NCERT Solutions for 12th Class Maths: Chapter 7-Integrals

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NCERT Solutions for 12th Class Maths: Chapter 7-Integrals

Class 12: Maths Chapter 7 solutions. Complete Class 12 Maths Chapter 7 Notes.

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Class 12: Maths Chapter 7 solutions. Complete Class 12 Maths Chapter 7 Notes.

Ex 7.1 Class 12 Maths Question 1.

sin 2x

Solution:

$$\int sin2x \quad dx = -\frac{cos2x}{2} + C$$

Ex 7.1 Class 12 Maths Question 2.

cos 3x

Solution:

$$\int cos3x \quad dx = \frac{sin3x}{3} + C$$

Ex 7.1 Class 12 Maths Question 3.

e^{2x}

Solution:

$$\int e^{2x} dx = \frac{e^{2x}}{2} + C$$

Ex 7.1 Class 12 Maths Question 4.

$(ax + c)^2$

Solution:

$$\int (ax + b)^2 dx = \frac{(ax+b)^3}{3a} + C$$

Ex 7.1 Class 12 Maths Question 5.

$sin \quad 2x - 4e^{3x}$

Solution:

$$\int \left(sin2x - 4e^{3x}\right) dx = -\frac{cos2x}{2} - \frac{4e^{3x}}{3} + C$$

Find the following integrals in Exercises 6 to 20 :

Ex 7.1 Class 12 Maths Question 6.

$$\int \left(4e^{3x}+1\right) dx$$

Solution:

$$= \int 4e^{3x} dx + \int dx = \frac{4}{3}e^{3x} + x + c$$

Ex 7.1 Class 12 Maths Question 7.

$$\int x^2 \left(1-\frac{1}{x^2}\right) dx$$

Solution:

$$= \int x^2 \left(1-\frac{1}{x^2}\right) dx = \frac{x^3}{3} - x + C$$

Ex 7.1 Class 12 Maths Question 8.

$$\int (ax^2 + bx + c)dx$$

Solution:

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$$

Ex 7.1 Class 12 Maths Question 9.

$$\int \left(2x^2+e^x\right) dx$$

Solution:

$$= \frac{2x^3}{3} + e^x + c$$

Ex 7.1 Class 12 Maths Question 10.

$$\int \left[\sqrt{x} - \frac{1}{\sqrt{x}}\right]^2 dx$$

Solution:

$$= \frac{x^2}{2} + logx - 2x + C$$

Ex 7.1 Class 12 Maths Question 11.

$$\int \frac{x^3+5x^2-4}{x^2} dx$$

Solution:

$$\int \left(\frac{x^3}{x^2} + \frac{5x^2}{x^2} - \frac{4}{x^2}\right)$$

$$= \int xdx + 5\int 1dx - 4\int x^2dx$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + c$$

Ex 7.1 Class 12 Maths Question 12.

$\int \frac{x^3+3x+4}{\sqrt{x}} dx$

Solution:

$= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\right) dx$

$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + c$

Ex 7.1 Class 12 Maths Question 13.

$\int \frac{x^3-x^2+x-1}{x-1} dx$

Solution:

$= \int \frac{x^2(x-1)+(x-1)}{x-1} dx$

$= \int \left(x^2 + 1\right) dx = \frac{x^3}{3} + x + c$

Ex 7.1 Class 12 Maths Question 14.

Solution:

$= \int x^{\frac{1}{2}} - x^{\frac{3}{2}} dx \quad = \quad \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}$

Ex 7.1 Class 12 Maths Question 15.

$\int \sqrt{x}\left(3x^2+2x+3\right)dx$

Solution:

$= \int \left(3x^{\frac{5}{2}}+2^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$

$= \frac{6}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+\frac{6}{3}x^{\frac{3}{2}}+c$

Ex 7.1 Class 12 Maths Question 16.

$\int (2x-3cosx+e^x)dx$

Solution:

$= \frac{2x^2}{2}-3sinx+e^x+c$

$= x^2-3sinx+e^x+c$

Ex 7.1 Class 12 Maths Question 17.

$\int \left(2x^2-3sinx+5\sqrt{x}\right)dx$

Solution:

$= \frac{2x^3}{3}+3cosx+5\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+c$

$= \frac{2}{3}x^3 + 3cosx + \frac{10}{3}x^{\frac{3}{2}} + c$

Ex 7.1 Class 12 Maths Question 18.

$\int secx(secx + tanx)dx$

Solution:

$= \int (sec^2x + secxtanx)dx$

= tanx + secx + c

Ex 7.1 Class 12 Maths Question 19.

$\int \frac{sec^2x}{cosec^2x}dx$

Solution:

$= \int \frac{1}{cos^2x}sin^2xdx$

$= \int tan^2xdx \quad = \int (sec^2x - 1)dx \quad = tanx - x + c$

Ex 7.1 Class 12 Maths Question 20.

$\int \frac{2-3sinx}{cos^2x}dx$

Solution:

$$= \int \left(\frac{2}{cos^2x} - 3\frac{sinx}{cos^2x}\right) dx = \int (2sec^2x - 3secxtanx)dx$$

= 2tanx – 3secx + c

Choose the correct answer in Exercises 21 and 22.

Ex 7.1 Class 12 Maths Question 21.

The antiderivative

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$$

equals

(a)

$$\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + c$$

(b)

$$\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^{2} + c$$

(c)

$$\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

(d)

$$\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + c$$

Solution:

(c)

$$\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right) dx$$

$$=\int\left(x^{\frac{1}{2}}+x^{\frac{1}{2}}\right) dx$$

$$=\frac{2}{3} x^{\frac{3}{2}}+2 x^{\frac{1}{2}}+c$$

Ex 7.1 Class 12 Maths Question 22.

If

$$\frac{d}{d x} f(x)=4 x^{3}-\frac{3}{x^{4}}$$

such that f(2)=0 then f(x) is

(a)

$$x^{4}+\frac{1}{x^{3}}-\frac{129}{8}$$

(b)

$$x^{3}+\frac{1}{x^{4}}+\frac{129}{8}$$

(c)

$x^4+\frac{1}{x^3}+\frac{129}{8}$

(d)

$x^3+\frac{1}{x^4}-\frac{129}{8}$

Solution:

(a)

$$f(x)=\int\left(4x^3-\frac{3}{x^4}\right)dx$$

$$=x^4+\frac{1}{x^3}+c$$

$$\therefore f(2)=(2)^4+\frac{1}{(2)^3}+c=0=-\frac{129}{8}$$

Ex 7.1 Class 12 Maths Question 1.

$\frac{2x}{1+x^2}$

Solution:

Let $1+x^2=t$

$\Rightarrow 2xdx=dt$

$$\therefore\int\frac{2x}{1+x^2}dx \quad =\int\frac{dt}{t} \quad =logt+C \quad =log(1+x^2)+C$$

Ex 7.2 Class 12 Maths Question 2.

$\frac{(logx)^2}{x}$

Solution:

Let logx = t

⇒

$\frac{1}{x}dx = dt \therefore \int \frac{(logx)^2}{x}dx \quad = \int t^2 dt \quad = \frac{t^3}{3} + c \quad = \frac{1}{3}(logx)^3 + c$

Ex 7.2 Class 12 Maths Question 3.

$\frac{1}{x+xlogx}$

Solution:

Put 1+logx = t

∴

$\frac{1}{x}dx = dt \int \frac{1}{x(1+logx)}dx = \int \frac{1}{t}dt = log|t| + c$

= log|1+logx|+c

Ex 7.2 Class 12 Maths Question 4.

sinx sin(cosx)

Solution:

Put cosx = t, -sinx dx = dt

$$\int sinx \quad sin(cosx)dx = -\int sin(cosx)(-sinx)dx$$

$$= -\int sint \quad dt \quad = cost + c \quad = cos(cosx) + c$$

Ex 7.2 Class 12 Maths Question 5.

sin(ax+b) cos(ax+b)

Solution:

let sin(ax+b) = t

⇒ cos(ax+b)dx = dt

$$\therefore \int sin(ax+b)cos(ax+b)dx = \frac{1}{a}\int t \quad dt$$

$$= \frac{1}{a}.\frac{t^2}{2} + c \quad = \frac{1}{2a}sin^2(ax+b) + C$$

Ex 7.2 Class 12 Maths Question 6.

$$\sqrt{ax+b}$$

Solution:

$$\int \sqrt{ax+b}dx \quad = \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$$

Ex 7.2 Class 12 Maths Question 7.

$$x\sqrt{x+2}$$

Solution:

Let x+2 = t²

⇒ dx = 2t dt

$$\therefore \quad \int x\sqrt{x+2}\,dx = \int (t^2-2)\,t\,(2t)\,dt$$
$$= \quad 2\int (t^4-2t^2)\,dt = 2\frac{t^5}{5} - 4\frac{t^3}{3} + C$$
$$= \quad \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

Ex 7.2 Class 12 Maths Question 8.

$$x\sqrt{1+2x^2}$$

Solution:

let 1+2x² = t²

⇒ 4x dx = 2t dt

$$= \frac{1}{2}\int t^2 dt = \frac{t^3}{6} + c = \frac{1}{6}(1+2x^2)^{\frac{3}{2}} + c$$

Ex 7.2 Class 12 Maths Question 9.

$$(4x+2)\sqrt{x^2+x+1}$$

Solution:

let x²+x+1 = t

⇒(2x+1)dx = dt

$$\therefore \int (4x+1)\sqrt{x^2+x+1}dx = 2\int \sqrt{t}dt$$

$$= \frac{2t^{\frac{3}{2}}}{\frac{3}{2}} + c \quad = \frac{4}{3}t^{\frac{3}{2}} + c \quad = \frac{4}{3}(x^2+x+1)^{\frac{3}{2}} + c$$

Ex 7.2 Class 12 Maths Question 10.

$$\frac{1}{x-\sqrt{x}}$$

Solution:

$$\int \frac{1}{x-\sqrt{x}}dx = \int \frac{1}{\sqrt{x}(\sqrt{x}-1)}dx = I$$

Let √x-1 = t

$$\frac{1}{2}x^{-\frac{1}{2}}dx = dt \ I = 2\int \frac{dt}{t}$$

= 2logt + c

= 2log(√x-1)+c

Ex 7.2 Class 12 Maths Question 11.

$$\frac{x}{\sqrt{x+4}}, x > 0$$

Solution:

let x+4 = t

⇒ dx = dt, x = t-4

$$\therefore \int \frac{x}{\sqrt{x+4}} dx = \int \frac{t-4}{\sqrt{t}} dt = \int \left(t^{1/2} - 4t^{-\frac{1}{2}} \right) dt$$

$$= \frac{2}{3} t^{3/2} - 4 \times 2t^{1/2} + C$$

$$= \frac{2}{3} (x+4)^{3/2} - 8(x+4)^{1/2} + C$$

Ex 7.2 Class 12 Maths Question 12.

$(x^3 - 1)^{\frac{1}{3}} . x^5$

Solution:

$$\int (x^3 - 1)^{\frac{1}{3}} . x^5 . dx \quad = \tfrac{1}{7} (x^3 - 1)^{\frac{7}{3}} + \tfrac{1}{4} (x^3 - 1)^{\frac{4}{3}} + c$$

Ex 7.2 Class 12 Maths Question 13.

$\frac{x^2}{(2+3x^3)^3}$

Solution:

Let $2+3x^3 = t$

⇒ $9x^2$ dx = dt

$$\therefore \int \frac{x^2}{(2+3x^3)^3}dx = \frac{1}{9}\int \frac{dt}{t^3} = \frac{1}{9}\int t^{-3}dt$$

$$= -\frac{1}{18t^2} + c \quad = -\frac{1}{18(2+3x^3)^2} + c$$

Ex 7.2 Class 12 Maths Question 14.

$$\frac{1}{x(logx)^m}, x > 0$$

Solution:

Put log x = t, so that

$$\frac{1}{x}dx = dt$$

$$\therefore \int \frac{1}{x(logx)^m}dx = \int \frac{dt}{t^m} = \frac{t^{-m+1}}{-m+1} + c$$

$$= \frac{(logx)^{1-m}}{1-m} + c$$

Ex 7.2 Class 12 Maths Question 15.

$$\frac{x}{9-4x^2}$$

Solution:

put $9-4x^2 = t$, so that $-8x\ dx = dt$

$$\therefore \int \frac{x}{9-4x^2}dx = -\frac{1}{8}\int \frac{dt}{t} = -\frac{1}{8}log|t| + c$$

$$= \frac{1}{8}log\frac{1}{|9-4x^2|} + c$$

Ex 7.2 Class 12 Maths Question 16.

$$e^{2x+3}$$

Solution:

put 2x+3 = t

so that 2dx = dt

$$\int e^{2x+3}dx \quad = \frac{1}{2}\int e^t dt \quad = \frac{1}{2}e^t + c \quad = \frac{1}{2}e^{2x+3} + c$$

Ex 7.2 Class 12 Maths Question 17.

$$\frac{x}{e^{x^2}}$$

Solution:

Let x^2 = t

⇒ 2xdx = dt ⇒

$$xdx = \frac{dt}{2} \therefore \int \frac{x}{e^{x^2}}dx \quad = \frac{1}{2}\int \frac{dt}{e^t} \quad = \frac{1}{2}\int e^{-t}dt$$

$$= -\frac{1}{2}e^{-x^2} + c$$

Ex 7.2 Class 12 Maths Question 18.

$\frac{e^{tan^{-1}x}}{1+x^2}$

Solution:

$let \quad tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}dx = dt$

$\therefore \int \frac{e^{tan^{-1}x}}{1+x^2}dx \quad = \int e^t dt \quad = e^{tan^{-1}x} + c$

Ex 7.2 Class 12 Maths Question 19.

$\frac{e^{2x}-1}{e^{2x}+1}$

Solution:

$\int \frac{e^{2x}-1}{e^{2x}+1}dx \quad = \int \frac{e^x(e^x-e^{-x})}{e^x(e^x+e^{-x})}dx = I$

put e^x+e^{-x} = t

so that $(e^x-e^{-x})dx$ = dt

$\therefore I = \int \frac{dt}{t} = log|t| + c = log|e^x + e^{-x}| + c$

Ex 7.2 Class 12 Maths Question 20.

$\frac{e^{2x}-e^{2x}}{e^{2x}+e^{-2x}}$

Solution:

put e^{2x}-e^{-2x} = t

so that ($2e^{2x}$-$2e^{-2x}$)dx = dt

$$\therefore \int \frac{e^{2x}-e^{2x}}{e^{2x}+e^{-2x}} dx = \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} log|t| + c$$

$$= \frac{1}{2} log + |e^{2x} + e^{-2x}| + c$$

Ex 7.2 Class 12 Maths Question 21.

$\tan^2(2x-3)$

Solution:

$\int \tan^2(2x-3)dx = \int[\sec^2(2x-3)-1]dx$ = I

put 2x-3 = t

so that 2dx = dt

I =

$\frac{1}{2}$

$\int \sec^2 t$ dt-x+c

=

$\frac{1}{2}t - x + c$

=

$\frac{1}{2}tan(2x-3)-x+c$

Ex 7.2 Class 12 Maths Question 22.

$sec^2(7-4x)$

Solution:

$\int sec^2(7-4x)dx$

=

$\frac{tan(7-4x)}{-4}+c$

Ex 7.2 Class 12 Maths Question 23.

$\frac{sin^{-1}x}{\sqrt{1-x^2}}$

Solution:

$let \quad sin^{-1}x=t \quad \Rightarrow \frac{1dx}{\sqrt{1-x^2}}=dt$

$\int \frac{sin^{-1}x}{\sqrt{1-x^2}}dx=\int t \quad dt=\frac{1}{2}t^2+c=\frac{1}{2}(sin^{-1}x)^2+c$

Ex 7.2 Class 12 Maths Question 24.

$\frac{2cosx-3sinx}{6cosx+4sinx}$

Solution:

put 2sinx+4cosx = t

⇒ (2cosx-3sinx)dx = dt

$$\frac{1}{2}\int\frac{2cosx-3sinx}{2sinx+3cosx}dx=\frac{1}{2}\int\frac{dt}{t}=\frac{1}{2}log|t|+c$$

$$\frac{1}{2}log|2sinx+3cosx|+c$$

Ex 7.2 Class 12 Maths Question 25.

$$\frac{1}{cos^2x(1-tanx)^2}$$

Solution:

put 1-tanx = t

so that -sec²x dx = dt

$$\therefore\int\frac{1}{cos^2x(1-tanx)^2}dx=\int\frac{sec^2x}{(1-tanx)^2}dx$$

$$=-\int\frac{dt}{t^2}=\frac{1}{t}+c=\frac{1}{(1-tanx)}+c$$

Ex 7.2 Class 12 Maths Question 26.

$$\frac{cos\sqrt{x}}{\sqrt{x}}$$

Solution:

$put \quad \sqrt{x}=t, so \quad that \frac{1}{2\sqrt{x}}dx=dt$

$\therefore \int \frac{cos\sqrt{x}}{\sqrt{x}}dx \quad = \quad 2 \quad = \int cost \quad dt \quad =2sint+c$

= 2sin√x+c

Ex 7.2 Class 12 Maths Question 27.

$\sqrt{sin2x}cos2x$

Solution:

put sin2x = t^2

⇒ cos2x dx = t dt

$\therefore \int \sqrt{sin2x}.cos2x \quad dx \quad = \int t.tdt = \frac{t^3}{3}+c$

$= \frac{(sin2x)^{\frac{3}{2}}}{3}+c$

Ex 7.2 Class 12 Maths Question 28.

$\frac{cosx}{\sqrt{1+sinx}}$

Solution:

put 1+sinx = t^2

⇒cosx dx = 2t dt

$\therefore \int \frac{cosx}{\sqrt{1+sinx}}dx = 2\int dt = 2t+c$

$$= 2\sqrt{1+sinx}+c$$

Ex 7.2 Class 12 Maths Question 29.

cotx log sinx

Solution:

put log sinx = t,

⇒ cot x dx = dt

$$\therefore \int cot \quad logsinx \quad dx = \int tdt \quad = \frac{t^2}{2}+c$$

$$= \frac{1}{2}(log \quad sinx)^2+c$$

Ex 7.2 Class 12 Maths Question 30.

$$\frac{sinx}{1+cosx}$$

Solution:

put 1+cosx = t

⇒ -sinx dx = dt

$$\therefore \int \frac{sinx}{1+cosx}dx = \int -\frac{dt}{t} = -logt+c$$

=-log(1+cosx)+c

Ex 7.2 Class 12 Maths Question 31.

$\frac{sinx}{(1+cosx)^2}$

Solution:

put 1+cosx = t

so that -sinx dx = dt

$$\therefore \int \frac{sinx}{(1+cosx)^2} dx = -\int \frac{dt}{t^2}$$

$$= \frac{1}{t} + c = \frac{1}{1+cosx} + c$$

Ex 7.2 Class 12 Maths Question 32.

$\frac{1}{1+cotx}$

Solution:

$$\int \frac{1}{1+\frac{cosx}{sinx}} dx = \frac{1}{2} \int \frac{2sinx \quad dx}{sinx+cosx}$$

$$= \frac{1}{2} \int \frac{(\sin x + \cos x) - (\cos x - \sin x)}{(\sin x + \cos x)} dx$$

$$= \frac{1}{2} \int 1 dx - \frac{1}{2} \int \frac{\cos x - \sin x}{\sin x + \cos x} dx = I \text{ (say)}$$

Put $\sin x + \cos x = t$, so that $(\cos x - \sin x) dx = dt$

$$I = \frac{x}{2} - \frac{1}{2} \int \frac{1}{t} dt + c = \frac{x}{2} - \frac{1}{2} \log |t| + c$$

$$= \frac{x}{2} - \frac{1}{2} \log |\sin x + \cos x| + c.$$

Ex 7.2 Class 12 Maths Question 33.

$\frac{1}{1-tanx}$

Solution:

$$\int \frac{1}{1-tanx} dx = \frac{1}{2} \int \frac{2cosx \quad dx}{cosx-sinx}$$

$$= \frac{1}{2} \int \frac{\cos x + \sin x + \cos x - \sin x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put $\cos x - \sin x = t \Rightarrow (\sin x + \cos x) dx = -dt$

$$\Rightarrow I = \frac{1}{2} \int dx + \frac{1}{2} \int \frac{-dt}{t}$$

$$= \frac{x}{2} - \frac{1}{2} \log (\cos x - \sin x) + C$$

Ex 7.2 Class 12 Maths Question 34.

$\frac{\sqrt{tanx}}{sinxcosx}$

Solution:

$$\int\frac{\sqrt{tanx}}{sinxcosx}dx=\int\frac{\sqrt{tanx}}{tanx}.sec^2xdx$$

$$=\int(\tan x)^{-1/2}\sec^2 x\,dx=\mathrm{I}\,(\text{say})$$

Put $\tan x = t$, so that $\sec^2 x\,dx = dt$

$$\therefore \mathrm{I}=\int t^{-\frac{1}{2}}dt=\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+\mathrm{C}\;=2\sqrt{\tan x}+\mathrm{C}$$

Ex 7.2 Class 12 Maths Question 35.

$\frac{(1+logx)^2}{x}$

Solution:

let 1+logx = t

⇒

$$\frac{1}{x}dx=dt\;\int\frac{(1+logx)^2}{x}dx=\int t^2dt=\frac{t^3}{3}+c$$

$$= \frac{1}{3}(1 + logx)^3 + c$$

Ex 7.2 Class 12 Maths Question 36.

$$\frac{(x+1)(x+logx)^2}{x}$$

Solution:

put x+logx = t

$$\left(\frac{x+1}{x}\right) dx = dt \therefore \int \frac{(x+1)(x+logx)^2}{x} dx = \int t^2 dt$$

$$= \frac{(x+logx)^3}{3} + c$$

Ex 7.2 Class 12 Maths Question 37.

$$\frac{x^3 sin(tan^{-1}x^4)}{1+x^8} dx$$

Solution:

$$put \quad tan^{-1}x^4 = t \quad so \quad that \frac{1}{1+x^8} . 4x^3 dx = dt$$

$$\therefore \int \frac{x^3 sin(tan^{-1}x^4)}{1+x^8} dx = \frac{1}{4} \int sint \quad dt$$

$$= \frac{1}{4}(-cost) + c = -\frac{1}{4}cos(tan^{-1}x^4) + c$$

Choose the correct answer in exercises 38 and 39

Ex 7.2 Class 12 Maths Question 38.

$$\int \frac{10x^9 + 10^x loge^{10}}{x^{10} + 10^x} dx$$

(a) $10^x - x^{10} + C$

(b) $10^x + x^{10} + C$

(c) $(10^x - x^{10}) + C$

(d) log $(10^x + x^{10}) + C$

Solution:

(d)

$$\int \frac{10x^9 + 10^x loge^{10}}{x^{10} + 10^x} dx$$

= log $(10^x + x^{10}) + C$

Ex 7.2 Class 12 Maths Question 39.

$$\int \frac{dx}{sin^2x \quad cos^2x} =$$

(a) tanx + cotx + c

(b) tanx – cotx + c

(c) tanx cotx + c

(d) tanx – cot2x + c

Solution:

(c)

$$\int \frac{dx}{sin^2x \quad cos^2x} = = \int \left(sec^2x + cosec^2x\right) dx$$

= tanx – cotx + c

Ex 7.3 Class 12 Maths Question 1.

$sin^2(2x+5)$

Solution:

$\int sin^2(2x+5)dx$

= $\frac{1}{2}\int$[1-cos2(2x+5)]dx

= $\frac{1}{2}\int$[1-cos(4x+10)]dx

=

$$\frac{1}{2}\left[x - \frac{sin(4x+10)}{4}\right] + c$$

Ex 7.3 Class 12 Maths Question 2.

sin3x cos4x

Solution:

$\int$sin3x cos4x

= $\frac{1}{2}\int$[sin(3x+4x)+cos(3x-4x)]dx

= $\frac{1}{2}\int$[sin7x+sin(-x)]dx

=

$$-\frac{1}{14}cos7x + \frac{1}{2}cosx + c$$

Ex 7.3 Class 12 Maths Question 3.

∫cos2x cos4x cos6x dx

Solution:

$\frac{1}{2}$ **∫cos2x cos4x cos6x dx**

= $\frac{1}{2}$ **∫(cos6x+cos2x) cos6x dx**

$$= \frac{1}{4}\int (1 + \cos 12x)\,dx + \frac{1}{4}\int (\cos 8x + \cos 4x)\,dx$$

$$= \frac{1}{4}\left[x + \frac{1}{12}\sin 12x + \frac{1}{8}\sin 8x + \frac{1}{4}\sin 4x\right] + c$$

Ex 7.3 Class 12 Maths Question 4.

∫sin³(2x+1)dx

Solution:

= $\frac{1}{4}$ **∫[3sin(2x+1)-sin3(2x+1)]dx**

=

$$-\frac{3}{8}cos(2x+1) + \frac{1}{24}[4cos^3(2x+1) - 3cos(2x+1)] + c$$

=

$$-\frac{1}{2}cos(2x+1) + \frac{1}{6}cos^3(2x+1) + c$$

Ex 7.3 Class 12 Maths Question 5.

$\sin^3 x \cos^3 x$

Solution:

put sin x = t

⇒ cos x dx = dt

$$\therefore \int sin^3 x cos^3 x dx = \int t^3(1-t^2)dt$$

$$\frac{t^4}{4} - \frac{t^6}{6} + c = \frac{(sinx)^4}{4} - \frac{(sinx)^6}{6} + c$$

Ex 7.3 Class 12 Maths Question 6.

sinx sin2x sin3x

Solution:

∫sinx sin2x sin3x dx

= $\frac{1}{2}$ ∫ 2sin x sin 2x sin 3x dx

= $\frac{1}{2}$ ∫ (cosx – cos3x)sin 3x dx

= $\frac{1}{2}$ ∫ (sin 4x + sin 2x – sin 6x)dx

=

$$\frac{1}{4}\left\{\frac{-cos4x}{4} - \frac{cos2x}{2} + \frac{cos6x}{6}\right\} + c$$

Ex 7.3 Class 12 Maths Question 7.

sin 4x sin 8x

Solution:

$\frac{1}{2}$

∫sin 4x sin 8xdx

=

$\frac{1}{2}$

∫(cos 4x – cos 12x)dx

=

$\frac{1}{2}\left[\frac{sin4x}{4}-\frac{sin12x}{12}\right]+c$

Ex 7.3 Class 12 Maths Question 8.

$\frac{1-cosx}{1+cosx}$

Solution:

$$\int\frac{1-cosx}{1+cosx}dx\int\frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}dx=\int tan^2\frac{x}{2}dx$$

$$=\int\left[sec^2\frac{x}{2}-1\right]dx\quad=2tan\frac{x}{2}-x+c$$

Ex 7.3 Class 12 Maths Question 9.

$\frac{cosx}{1+cosx}$

Solution:

$$\int \frac{cosx}{1+cosx}dx = \int 1dx - \int \frac{1}{1+cosx}dx$$

$$= x - \frac{1}{2}\int sec^2\frac{x}{2}dx + c \quad = x - tan\frac{x}{2} + c$$

Ex 7.3 Class 12 Maths Question 10.

∫sinx⁴ dx

Solution:

$$\int \left(\frac{1-cos2x}{2}\right)^2 dx \quad = \frac{1}{4}\int \left(1 + cos^2 2x - 2cos2x\right) dx$$

$$=\frac{1}{4}\int\left[1+\frac{1+\cos 4x}{2}-2\cos 2x\right]dx$$

$$=\frac{3}{8}x+\frac{1}{32}\sin 4x-\frac{1}{4}\sin 2x+c$$

Ex 7.3 Class 12 Maths Question 11.

$\cos^4 2x$

Solution:

$\int \cos^4 2x\, dx$

$$\int \left(\frac{1+cos4x}{2}\right)^2 dx$$

$$= \frac{1}{4} \int (1 + \cos^2 4x + 2\cos 4x)\, dx$$

$$= \frac{1}{4} \int \left[1 + \frac{1 + \cos 8x}{2} + 2\cos 4x\right] dx$$

$$= \frac{3}{8} x + \frac{1}{64} \sin 8x + \frac{1}{8} \sin 4x + C$$

Ex 7.3 Class 12 Maths Question 12.

$\frac{sin^2 x}{1+cos x}$

Solution:

$$\int \frac{sin^2 x}{1+cos x} dx \quad = \int \frac{1-cos^2 x}{1+cos x} dx$$

$$\int (1 - cos x) dx \quad = x - sin x + c$$

Ex 7.3 Class 12 Maths Question 13.

$\frac{cos 2x - cos 2\alpha}{cos x - cos\alpha}$

Solution:

let I =

$$\int \frac{\left(2cos^2 x - 1\right) - \left(2cos^2\alpha - 1\right)}{cos x - cos\alpha} dx \int \frac{2(cos x - cos\alpha) - (cos x + cos\alpha)}{cos x - cos\alpha} dx$$

= 2∫cos x dx + 2cos α∫dx

= 2(sinx+xcosα)+c

Ex 7.3 Class 12 Maths Question 14.

$\frac{cosx-sinx}{1+sin2x}$

Solution:

let I =

$$\int \frac{cosx-sinx}{1+sin2x}dx = \int \frac{cosx-sinx}{(cosx+sinx)^2}dx$$

put cosx+sinx = t

⇒ (-sinx+cosx)dx = dt

$$I = \int \frac{dt}{t^2} = -\frac{1}{t} + c \quad = \frac{-1}{cosx+sinx} + c$$

Ex 7.3 Class 12 Maths Question 15.

$$\int tan^3 2x \quad sec2x \quad dx = I$$

Solution:

I = ∫(sec^{2}2x-1)sec2x tan 2xdx

put sec2x=t,2 sec2x tan2x dx=dt

$$I = \frac{1}{2}\int (t^2 - 1)\, dt = \frac{1}{2}\left(\frac{t^3}{3} - t\right) + c$$

$$= \frac{1}{2}\left(\frac{1}{3}\sec^2 2x - 1\right)\sec 2x + c$$

Ex 7.3 Class 12 Maths Question 16.

$\tan^4 x$

Solution:

let I = ∫$\tan^4$ dx

= ∫$(\sec^2 x-1)^2$dx

$$= \int (\sec^4 x.dx - 2\int \sec^2 x dx + \int dx$$

$$\Rightarrow \quad I = I_1 - 2\tan x + x + C_1 \quad ...(i)$$

Now, $I_1 = \int \sec^4 x dx = \int (1 + \tan^2 x)\sec^2 x dx$

Put $\tan x = t$, so that $\sec^2 x\, dx = dt$

$\therefore I_1 = t + \frac{t^3}{3} + C_2 \Rightarrow I_1 = \tan x + \frac{1}{3}\tan^3 x + C_2$...(ii)

From (i) and (ii), we have.

$$I = \frac{1}{3}\tan^3 x - \tan x + x + C$$

Ex 7.3 Class 12 Maths Question 17.

$\frac{sin^3x+cos^3x}{sin^2xcos^2x}$

Solution:

$$\int \left(\frac{sin^3x}{sin^2xcos^2x} + \frac{cos^2x}{sinxcos^2x}\right) dx$$

= secx-cosecx+c

Ex 7.3 Class 12 Maths Question 18.

$\frac{cos2x+2sin^2x}{cos^2x}$

Solution:

$$I=\int\frac{\left(cos^2x-sin^2x\right)+2sin^2x}{cos^2x}dx$$

$$=\int\frac{\left(cos^2x-sin^2x\right)}{cos^2x}dx\quad=\int sec^2xdx\quad=tanx+c$$

Ex 7.3 Class 12 Maths Question 19.

$\frac{1}{sinxcos^3x}$

Solution:

$$I=\int\left(tanx+\frac{1}{tanx}\right)sec^2xdx$$

put tanx = t

so that sec^2x dx = dt

$$I=\int\left(t+\frac{1}{t}\right)dt\quad=\frac{t^2}{2}+log|t|+c$$

$$= log|tanx| + \frac{1}{2}tan^2x + c$$

Ex 7.3 Class 12 Maths Question 20.

$$\frac{cos2x}{(cosx+sinx)^2}$$

Solution:

$$I = \int \frac{cos^2x - sin^2x}{(cosx+sinx)^2}dx = \int \frac{cosx-sinx}{cosx+sinx}dx$$

put cosx+sinx=t

⇒(-sinx+cox)dx = dt

$$I = \int \frac{dt}{t} = log|t| + c \quad = log|cosx + sinx| + c$$

Ex 7.3 Class 12 Maths Question 21.

$\sin^{-1}$ (cos x)

Solution:

$$\int sin^{-1}(cosx)dx \quad = sin^{-1}\left[sin\left(\frac{\pi}{2} - x\right)\right]dx$$

$$\int \left(\frac{\pi}{2} - x\right)dx \quad = \frac{\pi x}{2} - \frac{x^2}{2} + c$$

Ex 7.3 Class 12 Maths Question 22.

$$\int \frac{1}{cos(x-a)cos(x-b)}dx$$

Solution:

$$\frac{1}{sin(a-b)}\int\frac{sin[(x-b)-(x-a)]}{cos(x-a)cos(x-b)}dx$$
$$=\frac{1}{sin(a-b)}\left[\int tan(x-b)dx-\int tan(x-a)dx\right]$$

$$=\frac{1}{sin(a-b)}log\left|\frac{cos(x-a)}{cos(x-b)}\right|+c$$

Ex 7.3 Class 12 Maths Question 23.

$$\int\frac{sin^2x-cos^2x}{sin^2xcos^2x}dx \quad is \quad equal \quad to$$

(a) tanx+cotx+c

(b) tanx+cosecx+c

(c) -tanx+cotx+c

(d) tanx+secx+c

Solution:

(a)

$$\int\frac{sin^2x-cos^2x}{sin^2xcos^2x}dx$$

= $\int(sec^2x-cosec^2x)dx$

= tanx+cotx+c

Ex 7.3 Class 12 Maths Question 24.

$$\int \frac{e^x(1+x)}{cos^2(e^x.x)}dx \quad is \quad equal \quad to$$

(a) -cot(e.x^x)+c

(b) tan(xe^x)+c

(c) tan(e^x)+c

(d) cot e^x+c

Solution:

(b)

$$\int \frac{e^x(1+x)}{cos^2(e^x.x)}dx$$

= ∫sec^2t dt

= tan t+c = tan(xe^x)+c

Ex 7.4 Class 12 Maths Question 1.

$$\frac{3x^2}{x^6+1}$$

Solution:

Let x^3 = t ⇒ $3x^2$dx = dt

$$\int \frac{3x^2}{x^6+1}dx = \int \frac{dt}{t^2+1} = tan^{-1}t + c$$

= $\tan^{-1}(x^3)+c$

Ex 7.4 Class 12 Maths Question 2.

$$\frac{1}{\sqrt{1+4x^2}}$$

Solution:

$$\frac{1}{2}\int\frac{dx}{\sqrt{\frac{1}{4}+x^2}}=\frac{1}{2}\int\frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2+x^2}}=\frac{1}{2}log\left|2x+\sqrt{1+4x^2}\right|+c$$

Ex 7.4 Class 12 Maths Question 3.

$$\frac{1}{\sqrt{(2-x)^2+1}}$$

Solution:

put (2-x)=t

so that -dx=dt

⇒ dx=-dt

$$\int\frac{dx}{\sqrt{(2-x)^2+1}}=-\int\frac{dt}{\sqrt{t^2+1}}=-log|t+\sqrt{t^2+1}|+c$$

$$=log\left|\frac{1}{(2-x)+\sqrt{x^2-4x+5}}\right|+c$$

Ex 7.4 Class 12 Maths Question 4.

$$\frac{1}{\sqrt{9-25x^2}}$$

Solution:

$$\int \frac{dx}{\sqrt{9-25x^2}} = \frac{1}{5}\int \frac{dx}{\sqrt{\left(\frac{3}{5}\right)^2-x^2}}$$

$$= \frac{1}{5}sin^{-1}\left(\frac{x}{\frac{3}{5}}\right)+c \quad = \frac{1}{5}sin^{-1}\left(\frac{5x}{3}\right)+c$$

Ex 7.4 Class 12 Maths Question 5.

$$\frac{3x}{1+2x^4}$$

Solution:

Put x²=t,so that 2x dx=dt

⇒x dx =

$$\frac{dt}{2} \therefore \int \frac{3x}{1+2x^4}dx = \frac{1}{2}\int \frac{dt}{1+2t^2} = \frac{3}{4}\int \frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2+t^2}$$

$$= \frac{3}{2\sqrt{2}}tan^{-1}(\sqrt{2}t)+c \quad = \frac{3}{2\sqrt{2}}tan^{-1}(\sqrt{2x^2})+c$$

Ex 7.4 Class 12 Maths Question 6.

$\frac{x^2}{1-x^6}$

Solution:

put x^3 = t,so that $3x^2dx$ = dt

$$\int \frac{x^2}{1-x^6}dx \quad = \frac{1}{3}\int \frac{dt}{1-t^2} \quad = \frac{1}{6}log\left|\frac{1+t}{1-t}\right| + c$$

$$= \frac{1}{6}log\left|\frac{1+x^3}{1-x^3}\right| + c$$

Ex 7.4 Class 12 Maths Question 7.

$\frac{x-1}{\sqrt{x^2-1}}$

Solution:

$$I = \int \frac{x-1}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx, I = I_1 - I_2$$

put x^2-1 = t,so that 2x dx = dt

$$I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\frac{t^{1/2}}{1/2} + c = \sqrt{x^2-1} + c_1$$

$$I_2 = \int \frac{1}{\sqrt{x^2-1}}dx = \log\left|x + \sqrt{x^2-1}\right|$$

$$\therefore \quad I = \sqrt{x^2-1} - \log\left|x + \sqrt{x^2-1}\right| + c$$

Ex 7.4 Class 12 Maths Question 8.

$$\frac{x^2}{\sqrt{x^6+a^6}}$$

Solution:

put $x^3 = t$

so that $3x^2dx = dt$

$$I = \frac{1}{3}\int \frac{dt}{t^2+\left(a^3\right)^2} = \frac{1}{3}log\left|t + \sqrt{t^2+a^6}\right| + c$$

$$= \frac{1}{3}log|x^3 + \sqrt{a^6+x^6}| + c$$

Ex 7.4 Class 12 Maths Question 9.

$$\frac{sec^2x}{\sqrt{tan^2x+4}}$$

Solution:

let tanx = t

sec x^2dx = dt

$$I = \int \frac{dt}{\sqrt{t^2+(2)^2}} = log|t + \sqrt{t^2+4}| + c$$

$$= log|tanx + \sqrt{tan^2x+4}| + c$$

Ex 7.4 Class 12 Maths Question 10.

$\frac{1}{\sqrt{x^2+2x+2}}$

Solution:

$$\int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{dx}{\sqrt{(x+1)^2+1}}$$

$$= log\left|(x+1)+\sqrt{x^2+2x+2}\right|+c$$

Ex 7.4 Class 12 Maths Question 11.

$\frac{1}{9x^2+6x+5}$

Solution:

$$\int \frac{1}{9x^2+6x+5} = \frac{1}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2}$$

$$= \frac{1}{6}tan^{-1}\left(\frac{3x+1}{2}\right)+c$$

Ex 7.4 Class 12 Maths Question 12.

$\frac{1}{\sqrt{7-6x-x^2}}$

Solution:

$$I = \int \frac{dx}{\sqrt{4^2-(x+3)^2}} \quad = sin^{-1}\left(\frac{x+3}{4}\right) + c$$

Ex 7.4 Class 12 Maths Question 13.

$$\frac{1}{\sqrt{(x-1)(x-2)}}$$

Solution:

$$\int \frac{1}{\sqrt{(x-1)(x-2)}}dx = \int \frac{dx}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$= log\left|x - \frac{3}{2} + \sqrt{x^2 - 3x + 2}\right| + c$$

Ex 7.4 Class 12 Maths Question 14.

$$\frac{1}{\sqrt{8+3x-x^2}}$$

Solution:

$$\int \frac{dx}{\sqrt{8+3x-x^2}} = \int \frac{dx}{\sqrt{8-\left(x^2-3x\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}} \quad = sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right) + c$$

Ex 7.4 Class 12 Maths Question 15.

$$\frac{1}{\sqrt{(x-a)(x-b)}}$$

Solution:

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}} = \int \frac{dx}{\left(x-\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2}$$

$$= log\left|\left(x-\frac{a+b}{2}\right)+\sqrt{(x-a)(x-b)}\right|+c$$

Ex 7.4 Class 12 Maths Question 16.

$$\frac{4x+1}{\sqrt{2x^2+x-3}}$$

Solution:

$$let \quad I = \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx$$

put $2x^2+x-3=t$

so that $(4x+1)dx=dt$

$$let \quad I = \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx$$

$$\therefore I = \int \frac{dt}{\sqrt{t}} = 2t^{\frac{1}{2}}+c \quad = 2\sqrt{2x^2+x-3}+c$$

Ex 7.4 Class 12 Maths Question 17.

$\frac{x+2}{\sqrt{x^2-1}}$

Solution:

$$\int \frac{x+2}{\sqrt{x^2-1}} dx \quad = \int \frac{x}{\sqrt{x^2-1}} dx + \int \frac{2}{\sqrt{x^2-1}} dx$$

$= I_1 + I_2 + C$ (say)

Put $x^2 - 1 = t, \Rightarrow 2x\, dx = dt$

$$I_1 = \int \frac{x}{x^2-1} dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \sqrt{t} = \sqrt{x^2-1}$$

$$\text{and } I_2 = \int \frac{2}{\sqrt{x^2-1}} dx = 2 \log|x + \sqrt{x^2-1}|$$

$$\text{Hence } I = \sqrt{x^2-1} + 2\log|x + \sqrt{x^2-1}| + C$$

Ex 7.4 Class 12 Maths Question 18.

$\frac{5x-2}{1+2x+3x^2}$

Solution:

put 5x-2=A

$\frac{d}{dx}$

(1+2x+3x²)+B

⇒ 6A=5, A=

$\frac{5}{6} - 2 = 2A + B$

, B=

$-\frac{11}{3}$

$$I = \int \frac{\frac{5}{6}(6x + 2)}{3x^2 + 2x + 1} dx - \frac{11}{3} \int \frac{dx}{3x^2 + 2x + 1}$$

$$= I_1 - \frac{11}{3} I_2 \text{ ; put } 3x^2 + 2x + 1 = t \therefore (6x + 2) dx = dt$$

$$I_1 = \frac{5}{6} \int \frac{dt}{t} = \frac{5}{6} \log t = \frac{5}{6} \log(3x^2 + 2x + 1) + c_1$$

$$\text{and } I_2 = \int \frac{dx}{3x^2 + 2x + 1} = \frac{1}{3} \int \frac{dx}{\left(x + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2}$$

$$\Rightarrow I_2 = \frac{1}{\sqrt{2}} \tan^{-1} \frac{3x + 1}{\sqrt{2}} + c$$

$$\therefore I = \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{3} \cdot \frac{1}{\sqrt{2}} \tan^{-1} \frac{3x + 1}{\sqrt{2}} + c$$

Ex 7.4 Class 12 Maths Question 19.

$\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

Solution:

$\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx = \int \frac{(6x+7)dx}{\sqrt{x^2-9x+20}}$

$$\text{Let } 6x+7 = A\times\frac{d}{dx}(x^2-9x+20)+B$$
$$\Rightarrow \quad 6x+7 = A(2x-9)+B$$
$$\Rightarrow 2A=6 \Rightarrow A=3 \ \& \ 7=-9A+B \Rightarrow B=34$$
$$\therefore I = 3\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx + 34\int\frac{dx}{\sqrt{x^2-9x+20}}$$
$$\text{Let } I = 3I_1+34I_2+C$$
$$\therefore I_1 = \int\frac{dt}{\sqrt{t}} = 2t^{1/2} = 2\sqrt{x^2-9x+20}$$
$$I_2 = \int\frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$
$$= \log\left|x-\frac{9}{2}+\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right|$$
$$\therefore I = 6\sqrt{x^2-9x+20}+34\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|+C$$

Ex 7.4 Class 12 Maths Question 20.

$\frac{x+2}{\sqrt{4x-x^2}}$

Solution:

$$I=\int \frac{x-2}{\sqrt{4-(x-2)^2}}dx+4\int \frac{dx}{\sqrt{4-(x-2)^2}}$$

$$=I_1+4\sin^{-1}\frac{x-2}{2}+C$$

For I_1, put $(x-2)^2=t \Rightarrow 2(x-2)\,dx=dt$

$$\therefore \quad I_1=\frac{1}{2}\int\frac{dt}{\sqrt{4-t}}=\sqrt{4-t}$$

$$\therefore \quad I=\sqrt{4-(x-2)^2}+4\sin^{-1}\frac{x-2}{2}+C$$

Ex 7.4 Class 12 Maths Question 21.

$$\frac{x+2}{\sqrt{x^2+2x+3}}$$

Solution:

$$I=\frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= I_1 + I_2 + C$$

$$I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2} \times 2t^{\frac{1}{2}} = \sqrt{x^2+2x+3}$$

$$I_2 = \int \frac{dx}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}} = \log\left|(x+1)+\sqrt{x^2+2x+3}\right|$$

$$.I = \sqrt{x^2+2x+3} + \log|(x+1) + \sqrt{x^2+2x+3}| + C$$

Ex 7.4 Class 12 Maths Question 22.

$\frac{x+3}{x^2-2x-5}$

Solution:

$$I = \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + \int \frac{dx}{x^2-2x-5}$$

$$= \frac{1}{2} I_1 + 4I_2 + C \text{ (say)}$$

Put $x^2 - 2x - 5 = t$, so that $(2x-2)\, dx = dt$

$$\therefore \quad I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2 - 2x - 5|$$

$$I_2 = \int \frac{dx}{(x-1)^2 - (\sqrt{6})^2} = \frac{1}{2\sqrt{6}} \log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|$$

$$\therefore I = \frac{1}{2}\log|x^2-2x-5| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

Ex 7.4 Class 12 Maths Question 23.

$\frac{5x+3}{\sqrt{x^2+4x+10}}$

Solution:

$$I=\int \frac{\frac{5}{2}(2x+4)+(3-10)}{\sqrt{x^2+4x+10}}dx$$

$$=\frac{5}{2}\int\frac{2x+4}{\sqrt{x^2+4x+10}}\,dx-7\int\frac{dx}{\sqrt{x^2+4x+10}}$$

$$=\frac{5}{2}I_1-7I_2+C\text{ (say)}$$

Put $x^2+4x+10=t, \Rightarrow (2x+4)\,dx=dt$

$$\therefore I_1=\int\frac{dt}{\sqrt{t}}=2\sqrt{t}=2\sqrt{x^2+4x+10}$$

$$I_2=\int\frac{dx}{\sqrt{(x+2)^2+(\sqrt{6})^2}}$$

$$=\log\left|x+2+\sqrt{x^2+4x+10}\right|$$

$$I=5\sqrt{x^2+4x+10}-7\log|x+2+\sqrt{x^2+4x+10}|+C$$

Ex 7.4 Class 12 Maths Question 24.

$\int \frac{dx}{x^2+2x+2} equals$

(a) $x\tan^{-1}(x+1)+c$

(b) $(x+1)\tan^{-1}x+c$

(c) $\tan^{-1}(x+1)+c$

(d) $\tan^{-1}x+c$

Solution:

(b)

$$let \quad I=\int\frac{dx}{x^2+2x+2}=\int\frac{dx}{(x+1)^2+1}$$

= $(x+1)\tan^{-1}x+c$

Ex 7.4 Class 12 Maths Question 25.

$$\int\frac{dx}{\sqrt{9x-4x^2}} equal s$$

(a)

$$\frac{1}{9}sin^{-1}\left(\frac{9x-8}{8}\right)+c$$

(b)

$$\frac{1}{2}sin^{-1}\left(\frac{8x-9}{9}\right)+c$$

(c)

$$\frac{1}{3}sin^{-1}\left(\frac{9x-8}{8}\right)+c$$

(d)

$$sin^{-1}\left(\frac{9x-8}{9}\right)+c$$

Solution:

(b)

$$\int\frac{dx}{\sqrt{9x-4x^2}}=\frac{1}{2}\left[\frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2-\left[x^2-\frac{9}{4}x+\left(\frac{9}{8}\right)^2\right]}}\right]$$

$$\frac{1}{2}sin^{-1}\left(\frac{8x-9}{9}\right)+c$$

Ex 7.5 Class 12 Maths Question 1.

$$\frac{x}{(x+1)(x+2)}$$

Solution:

let

$$\frac{x}{(x+1)(x+2)}$$

≡

$$\frac{A}{x+1}+\frac{B}{x+2}$$

⇒ x ≡ A(x+2)+B(x+1)….(i)

putting x = -1 & x = -2 in (i)

we get A = 1,B = 2

$$\therefore \int \frac{1}{(x+1)(x+2)}dx = \int \frac{-1}{x+1}dx + \int \frac{2}{x+2}dx$$

=-log|x+1| + 2log|x+2|+c

Ex 7.5 Class 12 Maths Question 2.

$$\frac{1}{x^2-9}$$

Solution:

let

$$\frac{1}{x^2-9} = \frac{1}{(x-3)(x+3)} \equiv \frac{A}{x-3} + \frac{B}{x+3}$$

⇒ x ≡ A(x+3)+B(x-3)…(i)

put x = 3, -3 in (i)

we get

$$A = \frac{1}{6}$$

&

$$B = -\frac{1}{6} \therefore \int \frac{1}{x^2-9}dx = \frac{1}{6}\int \left[\frac{1}{x-3} - \frac{1}{x+3}\right] dx$$

$= \frac{1}{6} log \left|\frac{x-3}{x+3}\right| + c$

Ex 7.5 Class 12 Maths Question 3.

$\frac{3x-1}{(x-1)(x-2)(x-3)}$

Solution:

Let

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

⇒ 3x-1 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(-2)…..(i)

put x = 1,2,3 in (i)

we get A = 1,B = -5 & C = 4

$\therefore I = \int \frac{1}{x-1} dx - 5 \int \frac{1}{x-2} dx + 4 \int \frac{1}{x-3} dx$

=log|x-1| – 5log|x-2| + 4log|x+3| + C

Ex 7.5 Class 12 Maths Question 4.

$\frac{x}{(x-1)(x-2)(x-3)}$

Solution:

let

$\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

⇒ x ≡ A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)…(i)

put x = 1,2,3 in (i)

$A = \frac{1}{2}, B = -2, C = \frac{3}{2} \therefore I = \frac{1}{2}\int \frac{dx}{x-1} - 2\int \frac{dx}{x-2} + \frac{3}{2}\int \frac{dx}{x-3}$

$= \frac{1}{2}log|x-1| - 2log|x-2| + \frac{3}{2}log|x-3| + c$

Ex 7.5 Class 12 Maths Question 5.

$\frac{2x}{x^2+3x+2}$

Solution:

let

$\frac{2x}{x^2+3x+2} = \frac{2x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

⇒ 2x = A(x+2)+B(x+1)…(i)

put x = -1, -2 in (i)

we get A = -2, B = 4

$\therefore \int \frac{2x}{x^2+3x+2}dx = -2\int \frac{dx}{x+1} + 4\int \frac{dx}{x+2}$

=-2log|x+1|+4log|x+2|+c

Ex 7.5 Class 12 Maths Question 6.

$$\frac{1-x^2}{x(1-2x)}$$

Solution:

$$\frac{1-x^2}{(x-2x^2)}$$

is an improper fraction therefore we

convert it into a proper fraction. Divide 1 – x^2 by x – $2x^2$ by long division.

$$\frac{1-x^2}{x-2x^2}=\frac{1}{2}\left[\frac{(2x^2-x)+(x-2)}{2x^2-x}\right]=\frac{1}{2}\left[1+\frac{x-2}{2x^2-x}\right]$$

$$\text{Now, } \frac{x-2}{2x^2-x}=\frac{x-2}{x(2x-1)}=\frac{A}{x}+\frac{B}{2x-1}$$

$$\Rightarrow x-2\equiv A(2x-1)+Bx \qquad \ldots(i)$$

$$\text{Put } x=0, \frac{1}{2} \text{ in (i), we get : } A=2\ B=-3$$

$$\therefore \int\frac{1-x^2}{x(1-2x)}dx=\frac{1}{2}x+\int\frac{1}{x}dx+\frac{3}{2}\int\frac{1}{1-2x}dx$$

$$=\frac{1}{2}x+\log|x|-\frac{3}{4}\log|1-2x|+c$$

Ex 7.5 Class 12 Maths Question 7.

$$\frac{x}{\left(x^2+1\right)(x-1)}$$

Solution:

let

$$\frac{x}{\left(x^2+1\right)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

⇒ x = A(x²+1)+(Bx+C)(x-1)

Put x = 1,0

⇒

$$A = \frac{1}{2}C = \frac{1}{2} \Rightarrow B = -\frac{1}{2} \therefore I = \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{x}{x^2+1}dx + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{2}log(x-1) - \frac{1}{4}log(x^2+1) + \frac{1}{2}tan^{-1}x + c$$

Ex 7.5 Class 12 Maths Question 8.

$$\frac{x}{(x-1)^2(x+2)}$$

Solution:

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

⇒ x ≡ A(x-1)(x+2)+B(x+2)+C(x-1)² …(i)

put x = 1, -2

we get

$$B=\frac{1}{3}, C=\frac{-2}{9} \therefore I=\frac{2}{9}\int\frac{1}{x-1}dx+\frac{1}{3}\int\frac{1}{(x-1)^2}dx-\frac{2}{9}\int\frac{1}{x+2}dx$$

$$=\frac{2}{9}log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+c$$

Ex 7.5 Class 12 Maths Question 9.

$$\frac{3x+5}{x^3-x^2-x+1}$$

Solution:

let

$$\frac{3x+5}{x^2(x-1)-1(x-1)}\ \frac{3x+5}{(x-1)^2(x+1)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}$$

⇒ 3x+5 = A(x-1)(x+1)+B(x+1)+C(x-1)

put x = 1,-1,0

we get

$$B=4, C=\frac{1}{2}, A=-\frac{1}{2} \therefore I=-\frac{1}{2}\int\frac{dx}{(x-1)}+4\frac{dx}{(x-1)^2}+\frac{1}{2}\int\frac{dx}{x+1}$$
$$=\frac{1}{2}log\left|\frac{x+1}{x-1}\right|-\frac{4}{x-1}+c$$

Ex 7.5 Class 12 Maths Question 10.

$$\frac{2x-3}{(x^2-1)(2x+3)}$$

Solution:

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x-1)(x+1)(2x+3)}$$

$$\equiv \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3}$$

$$\Rightarrow 2x-3 = A(x+1)(2x+3) + B(x-1)(2x+3) + C(x-1)(x+1) \quad ...(i)$$

Put x = 1, − 1 in (i), we get; $A = -\frac{1}{10}$ & $B = \frac{5}{2}$

Putting $x = -\frac{3}{2}$ in (i), we get : $C = \frac{-24}{5}$

$$\therefore I = -\frac{1}{10}\int \frac{dx}{x-1} + \frac{5}{2}\int \frac{dx}{x+1} - \frac{24}{5}\int \frac{dx}{2x+3}$$

$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + C$$

Ex 7.5 Class 12 Maths Question 11.

$$\frac{5x}{(x-1)(x^2-4)}$$

Solution:

let

$$\frac{5x}{(x-1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

$$\equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-2} \Rightarrow 5x \equiv A(x+2)(x-2)$$

$$+B(x+1)(x-2) + C(x+1)(x+2) \quad \ldots(i)$$

$$\text{Put } x = -1, -2, 2 \text{ in (i)} \Rightarrow A = \frac{5}{3}, B = -\frac{5}{2} \ \& \ C = \frac{5}{6}$$

$$\therefore I = \frac{5}{3}\int \frac{dx}{x+1} - \frac{5}{2}\int \frac{dx}{x+2} + \frac{5}{6}\int \frac{dx}{x-2}$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C.$$

Ex 7.5 Class 12 Maths Question 12.

$$\frac{x^3+x+1}{x^2-1}$$

Solution:

$$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{(x+1)(x-1)}$$

$$= \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

$$\Rightarrow 2x+1 = A(x-1) + B(x+1) \qquad ...(ii)$$

Put $x = -1, 1$ in (ii), we get: $A = \frac{1}{2}$ & $B = \frac{3}{2}$

$$\int \frac{x^3+x+1}{x^2-1} dx = \int x\, dx + \frac{1}{2}\int \frac{dx}{x+1} + \frac{3}{2}\int \frac{dx}{x-1}$$

$$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + c$$

Ex 7.5 Class 12 Maths Question 13.

$$\frac{2}{(1-x)(1+x^2)}$$

Solution:

$$\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$$

⇒ $2 = A(1+x^2) + (Bx+C)(1-x)$ …(i)

Putting $x = 1$ in (i), we get; $A = 1$

Also $0 = A - B$ and $2 = A + C \Rightarrow B = A = 1$ & $C = 1$

$$\therefore I = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\log|1-x| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C.$$

Ex 7.5 Class 12 Maths Question 14.

$\frac{3x-1}{(x+2)^2}$

Solution:

$\frac{3x-1}{(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{(x+2)^2}$

=>3x – 1 = A(x + 2) + B …(i)

Comparing coefficients A = -1 and B = -7

$\therefore \int \frac{3x-1}{(x+2)^2} dx = 3 \int \frac{dx}{x+2} - 7 \int \frac{dx}{(x+2)^2}$

$= 3log|x+2| + \frac{7}{x+2} + c$

Ex 7.5 Class 12 Maths Question 15.

$\frac{1}{x^4-1}$

Solution:

$\frac{1}{x^4-1} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$

⇒ $1 \equiv A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x+1)(x-1)$ ….(i)

Put $x = -1, 1$ in (i), we get : $A = \frac{-1}{4}$ & $B = \frac{1}{4}$

Comparing coefficients $C = 0$ and $D = -\frac{1}{2}$

$$I = -\frac{1}{4}\int\frac{dx}{x+1} + \frac{1}{4}\int\frac{1}{x-1}dx - \frac{1}{2}\int\frac{1}{(x^2+1)}dx$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C.$$

Ex 7.5 Class 12 Maths Question 16.

$$\frac{1}{x(x^n+1)}$$

[Hint : multiply numerator and denominator by x^{n-1} and put $x^n = t$]

Solution:

$$\frac{x^{n-1}}{x.x^{n-1}(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)}$$

Put $x^n = t$ so that $nx^{n-1}\,dx = dt$

$$\therefore \int \frac{dx}{x(x^n+1)} = \frac{1}{n}\int \frac{dt}{t(t+1)} \quad \text{...(i)}$$

$$\text{let } \frac{1}{t(t+1)} \equiv \frac{A}{t} + \frac{B}{t+1}$$

$$\Rightarrow 1 \equiv A(t+1) + Bt \quad \text{...(ii)}$$

Put $t = 0, -1$ in (i), we get : $\Rightarrow A = 1$ & $B = -1$

$$\therefore \int \frac{dx}{x(x^n+1)} = \frac{1}{n}\int \cdot \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$= \frac{1}{n}\,[\log|t| - \log|t+1|] + c = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c$$

Ex 7.5 Class 12 Maths Question 17.

$\frac{cosx}{(1-sinx)(2-sinx)}$

Solution:

put sinx = t

so that cosx dx = dt

$\therefore I = \int \frac{1}{(1-t)(2-t)} dt$

Let $\frac{1}{(1-t)(2-t)} \equiv \frac{A}{1-t}+\frac{B}{2-t}$

$\Rightarrow 1 \equiv A(2-t)+B(1-t)$ ....(ii)

Put, $t=1,2$ in (ii), we get : $A=1$ & $B=-1$

$\therefore I=\int \frac{1}{1-t}\, dt-\int \frac{dt}{2-t}$

$=\log\left|\frac{2-t}{1-t}\right|+C=\log\left|\frac{2-\sin x}{1-\sin x}\right|+C.$

Ex 7.5 Class 12 Maths Question 18.

$$\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}$$

Solution:

put $x^2=y$

$$I=1-\frac{2(2y+5)}{(y+3)(y+4)}$$

Let ; $\frac{2y+5}{(y+3)\,(y+4)}=\frac{A}{y+3}+\frac{B}{y+4}$,

$2y+5 \equiv A(y+4)+B(y+3)$

Put $y=-3$, $\therefore$ $A=-1$, Put $y=-4$, $\therefore$ $B=3$

$\therefore I=\int dx+2\int \frac{dx}{x^2+3}+6\int \frac{dx}{x^2+4}$

$=x+\frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-3\tan^{-1}\frac{x}{2}+c$

Ex 7.5 Class 12 Maths Question 19.

$\frac{2x}{(x^2+1)(x^2+3)}$

Solution:

put x²=y

so that 2xdx = dy

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int \frac{dy}{(y+1)(y+3)}$$

Let $\frac{1}{(y+1)(y+3)} = \frac{A}{y+1} + \frac{B}{y+3}$, we have

$1 = A(y+3) + B(y+1)$ ....(i)

Put $y = -1, -3$ in (i), we get; $A = \frac{1}{2}$ & $B = -\frac{1}{2}$

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)}dx = \frac{1}{2}\int \frac{dy}{y+1} - \frac{1}{2}\int \frac{dy}{y+3}$$

$$= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

Ex 7.5 Class 12 Maths Question 20.

$\frac{1}{x(x^4-1)}$

Solution:

put x^4 = t

so that $4x^3$ dx = dt

$$\therefore \quad I = \frac{1}{4}\int \frac{dt}{t(t-1)} \text{ ; Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$$

$$\Rightarrow \quad 1 = A(t-1) + Bt \qquad \ldots(i)$$

Put $t = 0$, 1 in (i), we get; A = –1 & B = 1

$$\therefore I = \frac{1}{4}\int \left(\frac{-1}{t} + \frac{1}{t-1}\right) dt$$

$$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C = \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C.$$

Ex 7.5 Class 12 Maths Question 21.

$$\therefore \quad I = \int \frac{1}{e^x - 1} dx = \int \frac{dt}{t(t-1)}$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \Rightarrow 1 = A(t-1) + Bt$$

$$\text{Let } t = 0 \Rightarrow A = -1 \text{ \& Let } t = 1 \Rightarrow B = 1$$

$$\therefore \quad I = \int\left(\frac{-1}{t} + \frac{1}{t-1}\right) dt$$

$$= -\log t + \log(t-1) + C = \log\left(\frac{e^x - 1}{e^x}\right) + C.$$

$$\frac{1}{e^x - 1}$$

Solution:

Let e^x = t ⇒ e^x dx = dt

⇒

$$dx = \frac{dt}{t}$$

Ex 7.5 Class 12 Maths Question 22.

choose the correct answer in each of the following :

$$\int \frac{xdx}{(x-1)(x-2)} equals$$

(a)

$$log\left|\frac{(x-1)^2}{x-2}\right|+c$$

(b)

$$log\left|\frac{(x-2)^2}{x-1}\right|+c$$

(c)

$$log\left|\left(\frac{x-1^2}{x-2}\right)\right|+c$$

(d) log|(x-1)(x-2)|+c

Solution:

(b)

$$\int \frac{x}{(x-1)(x-2)}dx=\int\left[\frac{-1}{x-1}+\frac{2}{x-2}\right]dx$$

$$log\left|\frac{(x-2)^2}{x-1}\right|+c$$

Ex 7.5 Class 12 Maths Question 23.

$\int \frac{dx}{x(x^2+1)} equals$

(a)

$log|x| - \frac{1}{2}log(x^2+1)+c$

(b)

$log|x| + \frac{1}{2}log(x^2+1)+c$

(c)

$-log|x| + \frac{1}{2}log(x^2+1)+c$

(d) $\frac{1}{2}log|x| + log(x^2+1)+c$

Solution:

(a) let

$\frac{1}{x\left(x^2+1\right)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

⇒ 1 = A(x^2+1)+(Bx+C)(x)

Let $x=0, 1=A \Rightarrow A=1$

Comparing coefficients of x^2 & x ; $B=-1$ & $C=0$

$$\therefore \int \frac{1}{x(x^2+1)}dx = \int\left[\frac{1}{x}+\frac{-x}{x^2+1}\right]dx$$

$$= \log x - \frac{1}{2}\log\,(x^2+1)+C$$

Ex 7.6 Class 12 Maths Question 1.

x sinx

Solution:

By part integration

∫x sinx dx = x(-cosx) – ∫1(-cosx)dx

=-x cosx + ∫cosxdx

=-x cosx + sinx + c

Ex 7.6 Class 12 Maths Question 2.

x sin3x

Solution:

∫x sin3x dx =

$$x\left(-\frac{cos3x}{3}\right) - \int 1.\left(\frac{-cos3x}{3}\right)dx$$

$$= -\frac{1}{3}x \quad cos3x + \frac{1}{9}sin3x + c$$

Ex 7.6 Class 12 Maths Question 3.

$$x^2e^x$$

Solution:

$$\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + c$$

$$= e^x\left(x^2 - 2x + 2\right) + c$$

Ex 7.6 Class 12 Maths Question 4.

x logx

Solution:

$$\int xlogx \quad dx = logx \int xdx - \int\left[\frac{d}{dx}(logx)\int xdx\right]dx$$

$$= \frac{x^2}{2}logx - \frac{1}{2}\int x \quad dx = \frac{x^2}{2}logx - \frac{1}{4}x^2 + c$$

Ex 7.6 Class 12 Maths Question 5.

x log2x

Solution:

$$\int x \quad log2xdx = (log2x)\frac{x^2}{2} - \int \frac{1}{2x}.2\left(\frac{x^2}{2}\right)dx$$

$$= \frac{x^2}{2}log|2x| - \frac{1}{2}\int xdx = \frac{x^2}{2}log|2x| - \frac{x^2}{4} + c$$

Ex 7.6 Class 12 Maths Question 6.

$$x^2 logx$$

Solution:

$$\int x^2 log x dx = log|x|\left(\frac{x^3}{3}\right) - \int \frac{1}{x}\left(\frac{x^3}{3}\right) dx$$

$$= \frac{x^3}{3} log|x| - \frac{1}{3}\int x^2 dx = \frac{x^3}{3} log|x| - \frac{x^3}{9} + c$$

Ex 7.6 Class 12 Maths Question 7.

$$x \quad sin^{-1} x$$

Solution:

$$I = x \quad sin^{-1} x.\left(\frac{x^2}{2}\right) - \int \frac{1}{\sqrt{1-x^2}}.\frac{x^2}{2} dx$$

$$= \frac{x^2}{2}\sin^{-1} x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2}{2}\sin^{-1} x - \frac{1}{2} I_1,$$

Put $x = \sin\theta$ so that $dx = \cos\theta \, d\theta$

$$\therefore \quad I_1 = \int \frac{\sin^2\theta}{\cos\theta}\cos\theta \, d\theta$$

$$= \frac{1}{2}\int (1 - \cos 2\theta) \, d\theta = \frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta + c$$

$$= \frac{1}{2}\sin^{-1} x - \frac{1}{2} x\sqrt{1-x^2} + c$$

$$\therefore \; I = \frac{1}{4}(\sin^{-1} x)\cdot(2x^2 - 1) + \frac{x\sqrt{1-x^2}}{4} + c$$

Ex 7.6 Class 12 Maths Question 8.

$x \quad tan^{-1}x$

Solution:

$$I = x \quad tan^{-1}x.\left(\frac{x^2}{2}\right) - \int \frac{1}{\sqrt{1+x^2}}.\frac{x^2}{2}dx$$
$$= \frac{x^2}{2}tan^{-1}x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2}{2}tan^{-1}x - \frac{1}{2}x + \frac{1}{2}tan^{-1}x + c$$

Ex 7.6 Class 12 Maths Question 9.

$x \quad cos^{-1}x$

Solution:

let I =

$$\int xcos^{-1}xdx = \int cos^{-1}x.xdx$$

$$= \cos^{-1} x \left(\frac{x^2}{2}\right) - \int \frac{-1}{\sqrt{1-x^2}} \left(\frac{x^2}{2}\right) dx$$

$$= \frac{x^2}{2} \cos^{-1} x + \frac{1}{2} I_1$$

Put $x = \cos\theta$, so that $dx = -\sin\theta\, d\theta$

$$\therefore\ I_1 = \int \frac{\cos^2\theta(-\sin\theta)}{\sqrt{1-\cos^2\theta}} d\theta = -\frac{1}{2}\int (1+\cos 2\theta) d\theta$$

$$= -\frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + C_1$$

$$= -\frac{1}{2}(\theta + \cos\theta\sqrt{1-\cos^2\theta}) + C_1$$

$$= -\frac{1}{2}(\cos^{-1} x + x\sqrt{1-x^2}) + C_1$$

$$I = \frac{\cos^{-1} x}{4}(2x^2-1) - \frac{x}{4}\sqrt{1-x^2} + C$$

Ex 7.6 Class 12 Maths Question 10.

$$(sin^{-1}x)^2$$

Solution:

$$put \quad sin^{-1}x = \theta \Rightarrow x = sin\theta \Rightarrow dx = cos\theta d\theta$$

$$\therefore \int (\sin^{-1} x)^2 dx = \int \theta^2 \cos\theta \, d\theta$$
$$= \theta^2 \sin\theta + 2\theta\cos\theta - 2\int \cos\theta \, d\theta + C$$
$$= \theta^2 \sin\theta + 2\theta\sqrt{1-\sin^2\theta} - 2\sin\theta + C$$
$$= x(\sin^{-1} x)^2 + 2\sin^{-1} x.\sqrt{1-x^2} - 2x + C$$

Ex 7.6 Class 12 Maths Question 11.

$\frac{x \quad cos^{-1}x}{\sqrt{1-x^2}}$

Solution:

$$put \quad cos^{-1}x = t \quad so \quad that \frac{x \quad cos^{-1}x}{\sqrt{1-x^2}} dx = dt$$

$$\therefore \text{ I} = -\int t\cos t \, dt = -[t(\sin t) - \int 1.(\sin t)\, dt$$
$$= -t\sin t - \cos t + \text{C} = -t\sqrt{1-\cos^2 t} - \cos t + \text{C}$$
$$= -[\cos^{-1} x.\sqrt{1-x^2} + x] + \text{C}.$$

Ex 7.6 Class 12 Maths Question 12.

x sec²x

Solution:

∫x sec²x dx =x(tanx)-∫1.tanx dx

= x tanx+log cosx+c

Ex 7.6 Class 12 Maths Question 13.

$tan^{-1}x$

Solution:

$$\int tan^{-1}xdx = xtan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$= xtan^{-1}x - \frac{1}{2}log|1+x^2| + c$$

Ex 7.6 Class 12 Maths Question 14.

x(logx)²

Solution:

∫x(logx)² dx

$$= \frac{x^2}{2}(logx)^2 - \left[(logx)\frac{x^2}{2} - \int \frac{1}{x}\frac{x^2}{2}dx\right]$$

$$= \frac{x^2}{2}(logx)^2 - \frac{x^2}{2}logx + \frac{1}{4}x^2 + c$$

Ex 7.6 Class 12 Maths Question 15.

(x²+1)logx

Solution:

∫(x²+1)logx dx

$$= logx\left(\frac{x^3}{3}+x\right) - \int \frac{1}{x}\left(\frac{x^3}{3}+x\right)dx$$

$$= \left(\frac{x^3}{3} + x\right) logx - \frac{x^3}{9} - x + c$$

Ex 7.6 Class 12 Maths Question 16.

$$e^x(sinx + cosx)$$

Solution:

$$put \quad e^x sinx = t \Rightarrow e^x(sinx + cosx)dx = dt$$
$$\therefore \int e^x(sinx + cosx)dx = \int dt = t + c = e^x sinx + c$$

Ex 7.6 Class 12 Maths Question 17.

$$\frac{xe^x}{(1+x)^2}$$

Solution:

$$\int \frac{xe^x}{(1+x)^2}$$

$$\int \frac{(x+1-1)\,e^x}{(1+x)^2}\,dx = \int e^x\left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right)dx$$

$$= I_1 - \int \frac{e^x}{(x+1)^2}\,dx \qquad \text{...(i)}$$

$$I_1 = \frac{1}{1+x}\int e^x dx - \int\left(\frac{d}{dx}\left(\frac{1}{1+x}\right)\int e^x dx\right)dx$$

$$= \frac{e^x}{1+x} + \int \frac{e^x}{(1+x)^2}\,dx, \therefore I = I_1 - \int \frac{e^x}{(1+x)^2}\,dx$$

$$\frac{e^x}{1+x} + \int \frac{e^x}{(1+x)^2}\,dx - \int \frac{e^x}{(1+x)^2}\,dx = \frac{e^x}{1+x} + c$$

Ex 7.6 Class 12 Maths Question 18.

$\frac{e^x(1+sinx)}{1+cosx}$

Solution:

$$I = \int e^x \left[\frac{1+2sin\frac{x}{2}cos\frac{x}{2}}{2cos^2\frac{x}{2}}\right] dx$$

$$= \int e^x\left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right]dx = \int e^x\left[\tan\frac{x}{2} + \frac{1}{2}\sec^2\frac{x}{2}\right]dx$$

Let $e^x \tan\frac{x}{2} = t \Rightarrow e^x\left(\tan\frac{x}{2} + \frac{1}{2}\sec^2\frac{x}{2}\right)dx = dt$

$$\therefore I = \int dt = t + C = e^x \tan\frac{x}{2} + C$$

Ex 7.6 Class 12 Maths Question 19.

$e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$

Solution:

put

$\frac{e^x}{x}=t \Rightarrow e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx=dt$

$\therefore I=\int dt=t+c=\frac{e^x}{x}+c$

Ex 7.6 Class 12 Maths Question 20.

$\frac{(x-2)e^x}{(x-1)^3}$

Solution:

$I=\int e^x\left[\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right]dx$

Put $\frac{e^x}{(x-1)^2}=t$

$\Rightarrow e^x\left[\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right]dx=dt$

$\therefore I=\int dt=t+C=\frac{e^x}{(x-1)^2}+C$

Ex 7.6 Class 12 Maths Question 21.

$e^{2x} sinx$

Solution:

let

$$I = \int e^{2x} sinx = e^{2x}(-cosx) - \int 2e^{2x}(-cosx)dx$$

$$= -e^{2x}\cos x + 2[e^{2x}(\sin x) - \int 2e^{2x}(\sin x)dx]$$
$$= e^{2x}(2\sin x - \cos x) - 4I$$
$$\Rightarrow I = \frac{e^{2x}}{5}(2\sin x - \cos x) + C$$

Ex 7.6 Class 12 Maths Question 22.

$sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Solution:

Put x = tan t

so that dx = sec² t dt

$$\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int \sin^{-1}\left(\frac{2\tan t}{1+\tan^2 t}\right)\sec^2 t\,dt$$

$$= \int \sin^{-1}(\sin 2t)\sec^2 t\,dt = \int 2t\sec^2 t\,dt$$

$$= 2\left[t\tan t - \int 1\cdot \tan t\,dt\right]$$

$$= 2t\tan t + 2\log|\cos t| + c$$

$$= 2x\tan^{-1}x - \log(1+x^2) + c.$$

Choose the correct answer in exercise 23 and 24

Ex 7.6 Class 12 Maths Question 23.

$\int x^2 e^{x^3} dx \quad equals$

(a)

$\frac{1}{3}e^{x^3} + c$

(b)

$\frac{1}{3} + e^{x^2} + c$

(c)

$\frac{1}{2}e^{x^3} + c$

(d)

$\frac{1}{2}e^{x^2}+c$

Solution:

(a) let $x^3 = t$

$\Rightarrow 3x^2\, dx = dt$

$$\therefore \int x^2 e^{x^3} dx = \frac{1}{3}\int e^t dt = \frac{1}{3}e^t + c = \frac{1}{3}e^{x^3} + c$$

Ex 7.6 Class 12 Maths Question 24.

$$\int e^x secx(1+tanx)dx \quad equals$$

(a)

$e^x cosx + c$

(b)

$e^x secx + c$

(c)

$e^x sinx + c$

(d)

$e^x tanx + c$

Solution:

(b)

$$\int e^x(secx + secx \quad tanx)dx = e^x secx + c$$

Ex 7.7 Class 12 Maths Question 1.

$$\sqrt{4-x^2}$$

Solution:

$$let \quad I = \int \sqrt{4-x^2}dx = \int \sqrt{(2)^2 - x^2}dx$$
$$= \frac{x\sqrt{4-x^2}}{2} + 2sin^{-1}\left(\frac{x}{2}\right) + c$$

Ex 7.7 Class 12 Maths Question 2.

$$\sqrt{1-4x^2}$$

Solution:

$$\int \sqrt{1-4x^2}dx = 2\int \sqrt{\left(\frac{1}{2}\right)^2 - x^2}dx$$

$$= \frac{x\sqrt{1-4x^2}}{2} + \frac{1}{4}sin^{-1}(2x) + c$$

Ex 7.7 Class 12 Maths Question 3.

$$\sqrt{x^2+4x+6}$$

Solution:

$$\int\sqrt{x^2+4x+6}dx=\int\sqrt{(x+2)^2+(\sqrt{2})^2}dx$$

$$=\tfrac{x+2}{2}\sqrt{x^2+4x+6}+log\left|(x+2)+\sqrt{x^2+4x+6}\right|+c$$

Ex 7.7 Class 12 Maths Question 4.

$$\sqrt{x^2+4x+1}$$

Solution:

$$\int\sqrt{x^2+4x+1}dx=\int\sqrt{(x+2)^2-(\sqrt{3})^2}dx$$

$$=\tfrac{x+2}{2}\sqrt{x^2+4x+1}-\tfrac{3}{2}log\left|(x+2)+\sqrt{x^2+4x+1}\right|+c$$

Ex 7.7 Class 12 Maths Question 5.

$$\sqrt{1-4x-x^2}$$

Solution:

$$\int \sqrt{1-4x-x^2}dx = \int \sqrt{(5)^2-(x+2)^2}dx$$

$$= \frac{x+2}{2}\sqrt{5-(x+2)^2}dx$$

Ex 7.7 Class 12 Maths Question 6.

$$\sqrt{x^2+4x-5}$$

Solution:

$$\int \sqrt{x^2+4x-5}dx = \int \sqrt{(x+2)^2-(3)^2}dx$$

$$= \frac{x+2}{2}\sqrt{x^2+4x-5} - \frac{9}{2}log|x+2+\sqrt{x^2+4x-5}| + c$$

Ex 7.7 Class 12 Maths Question 7.

$$\sqrt{1+3x-x^2}$$

Solution:

$$\int \sqrt{1-(x^2-3x)}dx = \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-(x-\frac{3}{2})^2}dx$$

$$= \frac{2x-3}{4}\sqrt{1+3x-x^2} + \frac{13}{8}sin^{-1}\left[\frac{2x-3}{\sqrt{3}}\right] + c$$

Ex 7.7 Class 12 Maths Question 8.

$$\sqrt{x^2+3x}$$

Solution:

$$\int \sqrt{x^2+3x}dx = \int \sqrt{\left(x+\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2}dx$$

$$= \frac{2x+3}{4}\sqrt{x^2+3x} - \frac{9}{8}log\left|x+\frac{3}{2}+\sqrt{x^2+3x}\right| + c$$

Ex 7.7 Class 12 Maths Question 9.

$$\sqrt{1+\frac{x^2}{9}}$$

Solution:

$$\int \sqrt{1+\frac{x^2}{9}}dx = \frac{1}{3}\int \sqrt{x^2+3^2}$$

$$= \frac{1}{6}\left[x\sqrt{x^2+9} + 9log|x+\sqrt{x^2+9}|\right] + c$$

Choose the correct answer in the Exercises 10 to 11:

Ex 7.7 Class 12 Maths Question 10.

$\int \sqrt{1+x^2}dx \quad is \quad equal \quad to$

(a)

$\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}log|x+\sqrt{1+x^2}|+c$

(b)

$\frac{2}{3}\left(1+x^2\right)^{\frac{3}{2}}+c$

(c)

$\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+c$

(d)

$\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2log\left|x+\sqrt{1+x^2}\right|+c$

Solution:

(a)

$\int \sqrt{1+x^2}dx$

$= \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}log(x+\sqrt{1+x^2}) + c$

Ex 7.7 Class 12 Maths Question 11.

$\int \sqrt{x^2-8x+7}dx \quad is \quad equal \quad to$

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+\log|x-4+\sqrt{x^2-8x+7}|+C$

(b) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log|x+4+\sqrt{x^2-8x+7}|+C$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log|x-4+\sqrt{x^2-8x+7}|+C$

(d) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log|x-4+\sqrt{x^2-8x+7}|+C$

Solution:

(d)

$\int \sqrt{x^2-8x+7}dx = \int \sqrt{(x-4)^2-(3)^2}dx$

$= \frac{x-4}{2}\sqrt{x^2-8x+7} - \frac{9}{2}log\left|(x-4)+\sqrt{x^2+8x+7}\right| + c$

Ex 7.8 Class 12 Maths Question 1.

$\int_a^b x \quad dx$

Solution:

on comparing

$$\int_a^b x \quad dx \quad with \quad \int_a^b f(x)dx$$

we have

$$f(a) = a,\ f(a+h) = a+h,\ f(a+2h) = a+2h.....$$

$$f(a+\overline{n-1}h) = a+\overline{n-1}h$$

$$\int_a^b f(x)dx = \underset{h\to 0}{\text{Lt}}\ h[f(a)+f(a+h)+....f(a+\overline{n-1}h)]$$

where $nh = b-a$

$$\therefore \int_a^b x\,dx = \underset{h\to 0}{\text{Lt}}\ h[a+(a+h)+\ ....+(a+\overline{n-1}h)]$$

$$= \underset{h\to 0}{\text{Lt}}\ h[na+h(1+2+....\overline{n-1})]$$

$$= \underset{h\to 0}{\text{Lt}}\ h\left[na+h.\frac{n-1}{2}(1+\overline{n-1})\right]$$

Applying limit and use $nh = b-a$

$$= (b-a)a+\frac{(b-a)(b-a)}{2}$$

$$= (b-a)\left[a+\frac{b-a}{2}\right] = \frac{b^2-a^2}{2}$$

Ex 7.8 Class 12 Maths Question 2.

$$\int_0^5 (x+1)dx$$

Solution:

on comparing

$\int_0^5 (x+1)dx \quad with \quad \int_0^5 f(x)dx$

we have f(x) = x+1, a = 0, b = 5

and nh = b-a = 5-0 = 5

$$f(a)=f(0)=1;\ f(a+h)=f(h)=1+h$$
$$f(a+2h)=f(2h)=2h+1=1+2h$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$f(a+\overline{n-1}h)=f(\overline{n-1}h)=1+(n-1)h=1+(n-1)h$$

By definition

$$\int_a^b f(x)\,dx=\underset{h\to 0}{\text{Lt}}\ h[f(a)+f(a+h)+\dots+$$
$$f(a+\overline{n-1}h)];\ \text{where } nh=b-a=5$$
$$\therefore \quad \int_0^5 (x+1)\,dx=\underset{h\to 0}{\text{Lt}}\ h[1+(1+h)$$
$$+\dots+(1+\overline{n-1}h)]$$
$$=\underset{h\to 0}{\text{Lt}}\ h[1+1+1+1\ \dots \text{to } n \text{ terms})$$
$$+h[1+2+3+\dots+\overline{n-1})]$$
$$=\underset{h\to 0}{\text{Lt}}\left[nh+\frac{(nh-h)nh}{2}\right]=\underset{h\to 0}{\text{Lt}}\left(5+\frac{(5-h)5}{2}\right)=\frac{35}{2}.$$

Ex 7.8 Class 12 Maths Question 3.

$\int_2^3 x^2 dx$

Solution:

compare

$\int_2^3 x^2dx \quad with \quad \int_a^b f(x)dx$

we have

$f(x)=x^2$, and $a=2, b=3$ Also, $f(a)=f(2)=2^2$
$f(a+h)=f(2+h)=(2+h)^2=4+h^2+4h$
$f(a+2h)=f(2+2h)=(2+2h)^2=4+4h^2+8h$
....
....

$f(a+\overline{n-1}h) = f[2+(\overline{n-1})h)] = [2+(\overline{n-1})h]^2$
$=4+(n-1)^2h^2+4(n-1)h$
By definition,
$\because \int_a^b f(x)\,dx = \underset{h\to 0}{\text{Lt}}\ h[f(a)+f(a+h)+...+$
$f(a+\overline{n-1}h)]$; where $nh=b-a=1$
$\therefore \int_2^3 x^2\,dx = \underset{h\to 0}{\text{Lt}}\ h[4n+h^2(1+4+9+... \quad +$

Ex 7.8 Class 12 Maths Question 4.

$\int_1^4 (x^2-x)dx$

Solution:

compare

$\int_1^4 (x^2-x)dx \quad with \quad \int_a^b f(x)dx$

we have f(x) = x^2-x and a = 1, b = 4

$$\text{Also } f(a) = f(1) = 1 - 1 = 0;$$

$$f(a+h) = f(1+h) = h^2 + h$$

$$f(a+2h) = f(1+2h) = 4h^2 + 2h$$

$$f(a+3h) = 9h^2 + 3h$$

....

....

$$f(a+\overline{n-1}h) = f(1+\overline{n-1}h) = (n-1)^2h^2 + (n-1)h$$

By definition, we have; $nh = b - a = 3$

$$\therefore \int_1^4 (x^2 - x)dx = \underset{h\to 0}{\text{Lt}}\ h[0 + (h^2 + h) + (4h^2 + 2h)$$

$$+(9h^2 + 3h) + ... + \{(n-1)^2h^2 + (n-1)h\}]$$

$$= \underset{h\to 0}{\text{Lt}}\ h[h^2\{1 + 4 + 9 + ... + (n-1)^2\}$$

$$+h\{1 + 2 + ... + (n-1)\}]$$

$$= \underset{h\to 0}{\text{Lt}}\left[\frac{(3-h)3(6-h)}{6} + \frac{(3-h)3}{2}\right] = \frac{27}{2}.$$

Ex 7.8 Class 12 Maths Question 5.

$$\int_{-1}^{1} e^x dx$$

Solution:

compare

$\int_{-1}^{1} e^x dx \quad with \quad \int_a^b f(x)dx$

we have

$f(x) = e^x$, a = –1, b = 1 and nh = b – a = 1 + 1 = 2

Now, f(a) = f(– 1) = e^{-1}; f(a + h) = f(– 1 + h) = e^{-1+h}

f(a + 2h) = f(– 1 – 2h) = e^{-1+2h}

....

....

$f(a+\overline{n-1}h) = f(-1+\overline{n-1}h) = e^{-1+\overline{n-1}h}$

By definition, we have

$$\int_{-1}^{1} e^x dx = \underset{h\to0}{\text{Lt}}\ h[e^{-1}(1+e^h+e^{2h}+....+e^{(n-1)h}]$$

$$= \underset{h\to0}{\text{Lt}}\ h.\frac{1}{e}.\frac{1(e^{nh}-1)}{e^h-1} = e-\frac{1}{e}$$

Ex 7.8 Class 12 Maths Question 6.

$\int_0^4 (x + e^{2x})dx$

Solution:

let f(x) = x + e^{2x},

a = 0, b = 4

and nh = b – a = 4 – 0 = 4

Ex 7.9 Class 12 Maths Question 1.

$\int_{-1}^{1} (x + 1)dx$

Solution:

$$= \left[\frac{x^2}{2} + x\right]_{-1}^{1} = \frac{1}{2}(1-1) + (1+1) \quad = 2$$

Ex 7.9 Class 12 Maths Question 2.

$$\int_2^3 \frac{1}{x} dx$$

Solution:

$$= [log \quad x]_2^3 \quad = log3 - log2 \quad = log\frac{3}{2}$$

Ex 7.9 Class 12 Maths Question 3.

$$\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx$$

Solution:

$$= \left[\frac{4x^4}{4} - \frac{5x^3}{3} + \frac{6x^2}{2} + 9x\right]_1^2$$

$$= \left[x^4 - \frac{5}{3}x^3 + 3x^2 + 9x\right]_1^2 \quad = \frac{64}{3}$$

Ex 7.9 Class 12 Maths Question 4.

$$\int_0^{\frac{\pi}{4}} sin2x \quad dx$$

Solution:

$= \left[-\frac{1}{2}cos2x\right]_0^{\frac{\pi}{4}} \quad = \frac{1}{2}$

Ex 7.9 Class 12 Maths Question 5.

$\int_0^{\frac{\pi}{2}} cos2x \quad dx$

Solution:

$= \left[\frac{1}{2}sin2x\right]_0^{\frac{\pi}{2}} \quad = 0$

Ex 7.9 Class 12 Maths Question 6.

$\int_4^5 e^x dx$

Solution:

$= \left[e^x\right]_4^5 \quad = e^5 - e^4$

Ex 7.9 Class 12 Maths Question 7.

$\int_0^{\frac{\pi}{4}} tanx \quad dx$

Solution:

$= \left[log \quad secx\right]_0^{\frac{\pi}{4}} \quad = \frac{1}{2}log2$

Ex 7.9 Class 12 Maths Question 8.

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cosec \quad xdx$$

Solution:

$$= log(cosecx - cotx)_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= log(\sqrt{2} - 1) - log(2 - \sqrt{3}) \quad = log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$$

Ex 7.9 Class 12 Maths Question 9.

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Solution:

$$= sin^{-1}(1) - sin^{-1}(0) \quad = \frac{\pi}{2}$$

Ex 7.9 Class 12 Maths Question 10.

$$\int_0^1 \frac{dx}{1+x^2}$$

Solution:

$$= \left[tan^{-1}x\right]_0^1 \quad = tan^{-1}(1) - tan^{-1}(0) \quad = \frac{\pi}{4}$$

Ex 7.9 Class 12 Maths Question 11.

$\int_2^3 \frac{dx}{x^2-1}$

Solution:

$= \left[\frac{1}{2}log\left(\frac{x-1}{x+1}\right)\right]_2^3 \quad = \frac{1}{2}log\frac{3}{2}$

Ex 7.9 Class 12 Maths Question 12.

$\int_0^{\frac{\pi}{2}} cos^2 x dx$

Solution:

$= \int_0^{\frac{\pi}{2}} \frac{1+cos2x}{2}dx = \frac{1}{2}\left[x + \frac{sin2x}{2}\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

Ex 7.9 Class 12 Maths Question 13.

$\int_2^3 \frac{x}{x^2+1}dx$

Solution:

$= \frac{1}{2}\int_2^3 \frac{2x}{x^2+1}dx \quad = \frac{1}{2}\left[log\left(x^2+1\right)\right]_2^3 \quad = \frac{1}{2}log2$

Ex 7.9 Class 12 Maths Question 14.

$\int_0^1 \frac{2x+3}{5x^2+1}dx$

Solution:

$$= \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1}dx + \frac{3}{5}\int_0^1 \frac{dx}{x^2+\left[\frac{1}{\sqrt{5}}\right]^2}$$

$$= \frac{1}{5}\left[\log\left(5x^2+1\right)\right]_0^1 + \frac{3}{5} \times \frac{1}{1/\sqrt{5}}\left[\tan^{-1}\frac{x}{1/\sqrt{5}}\right]_0^1$$

$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$$

Ex 7.9 Class 12 Maths Question 15.

$$\int_0^1 xe^{x^2}dx$$

Solution:

let $x^2 = t \Rightarrow 2xdx = dt$

when x = 0, t = 0 & when x = 1,t = 1

$$\therefore I = \frac{1}{2}\int_0^1 e^t dt \quad = \frac{1}{2}(e^t)_0^1 \quad = \frac{1}{2}[e-1]$$

Ex 7.9 Class 12 Maths Question 16.

$$\int_1^2 \frac{5x^2}{x^2+4x+3}dx$$

Solution:

$$\frac{20x+15}{x^2+4x+3}=\frac{20x+15}{(x+1)(x+3)}=\frac{A}{x+1}+\frac{B}{x+3}$$

$$\Rightarrow \quad 20x+15=A(x+3)+B(x+1) \quad \text{...(i)}$$

Put $x=-1,-3$ in (i), we get : $A=\frac{-5}{2}$ & $B=\frac{45}{2}$

$$I=\int_1^2\left(5+\frac{5}{2(x+1)}-\frac{45}{2(x-3)}\right)dx$$

$$=\left[5x+\frac{5}{2}\log|x+1|-\frac{45}{2}\log|x+3|\right]_1^2$$

$$=5-\frac{5}{2}\left(9\log\frac{5}{4}-\log\frac{3}{2}\right).$$

Ex 7.9 Class 12 Maths Question 17.

$$\int_0^{\frac{\pi}{4}}\left(2sec^2x+x^3+2\right)dx$$

Solution:

$$=\left[2tanx+\frac{x^4}{4}+2x\right]_0^{\frac{\pi}{4}}$$

$$=2\left(\tan\frac{\pi}{4}-\tan 0\right)+\frac{1}{4}\left(\frac{\pi^4}{256}-0\right)+2\left(\frac{\pi}{4}-0\right)$$

$$=\frac{\pi^4}{1024}+\frac{\pi}{2}+2$$

Ex 7.9 Class 12 Maths Question 18.

$\int_0^\pi \left(sin^2\frac{x}{2} - cos^2\frac{x}{2}\right)dx$

Solution:

$= -\int_0^\pi cosxdx \quad = -[sinx]_0^\pi - (0-0) \quad = 0$

Ex 7.9 Class 12 Maths Question 19.

$\int_0^2 \frac{6x+3}{x^2+4}dx$

Solution:

$= \int_0^2 \frac{6x}{x^2+4}dx + \int_0^2 \frac{3}{x^2+4}dx$

$$= 3\int_0^2 \frac{2x}{x^2+4}dx + 3\times\frac{1}{2}\tan^{-1}\frac{x}{2}\Big]_0^2$$

Let $x^2+4=t \Rightarrow 2x\,dx = dt$

$$= 3\int_4^8 \frac{dt}{t} + \frac{3}{2}\tan^{-1}\frac{x}{2}\Big]_0^2 = 3\log(x^2+4) + \frac{3}{2}\tan^{-1}\frac{x}{2}\Big]_0^2$$

$$= 9\log 2 + \frac{3}{8}\pi - 6\log 2 = 3\log 2 + \frac{3}{8}\pi$$

Ex 7.9 Class 12 Maths Question 20.

$\int_0^1 \left(xe^x + sin\frac{\pi x}{4}\right)dx$

Solution:

$$= \int_0^1 xe^x dx + \int_0^1 sin\frac{\pi x}{4}dx$$

$$= \left[xe^x\right]_0^1 - \int_0^1 1\cdot e^x dx - \frac{4}{\pi}\left[\cos\frac{\pi x}{4}\right]_0^1$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

Ex 7.9 Class 12 Maths Question 21.

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} \quad equals$$

(a)

$\frac{\pi}{3}$

(b)

$\frac{2\pi}{3}$

(c)

$\frac{\pi}{6}$

(d)

$\frac{\pi}{12}$

Solution:

(d)

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} \quad = \left[tan^{-1}x\right]_1^{\sqrt{3}} \quad = \frac{\pi}{3} - \frac{\pi}{4} \quad = \frac{\pi}{12}$$

Ex 7.9 Class 12 Maths Question 22.

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} equals$$

(a)

$\frac{\pi}{6}$

(b)

$\frac{\pi}{12}$

(c)

$\frac{\pi}{24}$

(d)

$\frac{\pi}{4}$

Solution:

(c)

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} \quad = \frac{1}{9}\int_0^{\frac{2}{3}} \frac{dx}{\left(\frac{2}{3}\right)^2+x^2}$$

$$= \frac{1}{6}\left[tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{\frac{2}{3}} \quad = \frac{1}{6} \times \frac{\pi}{4} \quad = \frac{\pi}{24}$$

Ex 7.10 Class 12 Maths Question 1.

$$\int_0^1 \frac{x}{x^2+1}dx = I$$

Solution:

Let $x^2 + 1 = t$

$\Rightarrow 2xdx = dt$

when x = 0, t = 1 and when x = 1, t = 2

$$\therefore I = \frac{1}{2}\int_0^1 \frac{dt}{t} = \left[\frac{1}{2logt}\right]_1^2 \quad = \frac{1}{2}log2$$

Ex 7.10 Class 12 Maths Question 2.

$$\int_0^{\frac{\pi}{2}} \sqrt{sin\phi}cos^5\phi d\phi = I$$

Solution:

$$I = \int_0^{\frac{\pi}{2}} \sqrt{sin\phi}(1 - sin^2)^2 cos\phi d\phi$$

put sinφ = t,so that cosφdφ = dt

When $\phi = 0$, $t = 0$; when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore \quad I = \int_0^1 \sqrt{t}(1-t^2)^2 dt = \int_0^1 t^{1/2} + t^{9/2} - 2t^{5/2})dt$$

$$= \left[\frac{2}{3}t^{\frac{3}{2}} + \frac{2}{11}t^{\frac{11}{2}} - \frac{4}{7}t^{\frac{7}{2}}\right]_0^1 = \frac{64}{231}$$

Ex 7.10 Class 12 Maths Question 3.

$$\int_0^1 sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = I$$

Solution:

let x = tanθ =>dx = sec²θ dθ

when x = 0 => θ = 0

and when x = 1 => $\theta \frac{\pi}{4}$

$\frac{1}{2}$

$$\therefore \quad I = \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta d\theta = \int_0^{\frac{\pi}{4}} 2\theta \sec^2\theta d\theta$$

$$= 2\theta\tan\theta - 2\log\sec\theta\Big]_0^{\frac{\pi}{4}} = \frac{\pi}{2} - \log 2$$

Ex 7.10 Class 12 Maths Question 4.

$\int_0^2 x\sqrt{x+2}dx = I(say)(put \quad x+2=t^2)$

Solution:

let x+2 = t =>dx = dt

when x = 0,t = 2 and when x = 2, t = 4

$$\therefore \quad I=\int_2^4 (t-2)\sqrt{t}\,dt=\int_2^4 (t^{3/2}-2t^{1/2})\,dt$$

$$=\frac{2}{5}t^{5/2}-2\times\frac{2}{3}t^{3/2}\Big]_2^4=\frac{16}{15}(2+\sqrt{2}).$$

Ex 7.10 Class 12 Maths Question 5.

$\int_0^{\frac{\pi}{2}} \frac{sinx \quad dx}{1+cos^2x} = I$

Solution:

put cosx = t

so that -sinx dx = dt

when x = 0, t = 1; when

$x = \frac{\pi}{2}$

, t = 0

$\therefore I = \int_1^0 \frac{-dt}{1+t^2} = -\left[tan^{-1}t\right]_1^0 = \frac{\pi}{4}$

Ex 7.10 Class 12 Maths Question 6.

$$\int_0^2 \frac{dx}{x+4-x^2} = I$$

Solution:

$$\int_0^2 \frac{dx}{x+4-x^2} = I$$

$$I = \int_0^2 \frac{dx}{\frac{17}{4} - \left(x - \frac{1}{2}\right)^2};$$

Put $a^2 = \frac{17}{4}$, and $x \to x - \frac{1}{2}$

$$I = \frac{1}{2\frac{\sqrt{17}}{2}} \log \left[\frac{\frac{\sqrt{17}}{2} + \left(x - \frac{1}{2}\right)}{\frac{\sqrt{17}}{2} - \left(x - \frac{1}{2}\right)}\right]_0^2$$

$$= \frac{1}{\sqrt{17}} \log \frac{\left(5+\sqrt{17}\right)^2}{25-17} = \frac{1}{\sqrt{17}} \log \frac{21+5\sqrt{17}}{4}$$

Ex 7.10 Class 12 Maths Question 7.

$$\int_{-1}^1 \frac{dx}{x^2+2x+5} = I$$

Solution:

$$I=\int_{-1}^{1}\frac{dx}{(x+1)^2+2^2}=\frac{1}{2}\left[tan^{-1}\frac{x+1}{2}\right]_{-1}^{1} \quad =\frac{\pi}{8}$$

Ex 7.10 Class 12 Maths Question 8.

$$\int_1^2\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx=I$$

Solution:

let 2x = t ⇒ 2dx = dt

when x = 1, t = 2 and when x = 2, t = 4

$$I=\int_2^4 e^t\left(\frac{1}{t}-\frac{1}{t^2}\right)dt \quad =e^t\left[\frac{1}{t}\right]_2^4 \quad =\frac{e^2}{2}\left[\frac{e^2}{2}-1\right]$$

Choose the correct answer in Exercises 9 and 10

Ex 7.10 Class 12 Maths Question 9.

The value of integral

$$\int_{\frac{1}{3}}^{1}\frac{(x-x^3)^{\frac{1}{3}}}{x^4}dx$$

is

(a) 6

(b) 0

(c) 3

(d) 4

Solution:

(a) let I =

$$\int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx \quad = \int_{\frac{1}{3}}^{1} \frac{x^{\frac{1}{3}}(1-x^2)^{\frac{1}{3}}}{x^4} dx$$

Let $1 - x^2 = t^3 \Rightarrow x^2 = 1 - t^3 \Rightarrow -2x\,dx = 3t^2\,dt$

Consider $I = -\frac{1}{2}\int \frac{x^{1/3}(1-x^2)^{1/3}(-2x)}{x^4 . x} dx$

$$I = -\frac{1}{2}\int \frac{(1-t^3)^{1/3} . t}{(1-t^3)^2} 3t^2 dt = \frac{-3}{4(x)^2} \therefore I = \frac{-3}{4}x^2 \Big]_{\frac{1}{3}}^{1} = 6$$

Ex 7.10 Class 12 Maths Question 10.

$$If \quad f(x) = \int_0^x tsint, \quad then \quad f'(x) \quad is$$

(a) cosx+xsinx

(b) xsinx

(c) xcosx

(d) sinx+xcosx

Solution:

(b)

$$f(x) = \int_0^x tsint \quad dt = t(-cost) - \int 1[(-cost)dt]_0^x$$

=-x cox+sinx

Ex 7.11 Class 12 Maths Question 1.

$\int_0^{\frac{\pi}{2}} cos^2x \quad dx = I$

Solution:

$I = \frac{1}{2}\int_0^{\frac{\pi}{2}}(1+cos2x)dx = \frac{1}{2}\left[x+\frac{sin2x}{2}\right]_0^{\frac{\pi}{2}} \quad = \frac{\pi}{4}$

Ex 7.11 Class 12 Maths Question 2.

$\int_0^{\frac{\pi}{2}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}dx$

Solution:

let I =

$\int_0^{\frac{\pi}{2}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}dx$

$$I=\int_0^{\pi/2}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$$I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \qquad \text{...(ii)}$$

Adding equ (i) & (ii) we get

$$2I=\int_0^{\pi/2}1\cdot dx=[x]_0^{\pi/2}=\frac{\pi}{2} \quad \therefore \quad I=\frac{\pi}{4}1$$

Ex 7.11 Class 12 Maths Question 3.

$$\int_0^{\frac{\pi}{2}} \frac{sin^{\frac{3}{2}}xdx}{sin^{\frac{3}{2}}x+cos^{\frac{3}{2}}dx}dx$$

Solution:

let I =

$$\int_0^{\frac{\pi}{2}} \frac{sin^{\frac{3}{2}}xdx}{sin^{\frac{3}{2}}x+cos^{\frac{3}{2}}dx}dx$$

$$\text{Also } I=\int_0^{\pi/2} \frac{\sin^{3/2}\left(\frac{\pi}{2}-x\right)}{\sin^{3/2}\left(\frac{\pi}{2}-x\right)+\cos^{3/2}\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\pi/2} \frac{\cos^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x}dx \qquad ...(ii)$$

Adding (i) and (ii), we have

$$2I=\int_0^{\pi/2} 1\, dx=[x]_0^{\pi/2}=\frac{\pi}{2}-0=\frac{\pi}{2} \therefore\ I=\frac{\pi}{4}$$

Ex 7.11 Class 12 Maths Question 4.

$$\int_0^{\frac{\pi}{2}} \frac{cos^5xdx}{sin^5x+cos^5x}$$

Solution:

let I =

$\int_0^{\frac{\pi}{2}} \frac{cos^5xdx}{sin^5x+cos^5x}$

$$\text{Also } I=\int_0^{\pi/2} \frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\pi/2} \frac{\sin^5 x}{\cos^5 x+\sin^5 x}dx \qquad \ldots(ii)$$

Adding (*i*) and (*ii*), we have

$$2I=\int_0^{\pi/2} 1dx = x]_0^{\pi/2} = \frac{\pi}{2} \therefore I=\frac{\pi}{4}.$$

Ex 7.11 Class 12 Maths Question 5.

$\int_{-5}^5 |x+2|\, dx = I$

Solution:

$I = \int_{-5}^5 |x+2|\, dx + \int_{-2}^5 |x+2|\, dx$

at x = – 5, x + 2 < 0; at x = – 2, x + 2 = 0; at x = 5, x + 2>0;x + 2<0, x + 2 = 0, x + 2>0

$$I=-\int_{-5}^{-2} x+2\ dx+\int_{-2}^{5} x+2\ dx$$

$$=-\left[\frac{x^2}{2}+2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{5}=29$$

Ex 7.11 Class 12 Maths Question 6.

$\int_2^8 |x-5| dx = I$

Solution:

$\int_2^8 |x-5| dx = I$

$$I = \int_2^5 |x-5|\,dx + \int_5^8 |x-5|\,dx$$
$$= -\int_2^5 (x-5)\,dx + \int_5^8 (x-5)\,dx$$
$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8 = 9$$

Ex 7.11 Class 12 Maths Question 7.

$\int_0^1 x(1-x)^n dx = I$

Solution:

$\int_0^1 x(1-x)^n dx = I$

Let $I = \int_0^1 (1-x)[1-(1-x)]^n dx$

$= \int_0^1 (x^n - x^{n-1}) dx$

$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 = \left(\frac{1}{(n+1)(n+2)}\right).$

Ex 7.11 Class 12 Maths Question 8.

$$\int_0^{\frac{\pi}{4}} log(1 + tanx)dx$$

Solution:

let I =

$$\int_0^{\frac{\pi}{4}} log(1 + tanx)dx$$

Also $I = \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx$

$= \int_0^{\pi/4} \log\left(1 + \frac{1-\tan x}{1+\tan x}\right) dx = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right) dx$

$= \int_0^{\pi/4} \log 2\, dx - \int_0^{\pi/4} \log(1+\tan x)\, dx;$

$I = \log 2 \int_0^{\pi/4} 1\, dx - I$

$\Rightarrow \quad 2I = \log 2\, [x]_0^{\pi/4} = \frac{\pi}{4} \log 2 \Rightarrow I = \frac{\pi}{8} \log 2$

Ex 7.11 Class 12 Maths Question 9.

$\int_0^2 x\sqrt{2-x}dx = I$

Solution:

let 2-x = t

⇒ – dx = dt

when x = 0, t = 2 and when x = 2,t = 0

$\frac{1}{2}$

$$\therefore \quad I = -\int_2^0 (2-t)\sqrt{t}\,dt = \int_0^2 (2t^{1/2} - t^{3/2})\,dt$$

$$= \frac{4}{3}t^{3/2} - \frac{2}{5}t^{5/2}\Big]_0^2 = \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{16\sqrt{2}}{15}.$$

Ex 7.11 Class 12 Maths Question 10.

$\int_0^{\frac{\pi}{2}} (2logsinx - logsin2x)\,dx = I$

Solution:

$\int_0^{\frac{\pi}{2}} (2logsinx - logsin2x)\,dx = I$

$$I = \int_0^{\pi/2} [2\log\sin x - \log 2 - \log\sin x - \log\cos x]\, dx$$

$$= \int_0^{\pi/2} \log\sin x\ dx - (\log 2)\,[x]_0^{\pi/2}$$

$$= -\int_0^{\pi/2} \log\sin x\ dx = \frac{\pi}{2}\log\frac{1}{2}$$

Ex 7.11 Class 12 Maths Question 11.

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} sin^2xdx$$

Solution:

Let f(x) = sin² x

f(-x) = sin² x = f(x)

∴ f(x) is an even function

$$\therefore \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} sin^2xdx \quad = 2\int_0^{\frac{\pi}{2}} \left[\frac{1-cos2x}{2}\right] dx = \left[x - \frac{sin2x}{x}\right]_0^{\frac{\pi}{2}} \therefore I = \frac{\pi}{2}$$

Ex 7.11 Class 12 Maths Question 12.

$$\int_0^{\pi} \frac{xdx}{1+sinx}$$

Solution:

let I =

$$\int_0^{\pi} \frac{xdx}{1+sinx}$$

…(i)

$$I=\int_0^{\pi}\frac{\pi-x}{1+\sin(\pi-x)}dx=\int_0^{\pi}\frac{\pi-x}{1+\sin x}dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I=\pi\int_0^{\pi}\frac{1}{1+\sin x}dx==\pi\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}dx$$

$$=\pi\int_0^{\pi}(\sec^2 x-\tan x\sec x)\,dx$$

$$=\pi[\tan x-\sec x]_0^{\pi}=2\pi\Rightarrow I=\pi.$$

Ex 7.11 Class 12 Maths Question 13.

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} sin^7 xdx$$

Solution:

Let f(x) = $\sin^7$ xdx

⇒ f(-x) = -$\sin^7$ x = -f(x)

⇒ f(x) is an odd function of x

⇒

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} sin^7 xdx = 0$$

Ex 7.11 Class 12 Maths Question 14.

$$\int_0^{2\pi} cos^5 xdx$$

Solution:

let f(x) = $\cos^5 x$

⇒ f(2π – x) = $\cos^5 x$

$$\therefore \quad \int_0^{2\pi} \cos^5 x\,dx = 2\int_0^{\pi} \cos^5 x\,dx$$

$$\text{Now, } f(\pi - x) = -\cos^5 x = -f(x)$$

$$\therefore \quad \int_0^{\pi} \cos^5 x\,dx = 0$$

$$\Rightarrow \int_0^{2\pi} \cos^5 x\,dx = 2\int_0^{\pi} \cos^5 x\,dx = 0.$$

Ex 7.11 Class 12 Maths Question 15.

$$\int_0^{\frac{\pi}{2}} \frac{sinx - cosx}{1 + sinx \quad cosx} dx$$

Solution:

let I =

$$\int_0^{\frac{\pi}{2}} \frac{sinx - cosx}{1 + sinx \quad cosx} dx$$

…(i)

$$\text{Then } I=\int_0^{\pi/2}\frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\pi/2}\frac{\cos x-\sin x}{1+\cos x\sin x}dx \quad \ldots(ii)$$

Adding (i) and (ii), we get

$$2I=\int_0^{\pi/2}\frac{\sin x-\cos x+\cos x-\sin x}{1+\sin x\cos x}dx$$

$$=\int_0^{\pi/2}0\,dx=0\Rightarrow \quad I=\mathbf{0}.$$

Ex 7.11 Class 12 Maths Question 16.

$$\int_0^{\pi} log(1+cosx)dx$$

Solution:

let I =

$$\int_0^{\pi} log(1+cosx)dx$$

then I =

$\int_0^{\pi} log[1 + cos(\pi - x)]dx$

$$I = \int_0^{\pi} \log(1 - \cos x)\, dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\pi} \log(1 - \cos^2 x)\, dx = 2\int_0^{\pi} \log \sin x\, dx$$

$$\Rightarrow I = \int_0^{\pi} \log \sin x\, dx = 2\int_0^{\pi/2} \log \sin x\, dx = 2I_1$$

$$\text{Now, } I_1 = \int_0^{\pi/2} \log \sin x\, dx \qquad ...(iii)$$

$$I_1 = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx = \int_0^{\pi/2} \log \cos x\, dx. \quad ...(iv)$$

Adding (iii) and (iv), we get

$$2I_1 = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx$$

$$= \int_0^{\pi/2} \log \sin 2x\, dx - \int_0^{\pi/2} \log 2\, dx$$

$$= \int_0^{\pi/2} \log \sin 2x\, dx - \frac{\pi}{2} \log 2 = I_2 - \frac{\pi}{2} \log_2 \quad ... (v)$$

Put $2x = t$, so that $2dx = dt$

When $x = 0, t = 0$; when $x = \frac{\pi}{2}, t = \pi$

$$\therefore I_2 = \frac{1}{2}\int_0^{\pi} \log \sin t\, dt = \int_0^{\pi/2} \log \sin x dx = I_1$$

$\therefore$ From (v), we get; $I_1 = -\frac{\pi}{2}\log 2$

$$\therefore \quad I = 2\times\left(-\frac{\pi}{2}\log 2\right) = -\pi \log 2.$$

Ex 7.11 Class 12 Maths Question 17.

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

Solution:

let I =

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

…(i)

$$\text{Then, } I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}}dx$$

$$\Rightarrow I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \quad ...(ii)$$

Adding (i) and (ii), we get ;

$$2I = \int_0^a 1dx = [x]_0^a = a - 0 = a \quad \therefore I = \frac{a}{2}.$$

Ex 7.11 Class 12 Maths Question 18.

$$\int_0^4 |x-1|\,dx = I$$

Solution:

$$I = -\int_0^1 (x-1)dx + \int_1^4 (x-1)dx$$

$$= -\left[\frac{x^2}{2} - x\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4 = 5$$

Ex 7.11 Class 12 Maths Question 19.

show that

$$4\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$$

if f and g are defined as f(x)=f(a-x) and g(x)+g(a-x)=4

Solution:

let I =

$$\int_0^a f(x)g(x)dx$$

$$= \int_0^a f(a-x)[4-g(a-x)]\,dx$$

$$= 4\int_0^a f(a-x)\,dx - \int_0^a f(a-x)g(a-x)\,dx$$

Let $a - x = t \Rightarrow -dx = dt$

When $x = 0, t = a$ and when $x = a, t = 0$

$$I = -4\int_a^0 f(t)\,dt + \int_a^0 f(t)g(t)\,dt$$

$$= 4\int_0^a f(x)\,dx - \int_0^a f(x)g(x)\,dx$$

$$I = 4\int_0^a f(x)\,dx - I \Rightarrow I = 2\int_0^a f(x)\,dx$$

Ex 7.11 Class 12 Maths Question 20.

The value of

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(x^3 + xcosx + tan^5x + 1\right) dx$$

is

(a) 0

(b) 2

(c) π

(d) 1

Solution:

(c) let I =

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(x^3 + xcosx + tan^5x + 1\right) dx$$

is

$$I = \int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x)\,dx + \int_{-\pi/2}^{\pi/2} 1\,dx$$

$$I = I_1 + x]_{-\pi/2}^{\pi/2} = I_1 + \frac{\pi}{2} + \frac{\pi}{2} = I_1 + \pi$$

Let $f(x) = x^3 + x\cos x + \tan^5 x$

$\therefore \quad f(-x) = -x^3 - x\cos x - \tan^5 x = -f(x)$

$\therefore \quad f(x)$ is an odd function. Thus $I_1 = 0 \therefore\ I = \pi$

Ex 7.11 Class 12 Maths Question 21.

The value of

$$\int_0^{\frac{\pi}{2}} log\left[\frac{4+3sinx}{4+3sinx}\right] dx$$

is

(a) 2

(b)

$$\frac{3}{4}$$

(c) 0

(d) -2

Solution:

let I =

$$\int_0^{\frac{\pi}{2}} log\left[\frac{4+3sinx}{4+3sinx}\right] dx$$

$$= \int_0^{\pi/2} \log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx$$

$$= -\int_0^{\pi/2} \log\left[\frac{4+3\sin x}{4+3\cos x}\right]dx$$

$$\Rightarrow \quad I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

Chapterwise NCERT Solutions for Class 12 Maths :

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