NCERT Solutions for 12th Class Maths: Chapter 5-Continuity and Differentiability

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Class 12: Maths Chapter 5 solutions. Complete Class 12 Maths Chapter 5 Notes.

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Ex 5.1 Class 12 Maths Question 1.

Prove that the function f (x) = 5x – 3 is continuous at x = 0, at x = – 3 and at x = 5.

Solution:

(i) At x = 0. $\lim_{x\to 0} f(x) = \lim_{x\to 0} (5x - 3) = -3$ and

$f(0) = -3$

∴f is continuous at x = 0

(ii) At x = – 3, $\lim_{x\to 3} f(x) = \lim_{x\to -3} (5x - 3) = -18$

and $f(-3) = -18$

∴ f is continuous at x = – 3

(iii) At x = 5, $\lim_{x\to 5} f(x) = \lim_{x\to 5} (5x - 3) = 22$ and

$f(5) = 22$

∴ f is continuous at x = 5

Ex 5.1 Class 12 Maths Question 6.

$$f(x)=\begin{cases}2x+3, if & x\leq 2\\ 2x-3, if & x>2\end{cases}$$

Solution:

$$f(x)=\begin{cases}2x+3, if & x\leq 2\\ 2x-3, if & x>2\end{cases}$$

at x≠2

$$\text{L.H.L.} = \lim_{x\to 2^-}(2x+3) = 7,\ f(2) = 2\times 2+3 = 7$$

$$\text{R.H.L.} = \lim_{x\to 2^+}(2x-3) = 2\times 2-3 = 1$$

$\Rightarrow$ L.H.L. $\neq$ R.H.L.

$\therefore$ f is discontinuous at $x = 2$

at $x = c < 2$

$$\lim_{x\to c}(2x+3) = 2c+3 = f(c)$$

$\therefore$ f is continuous at $x = c < 2$

at $x = c > 2$, $\lim_{x\to c}(2x-3) = 2c-3 = f(c)$

$\therefore$ f is continuous at $x = c > 2$ $\Rightarrow$ Point of discontinuity is $x = 2$

Ex 5.1 Class 12 Maths Question 7.

$$f(x) = \begin{cases} |x|+3, if & x \le -3 \\ -2x, if & -3 < x < 3 \\ 6x+2, if & x \ge 3 \end{cases}$$

Solution:

$$f(x)=\begin{cases}|x|+3, if \quad x\leq -3\\ -2x, if \quad -3<x<3\\ 6x+2, if \quad x\geq 3\end{cases}$$

$$\text{at } x=c>3,\ \lim_{x\to c}(6x+2)=6c+2=f(c)$$

$$\Rightarrow \lim_{x\to c}f(x)=f(c)$$

$$\Rightarrow f \text{ is continuous at } x=c>3$$

Ex 5.1 Class 12 Maths Question 8.

Test the continuity of the function f (x) at x = 0

Solution:

We have;

(LHL at x=0)

$$= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} f(-h)$$

$$= \lim_{h \to 0} \frac{|-h|}{-h} = \lim_{h \to 0} \frac{h}{-h} = \lim_{h \to 0} -1 = -1 \text{ and,}$$

(RHL at $x = 0$)

$$= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} f(h)$$

$$= \lim_{h \to 0} \frac{|h|}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

Thus, we have $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

Hence, f(x) is not continuous at $x = 0$.

Ex 5.1 Class 12 Maths Question 9.

$$f(x) = \begin{cases} \frac{x}{|x|}; if \quad x < 0 \\ -1, if \quad x \geq 0 \end{cases}$$

Solution:

$$\text{L.H.L (at } x=0) = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0} f(-h) = \lim_{h\to 0} \frac{-h}{|-h|} = -1$$

and R.H.L (at $x=0$)

$$= \lim_{x\to 0^+} f(x) = \lim_{h\to 0}(f(0+h))$$

$$= \lim_{h\to 0} f(h) = \lim_{h\to 0}(-1) = -1$$

Also $f(0) = -1$

Thus, we have $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

Hence $f(x)$ is continuous at $x = 0$.

Ex 5.1 Class 12 Maths Question 10.

$$f(x) = \begin{cases} x+1, if \quad x \geq 1 \\ x^2+1, if \quad x < 1 \end{cases}$$

Solution:

At $x = 1$, L.H.L. $= \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2+1) = 2$

R.H.L. $= \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x+1) = 2$,

$f(1) = 1 + 1 = 2$

$\Rightarrow$ f is continuous at $x = 1$

At $x = c > 1$, $\lim_{x \to c} f(x) = \lim_{x \to c} (x+1) = c+1 = f(c)$

$\Rightarrow$ f is continuous at $x = c > 1$, At $x = c < 1$,

$\lim_{x \to c} f(x) = \lim_{x \to c} (x^2+1) = c^2+1 = f(c)$

$\Rightarrow$ f is continuous at $x = c < 1$, There is no point of discontinuity at any point $x \in R$

Ex 5.1 Class 12 Maths Question 11.

$$f(x) = \begin{cases} x^3 - 3, if & x \leq 2 \\ x^2 + 1, if & x > 2 \end{cases}$$

Solution:

At x = 2, L.H.L. $\lim_{x\to2^-} (x^3 - 3) = 8 - 3 = 5$

R.H.L. = $\lim_{x\to2^+} (x^2 + 1) = 4 + 1 = 5$

$f(2) = 2^3 - 3 = 8 - 3 = 5 \Rightarrow$ f is continuous at $x = 2$

At $x = c < 2$, $\lim_{x \to c} (x^3 - 3) = c^3 - 3 = f(c)$,

At $x = c > 2$, $\lim_{x \to c} (x^2 + 1) = c^2 + 1 = f(c)$

$\Rightarrow$ f is continuous for all $x \in R$.

$\therefore$ There is no point of discontinuity.

Ex 5.1 Class 12 Maths Question 12.

$$f(x) = \begin{cases} x^{10} - 1, if & x \leq 1 \\ x^2, if & x > 1 \end{cases}$$

Solution:

$$f(x) = \begin{cases} x^{10} - 1, if & x \leq 1 \\ x^2, if & x > 1 \end{cases}$$

At $x = 1$, L.H.L. $= \lim_{x \to 1^-} f(x) = x^{10} - 1 = \lim_{h \to 0} (1-h)^{10} - 1$

$= \lim_{h \to 0} \left[1 - 10h + \frac{10.9}{2} h^2 + ...\right] - 1 = 0$

R.H.L. $= \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2$

$= \lim_{x \to 0} (1+h)^2 = \lim_{x \to 0} 1^2 + h^2 + 2h = 1$

$f(1) = 1^{10} - 1 = 0$

$\therefore$ L.H.L. $\neq$ R.H.L. $\neq f(1)$

$\Rightarrow$ f is not continuous at $x = 1$, At $x = c < 1$,

$\lim_{x \to c} x^{10} - 1 = c^{10} - 1 = f(c)$

At $x = c > 1$, $\lim_{x \to c} x^2 = c^2 = f(c)$

$\Rightarrow$ f is continuous at all points $x \in R - \{1\}$

$\therefore$ Point of discontinuity is $x = 1$.

Ex 5.1 Class 12 Maths Question 13.

Is the function defined by

$$f(x) = \begin{cases} x+5, if & x \leq 1 \\ x-5, if & x > 1 \end{cases}$$

a continuous function?

Solution:

At x = 1,L.H.L.= $\lim_{x\to 1^-}$ f(x) = $\lim_{x\to 1^-}$ (x + 5) = 6,

R.HL. = $\lim_{x\to 1^+}$ f(x) = $\lim_{x\to 1^+}$ (x – 5) = – 4

f(1) = 1 + 5 = 6,

f(1) = L.H.L. ≠ R.H.L.

=> f is not continuous at x = 1

At x = c < 1, $\lim_{x\to c}$ (x + 5) = c + 5 = f(c)

At x = c > 1, $\lim_{x\to c}$ (x – 5) = c – 5 = f(c)

∴ f is continuous at all points x ∈ R except x = 1.

Discuss the continuity of the function f, where f is defined by

Ex 5.1 Class 12 Maths Question 14.

$$f(x) = \begin{cases} 3, if & 0 \leq x \leq 1 \\ 4, if & 1 < x < 3 \\ 5, if & 3 \leq x \leq 10 \end{cases}$$

Solution:

In the interval $0 \le x \le 1, f(x) = 3$; f is continuous in this interval.

At x = 1, L.H.L. = lim f(x) = 3,

R.H.L. = $\lim_{x \to 1^+} f(x) = 4$ => f is discontinuous at

x = 1

At x = 3, L.H.L. = $\lim_{x \to 3^-} f(x) = 4$,

R.H.L. = $\lim_{x \to 3^+} f(x) = 5$ => f is discontinuous at

x = 3

=> f is not continuous at x = 1 and x = 3.

Ex 5.1 Class 12 Maths Question 15.

$$f(x) = \begin{cases} 2x, if \quad x < 0 \\ 0, if \quad 0 \le x \le 1 \\ 4x, if \quad x > 1 \end{cases}$$

Solution:

$$f(x) = \begin{cases} 2x, if \quad x < 0 \\ 0, if \quad 0 \le x \le 1 \\ 4x, if \quad x > 1 \end{cases}$$

At x = 0, L.H.L. = $\lim_{x \to 0^-} 2x = 0$,

R.H.L. = $\lim_{x \to 0^+} (0) = 0$, f(0) = 0

=> f is continuous at x = 0

At x = 1, L.H.L. = $\lim_{x \to 1^-} (0) = 0$,

R.H.L. = $\lim_{x \to 1^+} 4x = 4$

f(1) = 0, f(1) = L.H.L.≠R.H.L.

∴ f is not continuous at x = 1

when x < 0 f (x) = 2x, being a polynomial, it is

continuous at all points x < 0. when x > 1. f (x) = 4x being a polynomial, it is

continuous at all points x > 1.

when 0 ≤ x ≤ 1, f (x) = 0 is a continuous function

the point of discontinuity is x = 1.

Ex 5.1 Class 12 Maths Question 16.

$$f(x) = \begin{cases} -2, if & x \leq -1 \\ 2x, if & -1 < x \leq 1 \\ 2, if & x > 1 \end{cases}$$

Solution:

$$f(x) = \begin{cases} -2, if & x \leq -1 \\ 2x, if & -1 < x \leq 1 \\ 2, if & x > 1 \end{cases}$$

At x = – 1,L.H.L. = $\lim_{x\to 1-}$ f(x) = – 2, f(-1) = – 2,

R.H.L. = $\lim_{x\to 1+}$ f(x) = – 2

=> f is continuous at x = – 1

At x= 1, L.H.L. = $\lim_{x\to 1-}$ f(x) = 2,f(1) = 2

∴ f is continuous at x = 1,

R.H.L. = $\lim_{x\to 1+}$ f(x) = 2

Hence, f is continuous function.

Ex 5.1 Class 12 Maths Question 17.

Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax+1, if & x \leq 3 \\ bx+3, if & x > 3 \end{cases}$$

is continuous at x = 3

Solution:

At x = 3, L.H.L. = $\lim_{x\to 3-}$ (ax+1) = 3a+1 ,

f(3) = 3a + 1, R.H.L. = $\lim_{x\to 3+}$ (bx+3) = 3b+3

f is continuous ifL.H.L. = R.H.L. = f(3)

3a + 1 = 3b + 3 or 3(a – b) = 2

a – b = $\frac{2}{3}$ or a = b + $\frac{2}{3}$, for any arbitrary value of b.

Therefore the value of a corresponding to the value of b.

Ex 5.1 Class 12 Maths Question 18.

For what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x), if \quad x \leq 0 \\ 4x + 1, if \quad x > 0 \end{cases}$$

continuous at x = 0? What about continuity at x = 1?

Solution:

At x = 0, L.H.L. = $\lim_{x \to 0^-} \lambda(x^2 - 2x) = 0$,

R.H.L. = $\lim_{x \to 0^+} (4x + 1) = 1$, f(0)=0

f (0) = L.H.L. ≠ R.H.L.

=> f is not continuous at x = 0,

whatever value of λ ∈ R may be

At x = 1, $\lim_{x \to 1} f(x) = \lim_{x \to 1} (4x + l) = f(1)$

=> f is not continuous at x = 0 for any value of λ but f is continuous at x = 1 for all values of λ.

Ex 5.1 Class 12 Maths Question 19.

Show that the function defined by g (x) = x – [x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

Solution:

Let c be an integer, [c – h] = c – 1, [c + h] = c, [c] = c, g(x) = x – [x].

At x = c, $\lim_{x \to c^-} (x - [x]) = \lim_{h \to 0} [(c - h) - (c - 1)]$

$= \lim_{h \to 0} (c - h - (c - 1)) = 1 [\because [c - h] = c - 1]$

R.H.L. = $\lim_{x \to c^+} (x - [x]) = \lim_{h \to 0} (c + h - [c + h])$

$= \lim_{h \to 0} [c + h - c] = 0$

f(c) = c – [c] = 0,

Thus L.H.L. ≠ R.H.L. = f (c) => f is not continuous at integral points.

Ex 5.1 Class 12 Maths Question 20.

Is the function defined by f (x) = x² – sin x + 5 continuous at x = π?

Solution:

Let f(x) = x² – sinx + 5,

$$\text{At } x=\pi, \text{L.H.L.} = \lim_{x\to\pi^-} (x^2-\sin x+5), \text{Put } x=\pi-h,$$

$$\therefore \quad \text{L.H.L.} = \lim_{h\to 0} [(\pi-h)^2-\sin(\pi-h)+5]$$

$$= \lim_{h\to 0} [\pi^2-2\pi h + h^2-\sin h+5] =$$

$$\pi^2+5$$

$$\text{R.H.L.} = \lim_{x\to\pi^+} (x^2-\sin x+5), \quad \text{Put } x=\pi+h,$$

$$\therefore \text{L.H.L.} = \lim_{h\to 0} [(\pi+h)^2-\sin(\pi+h)+5]$$

$$= \lim_{h\to 0} [\pi^2+2\pi h+h^2+\sin h+5] = \pi^2$$

$$+5,$$

$$f(\pi)=\pi^2+5, \quad \therefore \text{L.H.L.} = \text{R.H.L.} = f(\pi)$$

Hence, f is continuous at $x=\pi$

Ex 5.1 Class 12 Maths Question 21.

Discuss the continuity of the following functions:

(a) f (x) = sin x + cos x

(b) f (x) = sin x – cos x

(c) f (x) = sin x · cos x

Solution:

(a) f(x) = sinx + cosx

(b) $f(x) = \sin x - \cos x$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)$$

$$= \sqrt{2}\left(\sin x \cos\frac{\pi}{4} - \cos x \sin\frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(x - \frac{\pi}{4}\right),$$

At $x = c$, L.H.L.

$$= \lim_{x \to c^-} \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(c - \frac{\pi}{4}\right)$$

$$\text{R.H.L.} = \lim_{x \to c^+} \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(c - \frac{\pi}{4}\right) = f(c)$$

$\therefore$ f is continuous for all $x \in R$.

(c) $f(x) = \sin x \cos x = \frac{1}{2}(2\sin x \cos x)$

$= \frac{1}{2}\sin 2x$. Again f is continuous for all $x \in R$.

Ex 5.1 Class 12 Maths Question 22.

Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

Solution:

(a) Let $f(x) = \cos x$

At $x = c, c \in R, \lim_{x \to c} \cos x = \cos c = f(c)$

$\therefore$ f is continuous for all values of $x \in R$.

(b) Let $f(x) = \sec x$,

$\sec x$ is undefined at $x = (2n + 1)\frac{\pi}{2}, n \in z$.

Also at $x = \frac{\pi}{2}$,

$$\text{L.H.L.} = \lim_{x \to \frac{\pi}{2}^-} \sec x = \lim_{h \to 0} \sec\left(\frac{\pi}{2} - h\right)$$

$$= \lim_{h \to 0} \operatorname{cosec} h = \infty$$

$$\text{R.H.L.} = \lim_{x \to \frac{\pi}{2}^+} \sec x = \lim_{h \to 0} \sec\left(\frac{\pi}{2} + h\right)$$

$$= -\lim_{h \to 0} \operatorname{cosec} h = -\infty$$

R.H.L. $\neq$ L.H.L.

$\therefore$ f is not continuous at $x = \frac{\pi}{2}$

or at $x = (2n+1)\frac{\pi}{2}$

At $x = c \neq (2n+1)\frac{\pi}{2}$

$\lim_{x \to c} \sec x^2 = \sec c = f(c)$

Hence f is continuous at $x \in R$ except

at $x = (2n+1)\frac{\pi}{2}$, where $n \in z$.

(c) $f(x) = \text{cosec}\, x$, f is not defined at $x = n\pi$
$\Rightarrow$ f is not continuous at $x = n\pi$.

(d) $f(x) = \cot x$, f is not defined at $x = n\pi$, At

$x = \pi$, L.H.L. $= \lim_{x \to \pi^-} \cot x = \lim_{h \to 0} \cot(\pi - h)$

$= \lim_{h \to 0} (-\cot h) = -\infty$,

R.H.L. $= \lim_{x \to \pi^+} \cot x = \lim_{h \to 0} \cot(\pi + h)$

$= \lim_{h \to 0} \cot h = \infty$

Thus this $f(x)$ does not exist at $x = n\pi$

At $x = c \neq n\pi$, $\lim_{x \to c} \cot x = \cot c = f(c)$

Thus f is continuous at all points $x \in R$ except $x = n\pi$, where $n \in Z$.

Ex 5.1 Class 12 Maths Question 23.

Find all points of discontinuity of f, where

$$f(x) = \begin{cases} \frac{sinx}{x}, if \quad x < 0 \\ x + 1, if \quad x \geq 0 \end{cases}$$

Solution:

At x = 0

$$\text{L.H.L.} = \lim_{x \to 0^-} f(x) = \frac{\sin(-h)}{-h} = 1,$$

$$\therefore f(0) = 1, \text{R.H.L.} = \lim_{x \to 0^+} f(x) = \frac{\sin(-h)}{-h} = 1$$

$\therefore$ f is continuous at $x = 0$ when $x < 0$, $\sin x$ and x both are continuous

$\therefore \frac{\sin x}{x}$ is also continuous

when $x > 0$, $f(x) = x + 1$ is a polynomial

Thus f is continuous $\Rightarrow$ f is not discontinuous at any point.

Ex 5.1 Class 12 Maths Question 24.

Determine if f defined by

$$f(x) = \begin{cases} x^2 sin\frac{1}{x}, if \quad x \neq 0 \\ 0, if \quad x = 0 \end{cases}$$

is a continuous function?

Solution:

At x = 0

$$\text{L.H.L.} = \lim_{x \to 0^-}\left(x^2 \sin\frac{1}{x}\right) = \lim_{h \to 0}(-h)^2 \sin\frac{1}{-h}$$

$$= -\lim_{h \to 0} h^2\left(\sin\frac{1}{h}\right)$$

$\sin\frac{1}{h}$ lies between -1 and 1, a finite quantity

$\therefore\ h^2 \sin\frac{1}{h} \to 0$ as $h \to 0$

$\therefore$ L.H.L. $= 0$. Similarly $\lim_{x \to 0^+}\left(x^2 \sin\frac{1}{x}\right) = 0$

Also $f(0) = 0$ (given)

$\therefore$ L.H.L. $=$ R.H.L. $= f(0)$

Hence f is continuous for all $x \in R$.

Ex 5.1 Class 12 Maths Question 25.

Examine the continuity of f, where f is defined by

$$f(x) = \begin{cases} sinx - cosx, if \quad x \neq 0 \\ -1, if \quad x = 0 \end{cases}$$

Solution:

$$\text{L.H.L.} = \lim_{x \to 0^-} (\sin x - \text{cox})$$

$$= \lim_{h \to 0} [\sin(-h) - \cos(-h)]$$

$$= \lim_{h \to 0} (-\sin h - \cos h) = -1$$

$$\text{R.H.L.} = \lim_{x \to 0^+} (\sin x - \cos x)$$

$$= \lim_{h \to 0} (\sin h - \cos h) = -1, \ f(0) = -1$$

$$\therefore \ \text{L.H.L.} = \text{R.H.L.} = f(0)$$

Thus, f is continuous at $x = 0$

Find the values of k so that the function is continuous at the indicated point in Questions 26 to 29.

Ex 5.1 Class 12 Maths Question 26.

Solution:

At x = $\frac{\pi}{2}$

L.H.L =

$$\lim_{x \to \left(\frac{\pi}{2}\right)^-} \frac{k \quad \cos x}{\pi - 2x}$$

$$= \lim_{h \to 0} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} \left(\text{Putting } x = \frac{\pi}{2} - h\right)$$

$$= \lim_{h \to 0} \frac{k \sin h}{\pi - \pi + 2h} = \lim_{h \to 0} \frac{k}{2} \cdot \frac{\sin h}{h} = \frac{k}{2}$$

$$\text{R.H.L.} = \lim_{x \to \left(\frac{\pi}{2}\right)^+} \frac{k \cos x}{\pi - 2x} = \lim_{h \to 0} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$\left(\text{Putting } x = \frac{\pi}{2} + h\right)$

$$= \lim_{h \to 0} \frac{-k \sin h}{-2h} = \lim_{h \to 0} \frac{k}{2} \frac{\sin h}{h} = \frac{k}{2}, f\left(\frac{\pi}{2}\right) = 3$$

(given)

Hence f is continuous if $\frac{k}{2} = 3$ or $k = 6$.

Ex 5.1 Class 12 Maths Question 27.

$$f(x) = \begin{cases} kx^2, if \quad x \le 2 \quad at \quad x = 2 \\ 3, if \quad x > 2 \quad at \quad x = 2 \end{cases}$$

Solution:

$$f(x) = \begin{cases} kx^2, if \quad x \leq 2 \quad at \quad x = 2 \\ 3, if \quad x > 2 \quad at \quad x = 2 \end{cases}$$

$$\text{L.H.L.} = \lim_{x \to 2^-} (kx^2),$$

$$\text{Put } x = 2 - h, \ \text{L.H.L.} = \lim_{h \to 0} k(2-h)^2 = 4k,$$

$$f(2) = k \cdot 2^2 = 4k$$

$$\text{R.H.L.} = \lim_{x \to 2^+} f(x) = 3$$

$$f \text{ is continuous if L.H.L.} = \text{R.H.L.} = f(2)$$

$$\therefore \ 4k = 3 \ \Rightarrow k = \frac{3}{4}$$

Ex 5.1 Class 12 Maths Question 28.

Solution:

$$\text{At } x = \pi, \text{L.H.L.} = \lim_{x \to \pi^-} f(x) = \lim_{x \to \pi^-} -(kx+1)$$

$$= k\pi + 1, \ f(x) = k\pi + 1$$

$$\text{R.H.L.} = \lim_{x \to \pi^+} \cos x = -1,$$

$$\text{for continuity at } x = \pi, \ \text{L.H.S.} = \text{R.H.S.} = f(\pi)$$

$$\therefore \ k\pi + 1 = -1 \ \Rightarrow \ k = \frac{-2}{\pi}$$

Ex 5.1 Class 12 Maths Question 29.

$$f(x)=\begin{cases}kx+1, if \quad x\leq 5 \quad at \quad x=5\\ 3x-5, if \quad x>5 \quad at \quad x=5\end{cases}$$

Solution:

At $x=5$, L.H.L. $=\lim_{x\to 5^-}(kx+1)=5k+1$, $f(5)$

$=k.5+1$

$=5k+1$, R.H.L. $=\lim_{x\to 5^+}(3x-5)=10$

f is continuous if L.H.L. = R.H.L. = f(5)

$\therefore\ 5k+1=10 \Rightarrow k=\frac{9}{5}$

Ex 5.1 Class 12 Maths Question 30.

Find the values of a and b such that the function defined by

$$f(x)=\begin{cases}5, if \quad x\leq 2\\ ax+b, if \quad 2<x<10\\ 21, if \quad x\geq 10\end{cases}$$

to is a continuous function.

Solution:

At $x=2$, L.H.L. $= \lim_{x\to 2^-}(5)=5$, $f(2)=5$,

R.H.L. $= \lim_{x\to 2^+}(ax+b)=2a+b$

f is continuous at $x=2$, if $2a + b=5$ ... (i)

At $x=10$, L.H.L. $= \lim_{x\to 10^-} f(x) = \lim_{x\to 10^-}(ax+b)$

$=10a+b$ and R.H.L. $= \lim_{x\to 10^+} f(x) = \lim_{x\to 10^+}(21)=21$

f is continuous at $x=10$ if $10a + b=21$ ... (ii)

Subtracting (i) from (ii)

$8a=21-5=16 \Rightarrow a=2$

from (i) $b=1$ Hence, $a=2, b=1$

Ex 5.1 Class 12 Maths Question 31.

Show that the function defined by $f(x)=\cos(x^2)$ is a continuous function.

Solution:

Now, $f(x) = \cos x^2$, let $g(x)=\cos x$ and $h(x)$ x^2

$\therefore$ $goh(x) = g(h(x)) = \cos x^2$

Now g and h both are continuous $\forall$ $x \in R$.

$f(x) = goh(x) = \cos x^2$ is also continuous at all $x \in R$.

Ex 5.1 Class 12 Maths Question 32.

Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

Solution:

Let $g(x) =|x|$and $h(x) = \cos x$, $f(x) = goh(x) = g(h(x)) = g(\cos x) = |\cos x|$

Now $g(x) = |x|$ and $h(x) = \cos x$ both are continuous for all values of $x \in R$.

$\therefore$ $(goh)(x)$ is also continuous.

Hence, $f(x) = goh(x) = |\cos x|$ is continuous for all values of $x \in R$.

Ex 5.1 Class 12 Maths Question 33.

Examine that $\sin |x|$ is a continuous function.

Solution:

Let $g(x) = \sin x$, $h(x) = |x|$, $goh(x) = g(h(x))$

$= g(|x|) = \sin|x| = f(x)$

Now $g(x) = \sin x$ and $h(x) = |x|$ both are continuous for all $x \in R$.

$\therefore f(x) = goh(x) = \sin |x|$ is continuous at all $x \in R$.

Ex 5.1 Class 12 Maths Question 34.

Find all the points of discontinuity of f defined by $f(x) = |x|-|x+1|$.

Solution:

$f(x) = |x|-|x+1|$, when $x< -1$,

$f(x) = -x-[-(x+1)] = -x + x + 1 = 1$

when $-1 \leq x < 0$, $f(x) = -x - (x + 1) = -2x - 1$,

when $x \geq 0$, $f(x) = x - (x + 1) = -1$

$$\therefore \quad f(x)=\begin{cases}1 & , \text{if } x<-1 \\ -2x-1 & , \text{if } -1 \le x<0 \\ -1 & , \text{if } x \ge 0\end{cases}$$

$$\text{At } x=-1, \text{L.H.L.} = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1) = 1$$

$$\text{R.H.L.} = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (-x-1) = 1$$

$$\therefore \ f(-1)=-2(-1)-1=2-1=1$$

$$\therefore \ \text{L.H.L.} = \text{R.H.L.} = f(-1)$$

$\Rightarrow$ f is continuous at $x=-1$

$$\text{At } x=0, \text{L.H.L.} = \lim_{x \to 0^-} (-2x-1) = -1$$

$f(0)=-1$ (given)

$$\text{R.H.L.} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (-1) = -1$$

$\therefore$ L.H.L. = R.H.L. = f(0)

$\therefore$ f is continuous at $x=0$ $\Rightarrow$ There is no point of discontinuous. Hence f is continuous for all $x \in R$.

Ex 5.2 Class 12 Maths Question 1.

$\sin(x^2 + 5)$

Solution:

Let y = sin(x2 + 5),

put $x^2 + 5 = t$

y = sint

$t = x^2+5$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{dy}{dx} = cost.\frac{dt}{dx} = cos(x^2 + 5)\frac{d}{dx}(x^2 + 5)$$

= cos (x^2 + 5) × 2x

= 2x cos (x^2 + 5)

Ex 5.2 Class 12 Maths Question 2.

cos (sin x)

Solution:

let y = cos (sin x)

put sinx = t

∴ y = cost,

t = sinx

∴

$$\frac{dy}{dx} = -sin \quad t, \frac{dt}{dx} = cos \quad x$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = (-sint) \times cosx$$

Putting the value of t,

$$\frac{dy}{dx} = -sin(sinx) \times cosx$$

$\frac{dy}{dx} = -[sin(sinx)]cosx$

Ex 5.2 Class 12 Maths Question 3.

sin(ax+b)

Solution:

let = sin(ax+b)

put ax+bx = t

∴ y = sint

t = ax+b

$\frac{dy}{dt} = cost, \frac{dt}{dx} = \frac{d}{dx}(ax+b) = a$

$\frac{dy}{dx} = acos(ax+b)$

Ex 5.2 Class 12 Maths Question 4.

sec(tan(√x))

Solution:

let y = sec(tan(√x))

by chain rule

$\frac{dy}{dx} = sec(tan\sqrt{x})tan(tan\sqrt{x})\frac{d}{dx}(tan\sqrt{x})$
$\frac{dy}{dx} = sec(tan\sqrt{x}).tan(tan\sqrt{x})sec^2\sqrt{x}.\frac{1}{2\sqrt{x}}$

Ex 5.2 Class 12 Maths Question 5.

$\frac{sin(ax+b)}{cos(cx+d)}$

Solution:

y =

$\frac{sin(ax+b)}{cos(cx+d)}$

=

$\frac{v}{u}$

u = sin(ax+b)

$$\therefore \frac{du}{dx} = \cos(ax+b)\frac{d}{dx}(ax+b)$$
$$= a\cos(ax+b), \quad v = \cos(cx+d),$$
$$\frac{dv}{dx} = -\sin(cx+d)\frac{d}{dx}(cx+d) = -\sin(cx+d) \times c$$
$$\frac{dy}{dx} = -c(\sin(cx+d)$$

$$\text{Now, } y = \frac{u}{v}, \frac{dy}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$
$$\frac{dy}{dx} \Rightarrow \frac{a\cos(ax+b)\cos(cx+d) + c\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$$

Ex 5.2 Class 12 Maths Question 6.

cos x^3 . $\sin^2(x^5)$ = y(say)

Solution:

Let u = cos x^3 and v = $\sin^2 x^5$,

put x^3 = t

$$\therefore\ u=\cos t,\ t=x^3,\ \frac{du}{dt}=-\sin t,\ \frac{dt}{dx}=3x^2$$

$$\therefore\ \frac{du}{dx}=\frac{du}{dt}\times\frac{dt}{dx}=-\sin t\times 3x^2=-3x^2\sin x^3$$

$$\text{put } x^5=t,\ \sin t=s\quad \therefore\ v=s^2$$

$$\frac{dv}{ds}=2s,\ \frac{dt}{dx}=5x^4,\ \frac{ds}{dt}=\cos t,$$

$$\frac{dv}{dx}=\frac{dv}{ds}\times\frac{ds}{dt}\times\frac{dt}{dx}=10x^4\sin x^5\cos x^5$$

$$\text{Now } y=uv,\ \frac{dy}{dx}=\frac{du}{dx}\times v+u\times\frac{dv}{dx}$$

$$=-3x^2\sin x^3\sin^2 x^5+10x^4\cos x^3\sin x^5\cos x^5$$

$$\therefore\ \frac{dy}{dx}=x^2\sin x^5(-3\sin x^3\sin x^5+10x^2\cos x^3\cos x^5)$$

Ex 5.2 Class 12 Maths Question 7.

$$2\sqrt{cot(x^2)}=y(say)$$

Solution:

$$2\sqrt{cot(x^2)}=y(say)$$

$$\frac{dy}{dx}=\frac{2d\left(\cot\left(x^2\right)\right)^{1/2}}{dx}$$

$$=2.\frac{1}{2}\left\{\cot\left(x^2\right)\right\}^{\frac{-1}{2}}.\frac{d}{dx}\cot\left(x^2\right)$$

$$=\frac{1}{\sqrt{\cot\left(x^2\right)}}.\left\{-\operatorname{cosec}\left(x^2\right)\right\}\frac{dx^2}{dx}\ \frac{-2x\operatorname{cosec}(x)^2}{\sqrt{\cot(x)^2}}$$

Ex 5.2 Class 12 Maths Question 8.

cos(√x) = y(say)

Solution:

cos(√x) = y(say)

$$\frac{dy}{dx}=\frac{d}{dx}cos\left(\sqrt{x}\right)=-sin\sqrt{x}.\frac{d\sqrt{x}}{dx}$$

$$=-sin\sqrt{x}.\frac{1}{2}(x)^{-\frac{1}{2}}=\frac{-sin\sqrt{x}}{2\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{[(1+h)-1]-(1-1)}{h}=\lim_{h\to 0}\frac{h}{h}=1$$

$$\text{L.H.D., at } x=1 \ =\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$$

$$=\lim_{h\to 0}\frac{1-(1-h)-(1-1)}{-h}=\lim_{h\to 0}\frac{h}{-h}=-1$$

R.H.D. $\neq$ L.H.D. $\Rightarrow$ f is not differentiable at $x=1$.

Ex 5.2 Class 12 Maths Question 9.

Prove that the function f given by f (x) = |x – 1|,x ∈ R is not differential at x = 1.

Solution:

The given function may be written as

$$f(x)=\begin{cases}x-1, & if \quad x\geq 1\\ 1-x, & if \quad x<1\end{cases}$$

$$R.H.D \quad at \quad x=1 \quad =\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

Ex 5.2 Class 12 Maths Question 10.

Prove that the greatest integer function defined by f (x)=[x], 0 < x < 3 is not differential at x = 1 and x = 2.

Solution:

(i) At x = 1

$$R.H.D=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h \to 0} \frac{1-1}{h} = 0 \quad \because [1+h] = 1$$

$$\text{L.H.D.} = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{0-1}{-h} = \text{not defined} \quad \because [1-h] = 1$$

$\therefore$ f is not differentiable at x = 1.

(ii) At x = 3 $\text{R.H.D.} = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$

$$= \lim_{h \to 0} \frac{3-3}{h} = 0; \ \text{L.H.D.} = \lim_{h \to 0} \frac{f(3-h) - f(3)}{-h}$$

$$= \lim_{h \to 0} \frac{2-3}{h} = \lim_{h \to 0} \frac{-1}{h} = \text{Not defined.}$$

R.H.D. $\neq$ L.H.D. $\Rightarrow$ f is not differentiable at x = 3.

Ex 5.3 Class 12 Maths Question 1.

2x + 3y = sinx

Solution:

2x + 3y = sinx

Differentiating w.r.t x,

$$2 + 3\frac{dy}{dx} = cosx$$

=>

$$\frac{dy}{dx} = \frac{1}{3}(cosx - 2)$$

Ex 5.3 Class 12 Maths Question 2.

2x + 3y = siny

Solution:

2x + 3y = siny

Differentiating w.r.t x,

$$2 + 3.\frac{dy}{dx} = cosy\frac{dy}{dx}$$

=>

$$\frac{dy}{dx} = \frac{2}{cosy-3}$$

Ex 5.3 Class 12 Maths Question 3.

ax + by² = cosy

Solution:

ax + by² = cosy

Differentiate w.r.t. x,

=>

$$or \quad (2b + siny)\frac{dy}{dx} = -a => \frac{dy}{dx} = -\frac{a}{2b+siny}$$

Differentiating w.r.t. x , $a + 2 by \frac{dy}{dx} = - \sin y$

$\frac{dy}{dx}$

or $(2b + \sin y) \frac{dy}{dx} = - a$

$\therefore \frac{dy}{dx} = - \frac{a}{2b + \sin y}$

Ex 5.3 Class 12 Maths Question 4.

$xy + y^2 = \tan x + y$

Solution:

$xy + y^2 = \tan x + y$

Differentiating w.r.t. x,

$$\left(1 \cdot y + x \frac{dy}{dx}\right) + \left(2y \frac{dy}{dx}\right) = \sec^2 x + \frac{dy}{dx}$$

or $(x + 2y - 1) \frac{dy}{dx} = \sec^2 x - y$

$\therefore \frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}$

Ex 5.3 Class 12 Maths Question 5.

$x^2 + xy + y^2 = 100$

Solution:

$x^2 + xy + xy = 100$

Differentiating w.r.t. x,

$$2x+\left(1\cdot y+x\frac{dy}{dx}\right)+2y\frac{dy}{dx}=0$$

$$\text{or}\quad (x+2y)\frac{dy}{dx}=-2x-y\quad \therefore \frac{dy}{dx}=-\frac{2x+y}{x+2y}$$

Ex 5.3 Class 12 Maths Question 6.

$x^3 + x^2y + xy^2 + y^3 = 81$

Solution:

Given that

$x^3 + x^2y + xy^2 + y^3 = 81$

Differentiating both sides we get

$$3x^2+x^2\frac{dy}{dx}+y(2x)+y^2+x\left(2y\frac{dy}{dx}\right)+3y^2\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}[x^2+2xy+3y^2]=-\left(3x^2+2xy+y^2\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$$

Ex 5.3 Class 12 Maths Question 7.

$\sin^2 y + \cos xy = \pi$

Solution:

Given that

$\sin^2 y + \cos xy = \pi$

Differentiating both sides we get

$$2 \quad sin \quad y\frac{d \quad siny}{dx} + (-sinxy)\frac{d(xy)}{dx} = 0$$

$$\Rightarrow \quad 2\sin y\cos y\frac{dy}{dx} + (-\sin xy)\left[x\frac{dy}{dx} + y\right] = 0$$

$$\Rightarrow \frac{dy}{dx}\,[2 \sin y \cos y - x \sin xy] = y \sin xy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sin xy}{2\sin y\cos y - x\sin xy}$$

$$= \frac{y\sin xy}{\sin 2y - x\sin xy}$$

Ex 5.3 Class 12 Maths Question 8.

$\sin^2 x + \cos^2 y = 1$

Solution:

Given that

$\sin^2 x + \cos^2 y = 1$

Differentiating both sides, we get

$$2 \sin x\frac{d\sin x}{dx} + 2\cos y\frac{d\cos y}{dx} = 0$$

$$\Rightarrow \quad 2\sin x\cos x + 2\cos y(-\sin y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \sin 2x - \sin 2y\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

Ex 5.3 Class 12 Maths Question 9.

$$y = sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Solution:

$$y = sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

put x = tanθ

$$y = sin^{-1}\left(\frac{2tan\theta}{1+tan^2\theta}\right) = sin^{-1}(sin2\theta) = 2\theta$$

$$y = 2sin^{-1}x \quad \therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

Ex 5.3 Class 12 Maths Question 10.

$$y = tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Solution:

$$y = tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

put x = tanθ

$$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta) = 3\theta$$

$$y = 3\tan^{-1}x \quad \therefore \frac{dy}{dx} = \frac{3}{1+x^2}$$

Ex 5.3 Class 12 Maths Question 11.

$$y = cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

Solution:

$$y = cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

put x = tanθ

$$y = cos^{-1}\left(\frac{1-tan^2\quad\theta}{1+tan^2\quad\theta}\right) = cos^{-1}(cos2\theta) = 2\theta$$

Ex 5.3 Class 12 Maths Question 12.

Solution:

put x = tanθ

we get

Ex 5.3 Class 12 Maths Question 13.

$$y = cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$$

Solution:

$$y = cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$$

put x = tanθ

we get

$$y = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$y = \cos^{-1}(\sin 2\theta)$$

$$y = \cos^{-1}\left\{\cos\left(\frac{\pi}{2} - 2\theta\right)\right\}$$

$$y = \frac{\pi}{2} - 2\theta$$

$$\left[\begin{array}{l} \because\ -1 < x < 1 \Rightarrow -1 < \tan\theta < 1 \\ \Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4} \end{array}\right]$$

$$y = \frac{\pi}{2} - 2\tan^{-1} x \quad \left[\begin{array}{l} \Rightarrow -\frac{\pi}{2} < 2\theta < \frac{\pi}{2} \\ \Rightarrow 0 < \frac{\pi}{2} - 2\theta < \pi \end{array}\right]$$

$$\frac{dy}{dx} = -\frac{2}{1+x^2}$$

Ex 5.3 Class 12 Maths Question 14.

$$y = sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

Solution:

$$y = sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

put x = tanθ

we get

$$y = sin^{-1}(2sin\theta \quad cos\theta) \quad = sin^{-1}(sin2\theta) \quad = 2\theta$$

$$y = 2sin^{-1}x \quad \therefore \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

Ex 5.3 Class 12 Maths Question 15.

$$y = sin^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$$

Solution:

put x = tanθ

we get

Ex 5.4 Class 12 Maths Question 1.

Solution:

Ex 5.4 Class 12 Maths Question 2.

$e^{sin^{-1}x}$

Solution:

x=sint

Ex 5.4 Class 12 Maths Question 3.

$$e^{x^3} = y$$

Solution:

$$e^{x^3} = y \; Put \quad x^3 = t \quad \therefore \quad y = e^t, \frac{dy}{dt} = e^t, \frac{dt}{dx} = 3x^2$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = e^t \times 3x^2 = 3x^2 e^{x^3}$$

Ex 5.4 Class 12 Maths Question 4.

Solution:

$$sin\left(tan^{-1}e^{-x}\right) = y$$

Ex 5.4 Class 12 Maths Question 5.

Solution:

https://www.indcareer.com/schools/ncert-solutions-for-12th-class-maths-chapter-5-continuity-and-differentiability/

$$\frac{dy}{dx} = \frac{1}{cos \quad e^x}\left(-sine^x\right).e^x \quad = -tan\left(e^x\right)$$

Ex 5.4 Class 12 Maths Question 6.

$e^x + e^{x^2} +$

...

Solution:

$$let \quad u = e^{x^n}, put \quad x^n = t, u = e^t, t = x^n$$

 $...+e^{x^5} = y(say)$

Ex 5.4 Class 12 Maths Question 7.

$\sqrt{e^{\sqrt{x}}}, x > 0$

Solution:

y =

$\sqrt{e^{\sqrt{x}}}, x > 0$

Ex 5.4 Class 12 Maths Question 8.

log(log x),x>1

Solution:

y = log(log x),

put y = log t, t = log x,

differentiating

Ex 5.4 Class 12 Maths Question 9.

Solution:

let

$$y = \frac{cosx}{logx}$$

Ex 5.4 Class 12 Maths Question 10.

cos(log x+e^x),x>0

Solution:

y = cos(log x+e^x),x>0

put y = cos t,t = log x+e^x

$$\frac{dy}{dt} = -\sin t,\ \frac{dt}{dx} = \frac{1}{x} + e^x$$

$$\text{Now } \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \quad = -\sin t\left(\frac{1}{x} + e^x\right)$$

$$\therefore \quad \frac{dy}{dx} = -\frac{1}{x}(1 + x\,e^x)\sin(\log x + e^x)$$

Differentiate the functions given in Questions 1 to 11 w.r.to x

Ex 5.5 Class 12 Maths Question 1.

cos x. cos 2x. cos 3x

Solution:

Let y = cos x. cos 2x . cos 3x,

Taking log on both sides,

log y = log (cos x. cos 2x. cos 3x)

log y = log cos x + log cos 2x + log cos 3x,

Differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}\frac{d}{dx}\cos x + \frac{1}{\cos 2x}\frac{d}{dx}\cos 2x + \frac{1}{\cos 3x}\frac{d}{dx}\cos 3x$$

$$= \frac{-\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}$$

$$= -(\tan x + 2\tan 2x + 3\tan 3x)$$

$$\therefore \frac{dy}{dx} = -y\ (\tan x + 2\tan 2x + 3\tan 3x)$$

$$\therefore \frac{dy}{dx} = -\cos x \,.\, \cos 2x \,.\, \cos 3x\ (\tan x + 2\tan 2x + 3\tan 3x)$$

Ex 5.5 Class 12 Maths Question 2.

$$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Solution:

$$y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

taking log on both sides

log y = log

$$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

$$\log y=\frac{1}{2}[\log(x-1)+\log(x-2)$$
$$-\log(x-3)-\log(x-4)-\log(x-5)]$$

Differentiating w.r.t. x ,

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2}\left[\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right]$$

$$\therefore \quad \frac{dy}{dx}=\frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$
$$\left[\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right]$$

Ex 5.5 Class 12 Maths Question 3.

$(\log x)^{\cos x}$

Solution:

let $y = (\log x)^{\cos x}$

Taking log on both sides,

$\log y = \log (\log x)^{\cos x}$

$\log y = \cos x \log (\log x)$,

Differentiating w.r.t. x,

$$\frac{1}{y}\frac{dy}{dx} = (-\sin x)\log(\log x) + \cos x\,\frac{d}{dx}\log(\log x)$$

$$= -\sin x \log(\log x) + \frac{\cos x}{x \log x}$$

$$\therefore \frac{dy}{dx} = y\left[\sin x \log(\log x) + \frac{\cos x}{x \log x}\right]$$

$$= (\log x)^{\cos x}\left[-\sin \log(\log x) + \frac{\cos x}{x \log x}\right]$$

Ex 5.5 Class 12 Maths Question 4.

$x - 2^{\sin x}$

Solution:

let $y = x - 2^{\sin x}$,

$y = u - v$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} \quad u = x^x,$$

Taking log on both side

$$\log u = x \log x, \ \frac{1}{u}\frac{du}{dx} = 1 \cdot \log x + x \cdot \frac{1}{x}$$
$$= (1 + \log x)$$

$$\frac{du}{dx} = x^x(1 + \log x) \ \text{ and } \ v = 2^{\sin x},$$

Taking log on both side

$$\log v = \sin x \log 2, \ \frac{1}{v}\frac{dv}{dx} = \cos x \log 2,$$

$$\frac{dv}{dx} = 2^{\sin x} \cos x \log 2$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x) - 2^{\sin x} \cos x \log 2.$$

Ex 5.5 Class 12 Maths Question 5.

$(x+3)^2.(x + 4)^3.(x + 5)^4$

Solution:

let $y = (x + 3)^2.(x + 4)^3.(x + 5)^4$

Taking log on both side,

$\log y = \log [(x + 3)^2 \cdot (x + 4)^3 \cdot (x + 5)^4]$

$= \log (x + 3)^2 + \log (x + 4)^3 + \log (x + 5)^4$

$\log y = 2 \log (x + 3) + 3 \log (x + 4) + 4 \log (x + 5)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = y\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$$

$$= (x + 3)^2 \cdot (x + 4)^2 \cdot (x + 5)^4$$

$$\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right].$$

Ex 5.5 Class 12 Maths Question 6.

$$\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$$

Solution:

let

$$y = \left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$$

let

$$u = \left(x+\frac{1}{x}\right)^x and \quad v = x^{\left(1+\frac{1}{x}\right)}$$

$$\therefore \quad y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(i)$$

$$\text{Now } u = \left(x + \frac{1}{x}\right)^x$$

Taking log on both side, $\log u = x \log\left(x + \frac{1}{x}\right)$

$$\Rightarrow \log u = x \log\left(\frac{x^2+1}{x}\right)$$

$\log u = x\,[\log(x^2+1) - \log x]$,
Differentiating w.r.t. x , we get

$$\frac{1}{u}\frac{du}{dx} = 1\cdot[\log(x^2+1) - \log x] + x$$

$$\left[\frac{1}{x^2+1}\cdot 2x - \frac{1}{x}\right]$$

$$= \log\left(x + \frac{1}{x}\right) + \frac{x^2-1}{x^2+1} \quad ,$$

$$\frac{du}{dx} = u\left[\log\left(x+\frac{1}{x}\right)+\frac{x^2-1}{x^2+1}\right]$$

$$\frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\log\left(x+\frac{1}{x}\right)+\frac{x^2-1}{x^2+1}\right],$$

and $v = x^{\left(1+\frac{1}{x}\right)}$

Taking log on both sides, we get

$$\log v = \left(1+\frac{1}{x}\right)\log x\ ,$$

$$\frac{1}{v}\frac{dv}{dx} = -\frac{1}{x^2}\log x + \left(1+\frac{1}{x}\right)\cdot\frac{1}{x}$$

$$\frac{dv}{dx} = v\,\frac{(x+1-\log x)}{x^2}\ ,$$

$$\frac{dv}{dx} = \frac{x^{\left(1+\frac{1}{x}\right)}(x+1-\log x)}{x^2}$$

Put in (i)

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\log\left(x+\frac{1}{x}\right)+\frac{x^2-1}{x^2+1}\right] + \frac{x^{\left(1+\frac{1}{x}\right)}(x+1-\log x)}{x^2}$$

Ex 5.5 Class 12 Maths Question 7.

$(\log x)^x + x^{\log x}$

Solution:

let y = $(\log x)^x + x^{\log x}$ = u+v

where u = $(\log x)^x$

∴ log u = x log(log x)

$$\frac{1}{u}\cdot\frac{du}{dx} = x\frac{d}{dx}\log(\log x) + \log(\log x)$$

$$= x\cdot\frac{1}{\log x}\cdot\frac{1}{x} + \log(\log x)$$

$$= \frac{1}{\log x} + \log(\log x)$$

$$\frac{du}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] \text{ and}$$

$$v = x^{\log x}$$

$$\log v = \log x \log x = (\log x)^2$$

$$\frac{1}{v}\frac{dv}{dx} = 2\log x\cdot\frac{1}{x}$$

$$\frac{dv}{dx} = x^{\log x}\left[\frac{2\log x}{x}\right]$$

$$\therefore \quad y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{2\log x}{x}\right]$$

Ex 5.5 Class 12 Maths Question 8.

$(\sin x)^x + \sin^{-1}\sqrt{x}$

Solution:

Let $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

let $u = (\sin x)x$, $v = \sin^{-1}\sqrt{x}$

$$\therefore\ y = u + v \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}, \qquad ...(1)$$

Now $u = (\sin x)^x$

Taking log on both sides, $\log u = x \log(\sin x)$,

$$\frac{1}{u}\frac{du}{dx} = \log(\sin x) + x \cdot \frac{1}{\sin x}\cos x$$

$$= \log(\sin x) + x\frac{\cos x}{\sin x},$$

$$\frac{du}{dx} = (\sin x)^x\,[\log(\sin x) + x\cot x] \quad \text{and}$$

$v = \sin^{-1}\sqrt{x}$

$$\text{Taking } t = \sqrt{x}, \quad v = \sin^{-1} t, \quad \frac{dv}{dt} = \frac{1}{\sqrt{1-t^2}}$$

$$\text{and } \frac{dt}{dx} = \frac{1}{2\sqrt{x}} \qquad \therefore\ \frac{dv}{dx} = \frac{dv}{dt} \times \frac{dt}{dx}$$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$\text{Put (1) we get } \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = (\sin x)^x [\log(\sin x) + x \cot x]$$

$$+ \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

Ex 5.5 Class 12 Maths Question 9.

$x^{\sin x} + (\sin x)^{\cos x}$

Solution:

let $y = x^{\sin x} + (\sin x)^{\cos x} = u+v$

where $u = x^{\sin x}$

$\log u = \sin x \log x$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \sin x \frac{d \log x}{dx} + \log x \frac{d \sin x}{dx}$$

$$= \sin x \cdot \frac{1}{x} + \log x \cos x$$

$$\frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x\right]$$

and $v = (\sin x)^{\cos x}$

$\log v = \cos x \log \sin x$

$$\frac{1}{v}\frac{dv}{dx} = \cos x \frac{d}{dx} \log \sin x$$

$$+ \log \sin x \frac{d}{dx} \cos x$$

$$= \cos x \frac{1}{\sin x} . \cos x +$$

$$\log \sin x(-\sin x)$$

$$= \cos x \cot x - \sin x \log \sin x$$

$$\therefore \quad y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\therefore \quad \frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x\right]$$

$$+ (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$$

Ex 5.5 Class 12 Maths Question 10.

$$x^{x} \quad cosx + \frac{x^2+1}{x^2-1}$$

Solution:

$$y = x^{x} \quad cosx + \frac{x^2+1}{x^2-1}$$

y = u + v

$$\therefore \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots(1)$$

Now $u = x^{x\cos x}$

Taking log on both sides, $\log u = x \cos x \log x$,

$$\frac{du}{dx} = x^{x\cos x}[\cos x \log x - x \sin x \log x + \cos x]$$

$$\text{and } v = \frac{x^2+1}{x^2-1}, \quad \frac{dv}{dx} = \frac{-4x}{(x^2-1)^2}$$

$$\text{Put in (1).} \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\therefore \quad \frac{dy}{dx} = x^{x\cos x}[\cos x \log x - x \sin x \log x + \cos x]$$

$$-\frac{4x}{(x^2-1)^2}$$

Ex 5.5 Class 12 Maths Question 11.

Solution:

Let u = $(x \cos x)^x$

logu = x log(x cosx)

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(x\cos x) + \log(x\cos x)$$

$$= x\left[\frac{1}{x\cos x}\frac{d(x\cos x)}{dx}\right] + \log(x\cos x)$$

$$= x\left[\frac{1}{x\cos x}\left(x\frac{d\cos x}{dx} + \cos x\right)\right] + \log(x\cos x)$$

$$= x\left[\frac{1}{x\cos x}(x(-\sin x) + \cos x)\right] + \log(x\cos x)$$

$$= 1 - x\tan x + \log(x\cos x)$$

$$\frac{du}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)]$$

Now $v = (x\sin x)^{1/x}$

$$\log v = \frac{1}{x}\log(x\sin x)$$

Find of the functions given in Questions 12 to 15.

Ex 5.5 Class 12 Maths Question 12.

$x^y + y^x = 1$

Solution:

$x^y + y^x = 1$

let $u = x^y$ and $v = y^x$

$\therefore$ $u + v = 1$,

Now u = x

Ex 5.5 Class 12 Maths Question 13.

$y^x = x^y$

Solution:

y = x

x logy = y logx

Ex 5.5 Class 12 Maths Question 14.

$(\cos x)^y = (\cos y)^x$

Solution:

We have

$(\cos x)^y = (\cos y)^x$

=> y log (cosx) = x log (cosy)

Ex 5.5 Class 12 Maths Question 15.

$xy = e^{(x-y)}$

Solution:

$\log(xy) = \log e^{(x-y)}$

=> $\log(xy) = x – y$

=> $\log x + \log y = x – y$

Ex 5.5 Class 12 Maths Question 16.

Find the derivative of the function given by $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$ and hence find $f'(1)$.

Solution:

Let $f(x) = y = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$

Taking log both sides, we get

$\log y = \log [(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)]$

$\log y = \log(1 + x) + \log(1 + x^2) + \log(1 + x^4) + \log(1 + x^8)$

Ex 5.5 Class 12 Maths Question 17.

Differentiate $(x^2 – 5x + 8) (x^3 + 7x + 9)$ in three ways mentioned below:

(i) by using product rule

(ii) by expanding the product to obtain a single polynomial.

(iii) by logarithmic differentiation.

Do they all give the same answer?

Solution:

(i) By using product rule

$f' = (x^2 – 5x + 8) (3x^2 + 7) + (x^3 + 7x + 9) (2x – 5)$

$f = 5x^4 – 20x^3 + 45x^2 – 52x + 11.$

(ii) By expanding the product to obtain a single polynomial, we get

Ex 5.5 Class 12 Maths Question 18.

If u, v and w are functions of w then show that

in two ways-first by repeated application of product rule, second by logarithmic differentiation.

Solution:

Let y = u.v.w

=> y = u. (vw)

If x and y are connected parametrically by the equations given in Questions 1 to 10, without eliminating the parameter. Find .

Ex 5.6 Class 12 Maths Question 1.

x = 2at², y = at⁴

Solution:

$$\frac{dx}{dt} = 4at, \frac{dy}{dt} = 4at^3 \quad \therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4at^3}{4at} = t^2$$

Ex 5.6 Class 12 Maths Question 2.

x = a cosθ,y = b cosθ

Solution:

$$\frac{dx}{d\theta} = -asin\theta, \frac{dy}{d\theta} = -sinb \quad sin\theta => \frac{dy}{dx} = \frac{b}{a}$$

Ex 5.6 Class 12 Maths Question 3.

x = sin t, y = cos 2t

Solution:

$$\therefore \frac{dx}{dt} = cos \quad t \quad and \frac{dy}{dt} = -sin2t.2 = -2sin2t$$

$$\frac{dy}{dx} = \frac{-2sin2t}{cost} = \frac{-2.2sintcost}{cost} = -4sint$$

Ex 5.6 Class 12 Maths Question 4.

Solution:

$$\frac{dx}{dt} = 4; \frac{dy}{dt} = \frac{-4}{t^2} => \frac{dy}{dx} = \frac{-4}{t^2} \times \frac{1}{4} = \frac{-1}{t^2}$$

Ex 5.6 Class 12 Maths Question 5.

x = cos θ – cos 2θ, y = sin θ – sin 2θ

Solution:

Ex 5.6 Class 12 Maths Question 6.

x = a(θ – sinθ), y = a(1 + cosθ)

Solution:

Ex 5.6 Class 12 Maths Question 7.

Solution:

Ex 5.6 Class 12 Maths Question 8.

Solution:

Ex 5.6 Class 12 Maths Question 9.

x = a sec θ,y = b tan θ

Solution:

x = a sec θ,y = b tan θ

Ex 5.6 Class 12 Maths Question 10.

x = a(cosθ+θsinθ), y = a(sinθ-θcosθ)

Solution:

x = a(cosθ+θsinθ), y = a(sinθ-θcosθ)

Ex 5.6 Class 12 Maths Question 11.

If

show that

Solution:

Given that

Ex 5.7 Class 12 Maths Question 1.

x² + 3x + 2 = y(say)

Solution:

$$\frac{dy}{dx} = 2x + 3 \quad and \frac{d^2y}{dx^2} = 2$$

Ex 5.7 Class 12 Maths Question 2.

x^{20} = y(say)

Solution:

$$\frac{dy}{dx} = 20x^{19} \quad => \frac{d^2y}{dx^2} = 20 \times 19x^{18} = 380x^{18}$$

Ex 5.7 Class 12 Maths Question 3.

x.cos x = y(say)

Solution:

$$\frac{dy}{dx} = x(-sinx) + cosx.1, = -xsinx + cosx$$

$$\frac{d^2y}{dx^2} = -xcosx - sinx - sinx = -xcosx - 2sinx$$

Ex 5.7 Class 12 Maths Question 4.

log x = y (say)

Solution:

$$\frac{dy}{dx} = \frac{1}{x} => \frac{d^2y}{dx^2} = -\frac{1}{x^2}$$

Ex 5.7 Class 12 Maths Question 5.

x^3 log x = y (say)

Solution:

x^3 log x = y

Ex 5.7 Class 12 Maths Question 6.

e^x sin5x = y

Solution:

e^x sin5x = y

Ex 5.7 Class 12 Maths Question 7.

e^{6x} cos3x = y

Solution:

e^{6x} cos3x = y

Ex 5.7 Class 12 Maths Question 8.

$\tan^{-1} x = y$

Solution:

Ex 5.7 Class 12 Maths Question 9.

$\log(\log x) = y$

Solution:

$\log(\log x) = y$

Ex 5.7 Class 12 Maths Question 10.

$\sin(\log x) = y$

Solution:

$\sin(\log x) = y$

Ex 5.7 Class 12 Maths Question 11.

If y = 5 cosx – 3 sin x, prove that

Solution:

$$\frac{d^2y}{dx^2} = -5cosx + 3sinx = -y$$

Hence proved

Ex 5.7 Class 12 Maths Question 12.

If y = $\cos^{-1}$ x, Find

$$\frac{d^2y}{dx^2}$$

in terms of y alone.

Solution:

$$\frac{dy}{dx} = -\left(1 - x^2\right)^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{-cosy}{\left(sin^2y\right)^{\frac{3}{2}}} = -coty \quad cosec^2y$$

Ex 5.7 Class 12 Maths Question 13.

If y = 3 cos (log x) + 4 sin (log x), show that

$$x^2y_2 + xy_1 + y = 0$$

Solution:

Given that

y = 3 cos (log x) + 4 sin (log x)

Ex 5.7 Class 12 Maths Question 14.

Solution:

Given that

Ex 5.7 Class 12 Maths Question 15.

If y = $500e^{7x} + 600e^{-7x}$, show that

$$\frac{d^2y}{dx^2} = 49y$$

.

Solution:

we have

$y = 500e^{7x} + 600e^{-7x}$

Ex 5.7 Class 12 Maths Question 16.

If $e^y(x+1) = 1$,show that

Solution:

Ex 5.7 Class 12 Maths Question 17.

If $y=(\tan^{-1} x)^2$ show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$

Solution:

we have

$y=(\tan^{-1} x)^2$

$$\frac{dy}{dx} = y_1 = 2\tan^{-1}x \cdot \frac{1}{1+x^2}$$

$$y_1 = \frac{2\tan^{-1}x}{1+x^2} \Rightarrow (1+x^2)\,y_1 = 2\tan^{-1}x$$

$$\text{Again } (1+x^2)\,y_2 + y_1 \cdot 2x = \frac{2}{1+x^2}$$

$$\Rightarrow \quad (1+x^2)^2 y_2 + 2x(1+x^2)\,y_1 = 2. \text{ Hence proved.}$$

Ex 5.8 Class 12 Maths Question 5.

Verify Mean Value Theorem, if f $(x)=x^3 - 5x^2 - 3x$ in the interval [a, b], where a = 1 and b = 3. Find all c ∈ (1,3) for which f' (c) = 0.

Solution:

f $(x)=x^3 - 5x^2 - 3x$,

It is a polynomial. Therefore it is continuous in the interval [1,3] and derivable in the interval (1,3)

Also, $f'(x)=3x^2-10x-3$

$$\text{or} \quad f'(c) = 3c^2 - 10c - 3, \quad f'(c) = \frac{f(b) - f(a)}{b-a}$$

$$f(1) = 1 - 5 - 3 = -7, \quad f(3) = 27 - 45 - 9 = -27$$

$$\text{or} \quad (c-1)(3c-7) = 0 \;\therefore\; c = 1 \text{ and } \frac{7}{3}$$

$$\text{Now } \frac{7}{3} \in (1,3), \;\therefore\; c = \frac{7}{3}$$

$$f'(c) = 0 \;\Rightarrow 3c^2 - 10c - 3 \neq 0 \;\therefore\; c = \frac{5 \pm \sqrt{34}}{3}$$

$$c = 3{\cdot}61, -0{\cdot}28 \quad \text{None of these values} \in (1,3)$$

Ex 5.8 Class 12 Maths Question 6.

Examine the applicability of Mean Value theroem for all three functions given in the above Question 2.

Solution:

(i) F (x)= [x] for x ∈ [5,9], f (x) = [x] in the interval [5, 9] is neither continuous, nor differentiable.

(ii) f (x) = [x], for x ∈ [-2,2],

Again f (x) = [x] in the interval [-2,2] is neither continuous, nor differentiable.

(iii) f(x) = x^2-1 for x ∈ [1,2], It is a polynomial. Therefore it is continuous in the interval [1,2] and differentiable in the interval (1,2)

f (x) = 2x, f(1) = 1 – 1 = 0 ,

f(2) = 4 – 1 = 3, f'(c) = 2c

$$\therefore \quad f'(c) = \frac{f(b) - f(a)}{b - a}, \quad 2c = \frac{3-0}{2-1} = \frac{3}{1}$$

$$\therefore \quad c = \frac{3}{2} \text{ which belong to } (1, 3)$$

Chapterwise NCERT Solutions for Class 12 Maths :

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