1\text{H}$,

$$E_1 = \frac{6 \times 10^{26} \times 26}{4}\text{ MeV} = 39 \times 10^{26}\text{ MeV}$$

(b) Energy released per fission of U-235 is 200 MeV.

Number of atoms in 1 kg of U-235,

$$n = \frac{1000\text{ g} \times 6 \times 10^{23}}{235\text{ g}} = 25.53 \times 10^{23}\text{ atoms}$$

Total energy released for fission of 1 kg of uranium,

$$E_2 = 25.53 \times 10^{23} \times 200\text{ MeV} = 5.1 \times 10^{26}\text{ MeV}$$

$$\frac{E_1}{E_2} = \frac{39 \times 10^{26}}{5.1 \times 10^{26}} = 7.65 \approx 8$$

So the energy released in fusion of 1 kg of Hydrogen is nearly 8 times the energy released in fission of 1 kg of uranium-235.

Question 31.

Suppose India has a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which is to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (/.e., conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year? Take the heat energy per fission of ^{235}U to be about 200 MeV.

Solution:

10% of total power 200,000 MW to be obtained from nuclear power plant by 2020 AD.

So, power from nuclear plants

$= 2 \times 10^5 \times 0.1$ MW

$= 2 \times 10^4$ MW $= 2 \times 10^{10}$ W

With efficiency of power plants 25% only, the energy converted into electrical energy per fission $= \frac{25}{100} \times 200 = 50$ Mev

$= 50 \times 1.6 \times 10^{-13}$ Joule $= 8 \times 10^{-3}$

Total energy to be produced

$= 2 \times 10^4$ MW $= 2 \times 10^{10}$ joule/sec

$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365$ joule / year

$$= \frac{2 \times 10^{24} \times 36 \times 24 \times 365}{8}$$

Mass of 6.023×10^{23} atoms of 235 U = 235 g

$= 235 \times 10^{-3}$ kg

Mass of $\frac{2 \times 36 \times 24 \times 365}{8} \times 10^{24}$ atoms

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times \frac{2 \times 36 \times 24 \times 365 \times 10^{24}}{8}$$

$= 3.08 \times 10^4$ kg

Hence mass of uranium needed per year = 3.08×10^4 kg

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- Chapter-2: Electrostatic Potential and Capacitance
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