NCERT

National Council Of Educational Research And Training

NCERT Solutions for 12th Class Maths: Chapter 9-Differential Equations

IndCareer Schools

indCareer indCareer indCareer

# NCERT Solutions for 12th Class Maths: Chapter 9-Differential Equations

Class 12: Maths Chapter 9 solutions. Complete Class 12 Maths Chapter 9 Notes.

## NCERT Solutions for 12th Class Maths: Chapter 9-Differential Equations

Class 12: Maths Chapter 9 solutions. Complete Class 12 Maths Chapter 9 Notes.

**Ex 9.1 Class 12 Maths Question 1.**

$$\frac{d^4y}{dx^4} + (siny^{III}) = 0$$

https://www.indcareer.com/schools/ncert-solutions-for-12th-class-maths-chapter-9-differential-equations/

**Solution:**

Order of the equation is 4

It is not a polynomial in derivatives so that it

has not degree.

**Ex 9.1 Class 12 Maths Question 2.**

$$y^{I} + 5y = 0$$

**Solution:**

$$y^{I} + 5y = 0$$

It is a D.E. of order one and degree one.

**Ex 9.1 Class 12 Maths Question 3.**

$$\left(\frac{ds}{dt}\right)^4 + 3s\left(\frac{d^2s}{dt^2}\right) = 0$$

**Solution:**

Order of the equation is 2.

Degree of the equation is

**Ex 9.1 Class 12 Maths Question 4.**

$$\left(\frac{d^2y}{dx^2}\right)^2 + cos\left(\frac{dy}{dx}\right) = 0$$

**Solution:**

$$\left(\frac{d^2y}{dx^2}\right)^2 + cos\left(\frac{dy}{dx}\right) = 0$$

It is a D.E. of order 2 and degree undefined

**Ex 9.1 Class 12 Maths Question 5.**

$$\frac{d^2y}{dx^2} = cos3x + sin3x$$

**Solution:**

$$\frac{d^2y}{dx^2} = cos3x + sin3x$$

It is a D.E. of order 2 and degree 1.

**Ex 9.1 Class 12 Maths Question 6.**

$$\left(y^{III}\right)^2 + \left(y^{II}\right)^3 + \left(y^{I}\right)^4 + y^5 = 0$$

**Solution:**

Order of the equation is 3

Degree of the equation is 2

**Ex 9.1 Class 12 Maths Question 7.**

$$y^{III} + 2y^{II} + y^{I} = 0$$

**Solution:**

$$y^{III} + 2y^{II} + y^{I} = 0$$

The highest order derivative is y.

Thus the order of the D.E. is 3.

The degree of D.E is 1

**Ex 9.1 Class 12 Maths Question 8.**

$$y^{I} + y = e^{x}$$

**Solution:**

$$y^{I} + y = e^{x}$$

The order of the D. E. = 1 (highest order derivative)

The degree of the D.E. = 1.

**Ex 9.1 Class 12 Maths Question 9.**

$$y^{III} + (y^{I})^{2} + 2y = 0$$

**Solution:**

$$y^{III} + (y^{I})^2 + 2y = 0$$

The highest derivative is 2.

Order of the D.E. = 2.

Degree of the D. E = 1

**Ex 9.1 Class 12 Maths Question 10.**

$$y^{II} + 2y^{I} + siny = 0$$

**Solution:**

Order of the equation is 2

Degree of the equation is 1

**Ex 9.1 Class 12 Maths Question 11.**

The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + sin\left(\frac{dy}{dx}\right) + 1 = 0$$

(a) 3

(b) 2

(c) 1

(d) not defined

**Solution:**

$$\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+sin\left(\frac{dy}{dx}\right)+1=0$$

The degree not defined.

Because the differential equation can not be written as a polynomial in all the differential coefficients.

Hence option (d) is correct.

**Ex 9.1 Class 12 Maths Question 12.**

The order of the differential equation

$$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$$

(a) 2

(b) 1

(c) 0

(d) not defined

**Solution:**

$$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$$

Thus order of the D.E. = 2

Hence option (a) is correct.

**Ex 9.2 Class 12 Maths Question 1.**

**Solution:**

$$y = e^x + 1 : y^{II} - y^I = 0$$

$y = e^x + 1 \Rightarrow y' = e^x \Rightarrow y'' = e^x$
Now L.H.S. $= y'' - y' = e^x - e^x = 0$
Hence $y = e^x + 1$ is a solution of $y'' - y' = 0$.

**Ex 9.2 Class 12 Maths Question 2.**

$$y = x^2 + 2x + c : y^I - 2x - 2 = 0$$

**Solution:**

$$y = x^2 + 2x + c : y^I - 2x - 2 = 0$$

$y = x^2 + 2x + c$
$\therefore y' = 2x + 2 \Rightarrow y' - 2x - 2 = 0$
Hence $y = x^2 + 2x + c$ is a solution of
$y' - 2x - 2 = 0$.

**Ex 9.2 Class 12 Maths Question 3.**

$y = cosx + c : y^I + sinx = 0$

**Solution:**

$y = cosx + c : y^I + sinx = 0$

$y = \cos x + c \Rightarrow y' = -\sin x$

$\Rightarrow y' + \sin x = 0$

Hence $y = \cos x + c$ is a solution of

$y' + \sin x = 0.$

**Ex 9.2 Class 12 Maths Question 4.**

$y = \sqrt{1 + x^2} : y^I = \frac{xy}{1+x^2}$

**Solution:**

$y = \sqrt{1 + x^2} : y^I = \frac{xy}{1+x^2}$

$$y=\sqrt{1+x^2} \Rightarrow y'=\frac{1}{2}(1+x^2)^{-1/2}\times 2x$$

$$y'=\frac{x}{\sqrt{1+x^2}}$$

Thus $y'=\frac{xy}{1+x^2}$

Hence $y=\sqrt{1+x^2}$ is a solution of $y'=\frac{xy}{1+x^2}$.

**Ex 9.2 Class 12 Maths Question 5.**

$$y = Ax : xy^I = y(x \neq 0)$$

**Solution:**

$$y = Ax : xy^I = y(x \neq 0)$$

$y = Ax \Rightarrow y' = A$

L.H.S. $= xy' = xA = y =$ R.H.S.

Hence $y = Ax$ is a solution of $xy' = y$.

**Ex 9.2 Class 12 Maths Question 6.**

$$y = x \quad sinx; xy^I = y + x\sqrt{x^2 - y^2}(x \neq 0 \quad and \quad x > y \quad or \quad x < -y)$$

**Solution:**

$y = x \quad sinx; xy^I = y + x\sqrt{x^2 - y^2}(x \neq 0 \quad and \quad x > y \quad or \quad x < -y)$

$y = x \sin x$ ...(i)

Differentiating w.r.t. x $\quad y' = 1. \sin x + x \cos x$

$y' = \sin x + x \cos x \; [\because \sin^2 x + \cos^2 x = 1]$

from (i) $\sin x = \frac{y}{x} \Rightarrow y = x \sin x$

$\Rightarrow y' = \sin x + x \cos x$

$y' = \sin x + x\sqrt{1 - \sin^2 x}$ ... (ii)

from (i) & (ii)

$$y' = \frac{y}{x} + x\sqrt{1 - \frac{y^2}{x^2}} = \frac{y}{x} + x\sqrt{\frac{x^2 - y^2}{x^2}}$$

Multiplying by x, $xy' = y + x\sqrt{x^2 - y^2}$ which is the reqd. differential equation.

**Ex 9.2 Class 12 Maths Question 7.**

xy = logy + C,

$$(1 - xy)\frac{dy}{dx} = y^2$$

**Solution:**

xy = logy + C,

$$(1 - xy)\frac{dy}{dx} = y^2$$

$xy = \log y + c$ Differentiating w.r.t. x

$1 \cdot y + xy' = \frac{1}{y} \cdot y'$ or $y^2 + xyy' = y'$

or $y^2 = y' - xyy' = y'(1 - xy)$ $\therefore$ $y' = \frac{y^2}{1 - xy}$

This is the reqd. differential equation.

**Ex 9.2 Class 12 Maths Question 8.**

**Solution:**

$$y - cosy = x : (ysiny + cosy + x)y^I = y$$

$y - \cos y = x \Rightarrow y' + \sin y y' = 1$

$y'(1 + \sin y) = 1 \Rightarrow y' = \frac{1}{1 + \sin y}$

L.H.S. $= (y \sin y + \cos y + x)y'$

$= (y \sin y + y)y'$ ($\because$ $y = x + \cos y$)

$= y(1 + \sin y)y'$

$= (1 + \sin y)y.\frac{1}{1 + \sin y} = y$ = R.H.S.

Here $y - \cos y = x$ is a solution of

$(y \sin y + \cos y + x)y' = y.$

**Ex 9.2 Class 12 Maths Question 9.**

$$x + y = tan^{-1}y; y^2y^I + y^2 + 1 = 0$$

**Solution:**

$$x + y = tan^{-1}y; y^2y^I + y^2 + 1 = 0$$

$x + y = \tan^{-1} y$

Differentiating w.r.t. x $\quad 1 + y' = \frac{y'}{1 + y^2}$

$(1 + y^2) + y'(1 + y^2) = y'$

or $\quad y^2 y' + y^2 + 1 = 0$

which is the reqd. differential equation.

**Ex 9.2 Class 12 Maths Question 10.**

$$y = \sqrt{a^2 - x^2} x \in (-a, a); x + y\frac{dy}{dx} = 0, (y \neq 0)$$

**Solution:**

$$y = \sqrt{a^2 - x^2} x \in (-a, a); x + y\frac{dy}{dx} = 0, (y \neq 0)$$

$y = \sqrt{a^2 - x^2}$ Squaring both sides

$y^2 = a^2 - x^2 \quad$ or $\quad x^2 + y^2 = a^2$

Differentiating w.r.t. x: $2x + 2yy' = 0$

$\Rightarrow 2\left(x + y\frac{dy}{dx}\right) = 0$ is the required solution.

**Ex 9.2 Class 12 Maths Question 11.**

The number of arbitrary constants in the general solution of a differential equation of fourth order are:

(a) 0

(b) 2

(c) 3

(d) 4

**Solution:**

(b) The general solution of a differential equation of fourth order has 4 arbitrary constants.

Because it contains the same number of arbitrary constants as the order of differential equation.

**Ex 9.2 Class 12 Maths Question 12.**

The number of arbitrary constants in the particular solution of a differential equation of third order are:

(a) 3

(b) 2

(c) 1

(d) 0

**Solution:**

(d) Number of arbitrary constants = 0

Because particular solution is free from arbitrary constants.

**Ex 9.3 Class 12 Maths Question 1.**

$$\frac{x}{a}+\frac{y}{b}=1$$

**Solution:**

Given that

$\frac{x}{a} + \frac{y}{b} = 1$

…(i)

differentiating (i) w.r.t x, we get

$\frac{1}{a} + \frac{1}{b}y^{I} = 0$

…(ii)

again differentiating w.r.t x, we get

$\frac{1}{b}y^{II} = 0 \Rightarrow y^{II} = 0$

which is the required differential equation

**Ex 9.3 Class 12 Maths Question 2.**

$y^2 = a(b^2 – x^2)$

**Solution:**

given that

$y^2 = a(b^2 – x^2)$…(i)

Differentiating w.r.t x; $2yy' = -2ax$ ... (ii)
Again differentiating, $2(yy'' + y'y') = -2a$ ... (iii)
$\Rightarrow 2(y'^2 + yy'') = -2a$

Dividing (iii) by (ii): $\frac{2(y'^2 + yy'')}{2yy'} = \frac{-2a}{-2ax}$
$\Rightarrow x(y'^2 + yy'') = yy'$; i.e. the differential equation

$$xy\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

**Ex 9.3 Class 12 Maths Question 3.**

$y = ae^{3x}+be^{-2x}$

**Solution:**

Given that

$y = ae^{3x}+be^{-2x}$ …(i)

Differentiating w.r.t. x : $y' = 3ae^{3x} - 2be^{-2x}$ ...(ii)
Again differentiating: $y'' = 9ae^{3x} + 4be^{-2x}$ ... (iii)
Multiply equation (i) by 2 and add with (ii) we get;

*i.e.,* $2y = 2ae^{3x} + 2be^{-2x}$
$y' = 3ae^{3x} - 2be^{-2x}$
$2y + y' = 5ae^{3x}$ or $ae^{3x} = \frac{2y + y'}{5}$

Multiply (i) by 3 and subtract (ii) from

$3y - y' = 5be^{-2x}$ or $be^{-2x} = \frac{3y - y'}{5}$

Putting the values of a and b in (iii), we get;

$5y'' = 30y + 5y'$ or $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$

Which is the req. differential equation

**Ex 9.3 Class 12 Maths Question 4.**

$y = e^{2x}(a+bx)$

**Solution:**

y = $e^{2x}$ (a+bx)

$$y' = 2y + be^{2x} \quad \text{.............(ii)}$$
$$y'' = 2y' + 2be^{2x} \quad \text{.............(iii)}$$
Subtracting (iii) from (ii) we get :
$$2y' - y'' = 4y - 2y' \quad \Rightarrow \quad y'' - 4y' + 4y = 0$$

**Ex 9.3 Class 12 Maths Question 5.**

y = $e^x$(a cosx+b sinx)

**Solution:**

The curve y = $e^x$(a cosx+b sinx) …(i)

differentiating w.r.t x

$$y' = e^x (a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$$
$$y' = e^x [(a+b)\cos x - (a-b)\sin x] \quad \text{... (ii)}$$
$$y'' = e^x [a+b)\cos x - (a-b)\sin x] + e^x[-(a+b)\sin x - (a-b)\cos x]$$
$$= e^x[2b\cos x - 2a\sin x] = 2e^x[b\cos x - a\sin x]$$
$$\text{or } \frac{y''}{2} = e^x(b\cos x - a\sin x) \quad \text{... (iii)}$$
Adding (ii) and (iii)
$$y + \frac{y''}{2} = e^x[(a+b)\cos x - (a-b)\sin x] = y'$$
$$\text{or } 2y + y'' = 2y' \quad \Rightarrow y'' - 2y' + 2y = 0$$
$$\text{Hence the diff. equ. is } \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

**Ex 9.3 Class 12 Maths Question 6.**

Form the differential equation of the family of circles touching the y axis at origin

**Solution:**

The equation of the circle with centre (a, 0) and radius a, which touches y- axis at origin

$(x-a)^2 + y^2 = a^2$

or $x^2 + y^2 = 2ax$ ... (i)

Differentiating w.r.t. x

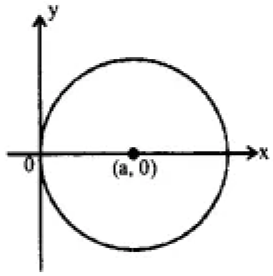
$2x + 2yy' = 2a$

or $x + yy' = a$

Putting value of a in (i)

$x^2 + y^2 = 2x(x + yy') = 2x^2 + 2xyy'$

$\therefore$ Reqd. diff. equation is: $2xy\frac{dy}{dx} + x^2 - y^2 = 0$




**Ex 9.3 Class 12 Maths Question 7.**

Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

**Solution:**

The equation of parabola having vertex at the origin and axis along positive y-axis is

$x^2 = 4ay$ ... (i)

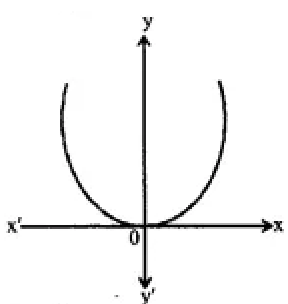
Differentiating w.r.t x

$2x = 4ay'$ ... (ii)

Dividing (ii) by (i)

$\frac{2x}{x^2} = \frac{4ay'}{4ay}$ $\therefore$ $xy' = 2y$

Regd. diff. eq. is: $x\frac{dy}{dx} - 2y = 0$



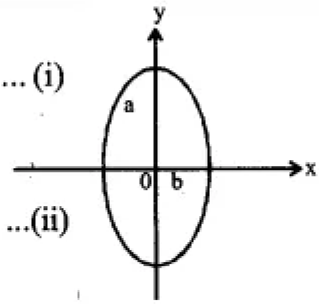

**Ex 9.3 Class 12 Maths Question 8.**

Form the differential equation of family of ellipses having foci on y-axis and centre at origin.

**Solution:**

The equation of family ellipses having foci at y- axis is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, a > b \quad \text{... (i)}$$




Differentiating w.r.t. x

$$\frac{x}{b^2} + \frac{yy'}{a^2} = 0 \quad \text{...(ii)}$$

Again Differentiating;

$$\frac{1}{b^2} + \frac{1}{a^2}(y'^2 + yy'') = 0 \text{ or } \frac{1}{b^2} = -\frac{y'^2 + yy''}{a^2}$$

Putting this value in (ii)

$$-\frac{x(y'^2 + y\,y'')}{a^2} + \frac{yy'}{a^2} = 0 \text{ or } xyy'' + xy'^2 - yy' = 0$$

$\therefore$ Diff. eq. is :

$$xy\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 0$$

**Ex 9.3 Class 12 Maths Question 9.**

Form the differential equation of the family of hyperbolas having foci on x-axis and centre at the origin.

**Solution:**

Equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Differentiating both sides w.r.t x

$$\Rightarrow \frac{x}{a^2} - \frac{y}{b^2}\frac{dy}{dx} = 0$$

Again differentiating, we get

$$\frac{1}{a^2} - \frac{1}{b^2}\left[y'^2 + y.y''\right] = 0 \Rightarrow \frac{b^2}{a^2} = yy'' + (y')^2$$

$$\Rightarrow yy' = xyy'' + x(y')^2$$

which is the req. differential eq. of the hyperbola.

**Ex 9.3 Class 12 Maths Question 10.**

Form the differential equation of the family of circles having centre on y-axis and radius 3 units

**Solution:**

Let centre be (0, a) and r = 3

Equation of circle is

$x^2 + (y – a)^2 = 9$ …(i)

Differentiating both sides, we get

$$(y-a)\frac{dy}{dx} = -x \Rightarrow y - a = \frac{-x}{y'} \quad \text{...........(ii)}$$

From (i) and (ii), we get

$$x^2 + \frac{x^2}{(y')^2} = 9 \Rightarrow x^2[(y')^2 + 1] = 9(y')^2$$

which is required equation

**Ex 9.3 Class 12 Maths Question 11.**

Which of the following differential equation has

$y = c_1 e^x + c_2 e^{-x}$

as the general solution ?

(a)

$\frac{d^2y}{dx^2} + y = 0$

(b)

$\frac{d^2y}{dx^2} - y = 0$

(c)

$\frac{d^2y}{dx^2} + 1 = 0$

(d)

$\frac{d^2y}{dx^2} - 1 = 0$

**Solution:**

(b)

$y = c_1e^x + c_2e^{-x} \Rightarrow \frac{dy}{dx} = c_1e^x - c_2e^{-x}$

$\frac{d^2y}{dx^2} = c_1e^x + c_2e^{-x} \Rightarrow \frac{d^2y}{dx^2} - y = 0$

**Ex 9.3 Class 12 Maths Question 12.**

Which of the following differential equations has y = x as one of its particular solution ?

(a)

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$$

(b)

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$$

(c)

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$$

(d)

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$$

**Solution:**

(c) y = x

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$$

**Ex 9.4 Class 12 Maths Question 1.**

$$\frac{dy}{dx} = \frac{1-cosx}{1+cosx}$$

**Solution:**

$$\frac{dy}{dx} = \frac{1-cosx}{1+cosx}\frac{dy}{dx} = \frac{1-cosx}{1+cosx} = \frac{2sin^2\left(\frac{x}{2}\right)}{2cos^2\left(\frac{x}{2}\right)} = tan^2\left(\frac{x}{2}\right)$$

integrating both sides, we get

$$\Rightarrow \int dy = \int \tan^2\frac{x}{2}dx \Rightarrow y = \int\left(\sec^2\frac{x}{2} - 1\right)dx$$

$$\Rightarrow y = 2\tan\frac{x}{2} - x + C \text{ (Required solution)}$$



$$\Rightarrow \int dy = \int \tan^2\frac{x}{2}dx \Rightarrow y = \int\left(\sec^2\frac{x}{2} - 1\right)dx$$

$$\Rightarrow y = 2\tan\frac{x}{2} - x + C \text{ (Required solution)}$$



**Ex 9.4 Class 12 Maths Question 3.**

$\frac{dy}{dx} + y = 1 (y \neq 1)$

**Solution:**

$$\frac{dy}{dx} + y = 1 \Rightarrow \int \frac{dy}{y-1} = -\int dx$$
$$\Rightarrow log(y-1) = -x + c \Rightarrow y = 1 + e^{-x}.e^{c}$$
$$Hence \quad y = 1 + Ae^{-x}$$

which is required solution

**Ex 9.4 Class 12 Maths Question 4.**

$\sec^2 x$ tany dx+$\sec^2 y$ tanx dy = 0

**Solution:**

we have

$\sec^2 x$ tany dx+$\sec^2 y$ tanx dy = 0

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = -\int \frac{\sec^2 x}{\tan x}$$
$$\Rightarrow \log(\tan y) = -\log(\tan x) + \log c$$
$$\Rightarrow \log |\tan x \tan y| = \log c \Rightarrow \tan x \tan y = c$$

**Ex 9.4 Class 12 Maths Question 5.**

$$\left(e^{x}+e^{-x}\right)dy-\left(e^{x}-e^{-x}\right)dx=0$$

**Solution:**

we have

$$\left(e^{x}+e^{-x}\right)dy-\left(e^{x}-e^{-x}\right)dx=0$$

Integrating on both sides

$$\int dy=\int\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} \qquad \int dy=\int\frac{dt}{t}$$

[Put $e^{x}+e^{-x}=t$ so that $(e^{x}-e^{-x})\,dx=dt$]

$\Rightarrow\ y=\log|t|+c \Rightarrow\ y=\log(e^{x}+e^{-x})+c$

**Ex 9.4 Class 12 Maths Question 6.**

$$\frac{dy}{dx}=\left(1+x^{2}\right)\left(1+y^{2}\right)$$

**Solution:**

$$\frac{dy}{1+y^{2}}=\left(1+x^{2}\right)dx$$

integrating on both side we get

$$tan^{-1}y=x+\frac{1}{3}x^{3}+c$$

which is required solution

**Ex 9.4 Class 12 Maths Question 7.**

y logy dx – x dy = 0

**Solution:**

$$\because \quad y \quad logy \quad dx = x \quad dy \Rightarrow \frac{dy}{y \quad logy} = \frac{dx}{x}$$

integrating we get

$$\int \frac{dy}{y \log y} = \frac{dx}{x} : \text{Put, } \log y = t \Rightarrow \frac{1}{y} dy = dt$$
$$\int \frac{dt}{t} = \log x + \log c \quad \text{or}$$
$$\log t = \log x + \log c \quad \log(\log y) = \log cx$$
$$\therefore \quad \log y = cx \Rightarrow y = e^{cx}.$$

**Ex 9.4 Class 12 Maths Question 8.**

**Solution:**

$$x^5 \frac{dy}{dx} = -y^5 \Rightarrow \int y^{-5} dy = -\int x^{-5} dx$$
$$\Rightarrow -\frac{1}{y^4} = \frac{1}{x^4} + 4c \Rightarrow x^{-4} + y^{-4} = k$$

**Ex 9.4 Class 12 Maths Question 9.**

solve the following

$\frac{dy}{dx} = sin^{-1}x$

**Solution:**

$\frac{dy}{dx} = sin^{-1}x \Rightarrow \int dy = \int sin^{-1}xdx$

integrating both sides we get

$$\Rightarrow \quad y = x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx$$
$$\text{Let } 1 - x^2 = t \quad \Rightarrow -2x\,dx = dt$$
$$\Rightarrow \quad y = x\sin^{-1}x + \frac{1}{2}\int \frac{1}{\sqrt{t}}dt = x\sin^{-1}x + \sqrt{t} + C$$
$$\Rightarrow \quad y = x\sin^{-1}x + \sqrt{1-x^2} + C \text{ (Required solution)}$$

**Ex 9.4 Class 12 Maths Question 10.**

$e^x tany \quad dx + (1 - e^x)sec^2 dy = 0$

**Solution:**

$e^x tany \quad dx + (1 - e^x)sec^2 dy = 0$

we can write in another form

or $(1-e^x)\sec^2 y\,dy = -e^x \tan y\,dx$

Dividing by $(1-e^x)\tan y$

$$\frac{\sec^2 y}{\tan y}dy = \frac{-e^x}{1-e^x}dx$$

Integrating, we get $\int \frac{\sec^2 y}{\tan y}dy = \int \frac{-e^x}{1-e^x}dx$

$\therefore$ $\log \tan y = \log(1-e^x) + \log c$

$\log \tan y = \log c(1-e^x) \Rightarrow \tan y = c(1-e^x)$

**Find a particular solution satisfying the given condition for the following differential equation in Q.11 to 14.**

**Ex 9.4 Class 12 Maths Question 11.**

$$\left(x^3+x^2+x+1\right)\frac{dy}{dx} = 2x^2+x; y=1, when \quad x=0$$

**Solution:**

here

$$dy = \frac{2x^2+x}{\left(x^3+x^2+x+1\right)}dx$$

integrating we get

$$\int dy = \int \frac{2x^2+x}{x^3+x^2+x+1} dx = \int \frac{2x^2+x}{(x+1)(x^2+1)} dx$$

Let $\frac{2x^2+x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$2x^2+x = A(x^2+1)+(Bx+C)(x+1)$

$= A(x^2+1)+B(x^2+x)+C(x+1)$

On Solving we get $A = \frac{1}{2}, B = \frac{3}{2} \& C = -\frac{1}{2}$

$$\therefore y = \frac{1}{2}\int \frac{1}{x+1} dx + \frac{3}{4}\int \frac{2x}{x^2+1} dx - \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{2}\log(x+1) + \frac{3}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + c$$

$1 = \frac{1}{2}\log 1 + \frac{3}{4}\log 1 - \frac{1}{2}\tan^{-1} 0 + c$

$[\because y = 1 \text{ when } x = 0] \quad \therefore 1 = 0 + c \Rightarrow c = 1$

Thus the solution is

$$y = \frac{1}{4}\log(x+1)^2(x^2+1)^3 - \frac{1}{2}\tan^{-1}x + 1$$

**Ex 9.4 Class 12 Maths Question 12.**

$$x\left(x^2-1\right)\frac{dy}{dx} = 1, y = 0 \quad when \quad x = 2$$

**Solution:**

$$x\left(x^2-1\right)\frac{dy}{dx} = 1, y = 0 \quad when \quad x = 2 \Rightarrow \int dy = \int \frac{dy}{x(x+1)(x-1)}$$

Let $\frac{1}{x(x+1)(x-1)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$

$\Rightarrow 1=A(x+1)(x-1)+Bx(x-1)+Cx(x+1)$

Let $x=0,1 \& -1 \Rightarrow A=-1,\ C=\frac{1}{2}\ B=\frac{1}{2}$

$\therefore \quad y=\int\left[\frac{-1}{x}+\frac{1/2}{x+1}+\frac{1/2}{x-1}\right]dx$

$=-\log x+\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)+C$

$y=\frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right)+\log C$

Now $x=2, y=0 \Rightarrow \log C=-\frac{1}{2}\log\frac{3}{4}$

Hence particular solution is :

$y=\frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right)-\frac{1}{2}\log\frac{3}{4}.$

**Ex 9.4 Class 12 Maths Question 13.**

$$cos\left(\frac{dy}{dx}\right)=a,(a\epsilon R),y=1 \quad when \quad x=0$$

**Solution:**

$$cos\left(\frac{dy}{dx}\right)=a \quad \therefore \frac{dy}{dx}=cos^{-1}a$$

or $dy=(\cos^{-1}a)\,dx \Rightarrow \int dy=\cos^{-1}a\ \int dx$

or $y=(\cos^{-1}a)\,x+c$ we have $y=2$ when $x=0$

$\Rightarrow c=2 \quad \therefore$ Solution is : $y=x(\cos^{-1}a)+2$

or $\cos^{-1}a=\frac{y-2}{x} \Rightarrow a=\cos\frac{y-2}{x}$ is the req. sol.

**Ex 9.4 Class 12 Maths Question 14.**

$$\frac{dy}{dx} = ytanx, y = 1 \quad when \quad x = 0$$

**Solution:**

$$\frac{dy}{dx} = ytanx \Rightarrow \int \frac{dy}{y} = \int tanx \quad dx$$

=> logy = logsecx + C

When x = 0, y = 1

=> log1 = log sec0 + C => 0 = log1 + C

=> C = 0

∴ logy = log sec x

=> y = sec x.

**Ex 9.4 Class 12 Maths Question 15.**

Find the equation of the curve passing through the point (0,0) and whose differential equation

$$y^I = e^x sinx$$

**Solution:**

$$y^I = e^x sinx \Rightarrow dy = e^x sinx \quad dx$$

Integrating, we get $\int dy = \int e^x \sin x\, dx$

$y = e^x(-\cos x) - \int e^x(-\cos x)\, dx$

$= -e^x \cos x + \int e^x \cos x\, dx$

Again integrating by parts taking $e^x$ as first functions

$= -e^x \cos x + e^x \sin x - \int e^x \sin x\, dx$

or $2y = -e^x \cos x + e^x \sin x + c$

$\therefore\ y = \frac{e^x}{2}[-\cos x + \sin x] + c$

Put $x = 0, y = 0 \Rightarrow 0 = -\frac{1}{2} + c \quad \therefore\ c = \frac{1}{2}$

$\therefore$ Solution is $y = \frac{e^x}{2}(\sin x - \cos x) + \frac{1}{2}$

**Ex 9.4 Class 12 Maths Question 16.**

For the differential equation

$$xy\frac{dy}{dx} = (x+2)(y+2)$$

find the solution curve passing through the point (1,-1)

**Solution:**

The differential equation is

$$xy\frac{dy}{dx} = (x+2)(y+2)$$

or xydy=(x + 2)(y+2)dx

$$\frac{y}{y+2}dy = \frac{x+2}{x}dx$$ Integrating, we get

$$\int y\frac{dy}{y+2} = \int\frac{x+2}{x}dx$$

$$\int\left(1-\frac{2}{y+2}\right)dy = \int\left(1+\frac{2}{x}\right)dx$$

$y - 2\log(y+2) = x + 2\log x + c$

The curve passes through $(1, -1)$

$\therefore\ -1 - 2\log 1 = 1 + 2\log 1 + c$

$(\log 1 = 0) \Rightarrow -1 = 1 + c \quad \Rightarrow c = -2$

Putting $c = -2 \Rightarrow y - 2\log(y+2) = x + 2\log x - 2$

or $y - x = 2[\log(y+2) + \log x] - 2 = 2\log x(y+2) - 2$

Solution is $y = x + 2\log x(y+2) - 2$.

**Ex 9.4 Class 12 Maths Question 17.**

Find the equation of a curve passing through the point (0, -2) given that at any point (pc, y) on the curve the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point

**Solution:**

According to the question

$$y\frac{dy}{dx} = x \Rightarrow \int ydy = \int xdx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$$

0, – 2) lies on it.c = 2

∴ Equation of the curve is : $x^2 - y^2 + 4 = 0$.

**Ex 9.4 Class 12 Maths Question 18.**

At any point (x, y) of a curve the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (-4,-3) find the equation of the curve given that it passes through (- 2,1).

**Solution:**

Slope of the tangent to the curve =

$$\frac{dy}{dx}$$

slope of the line joining (x, y) and (- 4, – 3)

$$= \frac{y+3}{x+4};\ \frac{dy}{dx} = 2\left(\frac{y+3}{x+4}\right)$$

$$\Rightarrow \frac{dy}{y+3} = \frac{2}{x+4}dx \Rightarrow \int \frac{dy}{y+3} = 2\int \frac{dx}{x+4}$$

or $\log(y+3) = 2\log(x+4) + \log c$

*i.e.*, $\log \frac{y+3}{(x+4)^2} = \log c \quad \therefore \quad \frac{y+3}{(x+4)^2} = c$

The curve passes through (–2, 1)

$$\frac{1+3}{(-2+4)^2} = c = \frac{4}{4} = 1$$

Equation of the curve is $\frac{y+3}{(x+4)^2} = 1$

or $y+3 = (x+4)^2$ or $y = (x+4)^2 - 3$.

**Ex 9.4 Class 12 Maths Question 19.**

The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and offer 3 seconds it is 6 units. Find the radius of balloon after t seconds.

**Solution:**

Let v be volume of the balloon.

$$\text{we have } \frac{dv}{dt} = k \quad \text{or} \quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k$$

$$\text{or } \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = k \quad \text{or} \quad 4\pi r^2 \frac{dr}{dt} = k$$

$\Rightarrow 4\pi r^2 \, dr = k \, dt$; Integrating, we get

$$4\pi \int r^2 \, dr = k \int dt \quad \text{or} \quad 4\pi \cdot \frac{r^3}{3} = kt + c \qquad \text{...(i)}$$

$$\text{when } t = 0, r = 3; \ \frac{4}{3}\pi (3)^3 = k.0 + c \ \Rightarrow 36\pi = c$$

$$\text{when } t = 3, r = 6; \ \frac{4}{3}(6)^3 = 3k + c = 3k + 36\pi$$

$$\Rightarrow \frac{216 \times 4}{3}\pi = 3k + 36\pi$$

$$\Rightarrow 3k = 72 \times 4\pi - 36\pi = 288\pi - 36\pi = 252\pi$$

$$\therefore \quad k = \frac{252}{3}\pi = 84\pi$$

$$\therefore \text{ in (i)}; \ \frac{4}{3}\pi r^3 = 84\pi + 36\pi$$

$$\Rightarrow r^3 = \frac{84 \times 3}{4} t + 36 \times \frac{3}{4}$$

$$r^3 = 63t + 27 \quad \text{or } r^3 = 9(7t + 3) \quad r = [9(7t + 3)]^{1/3}$$

**Ex 9.4 Class 12 Maths Question 20.**

In a bank principal increases at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years

**Solution:**

Let P be the principal at any time t.

According to the problem

$$\frac{dP}{dt}=\frac{r}{100}.P \Rightarrow \int\frac{dP}{P}=\int\frac{r}{100}dt$$

$$\log P=\frac{r}{100}t+C_1 \Rightarrow P=e^{rt/100}.e^{C_1}$$

$$\text{Now } P=100, \text{ when } t=0 \therefore P=e^{rt/100}\times 100$$

$$\text{When } P=200, t=10 \Rightarrow \log 2=\frac{r}{10}$$

$$\Rightarrow r=6.931\% \text{ per annum.}$$

**Ex 9.4 Class 12 Maths Question 21.**

In a bank principal increases at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years

**Solution:**

Let p be the principal Rate of interest is 5%

$$\frac{dp}{dt}=\frac{5}{100}p \quad \therefore \frac{dp}{p}=0\cdot05\,dt$$

$$\text{Integrating, } \log p=0\cdot05t+\log c \Rightarrow \log\frac{p}{c}=0\cdot05t$$

$$\frac{p}{c}=e^{0\cdot05t} \quad \therefore p=ce^{0\cdot05t} \quad ...(i)$$

$$\text{Initially, } p=₹\,1000,\ t=0 \quad 1000=c, e^{o}=c$$

$$\therefore c=1000 \quad \text{Putting this value in (i): } p=1000e^{0\cdot05t}$$

$$\text{when } t=10.\ p=1000\,e^{0\cdot05\times10}=1000e^{0\cdot5}$$

$$p=1000\times 1\cdot648 \Rightarrow p=1648\ (\because e^{0\cdot5}=1\cdot648)$$

After 10 years ₹ 1000 will amount to ₹ 1648.

**Ex 9.4 Class 12 Maths Question 22.**

In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000 if the rate of growth of bacteria is proportional to the number present

**Solution:**

Let y denote the number of bacteria at any instant t • then according to the question

$$\frac{dy}{dt} \alpha y \Rightarrow \frac{dy}{y} = k\,dt \qquad ...(i)$$

k is the constant of proportionality, taken to be + ve on integrating (i), we get

$$\log y = kt + c \qquad ...(ii)$$

Let $y_0$ be the initial number of bacteria *i.e.*, at $t = 0$ using this in (ii), $c = \log y_0$

$$\Rightarrow \quad \log y = kt + \log y_0$$

$$\Rightarrow \quad \log \frac{y}{y_0} = kt \qquad ...(iii)$$

$$\text{when } t = 2,\ y = \left(y_0 + \frac{10}{100} y_0\right) = \frac{11y_0}{10}$$

$$\text{So, from (iii), we get } \log \frac{\frac{11y_0}{10}}{y_0} = k\ (2)$$

$$\Rightarrow k = \frac{1}{2} \log \frac{11}{10} \qquad ...(iv)$$

$$\text{Using (iv) in (iii) } \log \frac{y}{y_0} = \frac{1}{2}\left(\log \frac{11}{10}\right) t \ ...(v)$$

let the number of bacteria become 1, 00, 000 to 2,00,000 in $t_1$ hours. *i.e.*, $y = 2y_0$ when $t = t_1$ hours. from (v)

$$\log \frac{2y_0}{y_0} = \frac{1}{2}\left(\log \frac{11}{10}\right) t_1 \quad \Rightarrow \quad t_1 = \frac{2 \log 2}{\log 11/10}$$

$$\text{Hence, the reqd. no. of hours} = \frac{2 \log 2}{\log 11/10}$$

**Ex 9.4 Class 12 Maths Question 23.**

The general solution of a differential equation

$$\frac{dy}{dx} = e^{x+y}$$

is

(a)

$e^{x}+e^{-y}=c$

(b)

$e^{x}+e^{y}=c$

(c)

$e^{-x}+e^{y}=c$

(d)

$e^{-x}+e^{-y}=c$

**Solution:**

(a)

$$\frac{dy}{dx}=e^{x}.e^{y}\Rightarrow\int e^{-y}dy=\int e^{x}dx$$
$$\Rightarrow e^{-y}=e^{x}+k\Rightarrow e^{x}+e^{-y}=c$$

**Ex 9.5 Class 12 Maths Question 1.**

$(x^2+xy)dy = (x^2+y^2)dx$

**Solution:**

$(x^2+xy)dy = (x^2+y^2)dx$

$$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy} = f(x,y) \quad ...(i)$$

$$f(x,y) = \frac{x^2+y^2}{x^2+xy}$$

Replacing x by $\lambda x$ and y by $\lambda y$

$$f(\lambda x, \lambda y) = \frac{\lambda^2 x^2 + \lambda^2 y^2}{\lambda^2 x^2 + \lambda^2 xy} = \frac{\lambda^2}{\lambda^2}\left(\frac{x^2+y^2}{x^2+xy}\right)$$

$$= \lambda^\circ \left(\frac{x^2+y^2}{x^2+xy}\right) = \lambda^\circ f(x,y)$$

Hence f (x, y) is a homogeneous function of degree zero to solve it put $y = vx$

or $\frac{dy}{dx} = v + x\frac{dv}{dx}$ Eq. (i)

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+(xvx)} = \frac{x^2(1+v^2)}{x^2(1+v)}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v \Rightarrow \frac{1+v}{1-v}dv = \frac{dx}{x}$$

Integrating, $\int \frac{1+v}{1-v}dv = \int \frac{dx}{x}$

or $\left(-1+\frac{2}{1-v}\right)dv = \int\frac{dx}{x}$

$$-v - 2\log(1-v) = \log|x| - \log c; \text{ Put } v = \frac{y}{x}$$

$$-\frac{y}{x} - 2\log\left|1-\frac{y}{x}\right| = \log|x| - \log c$$

or $\frac{y}{x} + 2\log\left|\frac{x-y}{x}\right| + \log|x| - \log c = 0$

$$\log\frac{(x-y)^2}{x^2}\times\frac{x}{c} = -\frac{y}{x} \quad \text{or} \quad \frac{(x-y)^2}{cx} = e^{-y/x}$$

$\therefore$ solution is $(x-y)^2 = cx\, e^{-y/x}$

**Ex 9.5 Class 12 Maths Question 2.**

$y^I = \frac{x+y}{x}$

**Solution:**

$y^I = \frac{x+y}{x}$

Given $\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x}$

Let $y = vx \Rightarrow v + x\frac{dv}{dx} = 1 + v \Rightarrow \int dv = \int \frac{dx}{x}$

$\Rightarrow v = \log x + C \Rightarrow y = x \log x + Cx.$

**Ex 9.5 Class 12 Maths Question 3.**

(x-y)dy-(x+y)dx=0

**Solution:**

$\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1+v}{1-v} \quad \Rightarrow \quad x\frac{dv}{dx} = \left(\frac{1+v^2}{1-v}\right)$$

$$\Rightarrow \quad \int \frac{v-1}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{2v}{v^2+1}dv - \int \frac{dv}{v^2+1} = -\log x$$

$$\Rightarrow \quad \frac{1}{2}\log\left(\frac{y^2}{x^2}+1\right) - \tan^{-1}\frac{y}{x} = -\log x + C$$

$$\Rightarrow \quad \tan^{-1}\frac{y}{x} = \frac{1}{2}\log(x^2+y^2) - C.$$

**Ex 9.5 Class 12 Maths Question 4.**

$(x^2-y^2)dx+2xy\ dy=0$

**Solution:**

$$\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2-1}{2v} - v \Rightarrow \frac{2v\,dv}{1+v^2} = -\frac{dx}{x}$$

$$\Rightarrow \int \frac{2v\,dv}{1+v^2} = -\int \frac{dx}{x}$$

$$\Rightarrow \log(v^2+1) = -\log x + \log C$$

$$\Rightarrow \log\left(\frac{x^2+y^2}{x^2}\right) + \log x = \log C$$

$$\Rightarrow \frac{x^2+y^2}{x} = C \Rightarrow x^2+y^2 = Cx.$$

**Ex 9.5 Class 12 Maths Question 5.**

$$x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$$

**Solution:**

$$\frac{dy}{dx} = 1 - 2\left(\frac{y}{x}\right)^2 + \frac{y}{x}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow x\frac{dv}{dx} = 1 - 2v^2 \Rightarrow \int \frac{dv}{2v^2 - 1} = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\int \frac{dv}{v^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = -\log x$$

$$\Rightarrow \frac{1}{2\sqrt{2}}\log\left(\frac{\sqrt{2}v - 1}{\sqrt{2}v + 1}\right) = \log\frac{C}{x}$$

$$\Rightarrow \frac{\sqrt{2}\left(\frac{y}{x}\right) - 1}{\sqrt{2}\frac{y}{x} + 1} = \left(\frac{C}{x}\right)^{2\sqrt{2}} \Rightarrow \frac{\sqrt{2}y - x}{\sqrt{2}y + x} = \left(\frac{C}{x}\right)^{2\sqrt{2}}.$$

**Ex 9.5 Class 12 Maths Question 6.**

$$xdy - ydx = \sqrt{x^2 + y^2}dx$$

**Solution:**

$$xdy - ydx = \sqrt{x^2 + y^2}dx$$

$$\frac{dy}{dx}=\frac{\sqrt{x^2+y^2}+y}{x}=\sqrt{\frac{x^2+y^2}{x^2}}+\frac{y}{x}=\sqrt{1+\left(\frac{y}{x}\right)^2}+\frac{y}{x}$$

let $y=vx \Rightarrow x\frac{dv}{dx}=\sqrt{1+v^2}$

$$\Rightarrow \int\frac{dv}{\sqrt{1+v^2}}=\int\frac{dx}{x}$$

$$\Rightarrow \log[v+\sqrt{1+v^2}]=\log x+\log C$$

$$\Rightarrow v+\sqrt{1+v^2}=Cx \Rightarrow y+\sqrt{x^2+y^2}=Cx^2$$

is the required solution.

**Ex 9.5 Class 12 Maths Question 7.**

$$\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}ydx=\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}xdy$$

**Solution:**

$$\left\{xcos\left(\frac{y}{x}\right)+ysin\left(\frac{y}{x}\right)\right\}ydx=\left\{ysin\left(\frac{y}{x}\right)-xcos\left(\frac{y}{x}\right)\right\}xdy$$

$$\frac{dy}{dx}=\frac{y\left[x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right]}{x\left[y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right]}=f(x,y)$$

Replacing x by $\lambda x$ and y by $\lambda y \cdot$ in f (x, y)

$$f(\lambda x,\lambda y)=\frac{\lambda y\left[\lambda x\cos\left(\lambda y/\lambda x\right)+\lambda y\sin\left(\lambda y/\lambda x\right)\right]}{\lambda x\left[\lambda x\sin\left(\lambda y/\lambda x\right)-\lambda y\cos\left(\lambda y/\lambda x\right)\right]}$$

$$=\lambda^0\frac{y\left[x\cos\left(y/x\right)+y\sin\left(y/x\right)\right]}{x\left[x\sin\left(y/x\right)-y\cos\left(y/x\right)\right]}=\lambda^0 f(x,y)$$

$\therefore$ f(x, y) is homogeneous function of degree zero.

Now, $\frac{dy}{dx}=\frac{y\left[x\cos\left(y/x\right)+y\sin\left(y/x\right)\right]}{x\left[y\cos\left(y/x\right)+x\sin\left(y/x\right)\right]}$ ...(i)

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Putting this value in (i)

$$v + x\frac{dv}{dx} = \frac{vx\left[x\cos\frac{vx}{x} + vx\sin\frac{vx}{x}\right]}{x\left[vx\sin\left(\frac{v}{x}\right) - x\cos\left(\frac{vx}{x}\right)\right]}$$

$$= \frac{vx^2\,[\cos v + v\sin v]}{x^2\,[v\sin v - \cos v]} = \frac{v(\cos v + v\sin v)}{v\sin v - \cos v}$$

Transposing v to R.H.S.

$$x\frac{dv}{dx} = \frac{v(\cos v + v\sin v)}{v\sin v - \cos v} - v$$

$$= \frac{v\cos v + v^2\sin v - v^2\sin v + v\cos v}{v\sin v - \cos v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2v\cos c}{v\sin v - \cos v}$$

$$\text{or } \left(\tan v - \frac{1}{v}\right)dv = \frac{2dx}{x} \text{ or}$$

$$\int\left(\tan v - \frac{1}{v}\right)dv = 2\int\frac{dx}{x}$$

Integrating, $\log\sec v - \log v = 2\log x + \log c$

$$\Rightarrow \log\frac{\sec v}{vx^2} = \log c \quad \textit{i.e.,}\ \frac{\sec v}{vx^2} = c$$

Putting $v = \frac{y}{x}$, $\frac{\sec y/x}{\frac{y}{x} \times x^2} = c$ or $\sec\frac{y}{x} = cxy$

**Ex 9.5 Class 12 Maths Question 8.**

$$x\frac{dy}{dx}-y+xsin\left(\frac{y}{x}\right)=0$$

**Solution:**

$$x\frac{dy}{dx}-y+xsin\left(\frac{y}{x}\right)=0\Rightarrow\frac{dy}{dx}=\frac{y}{x}-sin\frac{y}{x}$$

$$\text{Now let } y=vx \Rightarrow x\frac{dv}{dx}=-\sin v$$

$$\Rightarrow \int\frac{dv}{\sin v}=-\int\frac{dx}{x} \Rightarrow \int \operatorname{cosec} v\, dv=-\log x$$

$$\Rightarrow \log(\operatorname{cosec} v-\cot v)=-\log x+\log C$$

$$\Rightarrow \operatorname{cosec} v-\cot v=\frac{C}{x} \Rightarrow \operatorname{cosec}\frac{y}{x}-\cot\frac{y}{x}=\frac{C}{x}$$

$$x\left[\operatorname{cosec}\frac{y}{x}-\cot\frac{y}{x}\right]=C \text{ is the required solution.}$$

**Ex 9.5 Class 12 Maths Question 9.**

$$ydx+xlog\left(\frac{y}{x}\right)dy-2xdy=0$$

**Solution:**

$$\frac{dy}{dx}=\frac{y}{2x-xlog\frac{y}{x}}=\frac{\frac{y}{x}}{2-log\frac{y}{x}}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ ......(ii)

From (i) and (ii), we get

$$x\frac{dv}{dx} = \frac{-v + v\log v}{2 - \log v} \Rightarrow \frac{2 - \log v}{-v + v\log v}dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{v(\log v - 1)} - \int \frac{dv}{v} = \int \frac{dx}{x} \quad .....(iii)$$

For $\int \frac{1}{v(\log v - 1)}$ $\left[\text{Put} \log v - 1 = t \text{ so that } \frac{dv}{v} = dt\right]$

$$= \int \frac{dt}{t} = \log t = \log(\log v - 1)$$

$\therefore$ From (iii), we have

$$\log\left(\log\frac{y}{x} - 1\right) - \log y + \log x = \log x + \log C$$

$$\Rightarrow \log\left(\log\frac{y}{x} - 1\right) = \log y + \log C = \log Cy$$

$\Rightarrow \log\frac{y}{x} - 1 = Cy$ which is the required solution.

**Ex 9.5 Class 12 Maths Question 10.**

$$\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$$

**Solution:**

$$\frac{dx}{dy} = -\frac{e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)}{1+e^{\frac{x}{y}}} = \frac{\left(\frac{x}{y}-1\right)e^{\frac{x}{y}}}{1+e^{\frac{x}{y}}} = f(x,y)$$

$$\therefore\ f(x,y) = \frac{\left(\frac{x}{y}-1\right)e^{x/y}}{1+e^{x/y}}$$

Replace x by λx and y by λy

$$f(\lambda x, \lambda y) = \frac{\left(\frac{\lambda x}{\lambda y}-1\right)e^{\lambda x/\lambda y}}{1-e^{\lambda x/\lambda y}} = \lambda^{\circ}\frac{\left(\frac{x}{y}-1\right)e^{x/y}}{1+e^{x/y}}$$

Hence, f (x, y) is a homogeneous function of degree zero. Now,

$$\frac{dx}{dy} = \frac{\left(\frac{x}{y}-1\right)e^{x/y}}{1+e^{x/y}}\text{; Put } x = vy \text{ is } \frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\therefore\quad v + y\frac{dv}{dy} = \frac{\left(\frac{vy}{y}-1\right)e^{vy/y}}{1+e^{vy/y}} = \frac{(v-1)e^{v}}{1+e^{v}}$$

$$\text{or } y\frac{dv}{dy} = \frac{(v-1)\,e^{v}}{1+e^{v}} - v = \frac{ve^{v}-e^{v}-v-ve^{v}}{1+e^{v}}$$

$$= \frac{-(v+e^{v})}{1+e^{v}} \Rightarrow \left(\frac{1+e^{v}}{v+e^{v}}\right)dv = -\frac{dy}{y}$$

Integrating, $\int \frac{1+e^v}{v+e^v} dv = -\int \frac{dy}{y}$

Put $v + e^v = t$, $(1+e^v)\, dv = dt$

$\therefore \int \frac{dt}{t} = -\log y + \log c$

or $\log t = -\log y + \log c$ or $\log t + \log y = \log c$

$\therefore ty = c$ Putting $t = v + e^v = \frac{x}{y} + e^{x/y}$

$\therefore \left(\frac{x}{y} + e^{x/y}\right) y = c \therefore$ Req. sol. is : $x + y.\, e^{x/y} = cy.$

**For each of the following differential equation in Q 11 to 15 find the particular solution satisfying the given condition:**

**Ex 9.5 Class 12 Maths Question 11.**

$(x + y)\, dy + (x – y)dx = 0, y = 1$ when $x = 1$

**Solution:**

given

$(x + y)\, dy + (x – y)dx = 0$

$$\frac{dy}{dx}=\frac{\frac{y}{x}-1}{\frac{y}{x}+1};\ \text{let } \frac{y}{x}=v \Rightarrow x\frac{dv}{dx}=-\left(\frac{v^2+1}{v+1}\right)$$

$$\Rightarrow \int\frac{v+1}{v^2+1}dv=-\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\log(v^2+1)+\tan^{-1}v=-\log x+C$$

$$\Rightarrow \frac{1}{2}\log\left(\frac{y^2+x^2}{x^2}\right)+\tan^{-1}\frac{y}{x}=C$$

Now $x=1, y=1$

$$\Rightarrow \frac{1}{2}\log 2+\tan^{-1}(1)=C \Rightarrow \frac{1}{2}\log 2+\frac{\pi}{4}=C$$

$\therefore$ Particular solution is

$$\frac{1}{2}\log\left(\frac{y^2+x^2}{x^2}\right)+\tan^{-1}\left(\frac{y}{x}\right)=\frac{1}{2}\log 2+\frac{\pi}{4}.$$

**Ex 9.5 Class 12 Maths Question 12.**

$x^2dy+(xy+y^2)dx=0$, $y=1$ when $x=1$

**Solution:**

$$\frac{dy}{dx}=\frac{xy+y^2}{x^2}=f(x,y)$$

f(x,y) is homogeneous

∴ put y = vx

or $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = -\frac{x(vx) + (vx)^2}{x^2} = -\frac{x^2(v+v^2)}{x^2}$

$\therefore \quad \frac{1}{v^2 + 2v}dv = -\frac{dx}{x}$

Integrating, $\int \frac{1}{v^2 + 2v}dv = -\int \frac{dx}{x} + \log c$

or $\int \frac{1}{(v+1)^2 - 1}dv = -\log x + \log c$

or $\frac{1}{2}\log\frac{v+1-1}{v+1+1} = -\log x + \log c$

i.e., $\log\frac{\sqrt{v}}{\sqrt{v+2}} + \log x = \log c$

$\Rightarrow \quad \log\left|\frac{x\sqrt{v}}{\sqrt{v+2}}\right| = \log c \therefore \quad \frac{x\sqrt{v}}{\sqrt{v+2}} = c,$

Put $v = \frac{y}{x} \Rightarrow x^2y = c^2(y + 2x)$ ... (ii)

Putting $x = 1,\ y = 1;\ 1 = c^2(1+2) \Rightarrow c^2 = \frac{1}{3}$

Putting $c^2 = \frac{1}{3}$ in (i) $x^2y = \frac{1}{3}(y + 2x)$

Particular solution is : $3x^2y = y + 2x$.

**Ex 9.5 Class 12 Maths Question 13.**

$$\left(xsin^2\frac{y}{x} - y\right)dx + xdy = 0, y = \frac{\pi}{4}, when \quad x = 1$$

**Solution:**

$$\left(xsin^2\frac{y}{x} - y\right)dx + xdy = 0$$

$\frac{dy}{dx} = \frac{y}{x} - \sin^2 \frac{y}{x}$ which is homogeneous ...(i)

Put $y = vx$, $\therefore$ $v + x\frac{dv}{dx} = v - \sin^2 v$ from (i)

or $\frac{dv}{\sin^2 v} = -\frac{dx}{x}$

Integrating $\int \frac{dv}{\sin^2 v} = -\int \frac{dx}{x}$ $\int \operatorname{cosec}^2 v dx$

$= -\int \frac{dx}{x}$ $-\cot v = -\log x + c$

$\log x - \cot v = c$; $\log x - \cot \frac{y}{x} = c$

Putting $x = 1, y = \frac{\pi}{4}$; $c = -1$

Particular sol. is: $\cot \frac{y}{x} - \log x = 1$

**Ex 9.5 Class 12 Maths Question 14.**

$\frac{dy}{dx} - \frac{y}{x} + cosec\left(\frac{y}{x}\right) = 0, y = 0 \quad when \quad x = 1$

**Solution:**

$\frac{dy}{dx} - \frac{y}{x} + cosec\left(\frac{y}{x}\right) = 0$

which is a homogeneous differential equation

Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ ...(ii)

from (i) & (ii), we get: $\left(v + x\frac{dv}{dx}\right) - v + \operatorname{cosec} v = 0$

$x\frac{dv}{dx} + \operatorname{cosec} v = 0 \Rightarrow x\frac{dv}{dx} = -\operatorname{cosec} v$

$\Rightarrow -\frac{dv}{\operatorname{cosec} v} = \frac{dx}{x} \Rightarrow -\sin v\, dv = \frac{dx}{x}$

Integrating $\int -\sin v\, dv = \int \frac{dx}{x}$

$\Rightarrow \cos v = \log|x| + c \Rightarrow \cos\frac{y}{x} = \log|x| + c,$

$y(1) = 0$, i.e., when $x = 1, y = 0$

$\cos 0 = \log|1| + c \Rightarrow 1 = 0 + c, c = 1$

$\therefore \cos\frac{y}{x} = \log|x| + 1 \Rightarrow \log|x| = \cos\frac{y}{x} - 1$

which is reqd. solution.

**Ex 9.5 Class 12 Maths Question 15.**

$$2xy - y^2 - 2x^2\frac{dy}{dx} = 0, y = 2, when \quad x = 1$$

**Solution:**

$$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{2}\left(\frac{y}{x}\right)^2$$

…(i)

Put $y = vx$, $\therefore$ (i) becomes,

$$v + x\frac{dv}{dx} = v + \frac{1}{2}v^2 \Rightarrow \frac{2}{v^2}dv = \frac{dx}{x}$$

Integrating both sides, we get

$$-\frac{2}{v} = \log|x| + C \Rightarrow -\frac{2x}{y} = \log|x| + C$$

It is given that $y(1) = 2$ i.e., When $x = 1, y = 2$

$$\Rightarrow \quad y = \frac{2x}{1 - \log|x|}, (x \neq 0, \pm e)$$

which is the required solution.

**Ex 9.5 Class 12 Maths Question 16.**

A homogeneous equation of the form

$$\frac{dx}{dy} = h\left(\frac{x}{y}\right)$$

can be solved by making the substitution,

(a) y=vx

(b) v=yx

(c) x=vy

(d) x=v

**Solution:**

(c) option x = vy

**Ex 9.5 Class 12 Maths Question 17.**

Which of the following is a homogeneous differential equation?

(a) (a) (4x + 6y + 5)dy-(3y + 2x + 4)dx = 0

(b)

$$(xy)dx - (x^3 + y^3)dy$$

(c)

$$(x^3 + 2y^2)dx + 2xydy = 0$$

(d)

$$y^2dx + (x^2 - xy - y^2)dy = 0$$

**Solution:**

(d)

**Ex 9.6 Class 12 Maths Question 1.**

$$\frac{dy}{dx} + 2y = sinx$$

**Solution:**

Given equation is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

;

Here, P = 2, Q = sin x

I.F. $= e^{\int 2dx} = e^{2x}$ Solution to the diff. equation is

$ye^{2x} = \int(\sin x)\, e^{2x}\, dx = I_1$ ...(i)

$I_1 = \int e^{2x} \sin x\, dx = e^{2x}(-\cos x) - \int 2e^{2x}(-\cos x)\, dx$

$= -e^{2x}\cos x + 2\sin x\, e^{2x} - 4\int e^{2x}\sin x\, dx$

$I_1 = e^{2x}(2\sin x - \cos x) - 4I_1 \therefore 5I_1 = e^{2x}(2\sin x - \cos x)$

$I_1 = \frac{e^{2x}}{5}(2\sin x - \cos x) + c$

Putting the value of $I_1$ in (i), the general solution

$ye^{2x} = \frac{e^{2x}}{5}(2\sin x - \cos x) + c$

or $5y = 2\sin x - \cos x + 5ce^{-2x}$

**Ex 9.6 Class 12 Maths Question 2.**

$$\frac{dy}{dx} + 3y = e^{-2x}$$

**Solution:**

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Here P = 3,

$IF = e^{\int p.dx} = e^{3x}$

$$e^{3x}\frac{dy}{dx} + 3y\,e^{3x} = e^{-2x}\,e^{3x}$$

$$y.e^{3x} = \int e^{x}\,dx \Rightarrow y = e^{-2x} + Ce^{-3x}$$

which is required equation

https://www.indcareer.com/schools/ncert-solutions-for-12th-class-maths-chapter-9-differential-equations/

**Ex 9.6 Class 12 Maths Question 3.**

$\frac{dy}{dx} + \frac{y}{x} = x^2$

**Solution:**

$\frac{dy}{dx} + \frac{y}{x} = x^2 \; IF = e^{\int \frac{1}{x} dx} = e^{logx} = x$

$\therefore \quad x\frac{dy}{dx} + y = x^3 \Rightarrow y.x = \int x^3 dx$

$\Rightarrow \quad y.x = \frac{x^4}{4} + C \Rightarrow y = \frac{x^3}{4} + \frac{C}{x}$

**Ex 9.6 Class 12 Maths Question 4.**

$\frac{dy}{dx} + (secx)y = tanx \; \left(0 \leq x < \frac{\pi}{2}\right)$

**Solution:**

Here, P = secx, Q = tanx;

$IF = e^{\int p.dx} = e^{\int secx.dx} = e^{log|secx + tanx|}$

= sec x + tan x

i.e., The solu. is y.× I.F. = ∫Q × I.F. dx + c

or y × (secx+tanx) = ∫tanx(secx+tanx)dx+c

Reqd. sol. is

∴ y(secx + tanx) = (secx + tanx)-x + c

**Ex 9.6 Class 12 Maths Question 5.**

$$cos^2x\frac{dy}{dx}+y=tanx\left(0\le x\le\frac{\pi}{2}\right)$$

**Solution:**

$$\frac{dy}{dx}+y\quad sec^2x=sec^2x\quad tanx$$

⇒ integrating factor =

$$e^{\int sec^2xdx}=e^{tanx}$$

$$\therefore\ e^{\tan x}\frac{dy}{dx}+y\sec^2 x.e^{\tan x}=e^{\tan x}\sec^2 x\tan x$$

$$\Rightarrow\quad y.e^{\tan x}=\int e^{\tan x}\sec^2 x\tan x\,dx$$

Let $\tan x=t$

$$\Rightarrow\quad \sec^2 x\,dx=dt\Rightarrow y.e^{\tan x}=\int te^t\,dt$$

$$\Rightarrow\quad y.e^{\tan x}=te^t-t+C$$

$$\Rightarrow\quad y.e^{\tan x}=\tan x\,e^{\tan x}-e^{\tan x}+C$$

$$y=\tan x-1+Ce^{-\tan x}$$

**Ex 9.6 Class 12 Maths Question 6.**

$$x\frac{dy}{dx}+2y=x^2logx$$

**Solution:**

Here P =

$$\frac{2}{x}$$

and Q = x logx

$$\therefore \quad I.F. = e^{\int p dx} = e^{\int \frac{2}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2$$

The sol. of the given eq. is

$$y \times x^2 = \int (x \log x) x^2 dx + c = \int (x^3 \log x) dx + c$$

$$= \frac{1}{4} x^4 \log x - \frac{1}{4} \int x^3 dx + c$$

$$\text{or } y = \frac{x^2}{4} \log x - \frac{x^2}{16} + c.x^2$$

$$\text{or} \quad 16y = x^2(4\log x - 1) + 16c.x^2$$

**Ex 9.6 Class 12 Maths Question 7.**

$$xlogx\frac{dy}{dx} + y = \frac{2}{x}logx$$

**Solution:**

$$\frac{dy}{dx} + \frac{1}{xlogx}y = \frac{2}{x^2}$$

$$I.F = e^{\int \frac{1}{x\log x} dx} = e^{\log(\log x)} = \log x$$

$$\therefore \quad (\log x)\frac{dy}{dx} + \frac{1}{x} y = \frac{2}{x^2} \log x$$

$$\Rightarrow \quad y. \log x = 2\int (\log x)\,(x^{-2}) dx$$

$$\Rightarrow \quad y = -2\left[\frac{1}{x} + \frac{1}{x\log x}\right] + \frac{C}{\log x}$$

**Ex 9.6 Class 12 Maths Question 8.**

$(1+x^2)dy+2xy\,dx = \cot x\,dx (x \neq 0)$

**Solution:**

$(1+x^2)dy+2xy\,dx = \cot x\,dx$

$$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{\cot x}{1+x^2} \Rightarrow \text{IF}= e^{\int\frac{2x}{1+x^2}dx}=1+x^2$$

$$\Rightarrow y(1+x^2)=\int \cot x\,dx + C = \log\sin x + C$$

$$\Rightarrow y=\frac{\log\sin x}{1+x^2}+\frac{C}{1+x^2}$$

**Ex 9.6 Class 12 Maths Question 9.**

$$x\frac{dy}{dx}+y-x+xy \quad cotx=0(x\neq 0)$$

**Solution:**

$$x\frac{dy}{dx}+y-x+xy \quad cotx=0 \; x\frac{dy}{dx}+(1+xcotx)y=x$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{1 + x\cot x}{x}\right) y = 1$$

$$\Rightarrow \quad P = \frac{1 + x\cot x}{x} = \frac{1}{x} + \cot x$$

$$\Rightarrow IF = e^{\int P\,dx} = e^{\int\left(\frac{1}{x} + \cot x\right)dx} = e^{\log x + \log \sin x}$$
$$= e^{\log(x \sin x)} = x \sin x$$

$$\Rightarrow y \,.\, x\sin x = \int x \sin x\, dx + C$$

$$\Rightarrow y \,.\, x \sin x = -x\cos x + \sin x + C$$

$$\text{Here } y = -\cot x + \frac{1}{x} + \frac{C}{x\sin x}$$

**Ex 9.6 Class 12 Maths Question 10.**

$$(x + y)\frac{dy}{dx} = 1$$

**Solution:**

$$(x + y)\frac{dy}{dx} = 1 \; \frac{1}{(x+y)}\frac{dx}{dy} = 1 \Rightarrow \frac{dx}{dy} = x + y$$

$$\Rightarrow \frac{dx}{dy} - x = y \quad \text{Now P} = -1$$

$$\Rightarrow \text{IF} = e^{\int P\,dy} = e^{\int(-1)dy} = e^{-y}$$

$$\therefore e^{-y}\frac{dx}{dy} - xe^{-y} = ye^{-y} \Rightarrow \frac{d}{dy}[xe^{-y}] = ye^{-y}$$

$$\Rightarrow xe^{-y} = \int ye^{-y}dy + C = -ye^{-y} - e^{-y} + C$$

$$\Rightarrow x = -y - 1 + Ce^{y} \Rightarrow x + y + 1 = Ce^{y}$$

**Ex 9.6 Class 12 Maths Question 11.**

$$ydx + (x - y^2)dy = 0$$

**Solution:**

$$ydx + (x - y^2)dy = 0 \Rightarrow y\frac{dx}{dy} + x - y^2 = 0$$

$$\Rightarrow y.\frac{dx}{dy} + x = y^2 \Rightarrow \frac{dx}{dy} + \frac{1}{y}x = y \therefore \text{P} = \frac{1}{y}$$

$$\Rightarrow \text{Integrating factor} = e^{\int \frac{1}{y}dy} = e^{\log y} = y$$

$$\Rightarrow y\frac{dx}{dy} + x = y^2 \Rightarrow \frac{d}{dy}(xy) = y^2$$

$$\Rightarrow xy = \int y^2 dy + \text{C} \Rightarrow x = \frac{y^2}{3} + \frac{C}{y}$$

**Ex 9.6 Class 12 Maths Question 12.**

$$\left(x+3y^2\right)\frac{dy}{dx}=y(y>0)$$

**Solution:**

$$y\frac{dx}{dy}=x+3y^2 \quad or \quad \frac{dx}{dy}-\frac{x}{y}=3y$$

Here, $P=-\frac{1}{y}$, $Q=3y$, I. F. $=e^{\int pdy}$

$=e^{\int -\frac{1}{y}dy}=e^{-\log y}=e^{\log y^{-1}}=e^{\log\frac{1}{y}}=\frac{1}{y}$

The solution is $x\times \text{I.F.}=\int Q\times \text{I.F. } dy+c$

or $x\times\frac{1}{y}=\int 3y\times\frac{1}{y}dy+c=3y+c$

Hence, the reqd. solu. is $x=3y^2+cy$.

**For each of the following Questions 13 to is find a particular solution, satisfying the given condition:**

**Ex 9.6 Class 12 Maths Question 13.**

$$\frac{dy}{dx}+2ytanx=sinx, y=0 \quad when \quad x=\frac{\pi}{3}$$

**Solution:**

$$\frac{dy}{dx}+(2tanx)y=sinx, P=2tanx$$

$$\Rightarrow \text{IF} = e^{\int 2\tan x\,dx} = e^{2\log \sec x} = \sec^2 x$$

$$\therefore \sec^2 x\frac{dy}{dx} + \sec^2 x(2\tan x)y = \sec^2 x\sin x$$

$$\Rightarrow y.\sec^2 x = \int \sec^2 x\sin x\,dx = \int \sec x\tan x\,dx$$

$$\Rightarrow y.\sec^2 x = \sec x + C$$

When $x = \frac{\pi}{3}$ and $y = 0 \Rightarrow C = -2$

$\therefore\ y = \cos x - 2\cos^2 x$, is the req. sol.

**Ex 9.6 Class 12 Maths Question 14.**

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}, y = 0 \quad when \quad x = 1$$

**Solution:**

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{\left(1+x^2\right)^2}$$

Here, $P = \frac{2x}{x^2+1}$ and $Q = \frac{1}{(x^2+1)^2}$

$$I.F. = e^{\int pdx} = e^{\int \frac{2x}{x^2+1} dx} = e^{\log(x^2+1)} = x^2+1$$

$$\therefore \ y(x^2+1) = \int \frac{1}{(x^2+1)^2}(x^2+1)\,dx + c$$

$$\Rightarrow (x^2+1)\,y = \int \frac{1}{x^2+1}\,dx + c = \tan^{-1}x + c$$

$$\Rightarrow \ y = \frac{\tan^{-1}x}{x^2+1} + \frac{c}{x^2+1}, (x \in R)$$

$y = 0$ when $x = 1 \ \therefore \ c = -\tan^{-1}.1 = -\frac{\pi}{4}$

Req. particular solution is

$$y = \frac{\tan^{-1}x}{x^2+1} - \frac{\pi}{4(x^2+1)}$$

$$\text{or } y(x^2+1) = \tan^{-1}x - \frac{\pi}{4}$$

**Ex 9.6 Class 12 Maths Question 15.**

$$\frac{dy}{dx} - 3ycotx = sin2x, y = 2 \quad when \quad x = \frac{\pi}{2}$$

**Solution:**

Here P = -3cot x

Q = sin 2x

$\therefore$ IF $= e^{\int pdx} = e^{\log \operatorname{cosec}^3 x} = \operatorname{cosec}^3 x$

$\Rightarrow$ Solution is $y \times$ I.F. $= \int Q. \times$ I.F. $dx + c$

or $y \times \operatorname{cosec}^3 x = \int \sin 2x \operatorname{cosec}^3 x \, dx + c$

$$\int \frac{2 \sin x \cos x}{\sin^3 x} dx + c = 2 \int \operatorname{cosec} x \cot x \, dx + c$$

$= -2 \operatorname{cosec} x + c \Rightarrow y = -2\sin^2 x + c \sin^3 x$

Now, $y = 2, x = \frac{\pi}{2}$, $2 = -2 + c \quad \therefore \quad c = 4$

$\therefore$ Reqd particular solution is

$y = -2 \sin^2 x + 4 \sin^3 x \quad y = -2 \sin^2 x (1 - 2 \sin x)$

**Ex 9.6 Class 12 Maths Question 16.**

Find the equation of the curve passing through the origin given that the slope of the tangent to the curve at any point (x,y) is equal to the sum of the coordinates of the point

**Solution:**

$$\frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x \Rightarrow P = -1$$

$\Rightarrow$ IF $= e^{\int -dx} = e^{-x} \Rightarrow e^{-x} \frac{dy}{dx} - ye^{-x} = xe^{-x}$

$\Rightarrow y.e^{-x} = \int xe^{-x} dx = -xe^{-x} - e^{-x} + C$

$\Rightarrow y = -x - 1 + Ce^{x}$; When $x = 0, y = 0 \Rightarrow C = 1$

$\therefore y = -x - 1 + e^{x}$ (Particular solution).

**Ex 9.6 Class 12 Maths Question 17.**

Find the equation of the curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5

**Solution:**

By the given condition

$$x + y - \left|\frac{dy}{dx}\right| = 5$$

or $\left|\frac{dy}{dx}\right| = x + y - 5 \quad \therefore \frac{dy}{dx} = (x + y - 5)$

(i) Taking + ve: $\frac{dy}{dx} - y = x - 5$; I.F. $= e^{\int(-1)\,dx} = e^{-x}$

is $y \times e^{-x} = \int e^{-x} \times (x - 5)\,dx + c$

Integrating by parts taking (x – 5) as first function

$ye^{-x} = (x - 5)(-e^{-x}) - \int 1 . (-e^{-x})\,dx + c$

$= -(x - 5)e^{-x} - e^{-x} + c = 4 - x + c . e^{x}$

The curve passes through (0, 2)

$\therefore x = 0, y = 2 \Rightarrow 2 = 4 - 0 + c. e^{0} \;\therefore\; c = -2$

Req. eq. of the curve is $\quad y = 4 - x - 2e^{x}$

(ii) Taking – ve: $\frac{dy}{dx} = -(x + y - 5) = -x - y + 5$

or $\frac{dy}{dx} + y = -x + 5 \;\therefore\;$ I.F. $= e^{\int 1\,dx} = e^{x}$

Solution is $ye^{x} = \int(5 - x)e^{x}\,dx + c$

$= (5 - x)e^{x} - \int(-1). e^{x}dx + c = (5 - x)\,e^{x} + e^{x} + c$

or $y = 5 - x + 1 + ce^{x} = 6 - x + ce^{x}$

The curve passes through (0, 2)

$\therefore \; x = 0, y = 2 \Rightarrow 2 = 6 - 0 + c. e^{0}$ or $c = -4$

$\therefore$ Equation of the curve $\quad y = 6 - x - 4e^{-x}$

**Ex 9.6 Class 12 Maths Question 18.**

The integrating factor of the differential equation

$$x\frac{dy}{dx}-y=2x^2$$

(a)

$e^{-x}$

(b)

$e^{-y}$

(c)

$\frac{1}{x}$

(d) x

**Solution:**

(c)

$$P=\frac{-1}{x}\therefore IF=e^{-\int\frac{1}{x}dx}=e^{-logx}=\frac{1}{x}$$

**Ex 9.6 Class 12 Maths Question 19.**

The integrating factor of the differential equation

$$\left(1-y^2\right)\frac{dx}{dy}+yx=ay$$

(-1<y<1) is

(a)

$\frac{1}{y^2-1}$

(b)

$\frac{1}{\sqrt{y^2-1}}$

(c)

$\frac{1}{1-y^2}$

(d)

$\frac{1}{\sqrt{1-y^2}}$

**Solution:**

(d)

$$\left(1-y^2\right)\frac{dx}{dy}+yx=ay$$

$$P = \frac{y}{1-y^2}, \ IF = e^{\int \frac{y}{1-y^2} dx} = e^{-\frac{1}{2}\int \frac{(-2)y\,dy}{1-y^2}}$$

$$= e^{-\frac{1}{2}\log(1-y^2)} = (1-y^2)^{-1/2} = \frac{1}{\sqrt{1-y^2}}$$

# Chapterwise NCERT Solutions for Class 12 Maths :

# About NCERT

The National Council of Educational Research and Training is an autonomous organization of the Government of India which was established in 1961 as a literary, scientific, and charitable Society under the Societies Registration Act. The major objectives of NCERT and its constituent units are to: undertake, promote and coordinate research in areas related to school education; prepare and publish model textbooks, supplementary material, newsletters, journals and develop educational kits, multimedia digital materials, etc.Organise pre-service and in-service training of teachers; develop and disseminate innovative educational techniques and practices;collaborate and network with state educational departments, universities, NGOs and other educational institutions; act as a clearing house for ideas and information in matters related to school education; and act as a nodal agency for achieving the goals of Universalisation of Elementary Education.In addition to research, development, training, extension, publication and dissemination activities, NCERT is an implementation agency for bilateral cultural exchange programmes with other countries in the field of school education.Its headquarters are located at Sri Aurobindo Marg in New Delhi. Visit the Official NCERT website to learn more.