NCERT

National Council Of Educational Research And Training

NCERT Solutions for 12th Class Maths: Chapter 6-Application of Derivatives

IndCareer Schools

indCareer indCareer indCareer

NCERT Solutions for 12th Class Maths: Chapter 6-Application of Derivatives

Class 12: Maths Chapter 6 solutions. Complete Class 12 Maths Chapter 6 Notes.

NCERT Solutions for 12th Class Maths: Chapter 6-Application of Derivatives

Class 12: Maths Chapter 6 solutions. Complete Class 12 Maths Chapter 6 Notes.

Ex 6.1 Class 12 Maths Question 1.

Find the rate of change of the area of a circle with respect to its radius r when (a) r = 3 cm (b) r = 4 cm

https://www.indcareer.com/schools/ncert-solutions-for-12th-class-maths-chapter-6-application-of-derivatives/

Solution:

Let A be the area of the circle

$$\therefore\ A = \pi r^2 \text{ and } \frac{dA}{dr} = 2\pi r$$

$$\text{when } r = 3 \text{ or } 4. \text{ then } \frac{dA}{dr} = 2\pi \times 3 = 6\pi\,cm^2$$

$$\text{or } \frac{dA}{dr} = 2\pi \times 4 = 8\pi\,cm^2$$

Ex 6.1 Class 12 Maths Question 2.

The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of an edge is 12 cm?

Solution:

Let x be the length of the cube volume $V = x^3$,

$$S = 6x^2 \Rightarrow \frac{ds}{dx} = 12x \ \& \ V = x^3,$$

$$\therefore \frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \text{ We have } \frac{dV}{dt} = 8\ cm^3/sec.$$

$$\frac{dx}{dt} = \frac{8}{3x^2} \ \therefore \frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} = 2\frac{2}{3}\ cm^2/sec.$$

Ex 6.1 Class 12 Maths Question 3.

The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

Solution:

Let r be the radius of the circle.

Area of circle = πr^2 = A also

$\frac{dr}{dt}$

= 3 cm/ sec.

$$\therefore \frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \frac{d}{dr}(\pi r^2) \cdot \frac{dr}{dt} = 2\pi r \times 3, = 6\pi r.$$

$$\text{When } r = 10; \frac{dA}{dt} = 6\pi \times 10 = 60\,\pi\,cm^2/sec.$$

Ex 6.1 Class 12 Maths Question 4.

An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

Solution:

Let the edge of the cube = x cm

∴

$\frac{dx}{dt}$

= 3

$$\therefore \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = 3x^2 \times 3 = 9x^2\,m^3/sec.$$

$$\text{When } x = 10, \frac{dv}{dt} = 9 \times (10)^2 = 900\,cm^2/sec.$$

Ex 6.1 Class 12 Maths Question 5.

A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

Solution:

Let r be the radius of a wave circle:

$\frac{dx}{dt}$

= 5cm/sec.

$$\text{(A) Area} = \pi r^2, \ \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 5 = 10\pi r$$

$$\text{When } r = 8, \ \frac{dA}{dt} = 10\pi r = 10\pi \times 8 = 80\pi \, cm^2/sec.$$

Ex 6.1 Class 12 Maths Question 6.

The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

Solution:

The rate of change of circle w.r.t time t is given

to be 0.7 cm/sec. i.e.

$\frac{dr}{dt}$

= 0.7 cm/sec.

Now, circumference of the circle is c = 2πr

$$\therefore \frac{dc}{dt} = \left[\frac{d}{dr}(2\pi r) \cdot \frac{dr}{dt}\right] = 2\pi \frac{dr}{dt} = 1.4\pi cm/sec$$

Ex 6.1 Class 12 Maths Question 7.

The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rates of change of

(a) the perimeter, and

(b) the area of the rectangle.

Solution:

(a) The length x of a rectangle is decreasing at dx the rate of 5cm/min. =>

$\frac{dx}{dt}$

= – 5cm min …(i)

The width y is increasing at the rate of 4cm/min.

$\Rightarrow \frac{dy}{dt} = 4$ cm/ min. ...(ii)

The perimeter p of the rectangle is

$p = 2(x+y)$, $\frac{dp}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$

from (i) & (ii); $\frac{dp}{dt} = 2(-5+4) = -2$ cm/min.

$\therefore$ This shows the perimeter decreases at the rate of 2 cm/min.

(b) $A = xy$ $\therefore$ $\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$ ...(iii)

But $\frac{dx}{dt} = -5$ cm/min, $\frac{dy}{dt} = 4$cm/min.

and $x = 8, y = 6$; Put these values in (iii)

$\frac{dA}{dt} = (-5\times 6) + 8\times 4 = -30 + 32 = 2\,cm^2/min$

$\Rightarrow$ Area is increasing at the rate of $2\,cm^2/min$.

Ex 6.1 Class 12 Maths Question 8.

A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

Solution:

Volume of the spherical balloon = V =

$\frac{4}{3}\pi r^3$

$$\frac{dv}{dt}=\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\cdot\frac{dr}{dt} \quad =4\pi r^2\cdot\frac{dr}{dt}$$

$$\because \frac{dv}{dt}=900\ \text{cm}^3/\text{sec}. \quad \therefore\ 900=4\pi r^2\frac{dr}{dt}$$

When radius is 15cm, putting r = 15

$$\therefore\ 900=4\times\frac{22}{7}\times 15\times 15\times\frac{dr}{dt}\Rightarrow\frac{dr}{dt}=\frac{7}{22}$$

Hence, the radius of the balloon is increasing at the rate of $\frac{7}{22}$ cm/sec. i.e. $\frac{1}{\pi}$ cm/sec.

Ex 6.1 Class 12 Maths Question 9.

A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

Solution:

Let r be the variable radius of the balloon which is in the form of sphere Vol. of the sphere

$$V=\frac{4}{3}\pi r^3;\ \frac{dv}{dr}=\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)=4\pi r^2$$

When $r=10,\ \frac{dv}{dr}=4\pi\times 10\times 10=400\,\pi\,\text{cm}^3/\text{sec}.$

Ex 6.1 Class 12 Maths Question 10.

A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from die wall ?

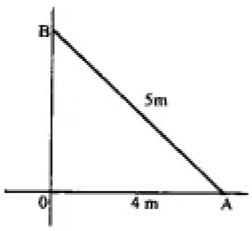
Solution:

Let AB be the ladder and OB be the wall. At an instant,

let OA = x, OB = y,

$x^2 + y^2 = 25$ …(i)

On differentiating,



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow \quad x\frac{dx}{dt} + y\frac{dy}{dt} = 0 \qquad \text{...(ii)}$$

When $x = 4$, then from (i), we have

$16 + y^2 = 15 \Rightarrow y = 3$

Now, $\frac{dx}{dt} = 0{\cdot}02$ m/sec. Put these values in (ii),

$$4 \times 0{\cdot}02 + 3 \times \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-0{\cdot}08}{3} = \frac{-2}{75}$$

Hence, the height of the ladder on the wall is decreasing at the rate of $\frac{2}{75}$ m/sec. $= \frac{8}{3}$ cm/sec.

Ex 6.1 Class 12 Maths Question 11.

A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Solution:

We have

$6y = x^3 + 2$…(i)

$$\Rightarrow \quad 2\frac{dy}{dt} = x^2\frac{dx}{dt} \qquad \text{...(ii)}$$

Now, y coordinate changes 8 times as fast as x coordinate i.e., $\frac{dy}{dt} = 8\frac{dx}{dt}$. $\therefore$ in (ii), we have

$x^2 = 16 \Rightarrow x = \pm 4$; When $x = 4$, then from (i); $y = 11$

When $x = -4$, then from (i); $y = \frac{1}{6}(-64+2) = -\frac{31}{3}$

Hence, the required points are $(4, 11), \left(-4, \frac{-31}{3}\right)$

Ex 6.1 Class 12 Maths Question 12.

The radius of an air bubble is increasing at the rate of

$\frac{1}{2}$

cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

Solution:

Let r be the radius then V =

$\frac{4}{3}\pi r^3$

$\frac{dr}{dt} = \frac{1}{2} cm/sec$

Hence, the rate of increase of volume when radius is 1 cm = 2π x 1² = 2π cm^3/sec.

Ex 6.1 Class 12 Maths Question 13.

A balloon, which always remains spherical, has a variable diameter

$\frac{3}{2}(2x+1)$

. Find the rate of change of its volume with respect to x.

Solution:

Dia of sphere =

$\frac{3}{2}(2x+1)$

∴ Radius =

$\frac{3}{4}(2x+1)$

$$\text{Vol.} = \frac{4}{3}\pi \cdot \frac{27}{64}(2x+1)^3 \Rightarrow V = \frac{9\pi}{16}(2x+1)^3$$

$$\therefore \; = \frac{dV}{dx} = \frac{27\pi}{8}(2x+1)^2$$

Ex 6.1 Class 12 Maths Question 14.

Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

Solution:

Let r and h be the radius and height of the sand

– cone at time t respectively, h =

$\frac{r}{6}$

…(i)

$$\therefore V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6h)^2 h = 12\pi h^3$$

$$\frac{dV}{dt} = 12\pi(3h^2).\frac{dh}{dt}, \quad \frac{dV}{dt} = 36\pi h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 12.cm^3/sec \Rightarrow 12 = 36\pi h^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{12}{36\pi h^2} = \frac{1}{3\pi h^2}$$

Hence the rate of increase of height of sand–cone w.r.t. – t.

When h=4 is $\left(\frac{dh}{dt}\right)_{h=4} = \frac{1}{3\pi(4)^2} = \frac{1}{48\pi}$ cm/sec.

Ex 6.1 Class 12 Maths Question 15.

The total cost C (x) in Rupees associated with the production of x units of an item is given by C (x) = 0.007 x^3 – 0.003 x^2 + 15x + 4000. Find the marginal cost when 17 units are produced.

Solution:

Marginal cost MC = Instantaneous rate of change

of total cost at any level of out put =

$\frac{dC}{dx}$

Now, $C(x) = 0{\cdot}007\,x^3 - 0{\cdot}003\,x^2 + 15x + 4000$

$\therefore \quad MC = \frac{dc}{dx} = 0{\cdot}021\,x^2 - 0{\cdot}006x + 15$

When $x = 17;\ \frac{dC}{dx} = 6{\cdot}069 - 0.102 + 15 = 20{\cdot}967$

$\therefore$ Marginal cost = ₹ 20·97.

Ex 6.1 Class 12 Maths Question 16.

The total revenue in Rupees received from the sale of x units of a product is given by

R (x) = $13x^2 + 26x + 15$. Find the marginal revenue when x = 7.

Solution:

Marginal Revenue (MR)

= Rate of change of total revenue w.r.t. the

number of items sold at an instant =

$\frac{dR}{dx}$

We know R(x) = $13x^2 + 26x + 15$,

MR =

$\frac{dR}{dx}$

= 26x + 26 = 26(x+1)

Now x = 7, MR = 26 (x + 1) = 26 (7 + 1) = 208

Hence, Marginal Revenue = Rs 208.

Choose the correct answer in Exercises 17 and 18.

Ex 6.1 Class 12 Maths Question 17.

The rate of change of the area of a circle with respect to its radius r at r = 6 cm is

(a) 10π

(b) 12π

(c) 8π

(d) 11π

Solution:

(b) ∵ $A = \pi r^2$ =>

$\frac{dA}{dr}$

= 2π x 6 = 12π cm^2/radius

Ex 6.1 Class 12 Maths Question 18.

The total revenue in Rupees received from the sale of x units of a product is given by R (x) = $3x^2 + 36x + 5$. The marginal revenue, when x = 15 is

(a) 116

(b) 96

(c) 90

(d) 126

Solution:

(d) $R(x) = 3x^2 + 36x + 5$,

MR =

$\frac{dR}{dx}$

$= 6x + 36$,

At $x = 15$;

$\frac{dR}{dx}$

$= 6 \times 15 + 36 = 90 + 36 = 126$

Ex 6.2 Class 12 Maths Question 1.

Show that the function given by $f(x) = 3x+17$ is strictly increasing on R.

Solution:

$f(x) = 3x + 17$

$\therefore f'(x) = 3>0 \ \forall \ x \in R$

$\Rightarrow$ f is strictly increasing on R.

Ex 6.2 Class 12 Maths Question 2.

Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

Solution:

We have $f(x) = e^{2x}$

$\Rightarrow f'(x) = 2e^{2x}$

Case I When $x > 0$, then $f'(x) = 2e^{2x}$

$$=2\left[1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\cdots\cdots\right]>0\ \forall\ x>0$$

Case II When $x=0$, then $f'(x)=2e^{\circ}=2>0$ for $x=0$

Case III When $x<0$: Let $x=-y$ where y is a positive quantity

$$\therefore f'(x)=2e^{-2y}=\frac{2}{e^{zy}}=\frac{2}{+\text{ve quantity}}>0 \text{ for } x<0$$

Thus $f'(x)>0\ \forall\ x\in R$. Hence, e^{2x} is strictly increasing on R.

Ex 6.2 Class 12 Maths Question 3.

Show that the function given by f (x) = sin x is

(a) strictly increasing in

$\left(0, \frac{\pi}{2}\right)$

(b) strictly decreasing in

$\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in (0, π)

Solution:

We have f(x) = sinx

∴ f' (x) = cosx

(a) f' (x) = cos x is + ve in the interval

$\left(0, \frac{\pi}{2}\right)$

⇒ f(x) is strictly increasing on

$$\left(0, \frac{\pi}{2}\right)$$

(b) f' (x) = cos x is a -ve in the interval

$$\left(\frac{\pi}{2}, \pi\right)$$

⇒ f (x) is strictly decreasing in

$$\left(\frac{\pi}{2}, \pi\right)$$

(c) f' (x) = cos x is +ve in the interval

$$\left(0, \frac{\pi}{2}\right)$$

while f' (x) is -ve in the interval

$$\left(\frac{\pi}{2}, \pi\right)$$

∴ f(x) is neither increasing nor decreasing in (0,π)

Ex 6.2 Class 12 Maths Question 4.

Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is

(a) strictly increasing

(b) strictly decreasing

Solution:

f(x) = $2x^2 - 3x$

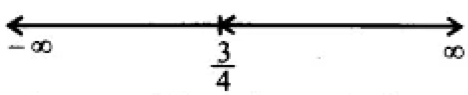
⇒ f’ (x) = 4x – 3

⇒ f’ (x) = 0 at x =

$\frac{3}{4}$

The point

divides the real



Line into two disjoint intervals viz.

$\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$.

In the interval, $\left(-\infty, \frac{3}{4}\right)$ f′ (x) is –ve

∴ f is strictly decreasing in $\left(-\infty, \frac{3}{4}\right)$

In the interval $\left(\frac{3}{4}, \infty\right)$, f′ is + ve.

∴ f is strictly increasing in the interval $\left(\frac{3}{4}, \infty\right)$

Ex 6.2 Class 12 Maths Question 5.

Find the intervals in which the function f given by f (x) = $2x^3 - 3x^2 - 36x + 7$ is

(a) strictly increasing

(b) strictly decreasing

Solution:

$f(x) = 2x^3 – 3x^2 – 36x + 7;$

$f (x) = 6 (x – 3) (x + 2)$

$\Rightarrow f' (x) = 0$ at $x = 3$ and $x = – 2$

The points $x = 3$, $x = – 2$, divide the real line into three disjoint intervals viz. $(-\infty,-2)$, $(-2,3)$, $(3,\infty)$

Now $f' (x)$ is +ve in the intervals $(-\infty, -2)$ and $(3,\infty)$. Since in the interval $(-\infty, -2)$ each factor $x – 3$, $x + 2$ is -ve.

$\Rightarrow f' (x) = +$ ve.

(a) f is strictly increasing in $(-\infty, -2) \cup (3,\infty)$

(b) In the interval $(-2,3)$, $x+2$ is +ve and $x-3$ is -ve.

$f (x) = 6(x – 3)(x + 2) = + \times – =$ -ve

$\therefore$ f is strictly decreasing in the interval $(-2,3)$.

Ex 6.2 Class 12 Maths Question 6.

Find the intervals in which the following functions are strictly increasing or decreasing:

(a) $x^2 + 2x – 5$

(b) $10 – 6x – 2x^2$

(c) $– 2x^3 – 9x^2 – 12x + 1$

(d) $6 – 9x – x^2$

(e) $(x + 1)^3(x – 3)^3$

Solution:

(c) Let $f(x) = -2x^3 - 9x^2 - 12x + 1$

∴ $f'(x) = -6x^2 - 18x - 12$

$= -6(x^2 + 3x + 2)$

$f'(x) = -6(x + 1)(x + 2)$, $f'(x) = 0$ gives $x = -1$ or $x = -2$

The points $x = -2$ and $x = -1$ divide the real line into three disjoint intervals namely $(-\infty, -2)$ $(-2, -1)$ and$(-1\ \infty)$.

In the interval $(-\infty,-2)$ i.e.,$-\infty<x<-2$ $(x+1)$ $(x+2)$ are -ve.

∴$f'(x) = (-)\ (-)\ (-) = -$ ve.

⇒ f (x) is decreasing in $(-\infty,-2)$

In the interval $(-2, -1)$ i.e., $-2 < x < -1$,

$(x + 1)$ is -ve and $(x + 2)$ is + ve.

∴ $f'(x) = (-)(-)\ (+) = +$ ve.

⇒ f (x) is increasing in $(-2, -1)$

In the interval $(-1,\infty)$ i.e.,$-1 <x<\infty$,$(x + 1)$ and $(x + 2)$ are both positive. $f'(x) = (-)\ (+)\ (+) =$ -ve.

⇒ f (x) is decreasing in $(-1, \infty)$

Hence, f (x) is increasing for $-2 < x < -1$ and decreasing for $x<-2$ and $x>-1$.

Ex 6.2 Class 12 Maths Question 7.

Show that

$$y = log(1 + x) - \frac{2x}{2+x}x > -1$$

, is an increasing function of x throughout its domain.

Solution:

let

$$f(x) = log(1+x) - \frac{2x}{2+x}x > -1$$

f' (x) =

$$\frac{x^2}{(x+1)(x+2)^2}$$

For f (x) to be increasing f' (x) > 0

$$\Rightarrow \frac{1}{x+1} > 0 \Rightarrow x > -1$$

Hence,

$$y = log(1+x) - \frac{2x}{2+x}$$

is an increasing function of x for all values of x > – 1.

Ex 6.2 Class 12 Maths Question 8.

Find the values of x for which y = $[x (x – 2)]^2$ is an increasing function.

Solution:

y = $x^4 – 4x^3 + 4x^2$

$\therefore \frac{dy}{dx}$ = $4x^3 – 12x^2 + 8x$

For the function to be increasing

$\frac{dy}{dx}$

>0

$4x^3 – 12x^2 + 8x>0$

⇒ 4x(x – 1)(x – 2)>0

For 0 < x < 1, $\frac{dy}{dx}$ = (+)(-)(-) = +ve and for x > 2,

$\frac{dy}{dx}$

= (+) (+) (+) = +ve

Thus, the function is increasing for 0 < x < 1 and x > 2.

Ex 6.2 Class 12 Maths Question 9.

Prove that

$$y=\frac{4sin\theta}{(2+cos\theta)}-\theta$$

is an increasing function of θ in

$$\left[0,\frac{\pi}{2}\right]$$

Solution:

$$\frac{dy}{dx}=\frac{8cos\theta+4}{(2+cos\theta)^2}-1=\frac{cos\theta(4-cos\theta)}{(2+cos\theta)^2}$$

For the function to be increasing

$\frac{dy}{dx}$

> 0

⇒ cosθ(4-cos²θ)>0

⇒ cosθ>0

⇒ θ∈

$\left[0, \frac{\pi}{2}\right]$ 1

Ex 6.2 Class 12 Maths Question 10.

Prove that the logarithmic function is strictly increasing on (0, ∞).

Solution:

Let f (x) = log x

Now, f' (x) =

$\frac{1}{x}$

; When takes the

values x > 0,

$\frac{1}{x}$ $\frac{1}{x}$

> 0, when x > 0,

∵ f' (x) > 0

Hence, f (x) is an increasing function for x > 0 i.e

Ex 6.2 Class 12 Maths Question 11.

Prove that the function f given by f (x) = $x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on (-1,1).

Solution:

Given

f (x) = $x^2 - x + 1$

$$f'(x) = 2x - 1 = 2\left(x - \frac{1}{2}\right) \text{Now} -1 < x < \frac{1}{2}$$

$$\Rightarrow \left(x - \frac{1}{2}\right) < 0 \Rightarrow 2\left(x - \frac{1}{2}\right) < 0$$

$$\Rightarrow f'(x) < 0 \text{ and } \frac{1}{2} < x < 1 \Rightarrow x - \frac{1}{2} > 0$$

$$\Rightarrow 2\left(x - \frac{1}{2}\right) > 0 \Rightarrow f'(x) > 0$$

∴ f (x) is neither increasing nor decreasing on (-1,1).

Ex 6.2 Class 12 Maths Question 12.

Which of the following functions are strictly decreasing on

$\left[0, \frac{\pi}{2}\right]$

(a) cos x

(b) cos 2x

(c) cos 3x

(d) tan x

Solution:

(a) We have f (x) = cos x

∴ f’ (x) = – sin x < 0 in

$\left[0, \frac{\pi}{2}\right]$

∴ f’ (x) is a decreasing function.

(b) We have $f(x) = \cos 2x \quad \therefore f'(x) = -2\sin 2x$

For $0 < x < \frac{\pi}{2}$ or $0 < 2x < \pi$, $\sin 2x$ is +ve

∴ f′ (x) is a decreasing function.

(c) We have $f(x) = \cos 3x \quad \therefore f'(x) = -3\sin 3x$

For $0 < x < \frac{\pi}{2} \Rightarrow 0 < 3x < \frac{3\pi}{2}$, $\sin 3x$ is +ve

in $0 < 3x < \pi \therefore f'(x) < 0 \Rightarrow f(x)$ is decreasing.

And $\sin 3x$ is – ve in $\pi < 3x < \frac{3\pi}{2}$

∴ $f'(x) > 0 \Rightarrow f(x)$ is increasing

∴ f ′ (x) is neither increasing nor decreasing in $\left(0, \frac{\pi}{2}\right)$

∴ f (x) is not a decreasing function in $\left(0, \frac{\pi}{2}\right)$.

(d) $f(x) = \tan x; f'(x) = \sec^2 x > 0 \ \forall \ x \in \left(0, \frac{\pi}{2}\right)$

∴ f (x) is an increasing function. Thus (a) cos x (b) cos 2 x are strictly decreasing function on $\left(0, \frac{\pi}{2}\right)$

Ex 6.2 Class 12 Maths Question 13.

On which of the following intervals is the function f given by f (x)= x^{100} + sin x – 1 strictly decreasing ?

Ex 6.2 Class 12 Maths Question 13.

On which of the following intervals is the function f given by f (x)= x^{100} + sin x – 1 strictly decreasing ?

(a) (0,1)

(b)

$\left[\frac{\pi}{2}, \pi\right]$

(c)

$\left[0, \frac{\pi}{2}\right]$

(d) none of these

Solution:

(d) f(x) = x^{100} + sin x – 1

∴ f’ (x)= $100x^{99}$+ cos x

(a) for(-1, 1)i.e.,- 1 <x< 1,-1 <x^{99}< 1

⇒ -100<$100x^{99}$<100;

Also 0 ⇒ f’ (x) can either be +ve or -ve on(-1, 1)

∴ f (x) is neither increasing nor decreasing on (-1,1).

(b) for (0,1) i.e. 0<x< 1 x^{99} and cos x are both +ve ∴ f’ (x) > 0

⇒ f (x) is increasing on(0,1)

(c) For $\left(\frac{\pi}{2}, \pi\right)$ i.e. $\frac{\pi}{2} < x < \pi$, x^{99} is +ve and $-1 < \cos x < 0$ $\therefore f'(x) > 0$

$\Rightarrow f(x)$ is increasing on $\left(\frac{\pi}{2}, \pi\right)$

(d) For $\left(0, \frac{\pi}{2}\right)$, i.e. $0 < x < \frac{\pi}{2}$, x^{99} and $\cos x$ are both –ve. $\therefore f'(x) < 0$ $f(x)$ is decreasing on $\left(0, \frac{\pi}{2}\right)$.

Ex 6.2 Class 12 Maths Question 14.

Find the least value of a such that the function f given by f (x) = x^2 + ax + 1 is strictly increasing on (1,2).

Solution:

We have f (x) = x^2 + ax + 1

∴ f' (x) = 2x + a.

Since f (x) is an increasing function on (1,2)

f' (x) > 0 for all 1 < x < 2 Now, f" (x) = 2 for all x ∈ (1,2) ⇒ f" (x) > 0 for all x ∈ (1,2)

⇒ f' (x) is an increasing function on (1,2)

⇒ f' (x) is the least value of f' (x) on (1,2)

But f' (x)>0 ∀ x∈ (1,2)

∴ f' (1)>0 =>2 + a>0

⇒ a > – 2 : Thus, the least value of a is – 2.

Ex 6.2 Class 12 Maths Question 15.

Let I be any interval disjoint from (-1,1). Prove that the function f given by

$f(x)=x+\frac{1}{x}$

is strictly increasing on I.

Solution:

Given

$f(x)=x+\frac{1}{x}$

$$f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2} \text{ Now } x\in I \Rightarrow x\notin(-1,1)$$
$$\Rightarrow x\leq-1 \text{ or } x\geq 1 \Rightarrow x^2\geq 1 \Rightarrow x^2-1\geq 0$$
$$\Rightarrow \frac{x^2-1}{x^2}\geq 0 \Rightarrow f'(x)\geq 0; \text{Thus } f'(x)\geq 0 \ \forall x\in I.$$

Hence, f’ (x) is strictly increasing on I.

Ex 6.2 Class 12 Maths Question 16.

Ex 6.2 Class 12 Maths Question 16.

Prove that the function f given by f (x) = log sin x is strictly increasing on

$\left(0,\frac{\pi}{2}\right)$

and strictly decreasing on

$\left(\frac{\pi}{2},\pi\right)$

Solution:

f’ (x) =

$$\frac{1}{sin \quad x}.cos \quad x \quad cot \quad x$$

when 0 < x <

$$\frac{\pi}{2}$$

, f’ (x) is +ve; i.e., increasing

When

$$\frac{\pi}{2}$$

< x < π, f’ (x) is – ve; i.e., decreasing,

∴ f (x) is decreasing. Hence, f is increasing on (0, π/2) and strictly decreasing on (π/2, π).

Ex 6.2 Class 12 Maths Question 17.

Prove that the function f given by f(x) = log cos x is strictly decreasing on

$$\left(0, \frac{\pi}{2}\right)$$

and strictly increasing on

$$\left(\frac{\pi}{2}, \pi\right)$$

Solution:

$$f(x) = log \quad cosx$$

f’ (x) =

$$\frac{1}{cosx}(-sinx) = -tanx$$

In the interval

$$\left(0, \frac{\pi}{2}\right)$$

,f’ (x) = -ve

∴ f is strictly decreasing.

In the interval

$$\left(\frac{\pi}{2}, \pi\right)$$

, f’ (x) is + ve.

∴ f is strictly increasing in the interval.

Ex 6.2 Class 12 Maths Question 18.

Prove that the function given by

f (x) = $x^3 - 3x^2 + 3x - 100$ is increasing in R.

Solution:

f’ (x) = $3x^2 - 6x + 3$

= 3 ($x^2 - 2x + 1$)

= 3 $(x - 1)^2$

Now x ∈ R, f'(x) = $(x - 1)^2 \geq 0$

i.e. f'(x)≥0 ∀ x∈R; hence, f(x) is increasing on R.

Ex 6.2 Class 12 Maths Question 19.

The interval in which $y = x^2 e^{-x}$ is increasing is

(a) $(-\infty,\infty)$

(b) $(-2,0)$

(c) $(2,\infty)$

(d) $(0,2)$

Solution:

(d) $f'(x) = 2xe^{-x} + x^2(-e^{-x}) = xe^{-x}(2-x) = e^{-x}x(2-x)$

Now e^{-x} is positive for all $x \in R$ $f'(x) = 0$ at $x = 0,2$

$x = 0$, $x = 2$ divide the number line into three disjoint intervals, viz. $(-\infty, 0)$, $(0,2)$, $(2, \infty)$

(a) Interval $(-\infty,0)$ x is +ve and $(2-x)$ is +ve

$\therefore f'(x) = e^{-x}x(2-x) = (+)(-)(+) = -ve$

$\Rightarrow$ f is decreasing in $(-\infty,0)$

(b) Interval $(0,2)$ $f'(x) = e^{-x}x(2-x)$

$= (+)(+)(+) = +ve$

$\Rightarrow$ f is increasing in $(0,2)$

(c) Interval $(2, \infty)$ $f'(x) = e^{-x}x(2-x) = (+)(+)(-)$

$= -ve$

$\Rightarrow$ f is decreasing in the interval $(2, \infty)$

Ex 6.3 Class 12 Maths Question 1.

Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

Solution:

The curve is $y = 3x^4 - 4x$

∴

$\frac{dy}{dx}$

$= 12x^3 - 4$

∴Req. slope =

$\left(\frac{dy}{dx}\right)_{x=4}$

$= 12 \times 4^3 - 4 = 764.$

Ex 6.3 Class 12 Maths Question 2.

Find the slope of the tangent to the curve

$$y = \frac{x-1}{x-2}, x \neq 2$$

at x = 10.

Solution:

The curve is

$$y = \frac{x-1}{x-2}$$

$$\frac{dy}{dx} = \frac{1(x-2) - (x-1) \cdot 1}{(x-2)^2} = \frac{-1}{(x-2)^2}$$

$$\therefore \text{Req. slope} = \left(\frac{dy}{dx}\right)_{x=10} = \frac{-1}{(10-2)^2} = \frac{-1}{8^2} = \frac{-1}{64}$$

Ex 6.3 Class 12 Maths Question 3.

Find the slope of the tangent to curve $y = x^3 – x + 1$ at the point whose x-coordinate is 2.

Solution:

The curve is $y = x^3 – x + 1$

$\frac{dy}{dx}$

$= 3x^2 – 1$

∴slope of tangent =

$\left(\frac{dy}{dx}\right)_{x=2}$

$= 3 \times 2^2 – 1$

$= 11$

Ex 6.3 Class 12 Maths Question 4.

Find the slope of the tangent to the curve $y = x^3 – 3x + 2$ at the point whose x-coordinate is 3.

Solution:

The curve is $y = x^3 – 3x + 2$

$\frac{dy}{dx}$

$= 3x^2 – 3$

∴slope of tangent =

$$\left(\frac{dy}{dx}\right)_{x=3}$$

= 3 x 3² – 3

= 24

Ex 6.3 Class 12 Maths Question 5.

Find the slope of the normal to the curve x = a cos³ θ, y = a sin³ θ at θ =

$$\frac{\pi}{4}$$

.

Solution:

$$\frac{dx}{d\theta} = -3a \quad cos^2\theta sin\theta, \frac{dy}{d\theta} = 3a \quad sin^2\theta cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\sin\theta}{\cos\theta}, \frac{dy}{dx} = -\tan\theta$$

$$\therefore \text{Req. slope} \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -\tan\frac{\pi}{4} = -1$$

$$\text{Hence slope of normal} = -\frac{1}{m} = \frac{-1}{-1} = 1$$

Ex 6.3 Class 12 Maths Question 6.

Find the slope of the normal to the curve x = 1 – a sin θ, y = b cos² θ at θ =

$$\frac{\pi}{2}$$

Solution:

$$\frac{dx}{d\theta} = -a \quad cos\theta \quad \& \quad \frac{dy}{d\theta} = 2b \quad cos\theta(-sin\theta)$$

$$\text{Now } \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-2b\sin\theta\cos\theta}{-a\cos\theta} = \frac{2b}{a}\sin\theta$$

$$\text{At } \theta = \frac{\pi}{2}, \frac{dy}{dx} = \frac{2b}{a}\sin\frac{\pi}{2} = \frac{2b}{a}\times 1 = \frac{2b}{a}$$

$$\therefore \text{slope of normal at } \theta = \frac{\pi}{2} \text{ is } \frac{-1}{(dy/dx)_{\theta=\frac{\pi}{2}}} = \frac{-a}{2b}$$

Ex 6.3 Class 12 Maths Question 7.

Find points at which the tangent to the curve y = x^3 – $3x^2$ – 9x + 7 is parallel to the x-axis.

Solution:

Differentiating w.r.t. x;

$$\frac{dy}{dx}$$

= 3 (x – 3) (x + 1)

Tangent is parallel to x-axis if the slope of tangent = 0

or

$$\frac{dy}{dx} = 0$$

⇒3(x + 3)(x + 1) = 0

⇒x = -1, 3

when x = -1, y = 12 & When x = 3, y = – 20

Hence the tangent to the given curve are parallel to x-axis at the points (-1, -12), (3, -20)

Ex 6.3 Class 12 Maths Question 8.

Find a point on the curve $y = (x – 2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

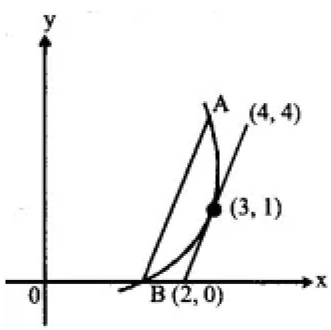
Solution:

The equation of the curve is $y = (x – 2)^2$

Differentiating w.r.t x

$$\frac{dy}{dx} = 2(x - 2)$$

The point A and B are (2,0) and (4,4) respectively.



Slope of AB =

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{4-2} = \frac{4}{2}$$

= 2 …(i)

Slope of the tangent = 2 (x – 2) ….(ii)

from (i) & (ii) 2 (x – 2)=2

∴ x – 2 = 1 or x = 3

when x = 3,y = $(3 – 2)^2$ = 1

∴ The tangent is parallel to the chord AB at (3,1)

Ex 6.3 Class 12 Maths Question 9.

Find the point on the curve y = x^3 – 11x + 5 at which the tangent is y = x – 11.

Solution:

Here, y = x^3 – 11x + 5

⇒

$\frac{dy}{dx}$

= $3x^2$ – 11

The slope of tangent line y = x – 11 is 1

∴ $3x^2$ – 11 = 1

⇒ $3x^2$ = 12

⇒ x^2 = 4, x = ±2

When x = 2, y = – 9 & when x = -2,y = -13

But (-2, -13) does not lie on the curve

∴ y = x – 11 is the tangent at (2, -9)

Ex 6.3 Class 12 Maths Question 10.

Ex 6.3 Class 12 Maths Question 10.

Find the equation of all lines having slope -1 that are tangents to the curve

$$y = \frac{1}{x-1}$$

, x≠1

Solution:

Here

$$y = \frac{1}{x-1}$$

⇒

$$\frac{dy}{dx} = \frac{-1}{(x-1)^2}$$

Slope of tangent $= -1 \therefore \frac{-1}{(x-1)^2} = -1$ or $(x-1)^2 = 1$

$\therefore \quad x - 1 = \pm 1$ or $x = 2, 0$ when $x = 2$,

$y = \frac{1}{x-1} = \frac{1}{1} = 1$ when $x = 0, y = \frac{1}{0-1} = -1$

The point where the tangents to the given curve have the slope -1 are $(2, 1)$ $(0, -1)$

$\therefore \quad$ Equation of the tangents at $(2, 1)$

$y - 1 = -1 \times (x - 2)$ or $x + y = 3$

Equation of tangent at $(0, 1)$,

$y + 1 = -1 \times (x - 0)$ or $x + y + 1 = 0$

Thus, the required lines are

$x + y - 3 = 0$,$x + y + 1 = 0$.

Ex 6.3 Class 12 Maths Question 11.

Find the equation of ail lines having slope 2 which are tangents to the curve

$$y = \frac{1}{x-3}$$

, x≠3.

Solution:

, x≠3.

Solution:

Here

$$y = \frac{1}{x-3}$$

$$\frac{dy}{dx} = (-1)(x-3)^{-2} = \frac{-1}{(x-3)^2}$$

∵ slope of tangent = 2

$$\frac{-1}{(x-3)^2} = 2 \Rightarrow (x-3)^2 = -\frac{1}{2}$$

Which is not possible as $(x - 3)^2 > 0$

Thus, no tangent to

$$y = \frac{1}{x-3}$$

has slope 2.

Ex 6.3 Class 12 Maths Question 12.

Find the equations of all lines having slope 0 which are tangent to the curve

$$y = \frac{1}{x^2-2x+3}$$

Solution:

Let the tangent at the point (x_1, y_1) to the curve

$$y=\frac{1}{x^2-2x+3} \quad ...(i)$$

$$\therefore \quad y_1=\frac{1}{x_1^2-2x_1+3} \quad ...(ii)$$

$$\therefore \frac{dy}{dx}=\frac{-(2x-2)}{(x^2-2x+3)^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)\text{at}(x_1\ y_1)=\frac{-(2x_1-2)}{(x_1^2-2x_1+3)^2}$$

$$\text{But slope at }(x_1, y_1)\text{ is }0 \ \therefore \ \frac{-(2x_1-2)}{(x_1^2-2x_1+3)^2}=0$$

$$\Rightarrow \ -2(x_1-1)=0 \ \Rightarrow \ x_1=1$$

$$\text{When } x_1=1\text{, then from (ii)} \quad y_1=\frac{1}{1-2+3}=\frac{1}{2}$$

The tangent at (x_1, y_1) i.e. (1, 1/2) having slope

$$0 \text{ is } y-\frac{1}{2}=0(x-1) \Rightarrow y-\frac{1}{2}=0 \Rightarrow y=\frac{1}{2}$$

Ex 6.3 Class 12 Maths Question 13.

Find points on the curve

$$\frac{x^2}{9}+\frac{y^2}{16}=1$$

at which the tangents are

(a) parallel to x-axis

(b) parallel to y-axis

Solution:

The equation of the curve is

$$\frac{x^2}{9}+\frac{y^2}{16}=1$$

…(i)

$$\Rightarrow \quad \frac{2x}{9}+\frac{2y}{16}\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{16}{9}\frac{x}{y}$$

(a) if the tangent is parallel to x-axis $\frac{dy}{dx}=0$

$\therefore \; -\frac{16}{9}\cdot\frac{x}{y}=0 \Rightarrow x=0;$ in (i) $\frac{y^2}{16}=1, \; \therefore \; y=\pm 4$

$\therefore$ Tangents are parallel to x-axis at $(0,4)$ & $(0,-4)$

(b) When tangents are parallel to y-axis, then denominator of $\frac{dy}{dx}$ is zero. $y=0$, Putting $y=0$ in (i) $\frac{x^2}{9}=1, x=\pm 3$

Ex 6.3 Class 12 Maths Question 14.

Find the equations of the tangent and normal to the given curves at the indicated points:

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0,5)

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (1,3)

(iii) $y = x^3$ at (1, 1)

(iv) $y = x^2$ at (0,0)

(v) x = cos t, y = sin t at t =

$$\frac{\pi}{4}$$

Solution:

$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$

Putting x = 0,

$\frac{dy}{dx}$

at (0,5) = – 10

Thus, the equation of tangent at P (0, 5) is
$\Rightarrow y - 5 = -10(x - 0) \Rightarrow y + 10x - 5 = 0$
and the equation of normal at P (0, 5)

$(x - x_1) + \left(\frac{dy}{dx}\right)_{at\,p}(y - y_1) = 0 \Rightarrow x - 10y + 50 = 0.$

(ii) $\left.\frac{dy}{dx}\right]_{(1,3)} = 4(1)^3 - 18(1)^2 + 26(1) - 10 = 2$

$\therefore$ Eq. of the tangent is : $2x - y + 1 = 0$

Eq. of the normal is: $x + 2y - 7 = 0$

(iii) $y = x^3 \Rightarrow \left.\frac{dy}{dx}\right]_{(1,1)} = 3x^2 = 3(1)^2 = 3$

The eq. of the tangent is $\Rightarrow 3x - y - 2 = 0$
The eq. of the normal is: $x + 3y - 4 = 0$

(iv) $y = x^2 \Rightarrow \left.\frac{dy}{dx}\right]_{(0,0)} = 2x = 2(0) = 0$

$\therefore$ The eq. of the tangent is: $y = 0$
$\therefore$ The eq. of the normal is: $x = 0$

(v) $\frac{dy}{dt} = \cos t,\ \frac{dx}{dt} = -\sin t \therefore \frac{dy}{dx} = -\cot t$

$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = -\cot\frac{\pi}{4} = -1$

Now $x = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $y = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\therefore x = \frac{1}{\sqrt{2}}$ and $y = \frac{1}{\sqrt{2}}$

$\therefore$ The equation of the tangent at the point

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is $y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad \sqrt{2}.x + \sqrt{2}.y - 2 = 0 \Rightarrow x + y = \sqrt{2}$

The equation of the normal at the point

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is $y - \frac{1}{\sqrt{2}} = 1\left(x - \frac{1}{\sqrt{2}}\right) \Rightarrow x - y = 0.$

Ex 6.3 Class 12 Maths Question 15.

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $5y - 15x = 13$.

Solution:

Equation of the curve is $y = x^2 - 2x + 7$ …(i)

$\frac{dy}{dx}$

$= 2x - 2 = 2(x - 1)$

(a) Slope of the line $2x - y + 9 = 0$ is 2

$\Rightarrow$ Slope of tangent =

$\frac{dy}{dx}$

= 2(x – 1) = 2

$\Rightarrow \quad x=2$

Putting x = 2 in (i) $y = 2^2 - 2 \cdot 2 + 7 = 7$

$\therefore$ Tangent parallel to $2x - y + 9 = 0$ at (2, 7) is

$y - 7 = 2(x - 2)$ or $2x - y + 3 = 0$

(b) Tangent is perpendicular to the line $5y - 15x = 13$

$\Rightarrow$ Slope of tangent x slope of line = – 1

$\Rightarrow x = 1 - \frac{1}{6} = \frac{5}{6}$; Put $x = \frac{5}{6}$ in (i) $y = \frac{217}{36}$

$\therefore$ Equation of the tangent which is perpendicular to $5y - 15x = 13$ at $\left(\frac{5}{6}, \frac{217}{36}\right)$ is

$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right) \quad \therefore \quad 12x + 36y - 227 = 0.$

Ex 6.3 Class 12 Maths Question 16.

Show that the tangents to the curve $y = 7x^3 + 11$ at the points where x = 2 and x = – 2 are parallel.

Solution:

Here, $y = 7x^3 + 11$

=> x

$\frac{dy}{dx}$

$= 21x^2$

Now m_1 = slope at x = 2 is

$\left(\frac{dy}{dx}\right)_{x=2}$

$= 21 \times 2^2 = 84$

and m_2 = slope at x = -2 is

$$\left(\frac{dy}{dx}\right)_{x=-2}$$

= 21 x (-2)² = 84

Hence, $m_1 = m_2$ Thus, the tangents to the given curve at the points where x = 2 and x = – 2 are parallel

Ex 6.3 Class 12 Maths Question 17.

Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point

Solution:

Let P (x_1, y_1) be the required point.

The given curve is: $y = x^3$

$$\Rightarrow \frac{dy}{dx} = 3x^2 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 3x_1^2$$

Since the slope of the tangent at $(x_1, y_1) = y_1$

$\therefore \quad 3x_1^2 = y_1$ ...(ii)

Also (x_1, y_1) lies on (i) so $y_1 = x_1^3$ ...(iii)

from (ii) & (iii), we have

$3x_1^2 = x_1^3 \Rightarrow x_1^2(3 - x_1) = 0 \Rightarrow x_1 = 0$ or $x_1 = 3$

When $x_1 = 0, y_1 = 0^3 = 0$ when $x_1 = 3, y_1 = 3^3 = 27$

$\therefore$ The required points are (0, 0) and (3, 27)

Ex 6.3 Class 12 Maths Question 18.

For the curve $y = 4x^3 – 2x^5$, find all the points at which the tangent passes through the origin.

Solution:

Let (x_1, y_1) be the required point on the given curve $y = 4x^3 – 2x^5$, then $y_1 = 4x_1^3 – 2x_1^5$ …(i)

$$\frac{dy}{dx}=12x^2-10x^4 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)}=12x_1^{\,2}-10x_1^{\,4}$$

Now, equation of the tangent at (x_1, y_1) is

$$y-y_1=\left(\frac{dy}{dx}\right)_{(x_1,y_1)}(x-x_1) \text{ or}$$

$y-y_1=(12x_1^{\,2}-10x_1^{\,4})(x-x_1)$

This passes through the origin

$\therefore \quad 0-y_1=(12x_1^{\,2}-10x_1^{\,4})(0-x_1^{\,4})$

$\Rightarrow y_1=12x_1^{\,3}-10x_1^{\,5}$ ...(ii)

Subtracting (ii) from (i), we get

$0=-8x_1^{\,3}+8x_1^{\,5} \quad \Rightarrow 8x_1^{\,3}(x_1^{\,2}-1)=0$

$\Rightarrow x_1=0$ or $x_1=\pm 1$

When $x_1=0$ from (ii) $y_1=0$, When $x_1=1$, from (ii)

$y_1=4.$

$1^3-2\cdot 1^5=4-2=2$; when $x_1=-1$, from (ii) $y_1=-2$

Hence, the req. points are $(0, 0)$, $(1, 2)$, $(-1, -2)$.

Ex 6.3 Class 12 Maths Question 19.

Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

Solution:

Here, $x^2 + y^2 - 2x - 3 = 0$

=>

$$\frac{dy}{dx}=\frac{1-x}{y}$$

Tangent is parallel to x-axis, if

$$\frac{dy}{dx}=0$$

i.e.

if 1 – x = 0

⇒ x = 1

Putting x = 1 in (i)

⇒ y = ±2

Hence, the required points are (1,2), (1, -2) i.e. (1, ±2).

Ex 6.3 Class 12 Maths Question 20.

Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

Solution:

Here, $ay^2 = x^3$

$$2ay\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

$$\text{At } (am^2, am^3), \frac{dy}{dx} = \frac{3m}{2}$$

$$\Rightarrow \text{Slope of the tangent at } (am^2, am^3) \text{ is } \frac{3m}{2} \text{ and}$$

$$\text{slope of the normal at } (am^2, am^3) \text{ is } \frac{-2}{3m}$$

$$\text{Equation of normal is; } 3my - 3am^4 = -2x + 2am^2$$

$$\Rightarrow 2x + 3my - am^2(2 + 3m^2) = 0.$$

Ex 6.3 Class 12 Maths Question 21.

Find the equation of the normal's to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

Solution:

Let the required normal be drawn at the point (x_1, y_1)

The equation of the given curve is $y = x^3 + 2x + 6$ …(i)

Differentiating w.r.t. x $\frac{dy}{dx} = 3x^2 + 2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 3x_1^2 + 2$

Since the normal at (x_1, y_1) is parallel to the line $x + 14y + 4 = 0$

$\therefore$ Slope of the normal at (x_1, y_1) = Slope of the line $x + 14y + 4 = 0$

$\Rightarrow \frac{-1}{(dy/dx)_{(x_1, y_1)}} = \frac{-1}{14} \Rightarrow 3x_1^2 = 12$

$\Rightarrow x_1^2 = 4 \Rightarrow x_1 = \pm 2 \quad \because (x_1, y_1)$ lies in (i)

$\therefore \; y_1 = x_1^3 + 2x_1 + 6$

When $x_1 = 2$, $y_1 = 18$; When $x_1 = -2$, $y_1 = -8 - 4 + 6 = -6$

Thus the co-ordinates of the points are (2, 18) and (–2, –6)

The eq. of the normal at (2, 18) is : $x + 14y - 254 = 0$

The equation of the normal at (–2, –6) is

$y + 6 = \frac{-1}{14}(x + 2) \Rightarrow x + 14y + 86 = 0$

Ex 6.3 Class 12 Maths Question 22.

Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

Solution:

Here, $y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(at^2, 2at)} = \frac{1}{t}$

The eq. of the tangent at $(at^2, 2at)$ is

$y - 2at = \frac{1}{t}(x - at^2)$

$ty - 2at^2 = x - at^2 \Rightarrow ty = x + at^2$ and the equation of the normal at $(at^2, 2at)$ is $y = -tx + 2at + at^3$

Ex 6.3 Class 12 Maths Question 23.

Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

Solution:

The given curves are $x = y^2$ …(i)

and $xy = k$ …(ii)

$\Rightarrow y = k^{1/3}$ from (i) $x = y^2 = (k^{1/3})^2 = k^{2/3}$

Thus the point of intersection is $(k^{2/3}, k^{1/3})$

Differentiating (i) w.r.t. x :

$1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$ at $(k^{2/3}, k^{1/3})$,

$\frac{dy}{dx} = \frac{1}{2k^{1/3}} \Rightarrow$ Slope of tangent at $(k^{2/3}, k^{1/3})$

Differentiating $xy = k$ w.r.t. x

$x\frac{dy}{dx} + y \cdot 1 = 0 \Rightarrow x\frac{dy}{dx} = -y \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$,

At $(k^{2/3}, k^{1/3})$, $\frac{dy}{dx} = \frac{-k^{1/3}}{k^{2/3}} = \frac{-1}{k^{1/3}}$

$\Rightarrow$ Slope of tangent at $(k^{2/3}, k^{1/3}) = \frac{-1}{k^{1/3}}$

Now the curves cut at right angles if the product of slopes of tangents to two curves at $(k^{2/3}, k^{1/3})$ is -1.

i.e., if $\left(\frac{1}{2k^{1/3}}\right)\left(\frac{-1}{k^{1/3}}\right) = -1 \Rightarrow -1 = 2k^{2/3} \Rightarrow 8k^2 = 1$

Ex 6.3 Class 12 Maths Question 24.

Find the equations of the tangent and normal to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point (x_0 ,y_0).

Solution:

$$\therefore \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}; \left(\frac{dy}{dx}\right)_{(x_0, y_0)} = \frac{b^2 x_0}{a^2 y_0}$$

$$\text{slope of the tangent} = \frac{b^2 x_0}{a^2 y_0}$$

$$\text{Since } (x_0, y_0) \text{ lies on } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\therefore \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1 \qquad \text{...(i)}$$

Equation of the tangent is

$$\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} \Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$$

$$\text{Now, Slope of normal} = \frac{-1}{(dy/dx)_{(x_0, y_0)}} = \frac{-a^2 y_0}{b^2 x_0}$$

Equation of the normal is

$$y - y_0 = \frac{-a^2 y_0}{b^2 x_0}(x - x_0) \Rightarrow \frac{y - y_0}{a^2 y_0} + \frac{x - x_0}{b^2 x_0} = 0.$$

Ex 6.3 Class 12 Maths Question 25.

Find the equation of the tangent to the curve

$$y = \sqrt{3x - 2}$$

which is parallel to the line 4x – 2y + 5 = 0.

Solution:

Let the point of contact of the tangent line parallel to the given line be P (x_1, y_1) The equation of the curve is

$$y = \sqrt{3x - 2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}} \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{3}{2\sqrt{3x_1 - 2}}$$

Since the tangent at (x_1, y_1) is parallel to the line $4x - 2y + 5 = 0$

$$\therefore \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \text{slope of line } 4x - 2y + 5 = 0$$

$$\Rightarrow 4\sqrt{3x_1 - 2} = 3 \Rightarrow x_1 = \frac{41}{48}$$

Since (x_1, y_1) lies on $y = \sqrt{3x - 2}$ $\therefore y_1 = \frac{3}{4}$

So, the point of contact is $\left(\frac{41}{48}, \frac{3}{4}\right)$ Hence, the required equation of the tangent is $48x - 24y = 23$.

Choose the correct answer in Exercises 26 and 27.

Ex 6.3 Class 12 Maths Question 26.

The slope of the normal to the curve y = 2x² + 3 sin x at x = 0 is

(a) 3

(b)

$\frac{1}{3}$

(c) -3

(d)

$-\frac{1}{3}$

Solution:

(d) ∵ y = 2x² + 3sinx

∴

$\frac{dy}{dx} = 4x + 3cosx$

at

x = 0,

$\frac{dy}{dx} = 3$

∴ slope = 3

⇒ slope of normal is =

$\frac{1}{3}$

Ex 6.3 Class 12 Maths Question 27.

The line y = x + 1 is a tangent to the curve $y^2 = 4x$ at the point

(a) (1,2)

(b) (2,1)

(c) (1,-2)

(d) (-1,2)

Solution:

(a) The curve is $y^2 = 4x$,

∴

$$\frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$$

Slope of the given line y = x + 1 is 1 ∴

$$\frac{2}{y} = 1$$

y = 2 Putting y= 2 in $y^2 = 4x$ $2^2 = 4x$

⇒ x = 1

∴ Point of contact is (1,2)

Ex 6.4 Class 12 Maths Question 1.

Using differentials, find the approximate value of each of the following up to 3 places of decimal.

(i)

$$\sqrt{25.3}$$

(ii)

$\sqrt{49.5}$

(iii)

$\sqrt{0.6}$

(iv)

$(0.009)^{\frac{1}{3}}$

(v)

$(0.999)^{\frac{1}{10}}$

(vi)

$(15)^{\frac{1}{4}}$

(vii)

$(26)^{\frac{1}{3}}$

(viii)

$(255)^{\frac{1}{4}}$

(ix)

$(82)^{\frac{1}{4}}$

(x)

$(401)^{\frac{1}{2}}$

(xi)

$(0.0037)^{\frac{1}{2}}$

(xii)

$(26.57)^{\frac{1}{3}}$

(xiii)

$(81.5)^{\frac{1}{4}}$

(xiv)

$(3.968)^{\frac{3}{2}}$

(xv)

$(32.15)^{\frac{1}{5}}$

Solution:

(i) y + Δy =

$\sqrt{25.3}$

=

$\sqrt{25 + 0.3}$

=

$\sqrt{x + \Delta x}$

∴ x = 25

Δx = 0.3

⇒ y = √x

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2 \times 5} = \frac{1}{10}$$

$$dy = \left(\frac{dy}{dx}\right) dx = \frac{1}{10} \times 0.3, = \frac{3}{100} = \Delta x$$

Hence $\sqrt{25.3} = y + \Delta y = 5 + 0.03 = 5.03.$

(ii) Let $y + \Delta y = \sqrt{49.5} = \sqrt{49 + 0.5} = \sqrt{x + \Delta x}$

$\therefore \quad x = 49, \Delta x = 0.5$

Now $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{14} = \frac{1}{2 \times 7}$

$$dy = \left(\frac{dy}{dx}\right) \Delta x = \frac{1}{14} \times 0.5 = \frac{1}{28}$$

$$\sqrt{49.5} = y + \Delta y = 7 + \frac{1}{28} = 7 + 0.035 = 7.035.$$

(iii) Let $y=\sqrt{x}$ Then $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$

$$\Rightarrow \frac{1}{2\sqrt{x}}.\delta x=\sqrt{x+\delta x}-\sqrt{x}$$

Putting $x=1$ and $\delta x=-0.4$, we get

$$\frac{1}{2\sqrt{1}}\times(-0.4)=\sqrt{0.6}-\sqrt{1}\Rightarrow\sqrt{0.6}=\frac{1.6}{2}=0.8$$

(iv) We know that $(0.008)^{1/3}=0.2$; Let $y=x^{1/3}$

$$\frac{dy}{dx}=\frac{1}{3x^{2/3}}\Rightarrow\frac{1}{3x^{2/3}}.\delta x=(x+\delta x)^{1/3}-x^{1/3}$$

Putting $x=0.008$ and $\delta x=0.001$

$$\Rightarrow (0.009)^{1/3}=(0.008)^{1/3}+\frac{0.01}{3\times(0.008)^{2/3}}$$

$=0.2+0.00833=0.20833.$

(v) We know that $(1)^{1/10}=1$ Let $y=x^{1/10}$

$$\frac{dy}{dx}=\frac{1}{10x^{9/10}}$$

$$\Rightarrow \frac{1}{10x^{9/10}}.\delta x=(x+\delta x)^{1/10}-x^{1/10}$$

Putting $x=1$ and $\delta x=-0.001$

$$\Rightarrow \frac{1}{10}(-0.001)=(0.999)^{1/10}-1$$

$$\Rightarrow \quad (0.999)^{1/10} = 1 - \frac{0.001}{10} = \frac{9.999}{10}$$

$= \mathbf{0.9999}$

(vi) We know that $(16)^{1/4} = 2$ Let $y = x^{1/4}$

$$\frac{dy}{dx} = \frac{1}{4x^{3/4}} \Rightarrow \left(\frac{dy}{dx}\right) . \delta x = (x + \delta x)^{1/4} - x^{1/4}$$

Putting $x = 16$ and $\delta x = -1$

$$\Rightarrow \quad (15)^{1/4} = \frac{64 - 1}{32} = \frac{63}{32} = 1.96875$$

(vii) We know that $(27)^{1/3} = 3$; Let $y = x^{1/3}$

$$\frac{dy}{dx} = \frac{1}{3x^{2/3}} \Rightarrow \left(\frac{dy}{dx}\right) . \delta x = (x + \delta x)^{1/3} - x^{1/3}$$

Putting $x = 27$ and $\delta x = -1$

$$\Rightarrow \quad (26)^{1/3} = 3 - \frac{1}{27} = \frac{81 - 1}{27} = \frac{80}{27} = 2.9629$$

(viii) We know that $(256)^{1/4} = 4$ Let $y = x^{1/4}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{4x^{3/4}} ; \Rightarrow \left(\frac{dy}{dx}\right) . \delta x = (x + \delta x)^{1/4} - x^{1/4}$$

Putting $x = 256$ and $\delta x = -1$

$$\Rightarrow \quad \frac{1}{4(256)^{3/4}} . (-1) = (255)^{1/4} - (256)^{1/4}$$

$$\Rightarrow \quad (255)^{1/4} = \frac{1024 - 1}{256} = \frac{1023}{256} = 3.9961$$

(ix) We know that $(81)^{1/4} = 3$ Let $y = x^{1/4}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{4x^{3/4}} \Rightarrow \left(\frac{dy}{dx}\right).\delta x = (x+\delta x)^{1/4} - x^{1/4}$$

Putting $x = 81$ and $\delta x = 1$

$$\frac{1}{4(81)^{3/4}}.(1) = (82)^{1/4} - (81)^{1/4} \Rightarrow (82)^{1/4} = 3.009$$

(x) We know that $(400)^{1/2} = 20$, Let $y = x^{1/2}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2x^{1/2}} \Rightarrow \left(\frac{dy}{dx}\right).\delta x = (x+\delta x)^{1/2} - x^{1/2}$$

Putting $x = 400$ and $\delta x = 1$

$$\frac{1}{2(400)^{1/2}}.(1) = (401)^{1/2} - 20 \Rightarrow (401)^{1/2} = 20.025$$

(xi) We know that $(0.0036)^{1/2} = 0.06$: Let $y = x^{1/2}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2x^{1/2}} \Rightarrow \left(\frac{dy}{dx}\right).\delta x = (x+\delta x)^{1/2} - x^{1/2}$$

Putting $x = 0.0036$ and $\delta x = 0.0001$

$$\frac{1}{2(0.0036)^{1/2}}(0.0001) = (0.0037)^{1/2} - (0.0036)^{1/2}$$

$$\Rightarrow \frac{0.0073}{0.12} = 0.06083$$

(xii) We know that $(27)^{1/3} = 3$; Let $y = x^{1/3}$

Then $\frac{dy}{dx} = \frac{1}{3x^{2/3}}$ Putting $x = 27$ and $\delta x = -0.43$

$$\Rightarrow \quad \frac{1}{3(27)^{2/3}} \cdot (-0.43) = (26.57)^{1/3} - 3$$

$$\Rightarrow \quad (26.57)^{1/3} = 3 - \frac{0.43}{27} = \frac{80.57}{27} = \mathbf{2.984}$$

(xiii) We know that $(81)^{1/4} = 3$; Let $y = x^{1/4}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{4x^{3/4}} \text{ Putting } x = 81 \text{ and } \delta x = 0.5$$

$$\Rightarrow \quad \frac{1}{4(81)^{3/4}} \cdot (0.5) = (81.05)^{1/4} - (81)^{1/4}$$

$$\Rightarrow \quad (81.05)^{1/4} = \frac{0.5 + 324}{108} = \frac{324.5}{108} = 3.004$$

(xiv) We know that $(4)^{3/2} = 8$; Let $y = x^{3/2}$

Then $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$ Putting $x = 4$ and $\delta x = -0.032$

$$\Rightarrow \quad \frac{3}{2}(4)^{1/2} \cdot (-0.0032) = (3.968)^{3/2} - (4)^{3/2}$$

$$\Rightarrow \quad (3.968)^{3/2} = 8 - 0.096 = 7.904$$

(xv) We know that $(32)^{1/5} = 2$; Let $y = x^{1/5}$

Then $\left(\frac{dy}{dx}\right) = \frac{1}{5x^{4/5}}$; Putting $x = 32$ and $\delta x = 0.15$

$$\frac{1}{5(32)^{4/5}} \cdot (0.15) = (32.15)^{1/5} - (32)^{1/5}$$

$$\Rightarrow \quad (32.15)^{1/5} = \frac{160.15}{80} = 2.00187$$

Ex 6.4 Class 12 Maths Question 2.

Find the approximate value of f (2.01), where f (x) = $4x^2 + 5x + 2$

Solution:

$f(x+\Delta x) = f(2.01)$, $f(x) = f(2) = 4.2^2 + 5.2 + 2 = 28$,

$f'(x) = 8x + 5$ Now, $f(x + \Delta x) = f(x) + \Delta f(x)$

$= f(x) + f'(x) \bullet \Delta x = 28 + (8x + 5)\Delta x$

= 28 + (16 + 5) x 0.01

= 28 + 21 x 0.01

= 28 + 0.21

Hence,f(2 x 01)

= 28 x 21.

Ex 6.4 Class 12 Maths Question 3.

Find the approximate value of f (5.001), where $f(x) = x^3 - 7x^2 + 15$.

Solution:

Let $x + \Delta x = 5.001$, $x = 5$ and $\Delta x = 0.001$,

$f(x) = f(5) = -35$

$f(x + \Delta x) = f(x) + \Delta f(x) = f(x) + f'(x).\Delta x$

$= (x^3 - 7x^2 + 15) + (3x^2 - 14x) \times \Delta x$

$f(5.001) = -35 + (3 \times 5^2 - 14 \times 5) \times 0.001$

$\Rightarrow f(5.001) = -35 + 0.005$

$= -34.995$.

Ex 6.4 Class 12 Maths Question 4.

Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

Solution:

The side of the cube = x meters.

Increase in side = 1% = 0.01 × x = 0.01 x

Volume of cube V= x^3

∴ ∆v =

$\frac{dv}{dx}$

× ∆x

= $3x^2 \times 0.01x$

= $0.03x^3 m^3$

Ex 6.4 Class 12 Maths Question 5.

Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

Solution:

The side of the cube = x m;

Decrease in side = 1% = 0.01 x

Increase in side = ∆x = – 0.01 x

Surface area of cube = $6x^2 m^2$ = S

∴

$\frac{ds}{dx}$

× ∆x = 12x × (- 0.01 x)

$= - 0.12\ x^2\ m^2$.

Ex 6.4 Class 12 Maths Question 6.

If the radius of a sphere is measured as 7m with an error of 0.02 m, then find the approximate error in calculating its volume.

Solution:

Radius of the sphere = 7m : $\Delta r = 0.02$ m.

Volume of the sphere V =

$$\frac{4}{3}\pi r^3$$

$$\Delta V = \frac{dV}{dr} \times \Delta r = \frac{4}{3}.\pi.3r^2 \times \Delta r$$

$= 4\pi \times 7^2 \times 0.02$

$= 3.92\ \pi m^3$

Ex 6.4 Class 12 Maths Question 7.

If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

Solution:

Radius of the sphere = 9 m: $\Delta r = 0.03$m

Surface area of sphere $S = 4\pi r^2$

Δs =

$$\frac{ds}{dr}$$

$\times \Delta r$

$= 8\pi r \times \Delta r$

$= 8\pi \times 9 \times 0.03$

$= 2.16\ \pi m^2$.

Ex 6.4 Class 12 Maths Question 8.

If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is

(a) 47.66

(b) 57.66

(c) 67.66

(d) 77.66

Solution:

(d) $x + \Delta x = 3.02$, where $x=30$, $\Delta x=.02$,

$\Delta f(x) = f(x + \Delta x) – f(x)$

$\Rightarrow f(x + \Delta x) = f(x) + \Delta f(x) = f(x) + f'(x)\Delta x$

Now $f(x) = 3\times2 + 15x + 5$; $f(3) = 77$, $f'(x) = 6x + 15$

$f'(3) = 33$

$\therefore f(3.02) = 87 + 33 \text{ x } 0\ 02 = 77.66$

Ex 6.4 Class 12 Maths Question 9.

The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

(a) $0.06\ x^3\ m^3$

(b) $0.6\ x^3\ m^3$

(c) $0.09\ x^3\ m^3$

(d) $0.9\ x^3\ m^3$

Solution:

(c) Side of a cube = x meters

Volume of cube = x^3,

for Δx. ⇒ 3% of x = 0.03 x

Let Δv be the change in v0l. $\Delta v = \frac{dv}{dx} \times \Delta x = 3x^2 \times \Delta x$

But, $\Delta x = 0.03\ x$

⇒ $\Delta v = 3x^2 \times 0.03\ x$

$= 0.09\ x^3 m^3$

Ex 6.5 Class 12 Maths Question 2.

Find the maximum and minimum values, if any, of the following functions given by

(i) $f(x) = |x + 2| - 1$

(ii) $g(x) = -|x + 1| + 3$

(iii) $h(x) = \sin 2x + 5$

(iv) $f(x) = |\sin(4x + 3)|$

(v) $h(x) = x + 1, x \in (-1,1)$

Solution:

(i) We have :$f(x) = |x + 2| - 1\ \forall x \in R$

Now $|x + 2| \geq 0\ \forall x \in R$

$|x + 2| - 1 \geq -1\ \forall x \in R$, So -1 is the min. value of f(x)

now f(x) = -1

⇒ |x + 2|-1

⇒ |x + 2| = 0

⇒ x = – 2

(ii) We have g(x) = -|x + 1| + 3 ∀x∈R

Now | x + 1| ≥ 0 ∀x∈R

-|x+ 1| + 3 ≤3 ∀x∈R

So 3 is the minimum value of f(x).

Now f(x) = 3

⇒ -|x+1| + 3

⇒ |x+1| = 0

⇒ x = – 1.

(iii) Thus maximum value of f(x) is 6 and minimum value is 4.

(iv) Let f(x) = |sin4x + 3|

Maximum value of sin 4x is 1

∴ Maximum value of |sin(4x+3)| is |1+3| = 4

Minimum value of sin 4x is -1

∴ Minimum value of f(x) is |-1+3| = |2|= 2

(v) Greatest value of f (x) is 2 and least value is 0.

Ex 6.5 Class 12 Maths Question 3.

Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:

(i) f(x) = x^2

(ii) g(x) = x^3 – 3x

(iii) h(x) = sinx+cosx,0<x<

$\frac{\pi}{4}$

(iv) f(x) = $\sin^4 x$ + $\cos^4 x$,0<x<

$\frac{\pi}{2}$

(v) f(x) = x^3 – 6x^2 + 9x:+15

(vi) g(x) =

$\frac{x}{2}+\frac{2}{x}$

, x>0

(vii) g(x) =

$\frac{1}{x^2+2}$

, x>0

(viii) f(x) =

$x\sqrt{1-x}$

, x>0

Solution:

(i) Let f(x) = x^2 ⇒ f'(x) = 2x

Now f'(x) = 0 ⇒ 2x = 0 i.e., x = 0

At x = 0; When x is slightly < 0, f' (x) is -ve When x is slightly > 0, f(x) is +ve

∴ f(x) changes sign from -ve to +ve as x increases through 0.

⇒ f' (x) has a local minimum at x = 0 local minimum value f(0) = 0.

(ii) Let $g(x) = x^3 - 3x \Rightarrow g'(x) = 3(x-1)(x+1)$
Now $f'(x) = 0 \Rightarrow$ either $x = 1$ or $x = -1$
At $x = 1$ When x is slightly < 1 $g'(x)$ is $(+)(-1)(+)$ *i.e.*, –ve.
When x is slightly > 1 $g'(x)$ is $(+)(+)(+)$ *i.e.*, +ve.
Thus $f'(x)$ changes sign from negative to positive as x increases through 1 and hence $x = 1$ is a point of local minimum. At $x = -1$
When x is slightly < -1 $g'(x)$ is $(+)(-)(-)$ *i.e.*, +ve.
When x is slightly > -1 $g'(x)$ is $(+)(-)(+)$ *i.e.*, –ve.
Thus $g'(x)$ changes sign from positive to negative as x increases through -1 and hence $x = -1$ is a point of local minimum.
Hence local min. value is $g(x) = g(1) = 1 - 3 = -2$
and local max. value is $g(x) = g(-1) = -1 + 3 = 2$.

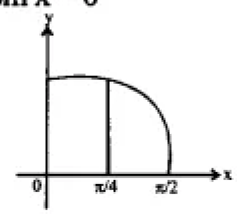
(iii) $h(x) = \sin x + \cos x$ $h'(x) = \cos x - \sin x$

$h'(x) = \cos x(1 - \tan x)$

$h'(x) = 0 \quad \Rightarrow \quad \cos x - \sin x = 0$

or $\tan x = 1,$

$\Rightarrow x = \frac{\pi}{4} \in \left(0, \frac{\pi}{4}\right)$



At $\alpha = \frac{\pi}{4}$,

$h'(x) = \cos x(1 - \tan x)$

(iv) $f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x$

$f'(x) = -4\sin x\cos x(\cos^2 x - \sin^2 x)$

$f'(x) = -2\sin 2x\cos 2x = -\sin 4x$

For local max. or local minimum, we have

$f'(x) = 0 \Rightarrow -\sin 4x = 0 \Rightarrow 4x = \pi$

$\Rightarrow x = \frac{\pi}{4}$: Now $f''(x) = -4\cos x$

$f''\left(\frac{\pi}{4}\right) = -4\cos\pi = (-4)(-1) = 4 > 0$

So $x = \frac{\pi}{4}$ is a point of local minimum and

local min. value is : $f\left(\frac{\pi}{4}\right) = \sin^4 x + \cos^4 x$

$= \sin^4\left(\frac{\pi}{4}\right) + \cos^4\left(\frac{\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{2}.$

(v) Let $f(x) = x^3 - 6x^2 + 9x - 8$

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

Now, $f'(x) = x = 1, 3$

At $x = 1$:

When x is slightly < 1, $f'(x) = (-)(-) = +ve$

When x is slightly > 1, $f'(x) = (+)(-) = -ve$

$\therefore$ $f'(x)$ changes sign from +ve to –ve as x increases through 1.

$\Rightarrow$ $f(x)$ has a local maximum at $x = 1$.

Local max. value $= f(1) = 1 - 6 + 9 + 15 = 19$

At $x = 3$:
When x is slightly < 3, $f'(x) = (+)(-) = -ve$
When x is slightly > 3, $f'(x) = (+)(+) = +ve$
$\therefore$ $f'(x)$ changes sign from $-ve$ to $+ve$ as x increases through 3.
$\Rightarrow$ $f(x)$ has a local minimum at $x = 3$.
Local min. value $= f(3) = (3)^3 - 6(3)^2 + 9(3) + 15$
$= 27 - 54 + 27 + 15 = 15$
Note : How to determine the change of sign of $f(x)$ as x increases through a particular point? Let $f'(x) = (x-a)(x-b)(x-c)$
At $x = a$, $x - a = a - a = 0$, $x - b = a - b$, $x - c = a - c$
Sign corresponding to $x - a$ changes from $-ve$ to $+ve$, sign corresponding to $x - c$ is that of $a - c$ and remains to be the same.
Let $f'(x) = (x-1)(x-3)$
At $x = 1$, $x - 1 = 1 - 1 = 0$; $x - 3 = 1 - 3 = -2 = -ve$
Factor $(x-1)(x-3)$
Signs when x is slightly < 1, $(-)(-) = +ve$
Signs when x is slightly > 1 $(+)(-) = -ve$
$\therefore$ $f'(x)$ changes from $+ve$ to $-ve$
At $x = 3$, $x - 1 = 3 - 1 = 2 = +ve$;
$x - 3 = 3 - 3 = 0$
Factor $(x-1)(x-3)$
Sign when x is slightly < 3, $(+)(-) = -ve$
Sign when x is slightly > 3, $(+)(+) = +ve$
$\therefore$ $f'(x)$ changes sign from $-ve$ to $+ve$

(vi) Let $g(x)=\frac{x}{2}+\frac{2}{x}, x>0;\quad g'(x)=\frac{x^2-4}{2x^2}$

Now $g'(x)=0 \Rightarrow x=\pm 2$ Since it is given that $x>0$ hence $x=-2$ is rejected.

At $x=2$: For x slightly <2, $g'(x)=\frac{-}{+}=-ve$

For x slightly >2, $g'(x)=\frac{+}{+}=+ve$

Thus $g'(x)$ changes sign from $-ve$ to $+ve$ as x increases through 2.

Hence $f(x)$ has a local minimum at $x=2$.

Hence local minimum value $=g(x)=2$

(vii) Let $g(x)=\frac{1}{x^2+2}\ \therefore\ g'(x)=-\frac{2x}{(x^2+2)^2}$

Now $g'(x)=0 \Rightarrow x=0$

At $x=0$

When x is slightly <0, $g'(x)=\frac{(-)(-)}{+}=+ve$

When x is slightly >0, $g'(x)=\frac{(-)(+)}{+}=-ve$

$\therefore$ $g'(x)$ changes sign from $+ve$ to $-ve$ as x increases through 0

$\therefore$ $f(x)$ has a local maximum at $x=0$.

$\therefore$ Local maximum value $f(0)=\frac{1}{2}$.

(viii) Let $f(x) = x\sqrt{1-x}$, $x > 0$

$$f'(x) = 1 = \frac{-3(x-2/3)}{2\sqrt{1-x}}$$

Now, $f'(x) = 0 \Rightarrow 2 - 3x = 0 \Rightarrow x = 2/3$

At $x = \frac{2}{3}, 1 - x = 1 - \frac{2}{3} = \frac{1}{3} = +\text{ve}$

When x is slightly $< 2/3 : f'(x) = \frac{(-)(-)}{(+)} = +\text{ve}$

When x is slightly $> \frac{2}{3}$; $f'(x) = \frac{(-)(+)}{(+)} = -\text{ve}$

$\therefore$ $f'(x)$ changes sign from (+ve) to (–)ve

as x increases through $x = \frac{2}{3}$

$\therefore$ $f(x)$ has a local maxima at $x = \frac{2}{3}$ local

maximum value $= f\left(\frac{2}{3}\right) = \frac{2\sqrt{3}}{9}$

Ex 6.5 Class 12 Maths Question 4.

Prove that the following functions do not have maxima or minima:

(i) $f(x) = e^x$

(ii) $f(x) = \log x$

(iii) $h(x) = x^3 + x^2 + x + 1$

Solution:

(i) $f'(x) = e^x$;

Since $f'(x) \neq 0$ for any value of x.

So $f(x) = e^x$ does not have a max. or min.

(ii) f’ (x) = $\frac{1}{x}$; Clearly f’ (x) ≠ 0 for any value of x.

So,f’ (x) = log x does not have a maximum or a minimum.

(iii) We have f(x) = $x^3 + x^2 + x + 1$

⇒f’ (x) = $3x^2 + 2x + 1$

Now, f’ (x) = 0 => $3x^2 + 2x + 1 = 0$

$$x = \frac{-2\pm\sqrt{4-12}}{6} = \frac{-1+\sqrt{-2}}{3}$$

i.e. f'(x) = 0 at imaginary points

i.e. f'(x) ≠ 0 for any real value of x

Hence, there is neither max. nor min.

Ex 6.5 Class 12 Maths Question 5.

Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:

(i) f(x) = x^3, x∈ [-2,2]

(ii) f(x) = sin x + cos x, x ∈ [0, π]

(iii) f(x) =

$$4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$$

(iv) f(x) =

$$(x-1)^2 + 3, x \in [-3, 1]$$

Solution:

(i) We have f’ (x) = x^3 in [-2,2]

∴ f'(x) = $3x^2$; Now, f’ (x) = 0 at x = 0, f(0) = 0

Now, f(-2) = $(-2)^3$ = – 8; f(0) = $(0)^2$ = 0 and f(0) = (2) = 8

Hence, the absolute maximum value of f (x) is 8 which it attained at x = 2 and absolute minimum value of f(x) = – 8 which is attained at x = -2.

(ii) We have f (x) = sin x + cos x in [0, π]

f’ (x) = cos x – sin x for extreme values f’ (x) = 0

$\Rightarrow \cos x - \sin x = 0 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

Now, we find f(x) at $x = 0, \frac{\pi}{4}, \pi$;

$f(0) = 1; f(\pi/4) = \sqrt{2}$

and $f(\pi) = \sin \pi + \cos \pi = 0 - 1 = -1$

$\therefore$ Absolute maximum value $= \sqrt{2}$

at $x = \pi/4$ and Absolute min. value $= -1$ at $x = \pi$.

(iii) We have $f(x) = 4x - \frac{x^2}{2}$ in $\left[-2, \frac{9}{2}\right]$

$\therefore f'(x) = 4 - \frac{1}{2} \cdot 2x = 4 - x$

For extreme values, $f'(x) = 0 \Rightarrow x = 4$

Now we find the values of f(x) at $x = -2, 4, \frac{9}{2}$

$f(-2) = -10, f(4) = 8, f\left(\frac{9}{2}\right) = 7.875$

At $x = 4$, absolute maximum value $= 8$

At $x = 2$, absolute minimum value $= -10$.

(iv) Let $f(x) = (x-1)^2 + 3 \Rightarrow f'(x) = 2(x-1)$

For external values $f(x) = 0$

$2(x-1) = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$

Now find $f(x)$ at $x = 1, -1, 0$

$f(1) = 3 = f(-3) = 19$ and $f(0) = 4$

$\therefore$ Absolute max. value 19 at $x = -3$ and absolute minimum value 3 at $x = 1$.

Ex 6.5 Class 12 Maths Question 6.

Find the maximum profit that a company can make, if the profit function is given by p(x) = 41 – 24x – 18x²

Solution:

Profit function in p(x) = 41 – 24x – 18x²

∴ p'(x) = – 24 – 36x = – 12(2 + 3x)

for maxima and minima, p'(x) = 0

Now, p'(x) = 0

⇒ – 12(2 + 3x) = 0

⇒ x =

$-\frac{2}{3}$

,

p'(x) changes sign from +ve to -ve.

⇒ p (x) has maximum value at x =

$-\frac{2}{3}$

Maximum Profit = 41 + 16 – 8 = 49.

Ex 6.5 Class 12 Maths Question 7.

Find both the maximum value and the minimum value of $3x^4 – 8x^3 + 12x^2 – 48x + 25$ on the interval [0,3].

Solution:

Let $f(x) = 3x^4 – 8x^3 + 12x^2 – 48x + 25$

$\therefore f'(x) = 12x^3 – 24x^2 + 24x – 48$

$= 12(x^2 + 2)(x – 2)$

For maxima and minima, f'(x) = 0

⇒ $12(x^2 + 2)(x – 2) = 0$

⇒ x = 2

Now, we find f (x) at x = 0,2 and 3, f (0) = 25,

$f(2) = 3(2^4) - 8(2^3) + 12(2^2) - 48(2) + 25 = -39$

and $f(3) = (3^4) - 8(3^3) + 12(3^2) - 48(3) + 25$

$= 243 - 216 + 108 - 144 + 25 = 16$

Hence at $x = 0$, Maximum value = 25

at $x = 2$, Minimum value = – 39.

Ex 6.5 Class 12 Maths Question 8.

At what points in the interval $[0, 2\pi]$, does the function sin 2x attain its maximum value?

Solution:

We have $f(x) = \sin 2x$ in $[0, 2\pi]$, $f'(x) = 2\cos 2x$

For maxima and minima $f'(x) = 0 \Rightarrow \cos 2x = 0$

$$\Rightarrow \quad 2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Now, we find f(x) at x

$$= 0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi, \quad f(0) = 0,$$

$f(\pi/4) = \sin \pi/2 = 1$ $f(3\pi/4) = \sin 3\pi/2$

$= -1$, $f(5\pi/4)$

$= \sin 5\pi/2 = 1$, $f(7\pi/4) = \sin 7\pi/2$

$= -1$ and $f(2\pi) = \sin 2\pi = 0$

Hence maxima value of $f(x) = 1$ at $x = \pi/4, 5\pi/4$

Ex 6.5 Class 12 Maths Question 9.

What is the maximum value of the function sin x + cos x?

Solution:

Consider the interval $[0, 2\pi]$,

Let $f(x) = \sin x + \cos x$,

f’ (x) = cosx – sinx

For maxima and minima, f’ (x) = 0

$$\Rightarrow \cos x - \sin x = 0 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$$

Now, we find $f(x)$ at $x = 0, \frac{\pi}{4}, \frac{5\pi}{4}, 2\pi$

$f(0) = 1;\ f(\pi/4) = \sqrt{2},$

$f(5\pi/4) = = -\sqrt{2}$

$f(2\pi) = 1$ Hence, max. value of $f(x) = \sqrt{2}$

Ex 6.5 Class 12 Maths Question 10.

Find the maximum value of $2x^3 - 24x + 107$ in the interval [1,3]. Find the maximum value of the same function in [-3, -1].

Solution:

∵ $f(x) = 2x^3 - 24x + 107$

∴$f(x) = 6x^2 - 24$,

For maxima and minima f'(x) = 0;⇒ x = ±2

For the interval [1,3], we find the values of f (x)

at x = 1,2,3; f(1) = 85, f(2) = 75, f(3) = 89

Hence, maximum f (x) = 89 at x = 3

For the interval [-3, -1], we find the values of f(x) at x = – 3, – 2, – 1;

f(-3) = 125;

f(-2) = 139

f(-1) = 129

∴ max.f(x) = 139 at x = – 2.

Ex 6.5 Class 12 Maths Question 11.

It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0,2]. Find the value of a.

Solution:

∵ $f(x) = x^4 - 62x^2 + ax + 9$

∴ $f'(x) = 4x^3 - 124x + a$

Now f’ (x) = 0 at x = 1

⇒ 4 – 124 + a = 0

⇒ a = 120

Now $f''(x) = 12x^2 - 124$:

At x = 1 f” (1) = 12 – 124 = – 112 < 0

⇒ f(x) has a maximum at x = 1 when a = 120.

Ex 6.5 Class 12 Maths Question 12.

Find the maximum and minimum values of x + sin 2x on [0,2π]

Solution:

∴f(x) = x + sin2x on[0,2π]

∴f’ (x) = 1+2 cos2x

For maxima and minima f’ (x) = 0

$$\Rightarrow \cos 2x = -\frac{1}{2} \Rightarrow 2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

$$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

Now, $f(x)$ at $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$

$f(0) = 0 + \sin 0 = 0,\ f(\pi/3) = \pi/3 + \sqrt{3}/2.$

$f(2\pi/3) = 2\pi/3 - \sqrt{3}/2,\ f(4\pi/3) = 4\pi/3 + \sqrt{3}/2$

$f(5\pi/3) = 5\pi/3 - \sqrt{3}/2$ and $f(2\pi) = 2\pi$

Hence, maximum $f(x) = 2\pi$ and minimum $f(x) = 0$.

Ex 6.5 Class 12 Maths Question 13.

Find two numbers whose sum is 24 and whose product is as large as possible.

Solution:

Let the required numbers hex and (24-x)

∴Their product,p = $x(24 – x) = 24x – x^2$

Now

$$\frac{dp}{dx}$$

= 0 ⇒24 – 2x = 0 ⇒ x = 12

Also

$$\frac{d^2p}{dx^2}$$

= -2<0: ⇒ p is max at x = 12

Hence, the required numbers are 12 and (24-12)i.e. 12.

Ex 6.5 Class 12 Maths Question 14.

Find two positive numbers x and y such that x + y = 60 and xy^3 is maximum.

Solution:

We have x + y = 60

⇒ y = 60 – x …(i)

Let $p = xy^3 = x(60-x)^3$; $\frac{dp}{dx} = (60-x)^2(60-4x)$

$\therefore \frac{dp}{dx} = 0 \Rightarrow (60-x)^2(60-4x) = 0 \Rightarrow x = 60$ or 15

The value x = 60 is rejected as it makes y = 0

At x = 15; When x is slightly < 15, $\frac{dp}{dx} = (+)(+) = +ve$,

When x is slightly > 15 $\frac{dp}{dx} = (+)(-) = -ve$

$\Rightarrow \frac{dp}{dx}$ changes sign from (+) ve to (–) ve as x increases through 15.

$\therefore$ p is maximum at x = 15

Hence, the req. numbers are 15 and (60 -15) i.e. 15 and 45.

Ex 6.5 Class 12 Maths Question 15.

Find two positive numbers x and y such that their sum is 35 and the product $x^2 y^5$ is a maximum.

Solution:

We have x + y = 35 ⇒ y = 35 – x

Product p = $x^2 y^5$

= $x^2 (35 – x)^5$

$$\therefore \quad \frac{dp}{dx} = x(35-x)^4[-5x+2(35-x)]$$
$$= x(35-x)^4(70-7x)$$

Now, $\frac{dp}{dx} = 0 \Rightarrow x = 0, 35, 10$

Only admissible value is $x = 10$, as $x = 0$ and 35 are rejected At $x = 10$

When x is slightly < 10, $\frac{dp}{dx} = (+)(+)(+) = (+)$ ve

When x is slightly > 10, $\frac{dp}{dx} = (+)(+)(-) = (-)$ ve

$\Rightarrow \frac{dp}{dx}$ changes sign from $(+)$ ve to $(-)$ ve as x increases through 10 $\Rightarrow$ p is maximum at $x = 10$, $y = 35 - 10 = 25$

Hence, the required numbers are 10 and 25.

Ex 6.5 Class 12 Maths Question 16.

Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

Solution:

Let two numbers be x and 16 – x

Let $s = x^3 + (16-x)^3 \quad \therefore \quad \frac{ds}{dx} = 3(32x - 256)$

Now $\frac{ds}{dx} = 0 \Rightarrow 3(32x - 256) = 0 \Rightarrow x = \frac{256}{32} = 8$

Also $\frac{d^2s}{dx^2} = 96 > 0 \Rightarrow$ s is minimum at 8

Hence, the required numbers are 8 and (16-8) i.e. 8 and 8.

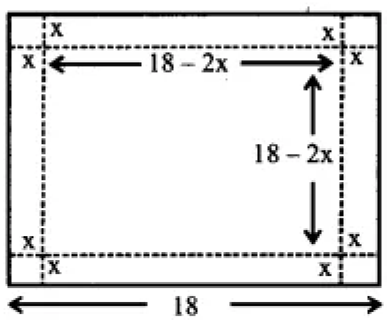
Ex 6.5 Class 12 Maths Question 17.

A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

Solution:

Let each side of the square to be cut off be x cm.

∴ for the box length = 18 – 2x: breadth = 18 – 2x and height = x



$\therefore$ Volume $= x(18-2x)^2$; $\frac{dv}{dx} = (18-2x)(18-6x)$

For maxima and minima,

$\frac{dv}{dx} = 0 \Rightarrow (18-2x)(18-6x) = 0 \Rightarrow x = 3, 9$

But $x = 9$ cm is not possible

Also $\frac{d^2v}{dx^2} = (18-2x)(-6) + (18-6x)(-2)$

At $x=3$, $\frac{d^2v}{dx^2} = (18-6)(-6) + (18-18)(-2) = -72 < 0$

$\therefore$ $x = 3$ attains for max^m volume

i.e. Square of side = 3 cm is cut from each corner.

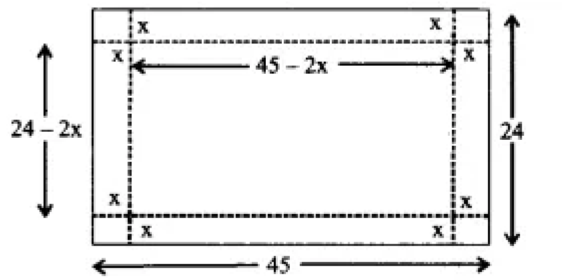
Ex 6.5 Class 12 Maths Question 18.

A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each comer and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum?

Solution:

Let each side of the square cut off from each comer be x cm.

∴ Sides of the rectangular box are (45 – 2x), (24 – 2x) and x cm.



Then, volume of the box

$V = (45-2x)(24-2x)(x) = 2(2x^3 - 69x^2 + 540x)$

$\Rightarrow \quad \frac{dV}{dx} = 12(x^2 - 23x + 90)$

For maxima and minima $\frac{dV}{dx} = 0 \Rightarrow x = 5, 18$

But x cannot be greater than 12

$\therefore \; x = 5, \; \frac{d^2V}{dx^2} = 12(10-23) = -ve \quad [\because x = 5]$

∴ V is maximum at x = 5 i.e. square of side 5 cm is cut off from each corner.

Ex 6.5 Class 12 Maths Question 19.

Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Solution:

Let the length and breadth of the rectangle inscribed in a circle of radius a be x and y respectively.

∴ $x^2 + y^2 = (2a)^2 \Rightarrow x^2 + y^2 = 4a^2$ …(i)

∴ Perimeter = 2 (x + y)

$$\Rightarrow P(x) = 2\left[x + \sqrt{4a^2 - x^2}\right]$$

$$\therefore P'(x) = 2\left[1 - \frac{x}{\sqrt{4a^2 - x^2}}\right] \quad \text{..(ii)}$$

and $P''(x) = \dfrac{-8a^2}{(4a^2 - x^2)^{3/2}}$ ...(iii)

For P (x) to be minimum $P'(x) = 0$ and $P''(x) < 0$

$\therefore$ from (i), $P'(x) = 0 \Rightarrow 4a^2 - x^2 = x^2 \Rightarrow x = a\sqrt{2}$

from (iii) $P''(x) = \dfrac{-8a^2}{(2a^2)^{3/2}} \Rightarrow P(x)$ is maximum

at $x = a\sqrt{2}$; from (i) $y = \sqrt{2}a = x$; Thus, $x = y$

Hence rectangle becomes square hence found.

Ex 6.5 Class 12 Maths Question 20.

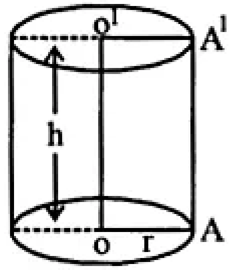
Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

Solution:

Let S be the given surface area of the closed cylinder whose radius is r and height h let v be the its Volume. Then

Surface area $S = 2\pi r^2 + 2\pi rh$, $h = \frac{S - 2\pi r^2}{2\pi r}$ ...(i)

$\therefore$ Volume $V = \pi r^2 h$



$= \pi r^2 \left(\frac{S - 2\pi r^2}{2\pi r}\right)$

$= \frac{1}{2}[Sr - 2\pi r^3]$

$\therefore \frac{dV}{dx} = \frac{1}{2} \times [S - 6\pi r^2]$ ...(ii)

For maxima and minima $\frac{dV}{dx} = 0$

$\therefore S = 6\pi r^2 \Rightarrow r = \sqrt{\frac{S}{6\pi}}$

from (i), $h = \frac{S - 2\pi r^2}{2\pi r}$ (Putting values of S)

Now, $\frac{d^2v}{dr^2} = -6\pi r < o$ at $r = \sqrt{\frac{s}{6\pi}}$

$\therefore$ V is maximum. Thus, volume is maximum when $h = 2r$ i.e.
when height of cylinder = diameter of the base.

Ex 6.5 Class 12 Maths Question 21.

Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimeters, find the dimensions of the can which has the minimum surface area ?

Solution:

Let r be the radius and h be the height of cylindrical can.

Volume $= \pi r^2 h = 100$ cc. $\quad \therefore \quad h = \dfrac{100}{\pi r^2}$

Total surface area of the can.

$$S = \frac{200}{r} + 2\pi r^2, \quad \frac{dS}{dr} = \frac{-200 + 4\pi r^3}{r^2}$$

Now, $\dfrac{dS}{dr} = 0 \Rightarrow r = \left(\dfrac{50}{\pi}\right)^{1/3}$

Also, $\dfrac{d^2S}{dr^2} = \dfrac{400}{r^3} + 4\pi$

At $\quad r = \left(\dfrac{50}{\pi}\right)^{1/3}, \quad \dfrac{d^2S}{dr^2} = \dfrac{400}{50/\pi} + 4\pi = 12\pi = +ve$

$\Rightarrow$ S is minimum or least when $r = \left(\dfrac{50}{\pi}\right)^{1/3}$

Hence, the total surface area is least when radius of base is $\left(\dfrac{50}{\pi}\right)^{1/3}$ cm and $h = \dfrac{100}{\pi r^2} = \dfrac{100}{\pi}\left(\dfrac{\pi}{50}\right)^{2/3}$ cm

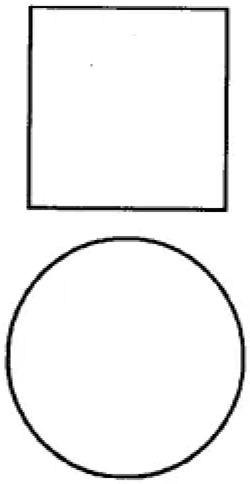
Ex 6.5 Class 12 Maths Question 22.

A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum ?

Solution:

Let one part be of length x, then the other part = 28 – x

Let the part of the length x be converted into a circle of radius r.



$\therefore\ 2\pi r = x \Rightarrow r = \dfrac{x}{2\pi}$

$\therefore$ Area of circle

$$= \pi r^2 = \frac{x^2}{4\pi}$$

Now, second part of length
$28 - x$ is
converted into a square.

$P = 4 \times \text{side} \therefore \text{side} = \dfrac{28 - x}{4}, 2\pi r = x \ \therefore r = \dfrac{x}{2\pi}$

$\therefore \quad \text{Side of square} = \dfrac{28 - x}{4},$

$$\text{Area of square} = \left(\frac{28-x}{4}\right)^2$$

$$\text{Total Area } A = \frac{x^2}{4\pi} + \left(\frac{28-x}{4}\right)^2$$

$$\therefore \frac{dA}{dx} = \frac{x}{2\pi} - \frac{28-x}{8} \quad \text{...(i)}$$

$$\frac{dA}{dx} = 0 \Rightarrow x = \frac{28\pi}{4+\pi}$$

$$\text{Other part} = 28 - x = 28 - \frac{28\pi}{4+\pi} = \frac{112}{4+\pi}$$

$$\text{Differentiating (i), we get } \frac{d^2A}{dx^2} = \frac{1}{2\pi} + \frac{1}{8} = +\text{ve}$$

$$\Rightarrow A \text{ is min. When } x = \frac{28\pi}{4+\pi} \text{ and } 28 - x = \frac{112}{4+\pi}$$

Ex 6.5 Class 12 Maths Question 23.

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is

$$\frac{8}{27}$$

of the volume of the sphere.

Solution:

Let a cone. VAB of greatest volume be inscribed in the sphere let AOC = θ

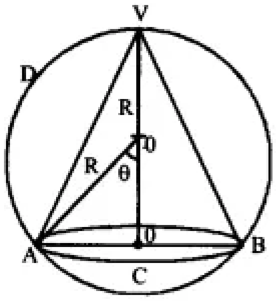
∴ AC, radius of the base of the cone = R sin θ

and VC = VO + OC = R(1 +cosθ)

= R + Rcosθ

= height of the cone.,

V, the volume of the cone.



$$V = \frac{1}{3}\pi (AC)^2 (VC)$$

$$\Rightarrow \quad V = \frac{1}{3}\pi R^3 \sin^2\theta\,(1+\cos\theta)$$

$$\therefore \quad \frac{dV}{d\theta} = \frac{1}{3}\pi R^3(-3\sin^3\theta + 2\sin\theta + 2\sin\theta\cos\theta)$$

For maximum and minimum, we have

$$\frac{dV}{d\theta} = 0 \Rightarrow \cos\theta = \frac{1}{3} \text{ or } \cos\theta = -1$$

But $\cos\theta \neq -1$ as $\cos\theta = -1 \Rightarrow \theta = \pi$,

which is not possible $\quad \therefore \cos\theta = \frac{1}{3}$

When $\cos\theta = \frac{1}{3}$, $\sin\theta = \sqrt{1-\cos^2\theta} = \frac{2\sqrt{2}}{3}$

$$\left(\frac{d^2V}{d\theta^2}\right) \text{at } \theta = \cos^{-1}\left(\frac{1}{3}\right) < 0$$

Hence V is maximum at $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

Now, $\cos\theta = \frac{1}{3}$, $\sin\theta = \frac{2\sqrt{2}}{3}$

$\therefore$ Maximum volume of cone. $= \frac{8}{27}\left(\frac{4}{3}\pi R^3\right)$

Max. volume $= \frac{8}{27} \times$ volume of the sphere of cone.

Ex 6.5 Class 12 Maths Question 24.

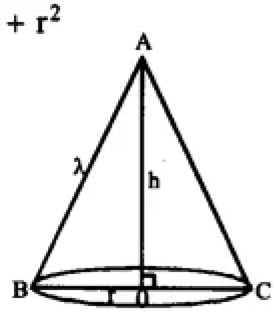
Show that die right circular cone of least curved surface and given volume has an altitude equal to √2 time the radius of the base.

Solution:

Let r and h be the radius and height of the cone.

$$\text{Volume } V = \frac{1}{3}\pi r^2 h = \frac{\pi k}{3} \Rightarrow h = \frac{k}{r^2} \quad \text{...(i)}$$

$$\text{Surface } S = \pi r \ell = \pi r\sqrt{h^2 + r^2}$$



$$\text{Put } h = \frac{k}{r^2}$$

$$\therefore S = \frac{\pi\sqrt{k^2 + r^6}}{r}$$

$$\Rightarrow \frac{dS}{dr} = \frac{2r^6 - k^2}{r^2\sqrt{r^6 + k^2}}$$

for minimum curved surface area

$$\Rightarrow \frac{dS}{dr} = 0 \Rightarrow k^2 = 2r^6 \Rightarrow r^3 = \frac{k}{\sqrt{2}} \quad \text{...(ii)}$$

$$\frac{d^2S}{dr^2} = \frac{r^2\sqrt{r^6+k^2}(12r^5) - (2r^6 - k^2)\left[\frac{r^2 \times 5r^5}{2\sqrt{r^6+k^2}} + 2r\sqrt{r^6+k^2}\right]}{r^4(r^6+k^2)}$$

$$\text{At } r^3 = \frac{k}{\sqrt{2}}, \frac{d^2S}{dr^2} > 0$$

So it attains minimum curved surface area.

Ex 6.5 Class 12 Maths Question 25.

Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

Solution:

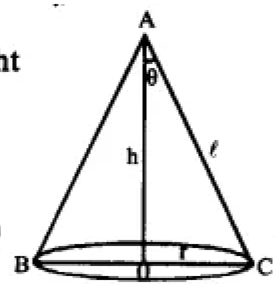
Let v be the volume, l be the slant height and 0 be the semi vertical angle of a cone.

$v = \frac{1}{3}\pi r^2 h$, Vertical height

$h = \ell \cos\theta$

and radius $= \ell \sin\theta$

$v = \frac{1}{3}\pi(\ell \sin\theta)^2(\ell\cos\theta)$



$\frac{dv}{d\theta} = \frac{1}{3}\pi\ell^3 \sin\theta(2\cos^2\theta - \sin^2\theta)$,

$= 0$ at $\tan\theta = \sqrt{2}$

$$\frac{dv}{d\theta} = -\frac{1}{3}\pi\ell^3 \sin\theta\cos^2\theta\left(\tan\theta - \sqrt{2}\right)\left(\tan\theta + \sqrt{2}\right)$$

When θ is slightly $< \tan^{-1}\sqrt{2}$ $\sin\theta\cos^2\theta = +ve$,

$\tan\theta - \sqrt{2} = -ve$; $\tan\theta + \sqrt{2} = +ve$

$\therefore \frac{dv}{d\theta} = +ve$,

when θ is slightly $> \tan^{-1}\sqrt{2}$

$\sin\theta\cos^2\theta = +ve$, $\tan\theta - \sqrt{2} = +ve$

$\tan\theta + \sqrt{2} = +ve$, $\therefore \frac{dv}{d\theta} = (-)(+)(+)(+) = -ve$

$\therefore \frac{dv}{d\theta}$ changes sign $+ve$ to $-ve$

$\therefore$ v is maximum at $\theta = \tan^{-1}\sqrt{2}$.

Ex 6.5 Class 12 Maths Question 26.

Show that semi-vertical angle of right circular cone of given surface area and maximum volume is

$sin^{-1}\left(\frac{1}{3}\right)$

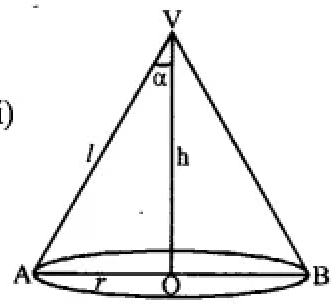
Solution:

https://www.indcareer.com/schools/ncert-solutions-for-12th-class-maths-chapter-6-application-of-derivatives/

Let r be radius, l be the slant height and h be the height of the cone of given surface area s.Then

$$s = \pi r^2 + \pi r l$$

$$\Rightarrow \quad l = \frac{s - \pi r^2}{\pi r} \quad ...(i)$$



Let V be the volume of cone,then

$$V = \frac{1}{3}\pi r^2 h$$

$$\therefore \; V^2 = \frac{1}{9}\pi^2 r^4 h^2 = \frac{1}{9}\pi^2 r^4 (l^2 - r^2) \text{ Using } (i)$$

$$\Rightarrow \; V^2 = \frac{\pi^2 r^4}{9}\left[\left(\frac{s - \pi r^2}{\pi r}\right)^2 - r^2\right] = \frac{1}{9}s(sr^2 - 2\pi r^4)$$

Let $z = V^2$. Then V is maximum or minimum according as z is maximum or minimum.

$$z = \frac{1}{9}s(sr^2 - 2\pi r^4)$$

$$\Rightarrow \frac{dz}{dr} = \frac{1}{9}s(2sr - 8\pi r^3) \qquad ...(ii)$$

For maximum or minimum, we must have $\frac{dz}{dr} = 0$

$$\Rightarrow \quad 2sr - 8\pi r^3 = 0 \quad \Rightarrow \quad s = 4\pi r^2 \qquad ...(iii)$$

Diff. (ii) w.r.t. r, we get $\frac{d^2z}{dr^2} = \frac{s}{9}(2s - 24\pi r^2)$

$$\frac{d^2z}{dr^2} = \frac{s}{9}\left[2s - 24\pi\frac{s}{4\pi}\right] = -\frac{4s^2}{9} < 0$$

So z is maximum when $s = 4\pi r^2$

Now $s = 4\pi r^2 \Rightarrow s = \pi rl + \pi r^2$

$\Rightarrow 4\pi r^2 = \pi rl + \pi r^2$

$\Rightarrow 3\pi r^2 = \pi rl \Rightarrow l = 3r \therefore \sin\alpha = \frac{r}{l} = \frac{r}{3r} = \frac{1}{3}$

Hence V is maximum when $\alpha = \sin^{-1}\frac{1}{3}$.

Choose the correct answer in the Exercises 27 and 29.

Ex 6.5 Class 12 Maths Question 27.

The point on die curve $x^2 = 2y$ which is nearest to the point (0,5) is

(a) $(2\sqrt{2},4)$

(b) $(2\sqrt{2},0)$

(c) (0,0)

(d) (2,2)

Solution:

(a) Let P (x, y) be a point on the curve The other point is A (0,5)

$Z = PA^2 = x^2 + y^2 + 25 - 10y$ [$\because x^2 = 2y$]

$$z = y^2 - 8y + 25 \therefore \frac{dz}{dy} = 2y - 8, \frac{d^2z}{dy^2} = 2 = +ve$$

$$\frac{dz}{dy} = 0 \Rightarrow y = 4 \quad \frac{d^2z}{dy^2} = +ve, \text{ z is minimum}$$

$$\therefore x^2 = 2y = 2 \times 4 = 8,$$

$$\therefore x = 2\sqrt{2} \Rightarrow \text{z is minimum at } (2\sqrt{2}, 4)$$

$$\Rightarrow \sqrt{z} \text{ is minimum at } (2\sqrt{2}, 4)$$

Ex 6.5 Class 12 Maths Question 28.

For all real values of x, the minimum value of

$\frac{1-x+x^2}{1+x+x^2}$

(a) 0

(b) 1

(c) 3

(d)

$\frac{1}{3}$

Solution:

(d) Let

$y = \frac{1-x+x^2}{1+x+x^2}$

$$\frac{dy}{dx}=\frac{(-1+2x)(1+x+x^2)-(1-x+x^2)(1+2x)}{(1+x+x^2)}$$

Numberator of $\frac{dy}{dx}=2(x-1)(x+1)$,

$$\Rightarrow\frac{dy}{dx}=\frac{2(x-1)(x+1)}{(x^2+x+1)^2};\quad \frac{dy}{dx}=0 \text{ at } x=1,-1$$

At $x=1$, $\frac{dy}{dx}$ changes sign from – ve to + ve

$\therefore$ y is minimum at $x=1$

Minimum value of $\frac{1-x+x^2}{1+x+x^2}=\frac{1-1+1}{1+1+1}=\frac{1}{3}$

Ex 6.5 Class 12 Maths Question 29.

The maximum value of

$$[x(x-1)+1]^{\frac{1}{3}}, 0\leq x\leq 1$$

is

(a)

$$\left(\frac{1}{3}\right)^{\frac{1}{3}}$$

(b)

$$\frac{1}{2}$$

(c) 1

(d) 0

Solution:

(c) Let y =

$$[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$$

$\therefore \frac{dy}{dx} = \frac{(2x-1)}{3[x(x-1)+1]^{2/3}}, \frac{dy}{dx} = 0 \text{ at } x = \frac{1}{2}$

$\frac{dy}{dx}$ Changes sign from –ve to + ve at $x = \frac{1}{2}$

$\therefore$ y is minimum at $x = \frac{1}{2}$

Value of y at $x = 0, (0+1)^{1/3} = 1^{1/3} = 1$

Value of y at $x = 1, (0+1)^{1/3} = 1^{1/3} = 1$

$\therefore$ The maximum value of y is 1.

Chapterwise NCERT Solutions for Class 12 Maths :

- Chapter 1 – Relations and Functions
- Chapter 2 – Inverse Trigonometric Functions.
- Chapter 3 – Matrices
- Chapter 4 – Determinants.
- Chapter 5 – Continuity and Differentiability.0.0
- Chapter 6 – Application of Derivatives.
- Chapter 7 – Integrals.
- Chapter 8 – Application of Integrals.
- Chapter 9: Differential Equations
- Chapter 10: Vector Algebra
- Chapter 11: Three Dimensional Geometry
- Chapter 12: Linear Programming
- Chapter 13: Probability

About NCERT

The National Council of Educational Research and Training is an autonomous organization of the Government of India which was established in 1961 as a literary, scientific, and charitable Society under the Societies Registration Act. The major objectives of NCERT and its constituent units are to: undertake, promote and coordinate research in areas related to school education; prepare and publish model textbooks, supplementary material, newsletters, journals and develop educational kits, multimedia digital materials, etc.Organise pre-service and in-service training of teachers; develop and disseminate innovative educational techniques and practices;collaborate and network with state educational departments, universities, NGOs and other educational institutions; act as a clearing house for ideas and information in matters related to school education; and act as a nodal agency for achieving the goals of Universalisation of Elementary Education.In addition to research, development, training, extension, publication and dissemination activities, NCERT is an implementation agency for bilateral cultural exchange programmes with other countries in the field of school education.Its headquarters are located at Sri Aurobindo Marg in New Delhi. Visit the Official NCERT website to learn more.