

NCERT Solutions for 12th Class Maths: Chapter 8-Application of Integrals

Class 12: Maths Chapter 8 solutions. Complete Class 12 Maths Chapter 8 Notes.

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Ex 8.1 Class 12 Maths Question 1.

Find the area of the region bounded by the curve $y^2 = x$ and the lines x = 1, x = 4, and the x-axis.

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Solution:

The curve $y^2 = x$ is a parabola with vertex at origin.Axis of x is the line of symmetry, which is the axis of parabola. The area of the region bounded by the curve, x = 1, x=4 and the x-axis. Area LMQP



Ex 8.1 Class 12 Maths Question 2.

Find the area of the region bounded by $y^2 = 9x$, x = 2, x = 4 and x-axis in the first quadrant

Solution:

The given curve is $y^2 = 9x$, which is a parabola with vertex at (0, 0) and axis along x-axis. It is symmetrical about x-axis, as it contains only even powers of y. x = 2 and x = 4 are straight lines parallel toy-axis at a positive distance of 2 and 4 units from it respectively.

: Required area = Area ABCD



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Ex 8.1 Class 12 Maths Question 3.

Find the area of the region bounded by $x^2 = 4y$, y = 2, y = 4 and the y-axis in the first quadrant.

Solution:

The given curve $x^2 = 4y$ is a parabola with vertex at (0,0). Also since it contains only even powers of x, it is symmetrical about y-axis.y = 2 and y = 4 are straight lines parallel to x-axis at a positive distance of 2 and 4 from it respectively.



Ex 8.1 Class 12 Maths Question 4.

Find the area of the region bounded by the ellipse



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Solution:

The equation of the ellipse is

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

The given ellipse is symmetrical about both axis as it contains only even powers of y and x.





Now, area bounded by the ellipse = 4 (area of ellipse in first quardant) = 4 (area OAC)

$$=4\int_{0}^{4} y dx = \int_{0}^{4} \frac{3}{4} \sqrt{16 - x^{2}} dx$$

Put x = 4 sin θ so that dx = 4 cos θ d θ ,

Now when
$$x = 0, \theta = 0$$
 and when $x = 4, \theta = \frac{\pi}{2}$

$$\therefore \operatorname{Req. area} = \frac{4 \times 3}{4} \int_{0}^{\frac{\pi}{2}} \sqrt{16 - 16 \sin^{2} \theta} \cdot 4 \cos \theta \, d\theta$$
$$= 3 \int_{0}^{\frac{\pi}{2}} 4 \sqrt{1 - \sin^{2} \theta} \cdot 4 \cos \theta \, d\theta$$
$$= 48 \int_{0}^{\frac{\pi}{2}} \cos^{2} \theta \, d\theta = 24 \int_{0}^{\frac{\pi}{2}} (1 + \cos 2\theta) \, d\theta$$
$$= 24 \left[\theta + \frac{\sin 2\theta}{2} \right]_{0}^{\frac{\pi}{2}} = 12\pi \operatorname{sq. units.}$$

Ex 8.1 Class 12 Maths Question 5.

Find the area of the region bounded by the ellipse

Solution:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

It is an ellipse with centre (0,0) and length of major axis = $2a = 2 \times 3 = 6$ and length of minor axis = $2b = 2 \times 2 = 4$.



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Its rough sketch is shown in Fig.

∴ Area bounded by ellipse = 4 × area of region AOB

$$= 4 \int_0^2 \frac{3}{2} \sqrt{4 - x^2} \, dx = 6 \int_0^2 \sqrt{4 - x^2} \, dx$$
$$= 6 \left[\frac{x\sqrt{4 - x^2}}{2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 = 6\pi \text{ sq. units.}$$

Ex 8.1 Class 12 Maths Question 6.

Find the area of the region in the first quadrant enclosed by x-axis, line x = $\sqrt{3}$ y and the circle x² + y² = 4.

Solution:

Consider the two equations $x^2 + y^2 = 4 \dots (i)$

and $x = \sqrt{3}y$ i.e.

$$y = \frac{1}{\sqrt{3}}x$$

(1) $x^2 + y^2 = 4$ is a circle with centre O (0,0) and radius = 2.



(2) $y = \frac{1}{\sqrt{3}} x$ is a straight line passing through (0, 0) and intersecting the circle at $B(\sqrt{3}, 1)$. required area = shaded region, = area OBL + area LBA $= \frac{1}{\sqrt{3}} \int_{0}^{\sqrt{3}} x dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2} dx$ $= \frac{1}{\sqrt{3}} \left[\frac{x^2}{2} \right]_{0}^{\sqrt{3}} + \left[\frac{x\sqrt{4 - x^2}}{2} + \frac{4}{2} \sin \frac{x}{2} \right]_{\sqrt{3}}^{2}$ $= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 2\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$ sq. units.

Ex 8.1 Class 12 Maths Question 7.

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line

$$x = \frac{a}{\sqrt{2}}$$

Solution:

The equation of the given curve are

 $x^2 + y^2 = a^2 \dots$ (i) and $x = \frac{a}{\sqrt{2}}$...(ii) Clearly, (i) represent a circle and (ii) is the equation of a straight line parallel to y-axis at a



distance $\frac{a}{\sqrt{2}}$ units to the right of y-axis. Solving (i) and (ii).



 $\therefore P\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right) \text{ is the point of interaction of } curve (1) and (2) in the first quardant.}$

The smaller region bounded by these two curve is the shaded portion shown in the figure.

... Required Area = shaded region

$$= 2 \int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2} \, dx$$

= $2 \left[\frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{x}{a} \right]_{\frac{a}{\sqrt{2}}}^{a} = \frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)$ sq.units.

Ex 8.1 Class 12 Maths Question 8.

The area between $x = y^2$ and x = 4 is divided into two equal parts by the line x = a, find the value of a.

Solution:

Graph of the curve $x = y^2$ is a parabola as given in the figure. Its vertex is O and axis is x-axis. QR is the ordinate along x = 4





Area of the region bounded by the curve and ordinate x = a is

$$A_2 = 2 \int_0^a y \, dx = 2 \int_0^a \sqrt{x} \, dy = 2 \cdot \frac{2}{3} \left[\frac{3}{x^2} \right]_0^a = \frac{4}{3} \cdot \left(\frac{3}{a^2} - 0 \right)$$

Now PS [x = a] devided the area of the region ORQ into two equal parts

$$\therefore \text{ Area of rregion ORQ} = 2 \text{ Area of the region OPS,}$$

$$\therefore A_1 = 2 A_2$$

$$\Rightarrow \frac{32}{3} = 2 \cdot \frac{4}{3} a^{\frac{3}{2}} \Rightarrow a^{\frac{3}{2}} = 4 \Rightarrow a = 4^{\frac{2}{3}} \text{ units.}$$

Ex 8.1 Class 12 Maths Question 9.

Find the area of the region bounded by the parabola $y = x^2$ and y = |x|.

Solution:

Clearly $x^2 = y$ represents a parabola with vertex at (0, 0) positive direction of y-axis as its axis it opens upwards.

y = |x| i.e., y = x and y = -x represent two lines passing through the origin and making an angle of 45° and 135° with the positive direction of the x-axis.

The required region is the shaded region as shown in the figure. Since both the curve are symmetrical about y-axis. So,

Required area = 2 (shaded area in the first quadrant)





$$= 2\int_0^1 (x - x^2) dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$
 sq. units.

Ex 8.1 Class 12 Maths Question 10.

Find the area bounded by the curve $x^2 = 4y$ and the line x = 4y - 2

Solution:

Given curve is $x^2 = 4y \dots (i)$

which is an upward parabola with vertex at (0,0) and it is symmetrical about y-axis

Equation of the line is x = 4y - 2...(ii)

Solving (i) and (ii) simultaneously, we get; $(4y - 2)^2 = 4y$

$$\Rightarrow$$
 16y² - 16y + 4 = 4y

 $\Rightarrow 4y^2 - 5y + 1 = 0$



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$$\Rightarrow (4y-1)(y-1) = 0 \Rightarrow y = \frac{1}{4}, 1$$

From (ii),
when $y = \frac{1}{4}, x = -1$;
When $y = 1, x = 2$
 $\therefore A\left(-1, \frac{1}{4}\right)$ and B
(2, 1) are the points
of intersection of
(i) and (ii).Clearly, from the figures the shaded
portion OBDAO is the region of the required area.
Required area = area LOMBDA – area LMBOAL
...(iii)
Now area LMBDA = $\int_{-1}^{2} y \, dx$, $\frac{1}{4} \int_{-1}^{2} (x+2) \, dx$
(where $x = 4y \Rightarrow y = \frac{1}{4} (x+2)$)
 $= \frac{1}{4} \left[\frac{x^2}{2} + 2x \right]_{-1}^{2} = \frac{1}{4} \left(6 + \frac{3}{2} \right) = \frac{15}{8}$ sq. units ...(iv)

and area LMBOAL =
$$\int_{-1}^{2} y \, dx$$
, where $x^2 = 4y$
= $\frac{1}{4} \int_{-1}^{2} x^2 \, dx = \frac{1}{4} \left[\frac{x^3}{3} \right]_{-1}^{2} = \frac{3}{4}$ sq. units. ...(v)

From (iii), (iv) and (v), the required area, = $\frac{15}{8} - \frac{3}{4} = \frac{9}{8}$ sq. units.

Ex 8.1 Class 12 Maths Question 11.

Find the area of the region bounded by the curve $y^2 = 4x$ and the line x = 3.

Solution:



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The curve $y^2 = 4x$ is a parabola as shown in the figure. Axis of the parabola is x-axis. The area of the region bounded by the curve $y^2 = 4x$ and the line x = 3 is

A = Area of region OPQ = 2 (Area of the region OLQ)



Choose the correct answer in the following Exercises 12 and 13:

Ex 8.1 Class 12 Maths Question 12.

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x = 2 is

(a) π

(b)

 $\frac{\pi}{2}$



 $\frac{\pi}{3}$

(d)

 $\frac{\pi}{4}$



Solution:

(a) $x^2 + y^2 = 4$. It is a circle at the centre (0,0) and r=2



Ex 8.1 Class 12 Maths Question 13.

Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line y = 3 is

(a) 2

(b)

 $\frac{9}{4}$

4

(C)

 $\frac{9}{3}$



(d)

 $\frac{9}{2}$

Solution:

(b) y² = 4x is a parabola



Ex 8.2 Class 12 Maths Question 1.

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

Solution:

Area is bounded by the circle $4x^2 + 4y^2 = 9$ and interior of the parabola $x^2 = 4y$.

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Putting x^2 = 4y in x^2 + y^2 = \frac{9}{4}
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We get $4y + y^2 =$





$$= 4 \int_{0}^{1/2} \sqrt{y} dy + 2 \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - y^2} dy$$

$$= 4 \times \frac{2}{3} \left[y^{\frac{3}{2}} \right]_{0}^{1/2} + 2 \left[\frac{y}{2} \sqrt{\frac{9}{4} - y^2} + \frac{9}{8} \sin^{-1} \left(\frac{y}{3/2} \right) \right]_{1/2}^{3/2}$$

$$= \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \left(1 \cdot \frac{2\sqrt{2}}{3} - 0 \right) = \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$$

Ex 8.2 Class 12 Maths Question 2.

Find the area bounded by curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

Solution:

Given circles are $x^2 + y^2 = 1 \dots (i)$

and $(x - 1)^2 + y^2 = 1$...(ii)

Centre of (i) is O (0,0) and radius = 1





Both these circle are symmetrical about x-axis solving (i) and (ii) we get, $-2x + 1 = 0 \Rightarrow x = \frac{1}{2}$

then $y^2 = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} \Rightarrow y = \frac{\sqrt{3}}{2}$ \therefore The points of intersection are

$$P\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$$
 and $Q\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

It is clear from the figure that the shaded portion in region whose area is required.

 \therefore Req. area = area OQAPO = 2 × area of the region OLAP = 2 × (area of the region OLPO + area of LAPL)

$$= 2 \left[\int_{0}^{1/2} \sqrt{1 - (x - 1)^2} \, dx + \int_{1/2}^{1} \sqrt{1 - x^2} \, dx \right]$$

$$= 2 \left[\frac{(x - 1)\sqrt{1 - (x - 1)^2}}{2} + \frac{1}{2} \sin^{-1} (x - 1) \right]_{0}^{1/2}$$

$$+ 2 \left[\frac{x\sqrt{1 - x^2}}{2} + \frac{1}{2} \sin^{-1} x \right]_{1/2}^{1}$$

$$= \frac{\sqrt{3}}{4} - \frac{\pi}{6} - \left(-\frac{\pi}{2} \right) + \frac{\pi}{2} - \frac{\sqrt{3}}{4} - \frac{\pi}{6} = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)$$

sq. units.

Ex 8.2 Class 12 Maths Question 3.



Find the area of the region bounded by the curves $y = x^2 + 2$, y = x, x = 0 and x = 3.

Solution:

Equation of the parabola is $y = x^2 + 2$ or $x^2 = (y - 2)$

Its vertex is (0,2) axis is y-axis.

Boundary lines are y = x, x = 0, x = 3.

Graphs of the curve and lines have been shown in the figure.

Area of the region PQRO = Area of the region OAQR-Area of region OAP

Ex 8.2 Class 12 Maths Question 4.

Using integration find the area of region bounded by the triangle whose vertices are (-1,0), (1,3) and (3,2).

Solution:

The points A (-1,0), B(1,3) and C (3,2) are plotted and joined.

Area of $\triangle ABC$ = Area of $\triangle ABL$ + Area of trap. BLMC – Area of $\triangle ACM \dots (i)$

The equation of the line joining the points





Ex 8.2 Class 12 Maths Question 5.

Using integration find the area of the triangular region whose sides have the equations y = 2x + 1, y = 3x + 1 and x = 4.

Solution:

The given lines are $y = 2x + 1 \dots (i)$

y = 3x + 1 ...(ii)

x = 4 ...(iii)

Subtract (i) from eq (ii) we get x = 0, Putting x = 0 in eq(i) y = 1

: Lines (ii) and (i) intersect at A (0,1) putting x = 4 in eq. (2) =>y = 12 + 1 = 13

The lines (ii) and (iii) intersect at B (4,13) putting x=4ineq. (i):y = 8 + 1 = 9

Lines (i) and (ii); Intersect at C (4,9),





Ex 8.2 Class 12 Maths Question 6.

Smaller area bounded by the circle $x^2 + y^2 = 4$ and the line x + y = 2

- (a) 2 $(\pi 2)$
- (b) π 2
- (c) 2π 1
- (d) $2(\pi + 2)$

Solution:

(b) A circle of radius 2 and centre at O is drawn. The line AB: x + y = 2 is passed through (2,0) and (0,2). Area of the region ACB

= Area of quadrant OAB – Area of $\triangle OAB \dots (i)$





$$= \int_{0}^{2} (2-x) dx = \left[2x - \frac{x^{2}}{2} \right]_{0} = \left(4 - \frac{4}{2} \right) - (0) = 2$$

Putting these values in (i), Area of region $ACB = \pi - 2$

Ex 8.2 Class 12 Maths Question 7.

Area lying between the curves $y^2 = 4x$ and y = 2x.

(a)

 $\frac{2}{3}$

(b)

 $\frac{1}{3}$

(C)

(d)



Solution:

(b) The curve is $y^2 = 4x ...(1)$

and the line is $y = 2x \dots (2)$

from (1) and (2): $(4x^2) = 4x$ $\Rightarrow x=0$, or x=1curves (1) and (2) intersect at O (0, 0), A (1, 2) Area of region OACO = Area of region OMACO - area of OMA ...(3) Now Area of region OMACO = Area of region bounded by OCA $= \int_0^1 \sqrt{4x} dx = 2 \cdot \frac{2}{3} [x^{3/2}]_0^1 = \frac{4}{3} \times 1 = \frac{4}{3} \text{ sq. units.}$ Area of $\Delta OMA = \text{Area of region bounded by OA}:$ y=2x, x=1 and x-axis. $= \int_0^1 2x dx = [x^2]_0^1 = 1$ Putting these values in (3), We get : \therefore Area of region OMACO $= \frac{4}{3} - 1 = \frac{1}{3} \text{ sq units.}$





Chapterwise NCERT Solutions for Class 12 Maths :

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- <u>Chapter 2 Inverse Trigonometric Functions.</u>
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