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Exercise 3.1

1. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write

(i) The order of the matrix (ii) The number of elements

(iii) Write the elements a_{13} , a_{21} , a_{33} , a_{24} & a_{23} .

Sol. (i) There are 3 horizontal lines (rows) and 4 vertical lines (columns) in the given matrix A.

\therefore Order of the matrix A is 3×4 .

(ii) The number of elements in this matrix A is $3 \times 4 = 12$.

(\because The number of elements in a $m \times n$ matrix is $m \cdot n$)

(iii) $a_{13} \Rightarrow$ Element in first row and third column = 19

$a_{21} \Rightarrow$

$a_{33} \Rightarrow$ Element in third row and third column = - 5

$a_{24} \Rightarrow$ Element in second row and fourth column = 12

$a_{23} \Rightarrow$ Element in second row and third column $\frac{5}{2}$.

2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Sol. We know that a matrix having mn elements is of order $m \times n$.

(i) Now $24 = 1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6$ and hence
 $= 24 \times 1, 12 \times 2, 8 \times 3, 6 \times 4$ also.

\therefore There are 8 possible matrices having 24 elements of orders
 $1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6, 24 \times 1, 12 \times 2, 8 \times 3, 6 \times 4$.

(ii) Again (prime number) $13 = 1 \times 13$ and 13×1 only.

\therefore There are 2 possible matrices of order 1×13 (Row matrix)
 and 13×1 (Column matrix)

3. If a matrix has 18 elements, what are the possible orders it can have? What if has 5 elements?

Sol. We know that a matrix having mn elements is of order $m \times n$.

(i) Now $18 = 1 \times 18, 2 \times 9, 3 \times 6$ and hence $18 \times 1, 9 \times 2,$
 6×3 also.

∴ There are 6 possible matrices having 18 elements of orders 1×18 , 2×9 , 3×6 , 18×1 , 9×2 and 6×3 .

(ii) Again (Prime number) $5 = 1 \times 5$ and 5×1 only.

∴ There are 2 possible matrices of order 1×5 and 5×1 .

4. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by:

$$(i) a_{ij} = \frac{(i+j)^2}{2} \quad (ii) a_{ij} = \frac{i}{j} \quad (iii) a_{ij} = \frac{(i+2j)^2}{2}$$

Sol. To construct a 2×2 matrix $A_{ij} = [a_{ij}]$

$$(i) \text{ Given: } a_{ij} = \frac{(i+j)^2}{2} \quad \dots(i)$$

In (i),

$$\text{Put } i = 1, j = 1, \quad \therefore a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$$

$$\text{Put } i = 1, j = 2, \quad \therefore a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2}$$

$$\text{Put } i = 2, j = 1; \quad \therefore a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

$$\text{Put } i = 2, j = 2; \quad \therefore a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$$

$$\therefore A_{2 \times 2} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

$$(ii) \text{ Given: } a_{ij} = \frac{i}{j} \quad \dots(ii)$$

In (ii),

$$\text{Put } i = 1, j = 1, \quad \therefore a_{11} = \frac{1}{1} = 1$$

$$\text{Put } i = 1, j = 2, \quad \therefore a_{12} = \frac{1}{2}$$

$$\text{Put } i = 2, j = 1; \quad \therefore a_{21} = \frac{2}{1} = 2$$

$$\text{Put } i = 2, j = 2; \quad \therefore a_{22} = \frac{2}{2} = 1$$

$$\therefore A_{2 \times 2} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

$$(iii) \text{ Given: } a_{ij} = \frac{(i+2j)^2}{2} \quad \dots(iii)$$

In (iii),

$$\text{Put } i = 1, j = 1: \quad \therefore a_{11} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2}$$

$$\text{Put } i = 1, j = 2: \quad \therefore a_{12} = \frac{(1+4)^2}{2} = \frac{5^2}{2} = \frac{25}{2}$$

$$\text{Put } i = 2, j = 1: \quad \therefore a_{21} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

$$\text{Put } i = 2, j = 2: \quad \therefore a_{22} = \frac{(2+4)^2}{2} = \frac{6^2}{2} = \frac{36}{2} = 18$$

$$\therefore A_{2 \times 2} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

5. Construct a 3×4 matrix, whose elements are given by:

$$(i) a_{ij} = \frac{1}{2} |-3i + j| \quad (ii) a_{ij} = 2i - j.$$

Sol. (i) To construct a 3×4 matrix say A.

$$\text{Given: } a_{ij} = \frac{1}{2} |-3i + j| \quad \dots(i)$$

In (i),

$$\text{Put } i = 1, j = 1,$$

$$\therefore a_{11} = \frac{1}{2} |-3 + 1| = \frac{1}{2} |-2| = \frac{1}{2} (2) = 1$$

$$\text{Put } i = 1, j = 2,$$

$$\therefore a_{12} = \frac{1}{2} |-3 + 2| = \frac{1}{2} |-1| = \frac{1}{2} (1) = \frac{1}{2}$$

$$i = 1, j = 3,$$

$$\therefore a_{13} = \frac{1}{2} |-3 + 3| = \frac{1}{2} |0| = \frac{1}{2} (0) = 0$$

$$i = 1, j = 4,$$

$$\therefore a_{14} = \frac{1}{2} |-3 + 4| = \frac{1}{2} |1| = \frac{1}{2} (1) = \frac{1}{2}$$

$$i = 2, j = 1,$$

$$\therefore a_{21} = \frac{1}{2} |-6 + 1| = \frac{1}{2} |-5| = \frac{5}{2}$$

$$i = 2, j = 2,$$

$$\therefore a_{22} = \frac{1}{2} |-6 + 2| = \frac{1}{2} |-4| = \frac{4}{2} = 2$$

$$i = 2, j = 3,$$

$$\therefore a_{23} = \frac{1}{2} |-6 + 3| = \frac{1}{2} |-3| = \frac{3}{2}$$

$$i = 2, j = 4,$$

$$\therefore a_{24} = \frac{1}{2} |-6 + 4| = \frac{1}{2} |-2| = \frac{2}{2} = 1$$

$$\begin{aligned}
 & i = 3, j = 1, \\
 \therefore a_{31} &= \frac{1}{2} |-9 + 1| = \frac{1}{2} |-8| = \frac{8}{2} = 4 \\
 & i = 3, j = 2, \\
 \therefore a_{32} &= \frac{1}{2} |-9 + 2| = \frac{1}{2} |-7| = \frac{7}{2} \\
 & i = 3, j = 3, \\
 \therefore a_{33} &= \frac{1}{2} |-9 + 3| = \frac{1}{2} |-6| = \frac{6}{2} = 3 \\
 & i = 3, j = 4, \\
 \therefore a_{34} &= \frac{1}{2} |-9 + 4| = \frac{1}{2} |-5| = \frac{5}{2} \\
 \therefore A_{3 \times 4} &= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 5 & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}
 \end{aligned}$$

(ii) Given: $a = 2i - j$

$$\begin{aligned}
 \therefore a_{11} &= 2 - 1 = 1, & a_{12} &= 2 - 2 = 0 \\
 a_{13} &= 2 - 3 = -1, & a_{14} &= 2 - 4 = -2 \\
 a_{21} &= 4 - 1 = 3, & a_{22} &= 4 - 2 = 2 \\
 a_{23} &= 4 - 3 = 1, & a_{24} &= 4 - 4 = 0 \\
 a_{31} &= 6 - 1 = 5, & a_{32} &= 6 - 2 = 4 \\
 a_{33} &= 6 - 3 = 3, & a_{34} &= 6 - 4 = 2
 \end{aligned}$$

$$\therefore A_{3 \times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

6. Find the values of x , y and z from the following equations:

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix} \quad (ii) \begin{bmatrix} x + y & 2 \\ 5 + z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$$(iii) \begin{bmatrix} x + y + z & 9 \\ x + z & 5 \\ y + z & 7 \end{bmatrix}$$

Sol. (i) Given: $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

By definition of Equal matrices, equating corresponding entries, we have $4 = y$, $3 = z$, $x = 1$, $5 = 5$
 $\therefore x = 1$, $y = 4$, $z = 3$.

(ii) Given: $\begin{bmatrix} x + y & 2 \\ 5 + z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Equating corresponding entries, we have

$$x + y = 6 \quad \dots(i)$$

$$5 + z = 5 \text{ i.e., } z = 5 - 5 = 0$$

and $xy = 8 \quad \dots(ii)$

Let us solve (i) and (ii) for x and y .

From (i), $y = 6 - x$

Putting this value of y in (ii), we have

$$x(6 - x) = 8 \text{ or } 6x - x^2 = 8$$

$$\text{or } -x^2 + 6x - 8 = 0 \text{ or } x^2 - 6x + 8 = 0$$

$$\text{or } x^2 - 4x - 2x + 8 = 0 \text{ or } x(x - 4) - 2(x - 4) = 0$$

$$\text{or } (x - 4)(x - 2) = 0$$

$$\therefore \text{ Either } x - 4 = 0 \text{ or } x - 2 = 0$$

$$\text{i.e., } x = 4 \text{ or } x = 2.$$

$$\text{When } x = 4, \text{ then } y = 6 - x = 6 - 4 = 2$$

$$\therefore x = 4, y = 2, z = 0.$$

$$\text{When } x = 2, \text{ then } y = 6 - x = 6 - 2 = 4$$

$$\therefore x = 2, y = 4, z = 0.$$

(iii) Given:
$$\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Equating corresponding entries, we have

$$x + y + z = 9 \quad \dots(i)$$

$$x + z = 5 \quad \dots(ii)$$

$$y + z = 7 \quad \dots(iii)$$

$$\text{Eqn. (i) - eqn. (ii) gives } y = 9 - 5 = 4$$

$$\text{Eqn. (i) - eqn. (iii) gives } x = 9 - 7 = 2$$

$$\text{Putting } x = 2 \text{ and } y = 4 \text{ in (i), } 2 + 4 + z = 9$$

$$\text{or } 6 + z = 9$$

$$\therefore z = 3$$

$$\text{Hence } x = 2, y = 4, z = 3.$$

7. Find the values of a, b, c and d from the equation

$$\begin{bmatrix} a - b & 2a + c \\ 2a - b & 3c + d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Sol. Equating corresponding entries of given equal matrices, we have

$$a - b = -1 \quad \dots(i)$$

$$2a - b = 0 \quad \dots(ii)$$

$$2a + c = 5 \quad \dots(iii)$$

and $3c + d = 13 \quad \dots(iv)$

$$\text{Eqn. (i) - eqn. (ii) gives } -a = -1 \text{ or } a = 1$$

$$\text{Putting } a = 1 \text{ in (i), } 1 - b = -1 \text{ or } -b = -2 \text{ or } b = 2$$

$$\text{Putting } a = 1 \text{ in (iii), } 2 + c = 5 \text{ } c = 5 - 2 = 3$$

$$\text{Putting } c = 3 \text{ in (iv), } 9 + d = 13 \text{ or } d = 13 - 9 = 4$$

$$\therefore a = 1, b = 2, c = 3, d = 4.$$

8. $A = [a_{ij}]_{m \times n}$ is a square matrix, if
 (A) $m < n$ (B) $m > n$ (C) $m = n$ (D) None of these.

Sol. (C) is the correct option.

(\because By definition of square matrix $m = n$)

9. Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

- (A) $x = \frac{-1}{3}, y = 7$ (B) Not possible to find
 (C) $y = 7, x = \frac{-2}{3}$ (D) $x = \frac{-1}{3}, y = \frac{-2}{3}$.

Sol. According to given, matrix $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \text{matrix} \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating corresponding entries, we have

$$3x + 7 = 0 \Rightarrow 3x = -7 \Rightarrow x = \frac{-7}{3} \quad \dots(i)$$

$$5 = y - 2 \Rightarrow 5 + 2 = y \Rightarrow y = 7$$

$$y + 1 = 8 \Rightarrow y = 8 - 1 = 7$$

$$\text{and } 2 - 3x = 4 \Rightarrow -3x = 2 \Rightarrow x = \frac{-2}{3} \quad \dots(ii)$$

The two values of $x = \frac{-7}{3}$ given by (i) and $x = \frac{-2}{3}$ given by (ii) are not equal.

\therefore No values of x and y exist to make the two matrices equal.

\therefore Option (B) is the correct answer.

10. The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

- (A) 27 (B) 18 (C) 81 (D) 512.

Sol. We know that general matrix of order 3×3 is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

This matrix has $3 \times 3 = 9$ elements.

The number of choices for a_{11} is 2 (as 0 or 1 can be used)

Similarly, the number of choices for each other element is 2.

Hence, total possible arrangements (matrices)

$$= \frac{2 \cdot 2 \cdot \dots \cdot 2}{9 \text{ times}} \quad (\text{By fundamental principle of counting})$$

$$= 2^9 = 512$$

\therefore Option (D) is the correct answer.

Exercise 3.2

1. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$.

Find each of the following:

- (i) $A + B$ (ii) $A - B$ (iii) $3A - C$
 (iv) AB (v) BA .

Sol. (i) $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

(ii) $A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

(iii) $3A - C = 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - C = \begin{bmatrix} 3 \cdot 2 & 3 \cdot 4 \\ 3 \cdot 3 & 3 \cdot 2 \end{bmatrix} - C$
 $= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

(iv) $AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

Performing row by column multiplication,

$$= \begin{bmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{bmatrix} = \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

(v) $BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

Performing row by column multiplication,

$$= \begin{bmatrix} 1(2)+3(3) & 1(4)+3(2) \\ (-2)(2)+5(3) & (-2)(4)+5(2) \end{bmatrix} = \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

Note. From solutions of part (iv) and (v), we can easily observe that AB need not be equal to BA i.e., matrix multiplication need not be commutative.

2. Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$$(iv) \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$\text{Sol. } (i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

$$(ii) \begin{bmatrix} a^2 + b^2 & b^2 + c^2 \\ a^2 + c^2 & a^2 + b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 + 2ab & b^2 + c^2 + 2bc \\ a^2 + c^2 - 2ac & a^2 + b^2 - 2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

$$(iii) \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

$$(iv) \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

3. Compute the indicated products:

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \quad (ii) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \ 3 \ 4]$$

$$(iii) \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \quad (iv) \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$(v) \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \quad (vi) \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Sol. (i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ is defined because the pre-matrix has

2 columns which is equal to the number of rows of the post-matrix.

Performing row by column multiplication,

$$= \begin{bmatrix} a^2 + b^2 & a(-b) + b^2 \\ (-b)a + ab & (-b)(-b) + a^2 \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & b^2 + a^2 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_3 \times \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}_{1 \times 3}$ is defined because the pre-matrix has one column which is equal to the number of rows of the post-matrix.

Performing row by column multiplication,

$$= \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}_{3 \times 3}$$

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1(1) + (-2)2 & 1(2) + (-2)3 & 1(3) + (-2)1 \\ 2(1) + 3(2) & 2(2) + 3(3) & 2(3) + 3(1) \end{bmatrix}$$

(Row by column multiplication)

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

Performing row by column multiplication

$$= \begin{bmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$ is defined because the pre-matrix has 2 columns which is equal to the number of rows of the post-matrix.

Performing row by column multiplication,

$$= \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ (-1)1+1(-1) & (-1)0+1(2) & (-1)1+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$(v) \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix}$$

(Row by column multiplication)

$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

4. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$,
 then compute $(A + B)$ and $(B - C)$. Also, verify that
 $A + (B - C) = (A + B) - C$.

Sol. $A + B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix}$

$$\Rightarrow A + B = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} \quad \dots(i)$$

Again $B - C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0+2 & 3-3 \end{bmatrix}$$

$$\Rightarrow B - C = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} \quad \dots(ii)$$

Putting the value of $(B - C)$ from (ii) in L.H.S.

$$= A + (B - C)$$

$$= \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-2 & -3+0 \\ 5+4 & 0-1 & 2+3 \\ 1+1 & -1+2 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \quad \dots(iii)$$

Putting the value of $(A + B)$ from (i) in R.H.S. $= (A + B) - C$

$$= \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 1-1 & -1-2 \\ 9-0 & 2-3 & 7-2 \\ 3-1 & -1+2 & 4-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \quad \dots (iv)$$

From (iii) and (iv), we have L.H.S. = R.H.S.

5. If $A = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 1 & 3 \\ 1 & 2 & 4 \\ 3 & 3 & 3 \\ 7 & 2 & 2 \\ 3 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 5 & 1 \\ 1 & 2 & 4 \\ 5 & 5 & 5 \\ 7 & 6 & 2 \\ 5 & 5 & 5 \end{bmatrix}$, then compute $3A - 5B$.

Sol. $3A - 5B = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 1 & 3 \\ 1 & 2 & 4 \\ 3 & 3 & 3 \\ 7 & 2 & 2 \\ 3 & 2 & 3 \end{bmatrix} - 5 \begin{bmatrix} 2 & 3 & 1 \\ 5 & 5 & 1 \\ 1 & 2 & 4 \\ 5 & 5 & 5 \\ 7 & 6 & 2 \\ 5 & 5 & 5 \end{bmatrix}$

Multiplying each entry of first matrix by 3 and each entry of second matrix by 5

$$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2-2 & 3-3 & 5-5 \\ 1-1 & 2-2 & 4-4 \\ 7-7 & 6-6 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Remark. Here answer is a zero matrix.

6. Simplify $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$.

Sol. $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

Multiplying each entry of first matrix by $\cos \theta$ and each entry of second matrix by $\sin \theta$

$$= \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix} + \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \sin \theta \cos \theta & \cos^2 \theta + \sin^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Remark. The answer matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ of this question is identity (unit) matrix I_2 .

7. Find X and Y if

$$(i) \ X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \text{ and } X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$(ii) \ 2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{ and } 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

Sol. (i) **Given:** $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$... (i)

and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$... (ii)

Adding eqns. (i) and (ii), we have

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} \frac{10}{2} & \frac{0}{2} \\ \frac{2}{2} & \frac{8}{2} \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Eqn. (i) - eqn. (ii) gives

$$2Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7-3 & 0-0 \\ 2-0 & 5-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\therefore Y = \frac{1}{2} \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{2} & \frac{0}{2} \\ \frac{2}{2} & \frac{2}{2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

(ii) **Given:** $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$... (i)

and $3X + 2Y = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$... (ii)

Multiplying equation (i) by 2, we have

$$4X + 6Y = 2 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \quad \dots (iii)$$

Multiplying equation (ii) by 3, we have

$$9X + 6Y = 3 \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -6 & -6 \\ -3 & 15 \end{bmatrix} \quad \dots (iv)$$

Equation (iv) - equation (iii) gives

$$5X = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} 6-4 & -6-6 \\ -3-8 & 15-0 \end{bmatrix} \\ = \begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix}$$

$$\therefore X = \frac{1}{5} \begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

Now from equation (j),

$$\begin{aligned} 3Y &= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2X \\ &= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2 \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} \\ &= \begin{bmatrix} 2 - \frac{4}{5} & 3 + \frac{24}{5} \\ 4 + \frac{22}{5} & 0 - 6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} \\ \Rightarrow Y &= \frac{1}{3} \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix} \end{aligned}$$

8. Find X if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$.

Sol. $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - Y$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

9. Find x and y, if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

Sol. Given: $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Equating corresponding entries, we have

$$2 + y = 5 \quad \text{and} \quad 2x + 2 = 8$$

$$\Rightarrow y = 5 - 2 = 3 \quad \text{and} \quad 2x = 8 - 2 = 6 \quad x = 3$$

$\therefore x = 3, y = 3.$

10. Solve the equation for x, y, z and t if

$$2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Sol. Given: $2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Since the two matrices are equal, so the corresponding elements are equal.

Thus, $2x + 3 = 9$

$$\Rightarrow 2x = 9 - 3 = 6 \Rightarrow x = 3$$

Also $2z - 3 = 15 \Rightarrow 2z = 18 \Rightarrow z = 9$

Also $2y = 12 \Rightarrow y = 6$

and $2t + 6 = 18$ and $2t = 12 \Rightarrow t = 6$

$\therefore x = 3, y = 6, z = 9$ and $t = 6$.

11. If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y .

Sol. Given: $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Equating corresponding entries, we have

$$2x - y = 10 \quad \dots(i)$$

$$\text{and } 3x + y = 5 \quad \dots(ii)$$

Adding eqns. (i) and (ii) we have $5x = 15$

or $x = \frac{15}{5} = 3$

Putting $x = 3$ in (ii), $9 + y = 5 \Rightarrow y = 5 - 9 = -4$

$\therefore x = 3, y = -4$.

12. Given: $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$; find the values of x, y, z and w .

Sol. Given: $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Equating corresponding entries, we have

$$3x = x + 4 \Rightarrow 2x = 4 \Rightarrow x = 2 \quad \dots(i)$$

and $3y = 6 + x + y \Rightarrow 2y = 6 + x = 6 + 2$ (By (i))

$$\Rightarrow 2y = 8 \Rightarrow y = 4 \quad \dots(ii)$$

$$\text{and } 3z = -1 + z + w \Rightarrow 2z - w = -1 \quad \dots(ii)$$

$$\text{and } 3w = 2w + 3 \Rightarrow w = 3.$$

Putting $w = 3$ in eqn. (ii),

$$2z - 3 = -1 \Rightarrow 2z = 2 \Rightarrow z = 1$$

$$\therefore x = 2, y = 4, z = 1, w = 3.$$

13. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y)$
 $= F(x+y)$.

Sol. Given: $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots(i)$

Changing x to y in (i), $F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{L.H.S.} = F(x)F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Performing row by column multiplication,

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0 + 0 + 0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [\because -\cos x \sin y - \sin x \cos y$$

$$= -(\cos x \sin y + \sin x \cos y) = -\sin(x+y)]$$

Now, changing x to $x+y$ in (i), we get

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Thus, L.H.S.} = \text{R.H.S.}$$

14. Show that:

$$(i) \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Sol. (i) L.H.S. = $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5(2) + (-1)3 & 5(1) + (-1)4 \\ 6(2) + 7(3) & 6(1) + 7(4) \end{bmatrix}$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \quad \dots(i)$$

$$\begin{aligned} \text{R.H.S.} &= \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix} \\ &= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

From (i) and (ii), we can say that L.H.S. \neq R.H.S.

(Because corresponding entries of matrix $\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$ and $\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$ are not same).

$$(ii) \text{ Let } A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

Here, matrices A and B are both of order 3×3 respectively, therefore AB and BA are both of same order 3×3 .

$$\text{Now, } AB = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

Performing row by column multiplication,

$$= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

$$\text{or } AB = \begin{bmatrix} -1+6 & 1-2+9 & 2+12 \\ 0 & -1 & 1 \\ -1 & 1-1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \dots(i)$$

$$\text{Again, } BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Performing row by column multiplication,

$$= \begin{bmatrix} (-1)1+1(0)+0(1) & (-1)2+1(1)+0(1) & (-1)3+1(0)+0(0) \\ 0(1)+(-1)0+1(1) & 0(2)+(-1)1+1(1) & 0(3)+(-1)0+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2+1 & -3 \\ 1 & -1+1 & 0 \\ 2+4 & 4+3+4 & 6 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \quad \dots(ii)$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$\therefore A^3 = A^2 \cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix} \text{ or } A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\text{L.H.S.} = A^3 - 6A^2 + 7A + 2I$$

$$= A^3 - 6A^2 + 7A + 2I_3$$

[Here I is I_3 because A, A^2, A^3 are matrices of order 3×3]

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} -9+9 & 0+0 & -14+14 \\ 0+0 & -16+16 & -7+7 \\ -14+14 & 0+0 & -23+23 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

= (zero matrix) O = R.H.S.

17. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

Sol. Given: $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Putting values of A and I in the given equation $kA - 2I$, we have

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Equating corresponding entries, we have

$$3k - 2 = 1 \Rightarrow 3k = 3 \Rightarrow k = 1 \text{ and } -2 = -2k \Rightarrow 2k = 1$$

$$\text{and } 4k = 4 \Rightarrow k = 1 \text{ and } -4 = -2k - 2 \Rightarrow 2k = -2 + 4 = 2$$

$$\Rightarrow k = 1$$

Therefore, value of $k = 1$ and is same from all the four equations. Therefore, k exists and $= 1$.

18. If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of

order 2, show that $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$.

Sol. $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2

$$\text{i.e., } I = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{L.H.S. } = I + A = I_2 + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix} \quad \dots(i)$$

$$\text{Again, } I - A = I_2 - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix}$$

$$\text{R.H.S.} = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Performing row by column multiplication,

$$= \begin{bmatrix} \cos \alpha + \sin \alpha \tan \frac{\alpha}{2} & -\sin \alpha + \cos \alpha \tan \frac{\alpha}{2} \\ -\cos \alpha \tan \frac{\alpha}{2} + \sin \alpha & \sin \alpha \tan \frac{\alpha}{2} + \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos \alpha + \sin \alpha \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} & -\sin \alpha + \cos \alpha \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} \\ -\cos \alpha \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} + \sin \alpha & \sin \alpha \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} + \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos \alpha \cos \frac{\alpha}{2} + \sin \alpha \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} & \frac{-\sin \alpha \cos \frac{\alpha}{2} + \cos \alpha \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} \\ \frac{-\cos \alpha \sin \frac{\alpha}{2} + \sin \alpha \cos \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} & \frac{\sin \alpha \sin \frac{\alpha}{2} + \cos \alpha \cos \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} \end{bmatrix}$$

Numerator of a_{21} is $-\left(\sin \alpha \cos \frac{\alpha}{2} - \cos \alpha \sin \frac{\alpha}{2}\right)$

$$= \begin{bmatrix} \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos \frac{\alpha}{2}} & \frac{-\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos \frac{\alpha}{2}} \\ \frac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos \frac{\alpha}{2}} & \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos \frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} \frac{\cos \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} & \frac{-\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} \\ \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} & \frac{\cos \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} \end{bmatrix}$$

[$\because \cos A \cos B + \sin A \sin B = \cos(A - B)$
and $\sin A \cos B - \cos A \sin B = \sin(A - B)$]

$$= \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix} \quad \dots (ii)$$

From equations (i) and (ii), we have L.H.S. = R.H.S.

$$\text{i.e., } I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}.$$

19. A trust fund has ₹ 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹ 30,000 in two types of bonds, if the trust fund must obtain an annual interest of

(a) ₹ 1800 (b) ₹ 2000.

Sol. Let the investment in first bond be ₹ x
then the investment in second bond = ₹ $(30,000 - x)$

Interest paid by first bond = $5\frac{5}{100}$ per rupee

Interest paid by second bond = $7\frac{7}{100}$ per rupee

Matrix of investment is $A = [x \quad 30000 - x]$

Matrix of annual interest per rupee is $\begin{bmatrix} 5 \\ 7 \\ 100 \end{bmatrix}$

Matrix of total annual interest is

$$AB = [x \quad 30000 - x] \begin{bmatrix} 5 \\ 7 \\ 100 \end{bmatrix} = \left[\frac{5x}{100} + \frac{7(30000 - x)}{100} \right]$$

$$= \left[\frac{5x + 210000 - 7x}{100} \right] = \left[\frac{210000 - 2x}{100} \right]$$

$$\therefore \text{Total annual interest} = \frac{2,10,000 - 2x}{100}$$

(a) total annual interest is given to be ₹ 1,800

$$\therefore \frac{2,10,000 - 2x}{100} = 1,800$$

$$\Rightarrow 2,10,000 - 2x = 1,80,000 \quad x = 15,000$$

Hence, investment in first bond = ₹ 15,000

and investment in second bond = ₹ $(30,000 - x)$
= ₹ $(30,000 - 15,000)$ = ₹ 15,000.

(b) Total annual interest is given to be ₹ 2,000

$$\therefore \frac{2,10,000 - 2x}{100} = 2,000$$

$$\Rightarrow 2,10,000 - 2x = 2,00,000 \quad \therefore x = 5,000$$

Hence, investment in first bond = ₹ 5,000 and investment in second bond = ₹ $(30,000 - x)$ = ₹ $(30,000 - 5,000)$ = ₹ 25,000.

20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹ 80, ₹ 60 and ₹ 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Sol. Let us represent the number of books as a 1×3 row matrix

$$B = \begin{bmatrix} 10 \text{ dozen} & 8 \text{ dozen} & 10 \text{ dozen} \\ 10 \times 12 = 120 & 8 \times 12 = 96 & 10 \times 12 = 120 \end{bmatrix}$$

Let us represent the selling prices of each book as a 3×1 column

$$\text{matrix } S = \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

\therefore [Total amount received by selling all books] $_{1 \times 1}$

$$\begin{aligned} &= BS = [120 \ 96 \ 120]_{1 \times 3} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}_{3 \times 1} \\ &= [120(80) + 96(60) + 120(40)] \\ &= [9600 + 5760 + 4800] = [20160] \end{aligned}$$

Equating corresponding entries,

Total amount received by selling all the books = ₹ 20160.

Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$ respectively. Choose the correct answer in Exercises 21 and 22.

21. The restriction on n , k and p so that $PY + WY$ will be defined are:

- (A) $k = 3$, $p = n$ (B) k is arbitrary, $p = 2$
 (C) p is arbitrary, $k = 3$ (D) $k = 2$, $p = 3$.

Sol. Given: Matrix $PY + WY$ is defined (\Rightarrow possible).

Matrix P is of order $p \times k$ and matrix Y is of order $3 \times k$ and matrix W is of order $n \times 3$.

Now $PY + WY = (P + W)Y$... (1)

We know that sum $P + W$ is defined if two matrices

$$\begin{array}{ccc} \downarrow & & \downarrow \\ p \times k & & n \times 3 \end{array}$$

P and W are of same order. Therefore $p = n$ and $k = 3$ and order of $P + W$ is $n \times 3$ (or $p \times k$)

Therefore from (1), $PY + WY = (P + W)Y$ is defined as

$$\begin{array}{ccc} \downarrow & & \downarrow \\ n \times 3 & & 3 \times k \\ \leftarrow & & \rightarrow \end{array}$$

Number of columns in $P + W$ is same as number of rows in Y .

$\therefore p = n$ and $k = 3$

\therefore Option (A) is the correct answer i.e., $k = 3$ and $p = n$.

22. If $n = p$, then order of the matrix $7X - 5Z$ is
(A) $p \times 2$ (B) $2 \times n$ (C) $n \times 3$ (D) $p \times n$.

Sol. Since $n = p$ (given), the order of matrices X and Z are equal.
 $\therefore 7X - 5Z$ is well defined and the order of $7X - 5Z$ is same as the order of X and Z.
 \therefore The order of $7X - 5Z$ is either equal to $2 \times n$ or $2 \times p$
($\because n = p$)
 \therefore The correct option is (B), i.e., the order of $7X - 5Z$ is $2 \times n$.

Exercise 3.3

1. Find the transpose of each of the following matrices:

$$(i) \begin{bmatrix} 5 \\ 1 \\ 2 \\ -1 \end{bmatrix} \quad (ii) \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \quad (iii) \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}.$$

Sol. (i) Let $A = \begin{bmatrix} 5 \\ 1 \\ 2 \\ -1 \end{bmatrix}$ (is a column matrix 3×1)

Changing column of A into a row, (row will automatically become column)

$$\text{Transpose of A (i.e., } A^T) = \begin{bmatrix} 5 & 1 & 2 & -1 \end{bmatrix}$$

(which is a row matrix 1×3)

(ii) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Changing rows of A to columns of A,
(columns will automatically become rows).

$$A^T \text{ or } A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}.$$

(iii) Let $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

(Making) changing rows of A as columns of the new matrix,

$$\text{we have } A^T \text{ or } A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}.$$

2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = A' - B'$.

Sol. (i) To verify $(A + B) = A' + B'$

$$\begin{aligned} A + B &= \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix} \end{aligned}$$

(Making) changing rows of $A + B$ as columns of the new matrix, we have

$$\text{L.H.S.} = (A + B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \quad \dots(i)$$

$$\begin{aligned} \text{R.H.S.} = A' + B' &= \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}' + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}' \\ &= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

From (i) and (ii), we have L.H.S. = R.H.S.

i.e., $(A + B) = A' + B'$

(ii) To verify $(A - B) = A' - B'$

$$\begin{aligned} A - B &= \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1+4 & 2-1 & 3+5 \\ 5-1 & 7-2 & 9-0 \\ -2-1 & 1-3 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix} \end{aligned}$$

(Making) changing rows of $A - B$ as columns of the new matrix, we have

$$\text{L.H.S.} = (A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \quad \dots(i)$$

$$\begin{aligned} \text{R.H.S.} &= A' - B' = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}' - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}' \\ &= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1+4 & 5-1 & -2-1 \\ 2-1 & 7-2 & 1-3 \\ 3+5 & 9-0 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

From (i) and (ii), we have L.H.S. = R.H.S.

Note $(A')' = A$.

3. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = A' - B'$.

Sol. Given: $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Making rows of A' as columns of the new matrix (transpose of

A' i.e., (A') i.e., $A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$$\begin{aligned} (i) \ A + B &= \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3-1 & -1+2 & 0+1 \\ 4+1 & 2+2 & 1+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix} \end{aligned}$$

$$\therefore \text{L.H.S.} = (A + B)' = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix} \quad \dots(i)$$

$$\text{R.H.S.} = A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}' + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}' \quad (\text{given})$$

$$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3-1 & 4+1 \\ -1+2 & 2+2 \\ 0+1 & 1+3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix} \quad \dots(ii)$$

From (i) and (ii), we have L.H.S. = R.H.S.

$$\begin{aligned}
 \text{(ii) } A - B &= \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 3+1 & -1-2 & 0-1 \\ 4-1 & 2-2 & 1-3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix} \\
 \therefore \text{L.H.S.} = (A - B)' &= \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} \quad \dots(i)
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S.} = A - B' &= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}' \\
 &\quad \text{(given)} \\
 &= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} \quad \dots(ii)
 \end{aligned}$$

From (i) and (ii), we have L.H.S. = R.H.S.

4. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)$

Sol. Given: $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

Making rows of A' as columns of the new matrix (transpose of A)

i.e., (A) i.e., $A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

$$\begin{aligned}
 \therefore A + 2B &= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2 \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} -2-2 & 1+0 \\ 3+2 & 2+4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}
 \end{aligned}$$

Making rows of this matrix as columns of new matrix, we have

$$(A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

5. For the matrices A and B , verify that $(AB)' = B'A'$, where

$$(i) A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = [-1 \ 2 \ 1] \quad (ii) A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = [1 \ 5 \ 7]$$

Sol. (i) Given: $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = [-1 \ 2 \ 1]$

$$\therefore AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3 \times 1} \quad [-1 \ 2 \ 1]_{1 \times 3} \text{ is a matrix of order}$$

$$3 \times 3 \text{ and } \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

(Using row by column multiplication rule)

$$\text{L.H.S.} = (AB)' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \quad \dots(i)$$

$$\text{R.H.S.} = 'AB = [-1 \ 2 \ 1] \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} [1 \ -4 \ 3] = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \quad \dots(ii)$$

From (i) and (ii), we have L.H.S. = R.H.S. *i.e.*, $(AB)' = B'A'$.

$$(ii) \text{ Given: } A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \text{ and } B = [1 \ 5 \ 7]$$

$$\therefore AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}_{3 \times 1} \quad [1 \ 5 \ 7]_{1 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\text{L.H.S.} = (AB) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix} \quad \dots(i)$$

$$\text{R.H.S.} = BA' = [1 \ 5 \ 7] \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}_{3 \times 1} \quad [0 \ 1 \ 2]_{1 \times 3}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix} \quad \dots(ii)$$

From (i) and (ii) we have L.H.S. = R.H.S.

i.e., $(AB)' = B'A'$.

Remark. Result to remember from this Q.No. 5:

$$(AB)' = B'A' \quad | \quad \text{Reversal Law}$$

6. (i) If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then verify that $AA = I$
- (ii) If $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$, then verify that $AA = I$.

Sol. (i) Given: $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

$$\begin{aligned} \therefore \text{L.H.S.} &= AA = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &\quad \text{(Row by Column Multiplication)} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 (= I) = \text{R.H.S.} \end{aligned}$$

(ii) Given: $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$

$$\begin{aligned} \therefore \text{L.H.S.} &= AA = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 (= I) = \text{R.H.S.} \end{aligned}$$

7. (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.
- (ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

Sol. (i) Given: $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$... (i)

(Making) changing rows of matrix A as the columns of the

$$\text{new matrix } A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A \quad [\text{By (i)}]$$

$$\therefore A' = A$$

\therefore By definition of symmetric matrix, A is a symmetric matrix.

$$(ii) \text{ Given: Matrix } A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \quad \dots(i)$$

$$\therefore A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

Taking (-1) common from R.H.S. of A we have

$$A' = - \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A \quad [\text{By (i)}]$$

\therefore By definition, matrix A is a skew-symmetric matrix.

8. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix.

(ii) $(A - A')$ is a skew symmetric matrix.

Sol. (i) Given: $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$$\begin{aligned} \text{Let } B = A + A' &= A + \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 1+1 & 5+6 \\ 6+5 & 7+7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

$$\therefore B' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = B \quad [\text{By (i)}]$$

$\therefore B$ i.e., $(A + A')$ is a symmetric matrix.

(ii) Given: $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$$\begin{aligned} \text{Let } B &= A - A' = A - \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}' \\ &= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1-1 & 5-6 \\ 6-5 & 7-7 \end{bmatrix} \end{aligned}$$

$$\text{or } B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \dots(i)$$

$$\therefore B' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Taking (-1) common from R.H.S. of B,

$$B' = - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -B \quad [\text{By (i)}]$$

\therefore Matrix B i.e., $A - A'$ is a skew symmetric matrix.

9. Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$ when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$.

Sol. Given: $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$\begin{aligned} \therefore A + A' &= \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & a-a & b-b \\ -a+a & 0+0 & c-c \\ -b+b & -c+c & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$\therefore \frac{1}{2}(A + A) = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \text{Again } A - A' &= \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0-0 & a+a & b+b \\ -a-a & 0-0 & c+c \\ -b-b & -c-c & 0-0 \end{bmatrix} = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} \end{aligned}$$

$$\therefore \frac{1}{2}(A - A) = \frac{1}{2} \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

Multiplying each entry by $\frac{1}{2}$, $= \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$.

10. Express the following matrices as the sum of a symmetric and skew symmetric matrix:

$$(i) \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

Note Formula. Every square matrix A can be expressed as the sum of a symmetric matrix $\frac{1}{2}(A + A')$ and skew symmetric matrix $\frac{1}{2}(A - A')$.

Sol. (i) **Given:** Matrix (say) $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

therefore, $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

By Formula above, symmetric matrix part of A

$$\begin{aligned} &= \frac{1}{2}(A + A') = \frac{1}{2} \left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} \quad \dots(i) \end{aligned}$$

and skew symmetric matrix part of A .

$$\begin{aligned} &= \frac{1}{2}(A - A') = \frac{1}{2} \left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3-3 & 5-1 \\ 1-5 & -1+1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(0) & \frac{1}{2}(4) \\ \frac{1}{2}(-4) & \frac{1}{2}(0) \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

\therefore Given matrix A is sum of matrices (i) and (ii)

= symmetric matrix $\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$ + skew symmetric matrix

$$\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

(ii) **Given:** matrix say $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

\therefore Symmetric part of A $\frac{1}{2}(A + A')$

$$\begin{aligned} &= \frac{1}{2} \left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \quad \dots(i) \end{aligned}$$

and skew symmetric part of A $\frac{1}{2}(A - A')$

$$\begin{aligned} &= \frac{1}{2} \left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 6-6 & -2+2 & 2-2 \\ -2+2 & 3-3 & -1+1 \\ 2-2 & -1+1 & 3-3 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

\therefore Given matrix A = sum of matrices (i) and (ii)

$$\begin{aligned} &= \text{symmetric matrix} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \\ &\quad + \text{skew symmetric matrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

(ii) **Given:** matrix say A = $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned}
 \therefore \text{Symmetric part of } A &= \frac{1}{2}(A + A') \\
 &= \frac{1}{2} \left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \right) \\
 &= \frac{1}{2} \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} \quad \dots(i)
 \end{aligned}$$

and skew symmetric part of $\frac{1}{2}(A - A')$

$$\begin{aligned}
 &= \frac{1}{2} \left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \right) \\
 &= \frac{1}{2} \begin{bmatrix} 3-3 & 3+2 & -1+4 \\ -2-3 & -2+2 & 1+5 \\ -4+1 & -5-1 & 2-2 \end{bmatrix} \\
 &= \frac{1}{2} \begin{bmatrix} 3-3 & 3+2 & -1+4 \\ -2-3 & -2+2 & 1+5 \\ -4+1 & -5-1 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} \quad \dots(ii)
 \end{aligned}$$

\therefore Given matrix A = sum of matrices (i) and (ii)

$$\begin{aligned}
 &= \text{symmetric matrix } \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} \\
 &\quad + \text{skew symmetric matrix } \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}
 \end{aligned}$$

(iv) **Given:** matrix say $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ $\therefore A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

\therefore Symmetric part of $A = \frac{1}{2}(A + A')$

$$= \frac{1}{2} \left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} \quad \dots(i)$$

and skew symmetric part of $A = \frac{1}{2}(A - A')$

$$= \frac{1}{2} \left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} \right) = \frac{1}{2} \left(\begin{bmatrix} 1-1 & 5+1 \\ -1-5 & 2-2 \end{bmatrix} \right)$$

$$= \frac{1}{2} \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} \quad \dots(ii)$$

\therefore Given matrix = Sum of matrices (i) and (ii)

$$= \text{Symmetric matrix} \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$+ \text{skew-symmetric matrix} \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}.$$

Choose the correct answer in Exercises 11 and 12

11. If A and B are symmetric matrices of same order, $AB - BA$ is a

- (A) Skew-symmetric matrix (B) Symmetric Matrix
(C) Zero matrix (D) Identity matrix.

Sol. Given: A and B are symmetric matrices

$$\Rightarrow A' = A \text{ and } B' = B \quad \dots(i)$$

$$\begin{aligned} \text{Now } (AB - BA)' &= (AB)' - (BA)' && [\because (P - Q)' = P' - Q'] \\ &= BA' - A'B' && [\text{Reversal Law}] \\ &= BA - AB && [\text{Using (i)}] \\ &= -(AB - BA) \end{aligned}$$

$\therefore (AB - BA)$ is a skew symmetric.

Thus, option (A) is the correct answer.

12. If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, then $A + A = I$, if the value of α is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) π (D) $\frac{3\pi}{2}$.

Sol. Given: $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Also given $A + A = I$

$$\Rightarrow \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = I = I_2$$

$$\Rightarrow \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \cos \alpha & 0 \\ 0 & 2 \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating corresponding entries, we have
 $2 \cos \alpha = 1$

$$\Rightarrow \cos \alpha = \frac{1}{2} = \cos \frac{\pi}{3} \quad \therefore \alpha = \frac{\pi}{3}$$

Thus, option (B) is the correct answer.

Exercise 3.4

Using elementary transformations, find the inverse of each of the matrices, if it exists in Exercises 1 to 6.

1. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

We shall find A^{-1} , if it exists; by elementary (**Row**) transformations (only)

So we must write $A = IA$ only and not $A = AI$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

(Here I is J because A is 2×2)

We shall reduce the matrix on left side to I

Here $a_{11} = 1$

Operate $R_2 \rightarrow R_2 - 2R_1$ to make $a_{21} = 0$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad \left| \begin{array}{l} R_2 \rightarrow 2 \quad 3 \\ 2R_1 \rightarrow 2 \quad -2 \\ \hline - \quad - \quad + \\ \hline \therefore R_2 - 2R_1 = 0 \quad 5 \\ R_2 \rightarrow 0 \quad 1 \\ 2R_1 \rightarrow 2 \quad 0 \\ \hline - \quad - \quad - \\ \hline \therefore R_2 - 2R_1 = -2 \quad 1 \end{array} \right.$$

Operate $R_2 \rightarrow \frac{1}{5}R_2$ to make $a_{22} = 1$

$$\therefore \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & \frac{1}{5} \end{bmatrix} A$$

Now operate $R_1 \rightarrow R_1 + R_2$ to make $a_{12} = 0$

$$\Rightarrow \begin{bmatrix} 1+0 & -1+1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{2}{5} & 0+\frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

$$\therefore \text{By definition of inverse of a matrix, } A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Note. Any row operation done on left hand side matrix must also be done on the prefactor of right hand side matrix.

Note. Definition of inverse of a square matrix. A square matrix B is said to be inverse of a square matrix A if $\mathbf{AB} = \mathbf{I}$ and $\mathbf{BA} = \mathbf{I}$. Then $B = A^{-1}$.

Remark. If the student is interested in finding A^{-1} by elementary column transformations, then he or she should start with $A = AI$ and apply only column operations.

2. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

We know that $A = I_2 A \Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Operate $R_1 \leftrightarrow R_2$ (to make $a_{11} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 2R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2-2 & 1-2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1-0 & 0-2 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A$$

Operate $R_2 \rightarrow (-1)R_2$ (to make $a_{22} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 - R_2$ (to make $a_{12} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$$

\therefore By definition of inverse of a square matrix, $A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$.

3. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

We know that $A = IA \Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Here $a_{11} = 1$. To make $a_{21} = 0$, let us operate $R_2 \rightarrow R_2 - 2R_1$.

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad \left| \begin{array}{l} R_2 \rightarrow 2 \quad 7 \\ 2R_1 \rightarrow 2 \quad 6 \\ \hline \therefore R_2 - 2R_1 = 0 \quad 1 \\ R_2 \rightarrow 0 \quad 1 \\ 2R_1 \rightarrow 2 \quad 0 \\ \hline \therefore R_2 - 2R_1 = -2 \quad 1 \end{array} \right.$$

Now $a_{22} = 1$. To make a_{12} as zero, operate $R_1 \rightarrow R_1 - 3R_2$.

$$\Rightarrow \begin{bmatrix} 1-0 & 3-3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+6 & 0-3 \\ -2 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$$

\therefore By definition, $A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$.

4. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Sol. Set $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

We know that $A = IA \Rightarrow \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Let us try to make $a_{11} = 1$. Operate $R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 5-4 & 7-6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0-2 & 1-0 \end{bmatrix} A \Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Now operate $R_1 \leftrightarrow R_2$ to make $a_{11} = 1$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 2R_1$ to make $a_{21} = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2-2 & 3-2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1+4 & 0-2 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 - R_2$ to make $a_{12} = 0$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} -2-5 & 1+2 \\ 5 & -2 \end{bmatrix}$$

$$\Rightarrow I_2 = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \Rightarrow A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

Remark. In the above solution to make $a_{12} = 0$, we could also operate $R_1 \rightarrow \frac{1}{2}R_1$. But for the sake of convenience and to avoid lengthy computations, we should avoid multiplying by fractions.

5. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

We know that $A = tA \Rightarrow \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Let us try to make $a_{11} = 1$. Operate $R_1 \rightarrow R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 7-6 & 4-3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0-3 & 1-0 \end{bmatrix} A \Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 - R_2$ to make $a_{21} = 1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} A$$

Now Operate $R_2 \rightarrow R_2 - R_1$ (to make $a_{12} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A$$

Now $a_{12} = 0$ and $a_{22} = 1$.

or $I_2 = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A$

\therefore By definition of inverse of a square matrix, $A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$.

6. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

We know that $A = \frac{1}{2}A \Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Operate $R_1 \leftrightarrow R_2$ to make $a_{11} = 1$;

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 2R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2-2 & 5-6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1-0 & 0-2 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A$$

Operate $R_2 \rightarrow (-1)R_2$ to make $a_{22} = 1$;

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 - 3R_2$ (to make $a_{12} = 0$)

$$\Rightarrow \begin{bmatrix} 1-0 & 3-3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+3 & 1-6 \\ -1 & 2 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

$$\therefore \text{By Definition, } A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

Using elementary transformations, find the inverse of each of the matrices, if it exists, in Exercises 7 to 14.

7. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

We know that $A = \frac{1}{2}A \Rightarrow \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Let us try to make $a_{11} = 1$.

Operate $R_1 \rightarrow 2R_1 \Rightarrow \begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} A$

Operate $R_1 \rightarrow R_1 - R_2$ (to make $a_{21} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 5R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0-10 & 1+5 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -10 & 6 \end{bmatrix} A$$

Operate $R_2 \rightarrow \frac{1}{2}R_2$ (to make $a_{22} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} A$$

Now a_{12} has already become zero. Therefore,

$$A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}.$$

8. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

We know that $A = I_2 A \Rightarrow \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Operate $R_1 \rightarrow R_1 - R_2$ (to make $a_{11} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 3R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3-3 & 4-3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1+3 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$$

Now a_{22} has already become 1.

Operate $R_1 \rightarrow R_1 - R_2$ (to make $a_{12} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} 1+3 & -1-4 \\ -3 & 4 \end{bmatrix} A$$

$$\Rightarrow I_2 = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A. \text{ Therefore, } A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}.$$

9. $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

We know that $A = I_2 A \Rightarrow \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Operate $R_1 \rightarrow R_1 - R_2$ (to make $a_{11} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 2R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2-2 & 7-6 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1+2 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A$$

Now $a_{22} = 1$. Operate $R_1 \rightarrow R_1 - 3R_2$ (to make $a_{12} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+6 & -1-9 \\ -2 & 3 \end{bmatrix} A$$

$$\Rightarrow I_2 = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \Rightarrow A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

10. $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

We know that $A = I_2 A \Rightarrow \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Let us try to make $a_{11} = 1$

Operate $R_1 \rightarrow R_1 + R_2$.

$$\Rightarrow \begin{bmatrix} 3-4 & -1+2 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+1 \\ 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} -1 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_1 \rightarrow (-1) R_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 + 4R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -4 & -3 \end{bmatrix} A$$

Operate $R_2 \rightarrow \left(-\frac{1}{2}\right) R_2$ (to make $a_{22} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 + R_2$ (to make $a_{12} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} A$$

\therefore By definition of inverse of a matrix; $A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$.

11. $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

We know that $A = I_2 A \Rightarrow \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

 Operate $R_1 \leftrightarrow R_2$ (to make $a_{11} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A$$

 Operate $R_2 \rightarrow R_2 - 2R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 2-2 & -6+4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1-0 & 0-2 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A$$

 Operate $R_2 \rightarrow \left(-\frac{1}{2}\right) R_2$ (to make $a_{22} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

 Operate $R_1 \rightarrow R_1 + 2R_2$ (to make $a_{12} = 0$)

$$\Rightarrow \begin{bmatrix} 1+0 & -2+2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0-1 & 1+2 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A \Rightarrow A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

12. $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

 Here, A is a 2×2 matrix. So, we start with $A = I_2 A$

or $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

 Operating $R_1 \rightarrow 1/6 R_1$ to make $a_{11} = 1$.

we have $\begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A$

 Operating $R_2 \rightarrow R_2 + 2R_1$ to make non-diagonal entries below a_{11} as zero,

we have $\begin{bmatrix} 1 & -\frac{1}{2} \\ -2+2 & 1-\frac{2}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0+\frac{2}{6} & 1+0 \end{bmatrix} A$

$$\text{or } \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A$$

Here, all entries in second row of left side matrix are zero.

$\therefore A^{-1}$ does not exist.

Note. If after doing one or more elementary row operations, we obtain all 0's in one or more rows of the left hand matrix A, then A^{-1} does not exist and we say A is not invertible.

13. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$$\text{We know that } A = \frac{1}{2}A \Rightarrow \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 + R_2$ (to make $a_{11} = 1$)

$$\Rightarrow \begin{bmatrix} 2-1 & -3+2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+1 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 + R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$$

Now $a_{22} = 1$. Operate $R_1 \rightarrow R_1 + R_2$ (to make $a_{12} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (= I_2) = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$$

$$\therefore \text{By definition; } A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}.$$

14. $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$$\text{We know that } A = \frac{1}{2}A \Rightarrow \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_1 \rightarrow \frac{1}{2}R_1$ (to make $a_{11} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 4R_1$ (to make $a_{21} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 4-4 & 2-2 \\ 0-2 & 1-0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0-2 & 1-0 \end{bmatrix} \quad A \Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -2 & 1 \end{bmatrix} A$$

Here one row (namely second row) of the matrix on L.H.S. contains zeros only.

Hence, A^{-1} does not exist.

Using elementary transformations, find the inverse of each of the matrices, if it exists, in Exercises 15 to 17.

15. $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

We know that $A \Rightarrow AI$ (we have taken I because matrix A is of order 3×3)

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Let us try to make $a_{11} = 1$

Operate $R_1 \rightarrow R_1 - R_3$

$$\Rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operate $R_1 \rightarrow (-1)R_1$ to make $a_{11} = 1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$ (to make $a_{21} = 0$ and $a_{31} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 2-2 & 2-2 & 3+2 \\ 3-3 & -2-3 & 2+3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0+2 & 1-0 & 0-2 \\ 0+3 & 0-0 & 1-3 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 5 \\ 0 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 0 & -2 \end{bmatrix} A$$

Operate $R_2 \leftrightarrow R_3$ (to make a_{22} non-zero)

$$16. \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

$$\text{Sol. Let } A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

We know that $A = I_3 A$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Here a_{11} is already 1.

Operate $R_2 \rightarrow R_2 + 3R_1$ and $R_3 \rightarrow R_3 - 2R_1$ (to make $a_{12} = 0$ and $a_{13} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3+3 & 0+9 & -5-6 \\ 2-2 & 5-6 & 0+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0+3 & 1+0 & 0+0 \\ 0-2 & 0-0 & 1-0 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Operate $R_2 \leftrightarrow R_3$ to make a_{22} simpler entry

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix} A$$

Operate $R_2 \rightarrow (-1)R_2$ to make $a_{22} = 1$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 - 3R_2$ to make $a_{12} = 0$ and $R_3 \rightarrow R_3 - 9R_2$ (to make $a_{32} = 0$)

$$\Rightarrow \begin{bmatrix} 1-0 & 3-3 & -2+12 \\ 0 & 1 & -4 \\ 0 & 9-9 & -11+36 \end{bmatrix} = \begin{bmatrix} 1-6 & 0-0 & 0+3 \\ 2 & 0 & -1 \\ 3-18 & 1-0 & 0+9 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 2 & 0 & -1 \\ -15 & 1 & 9 \end{bmatrix} A$$

Operate $R_3 \rightarrow \frac{1}{25}R_3$ to make $a_{33} = 1$.

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 2 & 0 & -1 \\ -\frac{15}{25} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 - 10R_3$ (to make $a_{13} = 0$) and $R_2 \rightarrow R_2 + 4R_3$ (to make $a_{23} = 0$).

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} (= I_3) = \begin{bmatrix} -5 + \frac{150}{25} & 0 - \frac{10}{25} & 3 - \frac{90}{25} \\ 2 - \frac{60}{25} & 0 + \frac{4}{25} & -1 + \frac{36}{25} \\ -\frac{15}{25} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\Rightarrow I_3 = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\therefore \text{By Definition, } A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

17. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

We know that $A^{-1} A = I \Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Let us try to make $a_{11} = 1$
Operate $R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operate $R_1 \leftrightarrow R_2$ (to make $a_{11} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operate $R_2 \rightarrow R_2 - 2R_1$ to make $a_{21} = 0$. Here a_{21} is already 0

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operate $R_2 \leftrightarrow R_3$ (to make $a_{22} = 1$)

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 0 \end{bmatrix} A$$

Operate $R_1 \rightarrow R_1 - R_2$ to make $a_{12} = 0$ and ${}_3R \rightarrow R_3 + 2R_2$ to make $a_{32} = 0$.

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

Now $a_{33} = 1$

Operate $R_1 \rightarrow R_1 + R_3$ (to make $a_{13} = 0$) and ${}_2R \rightarrow R_2 - 3R_3$ (to make $a_{23} = 0$)

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} (= I_3) = \begin{bmatrix} -2+5 & 1-2 & -1+2 \\ 0-15 & 0+6 & 1-6 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\text{or } I_3 = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore \text{By definition, } A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

18. Matrices A and B will be inverse of each other only if

- (A) $AB = BA$ (B) $AB = BA = 0$
 (C) $AB = 0, BA = I$ (D) $AB = BA = I$.

Sol. Option (D) i.e., $AB = BA = I$ is correct answer by definition of inverse of a square matrix.

Sol. Step I. When $n = 1$, $(aI + bA) = a^1I + na^{n-1}bA$
 $\Rightarrow (aI + bA)^1 = aI + 1^1bA \Rightarrow aI + bA = aI + bA$ which is true.
 \therefore The result is true for $n = 1$.

Step II. Suppose the result is true for $n = k$.
i.e., let $(aI + bA)^k = a^kI + k a^{k-1}bA$... (i)

Step III. To prove that the result is true for $n = k + 1$.

$$\begin{aligned} \text{Now } (aI + bA)^{k+1} &= (aI + bA)^k \cdot (aI + bA) \\ &= (aI + bA) (aI + k a^{k-1}bA) \quad [\text{Using (i)}] \\ &= a^{k+1}I^2 + k a^k bA + a bA I + k a^{k-1} bA^2 \\ &= a^{k+1}I + k a^k bA + k a bA + k a^{k-1} bA^2. \end{aligned}$$

[By distributive property]

$$\begin{aligned} \left[\because I^2 = I, IA = AI = A \text{ and } A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O \right] \\ = a^{k+1}I + (k+1) a^k bA + O = a^{k+1}I + (k+1) a^k bA \end{aligned}$$

\therefore The result is true for $n = k + 1$.

Hence, by the principle of mathematical induction, the result is true for all positive integers n .

2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ $n \in \mathbb{N}$.

Sol. We shall prove the result by using principle of mathematical induction.

Given: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$... (i)

Let $P(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$... (ii)

Step I. Putting $n = 1$ in (ii),

$$\text{Therefore, } P(1): A = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

which is given to be true by (i).

\therefore $P(1)$ is true *i.e.,* Eqn. (ii) is true for $n = 1$.

Step II. Let $P(k)$ be true *i.e.,* eqn. (ii) is true for $n = k$.

Putting $n = k$ in (ii), $A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$... (iii)

Step III. Multiplying corresponding sides of eqn. (iii) by eqn. (i)

$$A^k \cdot A^1 = \begin{bmatrix} 3^{k+1} & 3^{k+1} & 3^{k+1} \\ 3^{k+1} & 3^{k+1} & 3^{k+1} \\ 3^{k+1} & 3^{k+1} & 3^{k+1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Performing row by column multiplication on right side

$$\Rightarrow A^{k+1} = \begin{bmatrix} 3^{k+1} + 3^{k+1} + 3^{k+1} & 3^{k+1} + 3^{k+1} + 3^{k+1} & 3^{k+1} + 3^{k+1} + 3^{k+1} \\ 3^{k+1} + 3^{k+1} + 3^{k+1} & 3^{k+1} + 3^{k+1} + 3^{k+1} & 3^{k+1} + 3^{k+1} + 3^{k+1} \\ 3^{k+1} + 3^{k+1} + 3^{k+1} & 3^{k+1} + 3^{k+1} + 3^{k+1} & 3^{k+1} + 3^{k+1} + 3^{k+1} \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

$$(\because 3^{k+1} + 3^{k+1} + 3^{k+1} = 3 \cdot 3^k \therefore x + x + x = 3x)$$

$$= 3 \cdot 3^k = 3^{k+1}$$

\therefore Eqn. (iii) is true for $n = k + 1$ (\because on putting $n = k + 1$ in (ii), we get the above equation)

i.e., $P(k + 1)$ is true

$\therefore P(n)$ i.e., eqn. (ii) is true for all natural n by P.M.I.

3. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & 4n \\ n & 1+2n \end{bmatrix}$ where n is any positive integer.

Sol. We prove the result by mathematical induction.

Step I. When $n = 1$, $A^1 = \begin{bmatrix} 1+2n & 4n \\ n & 1+2n \end{bmatrix}$... (i)

$$\Rightarrow A^1 = \begin{bmatrix} 1+2 & 4 \\ 1 & 1+2 \end{bmatrix}$$

or $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ which is true. \therefore The result is true for $n = 1$.

Step II. Suppose that equation (i) is true for $n = k$.

$$\text{i.e., let } A^k = \begin{bmatrix} 1+2k & 4k \\ k & 1+2k \end{bmatrix} \quad \dots \text{(ii)}$$

Step III. To prove that the result is true for $n = k + 1$, we have to show that

(Putting $n = k + 1$ in (i))

$$A^{k+1} = \begin{bmatrix} 1+2(k+1) & 4(k+1) \\ (k+1) & 1+2(k+1) \end{bmatrix} = \begin{bmatrix} 3+2k & 4+4k \\ k+1 & 3+2k \end{bmatrix} \quad \dots \text{(iii)}$$

$$\text{Now } A^{k+1} = A^k A = \begin{bmatrix} 1+2k & 4k \\ k & 1+2k \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \quad [\text{Using (ii)}]$$

Performing row by column multiplication,

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix} = \begin{bmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{bmatrix}$$

which is the same as (ii).

∴ The result is true for $n = k + 1$.

Hence, by the principle of mathematical induction, the result is true for all positive integers n .

4. If A and B are symmetric matrices, prove that AB - BA is a skew symmetric matrix.

Sol. A and B are symmetric matrices

$$\Rightarrow A' = A \text{ and } B' = B$$

$$\begin{aligned} \text{Now } (AB - BA)' &= (AB)' - (BA)' && \dots(i) \\ &= BA' - AB' && [\because (PQ)' = Q'P'] \\ &= BA - AB && [\text{Reversal Law}] \\ &= -(AB - BA) && [\text{Using (i)}] \end{aligned}$$

∴ (AB - BA) is a skew symmetric matrix.

5. Show that the matrix B'AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Sol. Now, $(B'AB)' = [B'AB]'$

$$= (AB)B' \quad [\because (CD)' = D'C']$$

$$\text{or } (B'AB)' = BA'B \quad \dots(ii) \quad [\because (CD)' = D'C']$$

Case I. A is a symmetric matrix

$$\therefore A' = A$$

$$\text{Putting } A' = A \text{ in equation (i), } (B'AB)' = BAB$$

∴ B'AB is a symmetric matrix.

Case II. A is a skew symmetric matrix.

$$\therefore A' = -A$$

$$\text{Putting } A' = -A \text{ in equation (i), } (B'AB)' = B(-A)B = -BAB$$

∴ B'AB is a skew symmetric matrix.

6. Find the values of x, y, z if the matrix

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \text{ satisfies the equation } A' = I.$$

$$\text{Sol. Given: } A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

$$\text{Therefore, } A' = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$$\therefore A'A = I \text{ (given)}$$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(Here I is $\frac{1}{3}$ because) matrices A and Aare matrices of order 3×3)

$$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+xz & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equating corresponding entries, we have

$$\begin{aligned} 2x^2 &= 1, & 6y^2 &= 1, & 3z^2 &= 1 \\ \Rightarrow x^2 &= \frac{1}{2}, & y^2 &= \frac{1}{6}, & z^2 &= \frac{1}{3} \\ \Rightarrow x &= \pm \sqrt{\frac{1}{2}}, & y &= \pm \sqrt{\frac{1}{6}}, & z &= \pm \sqrt{\frac{1}{3}} \\ \therefore x &= \pm \frac{1}{\sqrt{2}}, & y &= \pm \frac{1}{\sqrt{6}}, & z &= \pm \frac{1}{\sqrt{3}}. \end{aligned}$$

7. For what value of x , $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix}$?

Sol. Given: $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix}$

Orders 1×3 3×3 3×1

Multiplying first matrix with second matrix.

$$\Rightarrow [1 \ 4 \ 12 \ 0 \ 00 \ 2 + \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow [6 \ 2 \ 4] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow [0 + 4 + 4x]_{1 \times 1} = 0 = [0]_{1 \times 1}$$

Equating corresponding entries $0 + 4 + 4x = 0 - 4$

$$\Rightarrow x = \frac{-4}{4} = -1.$$

8. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.

Sol. Given: $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

L.H.S. = $A^2 - 5A + 7I = A^2 - 5A + 7I_2$
 $(\because A \text{ is } 2 \times 2, \text{ therefore } I \text{ is } I_2)$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15 & 5-5 \\ -5+5 & 3-10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -7+7 & 0+0 \\ 0+0 & -7+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

9. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$.

Sol. Given: $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

\downarrow \downarrow \downarrow
 order 1×3 order 3×3 order 3×1
 Multiplying first matrix with second matrix

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x & -2 & -10 & 2x & -8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}$$

\downarrow \downarrow
 order 1×3 order 3×1
 $\Rightarrow [(x-2)x - 10(4) + (2x-8)1] = 0$
 $\Rightarrow [x^2 - 2x - 40 + 2x - 8] = 0 \Rightarrow [x^2 - 48]_{1 \times 1} = [0]_{1 \times 1}$
 Equating corresponding entries, $x^2 - 48 = 0$

$$\Rightarrow x^2 = 48 \Rightarrow x = \pm \sqrt{48} = \pm \sqrt{16 \cdot 3} = \pm 4\sqrt{3}$$

10. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	Products		
I	10,000	2,000	18,000
II	6,000	20,000	8,000

- (a) If unit sale prices of x , y and z are ₹2.50, ₹1.50 and ₹1.00, respectively, find the total revenue in each market with the help of matrix algebra.
- (b) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively. Find the gross profit.

Sol. The matrix showing the production of the three items in market I and II can be shown by a 2×3 matrix.

Let A be this matrix, then

$$A = \begin{matrix} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} \text{I} \\ \text{II} \end{matrix} & \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}_{2 \times 3} \end{matrix}$$

- (a) Let B be the column matrix representing sale price of each unit of products x , y , z .

$$\text{Then } B = \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}_{3 \times 1}$$

We know that revenue (= sale price \times number of items sold) in matrix form,

$$[\text{Revenue matrix}]_{2 \times 1} = A_{2 \times 3} \times B_{3 \times 1}$$

$$\begin{aligned} &\Rightarrow \begin{bmatrix} \text{Revenue from Market I} \\ \text{Revenue from Market II} \end{bmatrix} \\ &= \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}_{3 \times 1} \\ &= \begin{bmatrix} 25,000 & 3,000 & 18,000 \\ 15,000 & 30,000 & 8,000 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix} \end{aligned}$$

Equating corresponding entries, we have the revenue collected by sale of all items in Market I = ₹46,000 and the revenue collected by sale of all items in Market II = ₹53,000.

- (b) Let the cost matrix showing the cost of each unit of products x , y , z be given by the column matrix C (say)

$$C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}_{3 \times 1}$$

Thus, the total cost of three items for each market is given by: (In general form)

$$\begin{aligned} [\text{Cost matrix}] &= AC \\ &= \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}_{3 \times 1} \end{aligned}$$

$$= \begin{bmatrix} 20,000 & 2,000 & 9,000 \\ 12,000 & 20,000 & 4,000 \end{bmatrix}_2 \quad = \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix}_1$$

∴ The profit collected in two markets is given in matrix form as
 Profit matrix = Revenue matrix - Cost matrix

$$= \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix} - \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix} = \begin{bmatrix} 15,000 \\ 17,000 \end{bmatrix}$$

Hence, the gross profit in both the markets
 $= ₹ 15,000 + ₹ 17,000 = ₹ 32,000.$

11. Find the matrix X so that

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Sol. Given: $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$... (i)

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{order} & \text{order} & \text{order } 2 \times 3 \\ m \times n \text{ (say } 2 \times 3) & & \end{array}$$

∴ $n = 2$ (because numbers of columns in pre-matrix of product must be equal to number of rows in post matrix)

and so L.H.S. matrix is of order $m \times 3$. Again R.H.S. matrix is of order 2×3 . Therefore, $m = 2$ (By definition of equal matrices)

∴ Therefore, matrix X is of order $m \times n$ i.e., 2×2 .

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$... (ii)

Putting this value of X in eqn. (i),

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating corresponding entries, we have

$$\begin{array}{lll} a + 4b = -7 & \dots (iii) & c + 4d = 2 \quad \dots (vi) \\ 2a + 5b = -8 & \dots (iv) & 2c + 5d = 4 \quad \dots (vii) \\ 3a + 6b = -9 & \dots (v) & 3c + 6d = 6 \quad \dots (viii) \end{array}$$

Let us solve eqns. (iii) and (iv) for a and b

Eqn. (iii) $\times 2$ gives $2a + 8b = -14$

Eqn. (iv) is $2a + 5b = -8$

$$\begin{array}{r} - & - & + \\ \hline & & \end{array}$$

Subtracting, $3b = -6 \Rightarrow b = \frac{6}{3} = -2$

Putting $b = -2$ in (iii), $a - 8 = -7 \Rightarrow 7a = -7 + 8 = 1$

Putting $a = 1$ and $b = -2$ in eqn. (v), $3 - 12 = -9$

$\Rightarrow -9 = -9$ which is true. \therefore values of $a = 1$ and $b = -2$ exist.

Now let us solve eqns. (vi) and (vii) for c and d .

Eqn. (vi) $\times 2$ gives $2c + 8d = 4$

Eqn. (vii) is $2c + 5d = 4$

$$\begin{array}{r} \underline{\quad \quad \quad} \\ \underline{\quad \quad \quad} \\ \quad \quad \quad \end{array}$$

Subtracting, $3d = 0 \Rightarrow d = \frac{0}{3} = 0$

Putting $d = 0$ in (vi), $c = 2$

Putting $c = 2$ and $d = 0$ in (vii), $6 = 6$ which is true.

\therefore values of $c = 2$ and $d = 0$ exist.

Putting these values of a, b, c, d in (ii), matrix $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

12. If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $A^n B = B^n A$. Further, prove that $(AB)^n = A^n B^n$ for all $n \in \mathbb{N}$.

Sol. Given: $AB = BA$...(i)

Let $P(n): AB^n = B^n A$...(ii)

We have been asked to prove eqn. (ii) by P.M.I.

(Even if not asked, we would have proved it by P.M.I.)

Step I. For $n = 1$. From eqn. (i), $P(1):$ becomes $AB = BA$

which is given to be true by eqn. (i)

$\therefore P(1)$ is true i.e., eqn. (ii) is true for $n = 1$

Step II. Let $P(k)$ be true i.e., eqn. (ii) is true for $n = k$.

\therefore Putting $n = k$ in (ii), we have $AB^k = B^k A$...(iii)

Step III. Post-multiplying both sides of eqn. (iii) by B ,

We have $AB^k B = B^k AB$

or $A \cdot B^{k+1} = B^k AB$

Putting $AB = BA$ from (i) in R.H.S., we have

$A B^{k+1} = B^k BA \Rightarrow AB^{k+1} = B^{k+1} A$

\therefore Eqn. (ii) is true for $n = k + 1$

(\because On putting $n = k + 1$ in (ii), we get the above result)

$\therefore P(k + 1)$ is true.

$\therefore P(n)$ i.e., eqn. (ii) is true for all $n \in \mathbb{N}$ by P.M.I.

13. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$; then

(A) $1 + \alpha^2 + \beta\gamma = 0$ (B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$ (D) $1 + \alpha^2 - \beta\gamma = 0$

Sol. Given: $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = I (= I_2)$ $\therefore A$ is 2×2

$$\Rightarrow A \cdot A = I_2 \Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \gamma\alpha & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating corresponding entries, we have $\alpha^2 + \beta\gamma = 1$
 $\therefore 1 - \alpha^2 - \beta\gamma = 0$.

Therefore, option (C) is the correct answer.

14. If the matrix A is both symmetric and skew symmetric, then

- (A) A is a diagonal matrix (B) A is a zero matrix
 (C) A is a square matrix (D) None of these.

Sol. Because A is symmetric, therefore $A = A'$... (i)
 Because A is skew-symmetric, therefore $-A = A'$... (ii)
 Putting $A' = A$ from (i) in (ii), $A = -A \Rightarrow A + A = 0$

$$\Rightarrow 2A = 0 \Rightarrow A = \frac{0}{2} = 0$$

i.e., A is a zero matrix. \therefore Option (B) is correct answer.

Note: It may be noted that if A and B are square matrices of the same order, then

$$(A + B)^2 \neq A^2 + B^2 + 2AB \text{ always.}$$

But if matrices A and B commute i.e., $AB = BA$, then $(A + B)^2 = A^2 + B^2 + 2AB$ and also $(A + B)^3 = A^3 + B^3 + 3AB(A + B)$

15. If A is a square matrix such that $A^3 = A^2 = A$, then $(I + A)^7 - 7A$ is equal to

- (A) A (B) I - A (C) I (D) 3A.

Sol. Given: $A^3 = A^2 = A$... (i)

Multiplying both sides by A, $A^3 = A^2 = A$ (By (i)) ... (ii)

The given expression = $(I + A)^7 - 7A$
 $= I^7 + A^7 + 3IA^3 + 3IA(I + A) - 7A$

[We know that $AI = IA$, therefore using above note we can apply $(A + B)^3 = A^3 + B^3 + 3AB(A + B)$]

$$= I + A^3 + 3IA^3 + 3IA^2 - 7A$$

Putting $A^2 = A$ from (i) and $A^3 = A$ from (ii) and

$$I^3 = I \text{ and } I^7 = I \text{ (Because } I^n = I \text{ always for all } n \in \mathbb{N})$$

$$= I + A + 3IA + 3IA - 7A$$

$$= I + A + 3A + 3A - 7A$$

$$= I + 7A - 7A = I$$

Hence, option (C) is the correct answer.



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- Chapter 1 – Relations and Functions
- Chapter 2 – Inverse Trigonometric Functions.
- Chapter 3 – Matrices
- Chapter 4 – Determinants.
- Chapter 5 – Continuity and Differentiability.0.0
- Chapter 6 – Application of Derivatives.
- Chapter 7 – Integrals.
- Chapter 8 – Application of Integrals.
- Chapter 9: Differential Equations
- Chapter 10: Vector Algebra
- Chapter 11: Three Dimensional Geometry
- Chapter 12: Linear Programming
- Chapter 13: Probability

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