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NCERT Solutions for 12th Class Maths: Chapter 12-Linear Programming

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NCERT Solutions for 12th Class Maths: Chapter 12-Linear Programming

Class 12: Maths Chapter 12 solutions. Complete Class 12 Maths Chapter 12 Notes.

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Ex 12.1 Class 12 Maths Question 1.

Maximize Z = 3x + 4y

subject to the constraints:

x + y ≤ 4,x ≥ 0,y ≥ 0.

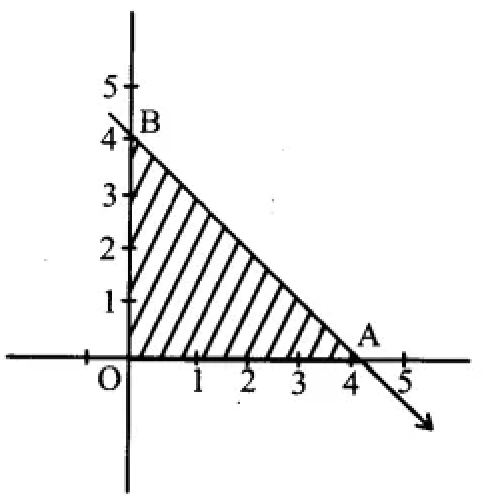
Solution:

As x ≥ 0, y ≥ 0, therefore we shall shade the other inequalities in the first quadrant only. Now consider x + y ≤ 4.

Let x + y = 4 =>

$$\frac{x}{4} + \frac{y}{4} = 1$$

Thus the line has 4 and 4 as intercepts along the axes. Now (0, 0) satisfies the inequation i.e., 0 + 0 ≤ 4. Now shaded region OAB is the feasible solution.



Now its corners are O (0,0), A (4, 0), B (0,4).

At O (0, 0) $Z = 0$

At A (4, 0), $Z = 3 \times 4 = 12$

At B (0, 4), $Z = 4 \times 4 = 16$

Now max $Z = 16$ at $x = 0, y = 4$.

Ex 12.1 Class 12 Maths Question 2.

Minimize Z = -3x+4y

subject to $x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0$

Solution:

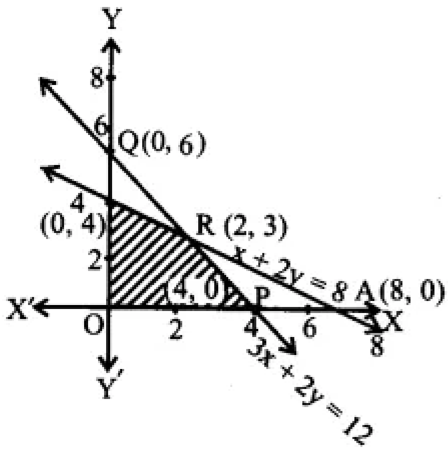
Objective function $Z = -3x + 4y$

constraints are $x+2y \leq 8$,

$3x + 2y \leq 12, x \geq 0, y \geq 0$

(i) Consider the line $x+2y = 8$. It pass through A (8,0) and B (0,4), putting $x = 0, y = 0$ in $x + 2y \leq 8, 0 \leq 8$ which is true.

=> region $x + 2y \leq 8$ lies on and below AB.



(ii) The line $3x + 2y = 12$ passes through P (4, 0), Q (0, 6) putting $x = 0, y = 0$ in $3x + 2y \leq 12$

$\Rightarrow$ $0 \leq 12$, which is true.

$\therefore$ Region $3x + 2y \leq 12$ lies on and below PQ.

(iii) $x \geq 0$ the region lies on and to the right of y-axis.

(iv) $y \geq 0$ lies on and above x-axis.

(v) Solving the equations $x + 2y = 8$ and $3x + 2y = 12$ we get $x = 2, y = 3$ $\Rightarrow$ R is (2,3) where AB and PQ. intersect.The shaded region OPRB is the feasible region.

At P (4,0)	$Z = -3x + 4y = -12 + 0 = -12$
At R (2,3)	$Z = -6 + 12 = 6$
At B (0,4)	$Z = 0 + 16 = 16$
At Q (0,0)	$Z = 0$

Thus minimum value of Z is -12 at P (4, 0)

Ex 12.1 Class 12 Maths Question 3.

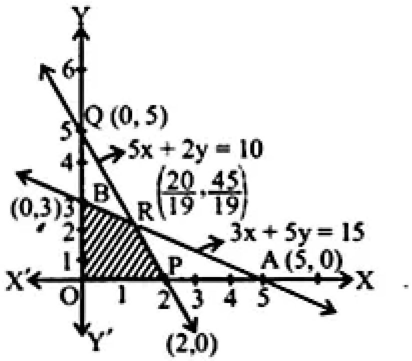
Maximize Z = 5x+3y

subject to $3x + 5y \leq 15, 5x + 2y \leq 10, x \geq 0, y \geq 0$

Solution:

The objective function is Z = 5x + 3y constraints

are $3x + 5y \le 15$, $5x + 2y \le 10, x \ge 0, y \ge 0$



(i) Consider the line $3x + 5y = 15$ which passes through A (5,0) and B (0,3) putting x = 0, y = 0 in $3x + 5y \le 15$

$\Rightarrow$ $0 \le 15$ which is true.

$\therefore$ Region $3x + 5y \le 15$ lies on and below AB.

(ii) The line $5x + 2y = 10$ passes through P (2, 0) and Q (0, 5), Put x = 0, y = 0 in $5x + 2y \le 10$

$\therefore$ $0 \le 10$ which is true.

$\therefore$ Region $5x + 2y \le 10$ lies on and below PQ.

(iii) $x \ge 0$ Region lies on and to the right of y-axis.

(iv) $y \ge 0$ lies on the above x-axis.

Ex 12.1 Class 12 Maths Question 4.

Minimize Z = 3x + 5y such that $x + 3y \ge 3$, $x + y \ge 2, x, y \ge 0$.

Solution:

For plotting the graph of x + 3y = 3, we have the following table:

x	0	3
y	1	0

Plot the points (0, 1) and (3, 0) on a graph paper and join them to get a line which represents the equation x +3y = 3. For equation x + y = 2, we have the following table:

x	1	0
y	1	2

Plot the points (1,1) and (0,2) on the graph paper and join them to get the line representing the equation
x + y = 2.

At C (3, 0), $Z = 3\times 3 + 5\times 0 = 9$
So, Z is minimum is = 7 when

$$x = \frac{3}{2} \text{ and } y = \frac{1}{2}.$$

Ex 12.1 Class 12 Maths Question 5.

Maximize Z=3x+2y subject to x+2y ≤ 10, 3x+y ≤ 15, x, y ≥ 0.

Solution:

Consider x + 2y ≤ 10

Let x + 2y = 10

=>

$$\frac{x}{10} + \frac{y}{5} = 1$$

Now (0,0) satisfies the inequation, therefore the half plane containing (0,0) is the required plane.

Again 3x+2y ≤ 15

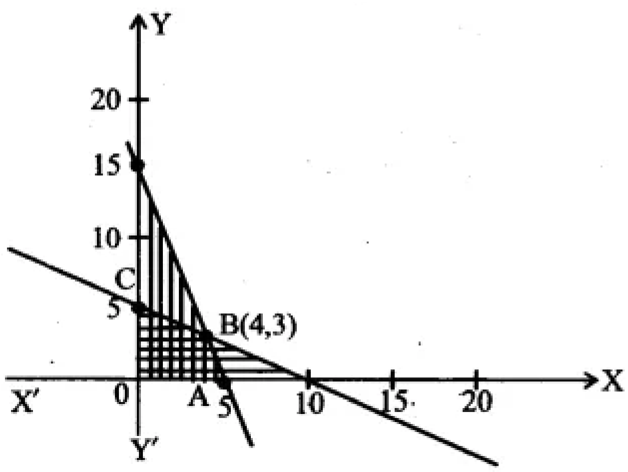
Let 3x + y = 15

=>

$$\frac{x}{5} + \frac{y}{15} = 1$$

It is also satisfies by (0,0) and its required half plane contains (0,0).

Now double shaded region in the first quadrant contains the solution.



Now OABC, represents the feasible region

$Z = 3x + 2y$

At O (0, 0), $Z = 0 + 0 = 0$

At A (5, 0), $Z = 15$

At B (4, 3), $Z = 18$

At C (0, 5), $Z = 10$

Now Max $Z = 18$ at $x = 4$, $y = 3$.

Ex 12.1 Class 12 Maths Question 6.

Minimize Z = x + 2y subject to 2x + y ≥ 3, x + 2y ≥ 6, x, y ≥ 0.

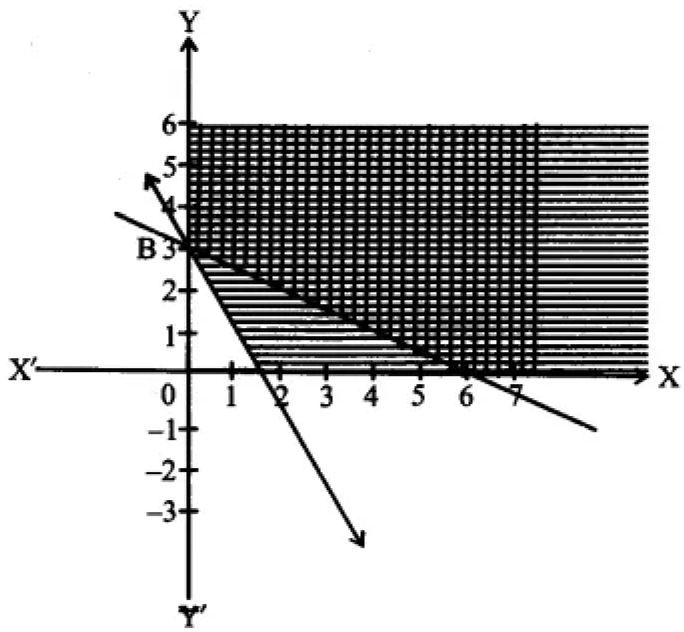
Solution:

Consider 2x + y ≥ 3

Let 2x + y = 3

⇒ y = 3 – 2x

	A	B	C
x	0	2	–1
y	3	–1	5



(0,0) is not contained in the required half plane as (0, 0) does not satisfy the in equation 2x + y ≥ 3.

Again consider x+2y≥6

Let x + 2y = 6

=>

$$\frac{x}{6}+\frac{y}{3}=1$$

Here also (0,0) does not contain the required half plane. The double-shaded region XABY' is the solution set. Its comers are A (6,0) and B (0,3). At A, Z = 6 + 0 = 6

At B, Z = 0 + 2 × 3 = 6

We see that at both points the value of Z = 6 which is minimum. In fact at every point on the line AB makes Z=6 which is also minimum.

Show that the minimum of z occurs at more than two points.

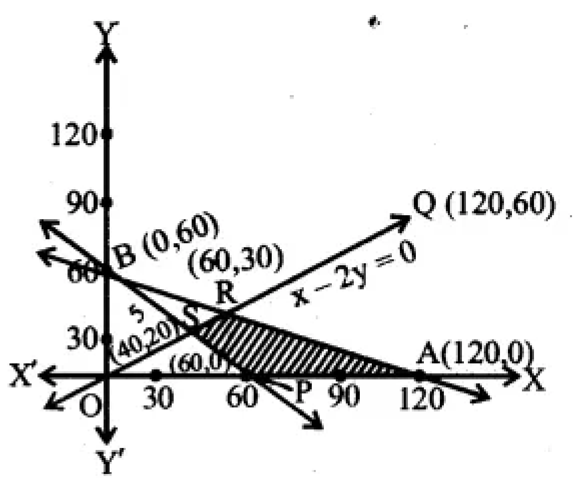
Ex 12.1 Class 12 Maths Question 7.

Minimise and Maximise Z = 5x + 10y

subject to x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x,y≥0

Solution:

The objective function is Z = 5x + 10y constraints are x + 2y≤120,x+y≥60, x-2y≥0, x,y≥0



(i) The line x + 2y = 120 passes through A (120, 0) and B (0, 60) putting x = 0, y = 0 is $x + 2y \leq 120$ we get $0 \leq 12$ which is true.

⇒ $x + 2y \leq 120$ lies on AB and below AB.

(ii) The line x + y = 60 passes through P (60, 0) and B (0, 60) putting x = 0 and y = 0 in $x + y \geq 60$, We get $0 \geq 60$ which is not true.

⇒ $x + y \geq 60$ lies on PB.

(iii) The line $x - 2y = 0$ passes through O (0, 0) and Q (120, 60) putting $x = 60, y = 0$ is $x - 2y \geq 0$, $60 \geq 0$ which is true.

$\Rightarrow$ $x - 2y \geq 0$ region is on OQ

(iv) $x \geq 0$ lies on y-axis and on its right.

(v) $y \geq 0$ lies on x-axis $\Rightarrow$ feasible region is PARS which has been shaded.

(a) Sloving OQ : $x - 2y = 0$ and AB : $x + 2y = 120$

$\Rightarrow x = 60, y = 30 \Rightarrow$ R is (60,30)

(b) Solving OQ : $x - 2y = 0$ and PB : $x + y = 60$, $x = 40, y = 20 \Rightarrow$ S is (40,20)

At A (120, 0),	$Z = 5x + 10y = 600$	max^m
At R (60, 30),	$Z = 300 + 300 = 600$	max^m
At S (40, 20),	$Z = 200 + 200 = 400$	
At P (60, 0),	$Z = 300 + 0 = 300$	min^m.

$\Rightarrow$ Minimum value of Z at P is 300 at P(60, 0) and maximum value of Z is 600 at all points joining A (120, 0) and R (60, 30).

Ex 12.1 Class 12 Maths Question 8.

Minimize and maximize Z = x + 2y subject to x + 2y ≥ 100,2x – y ≤ 0,2x + y ≤ 200;x,y ≥ 0.

Solution:

Consider x + 2y ≥ 100

Let x + 2y = 100

=>

$$\frac{x}{100} + \frac{y}{50} = 1$$

Now x + 2y ≥ 100 represents which does not include (0,0) as it does not made it true.

Again consider 2x – y ≤ 0

Let 2x – y = 0 or y = 2x

x	0	25	50	100
y	0	50	100	200

Now let the test point be (10,0)

$\therefore$ $2 \times 10 - 0 \leq 0$ which is false.

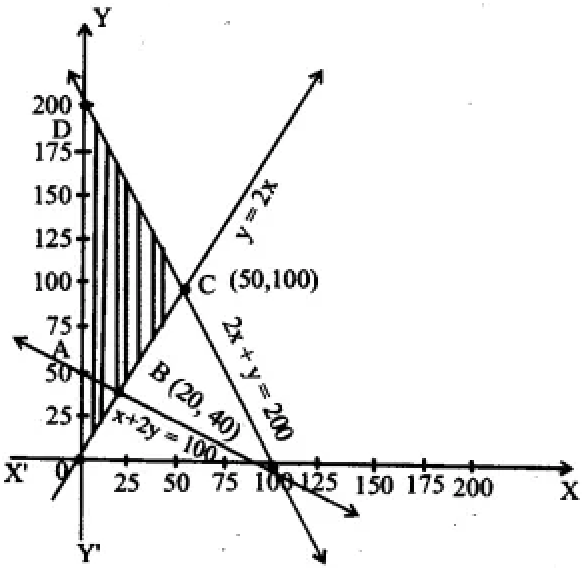
$\therefore$ The required half does not contain (10,0).

Again consider $2x + y \leq 200$

Let $2x + y = 200 \Rightarrow \frac{x}{100} + \frac{y}{100} = 1$

Now (0, 0) satisfies $2x + y \leq 200$

$\therefore$ The required half plane contains (0, 0).



Now triple shaded region is ABCDA which is the required feasible region.

At A (0, 50), $Z = x + 2y = 0 + 2 \times 50 = 100$

At B (20, 40), $Z = 20 + 2 \times 40 = 100$

At C (50, 100), $Z = 50 + 2 \times 100 = 250$

At D(0, 200), $Z = 0 + 2 \times 200 = 400$

Thus maximum Z = 400 at x = 0, y = 200 and minimum Z = 100 at x = 0, y = 50 or x = 20, y = 40

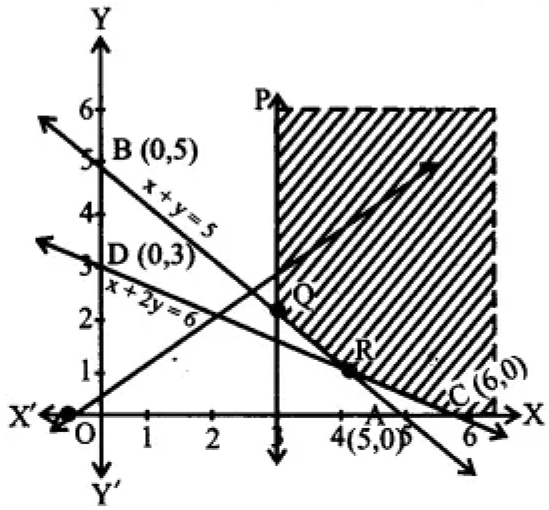
Ex 12.1 Class 12 Maths Question 9.

Maximize Z = -x + 2y, subject to the constraints: $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$

Solution:

The objective function is $Z = -x + 2y$.

The constraints are $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$



(i) The line x + y = 5 passes through A (5, 0), B (0, 5) putting x = 0, y = 0 in $x + y \geq 5$, we get $0 \geq 5$ which is not true

$\Rightarrow$ $x + y \geq 5$ liles on and above AB.

(ii) The line x + 2y = 6 passes through C (6, 0), D (0, 3) putting x = 0, y = 0 in $x + 2y \geq 6$. we get $0 \geq 6$ which is not true. $\Rightarrow x + 2y \geq 6$ lies on and above CD.

(iii) $x \geq 3$ lies on PQ or on the right of it.

(iv) $y \geq 0$ lies on and above x-axis.

(v) The feasible region is PQRCX.

(a) Solving x = 3 and x + y = 5, we get x = 3, y = 2

$\therefore$ these lines meet at Q (3,2)

(b) x + y = 5 and x + 2y = 6 meet at R *i.e.,* the point R is (4,1)

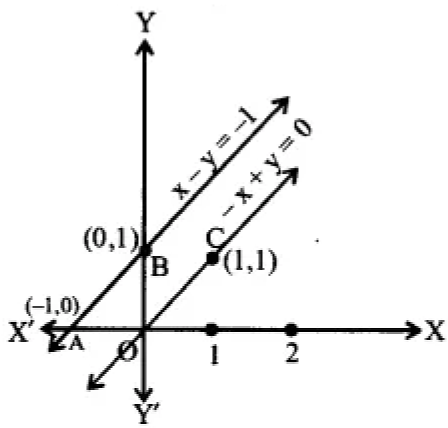
At Q (3, 2) $Z = -x + 2y = -3 + 4 = 1$ max^m
At R (4, 1) $Z = -4 + 2 = -2$
At C (6, 0) $Z = -6 + 0 = -6$ min^m
The maximum value of Z is 1 is but the feasible region is unbounded. Consider the inequality $-x + 2y > 1$.
The line $-x + 2y = 1$ passes through $(-1, 0)$ and $(0, 1/2)$ putting $x = 0, y = 0$ in $-x + 2y > 1$, we get $0 > 1$ which is not true.
$\Rightarrow$ $-x + 2y > 1$ lies above the line $-x + 2y = 1$ feasible region of $-x + 2y > 1$ have many points in common.
Therefore, there is no maximum value.

Ex 12.1 Class 12 Maths Question 10.

Maximize Z = x + y subject to x – y≤ -1, -x + y≤0,x,y≥0

Solution:

Objective function Z = x + y, constraints x – y≤ -1, -x + y≤0,x,y≥0



(i) The line $x - y = -1$ passes through $(-1, 0)$ and $(0, 1)$ putting $x = 0, y = 0$ in $x - y \leq -1$ we get $0 \leq -1$ which is not true.

$\Rightarrow$ $x - y \leq -1$ lies on and above AB, $x - y = -1$

(ii) The line $-x + y = 0$ passes through O $(0, 0)$ and C $(1, 1)$ putting $x = 0, y = 1$ in $-x + y \leq 0$ we get $1 \leq 0$ which is not true.

$\Rightarrow$ $-x + y \leq 0$ lies on and below OC.

(iii) $x \geq 0$ lies on and on the right of y-axis.

(iv) $y \geq 0$ lies on and above x-axis. There is no common region *i.e.,* there is no feasible region.

There is no maximum value of Z.

Ex 12.2 Class 12 Maths Question 1.

Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/ kg of Vitamin A and 5 units/ kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

Solution:

Let x kg of food P and y kg of food Q are mixed,

Food	Quantity	Vitamin A	Vitamin B	Cost
P	x kg	3 units/kg	5 units/kg	Rs. 60/kg
Q	y kg	4 units/kg	2 units/kg	Rs. 80/kg
least quantity of vitamin required		8 unit	11 unit	

Quantity of vitamin A in x kg food P and y kg food

$Q = 3x + 4y$

Quantity of vitamin required at least = 8 units

$\Rightarrow \quad Q = 3x + 4y \geq 8$

Quantity of vitamin B in x kg food P and y kg of food

$Q = 5x + 2y$

Quantity of vitamin required at least = 11 units

$\Rightarrow \quad 5x + 2y \geq 11$

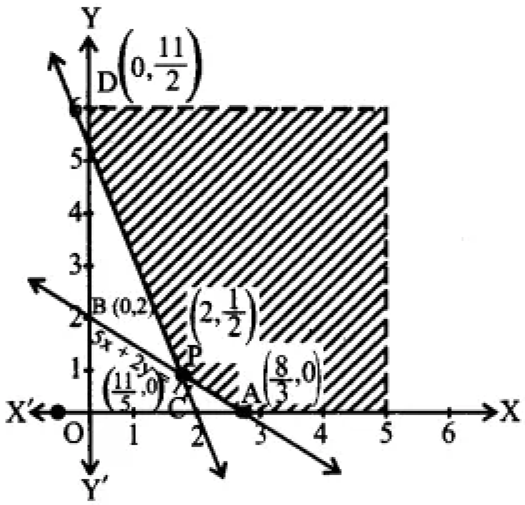
Cost of x kg of food P and y kg of food Q = 60x + 80y

$\Rightarrow$ Objective function Z = 60x + 80y, subject to Constraints are $3x + 4y \geq 8$ $5x + 2y \geq 11$ and $x, y \geq 0$

(i) The line 3x + 4y = 8 passes through

$A\left(\frac{8}{3}, 0\right)$ and

B (0, 2) putting x = 0, y = 0 in
$3x + 4y \geq 8$ we get $0 \geq 8$ which is not true



⇒ The region represented by $3x + 4y \geq 8$ lies on and above AB.

(ii) The line $5x + 2y = 11$ passes through $C\left(\frac{11}{5}, 0\right)$ and $D\left(0, \frac{11}{2}\right)$. Putting x = 0, y = 0 in $5x + 2y \geq 11$, $0 \geq 11$ which is not true

⇒ Region $5x + 2y \geq 11$ lies on and above CD.

(iii) $x \geq 0$ is the region on and to the right of y-axis.

(iv) $y \geq 0$ is the region on and above x-axis. The shaded area represents the feasible region YDPAX, where P is the intersection of AB and CD. These lines are

$3x + 4y = 8$ ...(i)

$5x + 2y = 11$ ...(ii)

Multiply (ii) by 2 and subtracting (i) from (ii)

$x = 2$

from (i) $4y = 8 - 3x = 8 - 6 = 2$

$\therefore\quad y = \frac{1}{2}$ the point P is $\left(2, \frac{1}{2}\right)$

Now objective function $Z = 60x + 80y$,

At D $\left(0, \frac{11}{2}\right)$, $Z = 80 \times \frac{11}{2} = 440$

At P $\left(2, \frac{1}{2}\right)$, $Z = 120 \times 40 = 160$ min^m cost,

At A $\left(\frac{8}{3}, 0\right)$, $Z = 60 \times \frac{8}{3} + 0 = 160 \rightarrow$ min^m cost

Minimum value of $Z = 160$. But feasible region is unbounded

$\therefore$ Consider the inequality $60x + 80y < 160$

or $3x + 4y < 8$

This shows that this region is below the line AD : $3x + 4y = 8$

$\Rightarrow$ There is no point in common between feasible region and $3x + 4y < 8$. Hence the minimum cost z is Rs 160 at all points lying on the segment joining A $\left(\frac{8}{3}, 0\right)$ and P $\left(2, \frac{1}{2}\right)$.

Ex 12.2 Class 12 Maths Question 2.

One kind of cake requires 200 g of flour and 25g of fat, and another kind of cake requires 100 g of flour and 50 g of fat Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients, used in making the cakes.

Solution:

Let number of cakes made of first kind are x and that of second kind is y.

∴ maximize $Z = x + y$

Subject to $200x + 100y \leq 5000$ and $25x + 50y \leq 1000$

$x \geq 0, y \geq 0.$

Consider $200x + 100y \leq 5000 \Rightarrow 2x + y \leq 50$

Let $2x + y = 50 \Rightarrow \frac{x}{25} + \frac{y}{50} = 1$

As (0, 0) satisfies the inequation, thus the required half plane contains (0, 0).

Again consider

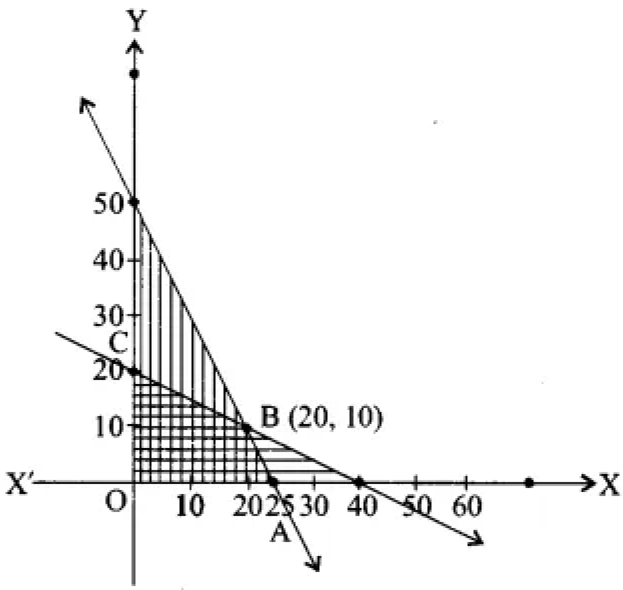
$25x + 50y \leq 1000 \Rightarrow x + 2y \leq 40$

Let $x + 2y = 40 \Rightarrow \frac{x}{40} + \frac{y}{20} = 1$

Now again (0, 0) also satisfies the inequation, thus the required half plane contains (0, 0).

Now the double shaded region is the feasible region i.e. solution set.

The corner points of this region are O (0, 0), A (25, 0), B (20, 10), C (0, 20),



Consider	$Z = x + y$
Now at O (0, 0),	$Z = 0 + 0 = 0$
At A (25,0),	$Z = 25 + 0 = 25$
At B (20, 10),	$Z = 20 + 10 = 30$
At C (0, 20),	$Z = 0 + 20 = 20$

Maximum number of cakes $Z = 30$ when $x = 20$, $y = 10$.

Ex 12.2 Class 12 Maths Question 3.

A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(i) What number of rackets and bats must be made if the factory is to work at full capacity?

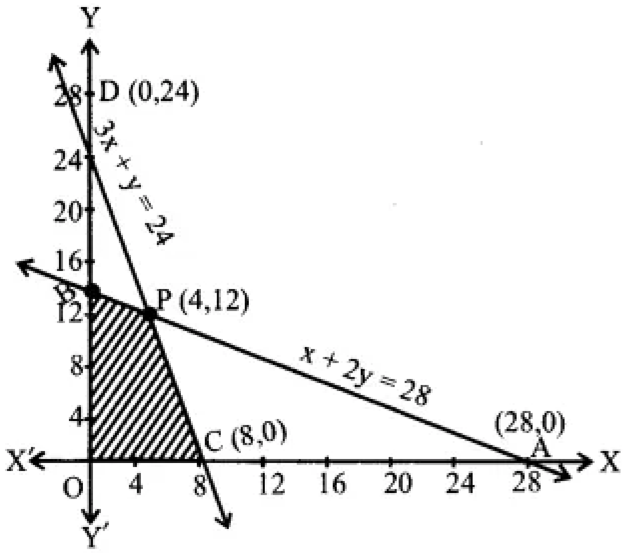
(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity. .

Solution:

Let x tennis rackets and y cricket bats are produced in one day in the factory.

Item	Number	Machine Hours	Craftman Hours	Profit
Tennis Rackets	x	1×5	3	₹ 20 per item
Cricket Bats	y	3	1	₹ 10 per item
Total time Available		42	24	

Total Machine hours $= 1\cdot5x + 3y$,
Maximum time available = 42 hours
$1\cdot5x + 3y \leq 42$ or $x + 2y \leq 28$ ...(i)
Craftman's hours $= 3x + y$
Maximum time available = 24
$3x + y \leq 24$ ...(ii)
Also $y \geq 0$
(i) $Z = x + y$ constraints are $x + 2y \leq 28$,
$3x + y \leq 24$, $x, y \geq 0$



(a) The line $x + 2y = 28$ passes through A (28,0) and B (0,14) putting $x = 0, y = 0$ in $x + 2y \leq 28$ represents the region on and below AB.

(b) The line $3x + y = 24$ passes through C (8, 0) and D (0, 24), putting $x = 0, y = 0$ in $3x + y \leq 24$ we get $0 \leq 24$ which is true.

∴ $3x + y \leq 24$ represents the region on and below the line CD.

(c) $x \geq 0$ represents the region on and to the right of y-axis.

(d) $y \geq 0$ represents the region on and above the x-axis.

(e) The shaded area BPCO is the feasible region. The two lines AB an CD are

$x + 2y = 28$ ...(i)

$3x + y = 24$ ...(ii)

Mulitply equ. (ii) by 2 and subtract (i) from (ii)

$5x = 48 - 28 = 20 \therefore x = 4$

from (ii) $12 + y = 24$, $y = 12$ Thus, these lines meet at P (4, 12)

(i) At B (0, 14) $Z = x + y = 0 + 14 = 14$

At P (4, 12) $Z = 4 + 12 = 16$

At C (8, 0) $Z = 8 + 0 = 8$

At O (0, 0) $Z = 0$

Maximum value of Z = 16 *i.e.,* 4 tennis racket and 12 cricket bats must be made so factory works at full capacity.

(ii) Profit function $Z = 20x + 10y$

At B (0, 14) $Z = 0 + 10 \times 14 = 140$

At P (4, 12) $Z = 80 + 120 = 200 \rightarrow$ max^m. Profit

At C (8, 0) $Z = 20 \times 8 + 0 = 160$

At D (0, 0) $Z = 0$

Thus maximum profit is ₹ 200, when 4 tennis racket and 12 cricket bats are produced.

Ex 12.2 Class 12 Maths Question 4.

A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours

Solution:

Let x nuts and y bolts are produced.

Item	Number	Machine A	Machine B	Profit
Nuts	x	1 hours	3 hours	17×50
Bolts	y	3 hours	1 hours	7×00
Max. Time available		12 hours	12 hours	

Machine A is used for x × 1 + y × 3 hours ,
Maximum time avaliable = 12 hours,

$\therefore \quad x + 3y \leq 12$

Machine B is used for 3 × x + 1× y hours.

Maximum time avaliable = 12 hours

$\Rightarrow \quad 3x + y \leq 12$

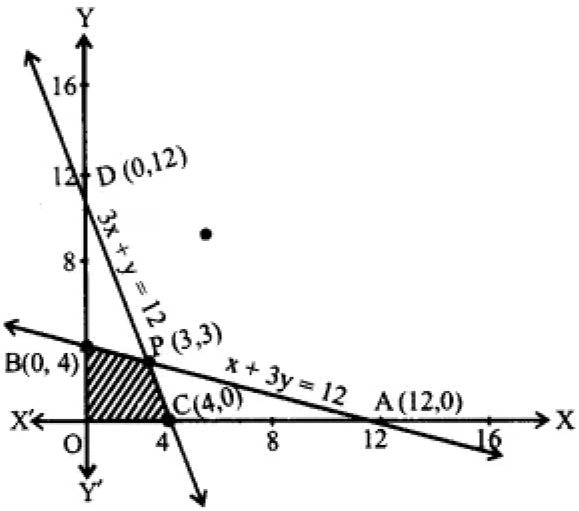
Profit function Z = (17·50) × x + (7·00) y. Thus the objective function Z =17·5 x + 7y, subject to constraints are x + 3 y ≤ 12, 3 x + y ≤ 12 and x, y ≤ 0

(i) The line x + 3y = 12 passes though A (12, 0), B (0, 4)

Putting x = 0, y = 0 in $x + 3y \leq 12$

we get $0 \leq 12$ which is true.

$\Rightarrow \quad x + 3y \leq 12$ lies on and below AB.



(ii) x + 3y = 12 passes thorugh C (4, 0) and D (0, 12) putting x = 0, y = 0 in $3x + y \leq 12$, we get $0 \leq 12$ which is true. $3x + y \leq 12$ lies on and below CD.

(iii) $x \geq 0$ is the region which lies on and to the right of y-axis.

(iv) $y \geq 0$ is the region which lies on and above the x-axis.

(v) The shaded area BPCO is the feasible region.

The point P is the intersection of the lines.

AB : x + 3y = 12 ...(i)

CD : 3x + y = 12 ...(ii)

Multiplying equ. (i) by 3 and subtracting (ii)

from (i) 8y = 36 – 12 = 24, ∴ y = 3

from (i) x = 12 – 3y = 12 – 9 = 3

∴ The point P is (3, 3) and z = 17·5x + 7y

At B (0, 4) Z = 0 + 7 × 4 = 28

At P (3, 3) Z = 17·5 × 3 + 7 × 3 = 73·5

At C (4, 0) Z = 17·5 × 4 + 0 = 70

At O (0, 0) Z = 0

Maximum profit is ₹ 73·50 when 3 nuts and 3 bolts packages are produced.

Ex 12.2 Class 12 Maths Question 5.

A factory manufactures two types of screws, A and B, Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.

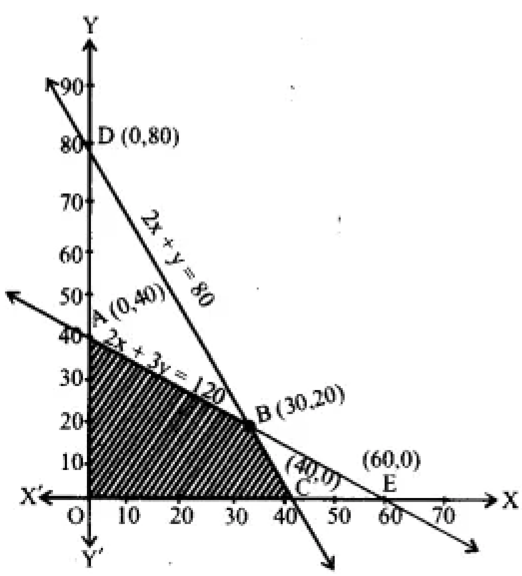
Solution:

Let the manufacturer produces x packages of screws A and y packages of screw B, then time taken by x packages of screw A and y packages of screw B on automatic machine = (4x + 6y) minutes.

And hand operated machine = (6x + 3y) minutes

As each machine is available for at the most 4 hours *i.e.*, $4 \times 60 = 240$ minutes.So,
we have $4x + 6y \leq 240$ *i.e.*, $2x + 3y \leq 120$ and $6x + 3y \leq 240$ *i.e.*, $2x + y \leq 80$
Profit on selling x packages of screws A and y packages of screws B is $Z = 7x + 10y$.
So,to find x and y such that $Z = 7x + 10y$ is maximum subject to $2x + 3y \leq 120$, $2x + y \leq 80$, $x \geq 0$ and $y \geq 0$. The feasible portion of the graph satisfying the inequalities $2x + 3y \leq 120$ and $2x + y \leq 80$ is OABC which is shaded in the figure. Coordinates of O, A, B and C are (0, 0), (0, 40), (30, 20) and (40, 0) respectively.

At A (0, 40), $Z = 7x + 10y = 0 + 10 \times 40 = 400$
At B (30, 20), $Z = 7 \times 30 + 10 \times 20 = 410$ max^m
At C (40, 0), $Z = 7 \times 40 + 0 = 280$
At O (0, 0), $Z = 0$



Hence, the maximum profit ₹410 when 30 screws of type A and 20 screws of type B are produced.

Ex 12.2 Class 12 Maths Question 6.

A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/ cutting machine and a sprayer. It takes 2 hours on grinding/ cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp, while it takes 1 hour on the grinding/ cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?

Solution:

Let the manufacturer produces x pedestal lamps and y wooden shades; then the time taken by x pedestal lamps and y wooden shades on grinding/ cutting machines = $(2x + y)$ hours and time taken by x pedestal lamps and y shades on the sprayer = $(3x + 2y)$ hours.

Since grinding/ cutting machine is available for at the most 12 hours, $2x + y \le 12$ and sprayer is available for at the most 20 hours.

We have: $3x + 2y \le 20$.

Profit from the sale of x lamps and y shades.

$Z = 5x + 3y$

So, our problem is to maximize $Z = 5x + 3y$ subject to constraints $3x + 2y \le 20, 2x + y \le 12, x, y \ge 0$.

Consider $3x + 2y \le 20$

Let $3x + 2y = 20$

	A	B	C
x	0	2	4
y	10	7	4

Now the area represented by $3x + 2y \leq 20$ is the half-plane containing (0, 0) as (0, 0) stisfies the inequations.

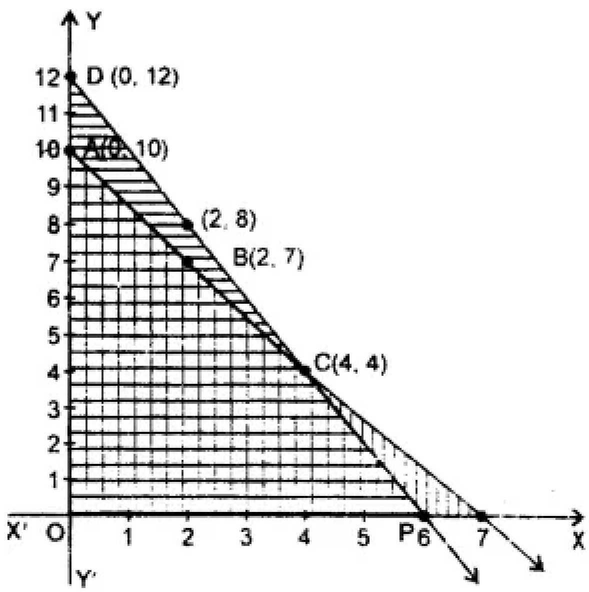
Again consider $2x + y \leq 12$

Let $2x + y = 12 \Rightarrow y = 12 - 2x$

	D	E	C
x	0	2	4
y	12	8	4

$y = \frac{20-3x}{2}$

The inequations consists of the half-plane containing (0, 0) as (0, 0) satisfies it.



The double shaded region OPCAO is our solution where O (0, 0), P (6, 0), C (4, 4), A (0, 10).

Now $Z = 5x + 3y$

At O (0, 0), $Z = 0$

At P (6, 0), $Z = 30$

At C (4, 4), $Z = 5 \times 4 + 3 \times 4 = 32$

At A (0 , 10), $Z = 5 \times 0 + 3 \times 10 = 30$

Now maximum $Z = 32$ at $x = 4, y = 4$.

Hence maximum Z = ₹ 32 when he manufacture 4 pedestal lamps and 4 wooden shades.

Ex 12.2 Class 12 Maths Question 7.

A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should be company manufacture in order to maximise the profit?

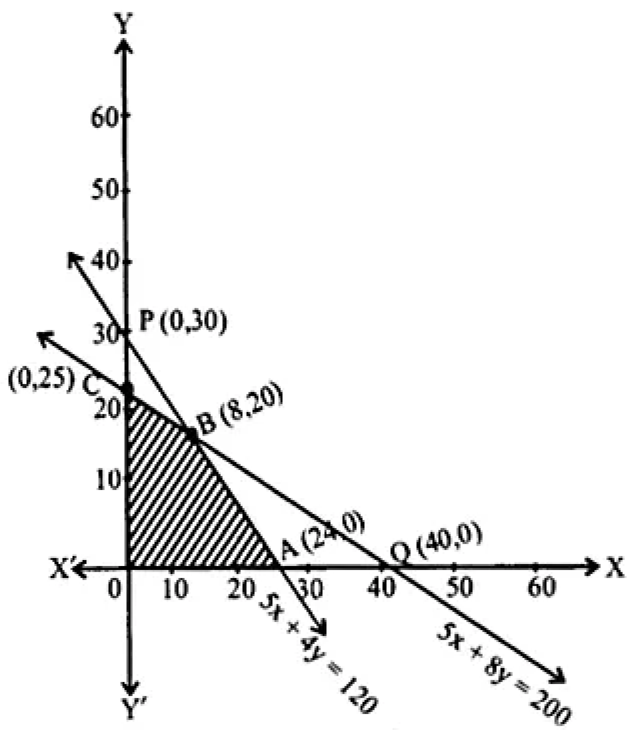
Solution:

Let the company manufactures x souvenirs of type A and y souvenirs of type B, then time taken for cutting x souvenirs of type A and y souvenirs of type A and y souvenirs of type B = (5x + 8y) minutes.

Since 3 hours 20 minutes i.e 200 minutes are available for cutting, so we should have $5x + 8y \le 200$

Also as 4 hours i.e., 240 minutes are available for assembling, so we have $10x + 8y \le 240$ i. e., $5x + 4y \le 120$

Thus our L.P.P is to maximize profit $Z = 5x + 6y$
i.e., $z = 5x + 6y$, subject to constraints are
$5x + 8y \le 200$, $5x + 4y \le 120$, $x, y \ge 0$
For equation $5x + 4y = 120$



x	0	24
y	30	0

For equation $5x + 8y = 200$

x	40	0
y	0	25

The inequalities $5x + 8y \leq 200$ and $5x + 4y \leq 120$ is OABC and in the shaded figure coordinate of points O, A, B and C are (0, 0), (24, 0), (8, 20) and (0, 25) respectively.

The objective function is $Z = 5x + 6y$

At A (24, 0) $Z = 5 \times 24 + 0 = 120$

At B (8, 20) $Z = 40 + 120 = 160 \rightarrow max^m$

At C (0, 25) $Z = 0 + 6 \times 25 = 150$

At D (0, 0) $Z = 0$

Thus, maximum profit is ₹160 when 8 souvenirs of type A and 20 souvenirs of type B are produced.

Ex 12.2 Class 12 Maths Question 8.

A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which die merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

Solution:

Let there be x desktop and y portable computers. Total monthly demand of computer does not exceed 250. => $x+y \leq 250$, cost of 1 desktop computer if Rs 25000 and 1 portable computer is Rs 40000

∴ cost of x desktop and y portable computer = Rs (25000 x+40000 y)

Maximum investment = Rs 70 lakhs = Rs 70,00,000

=> $25000x + 40000y \leq 7000000$ or $5x + 8y \leq 1400$

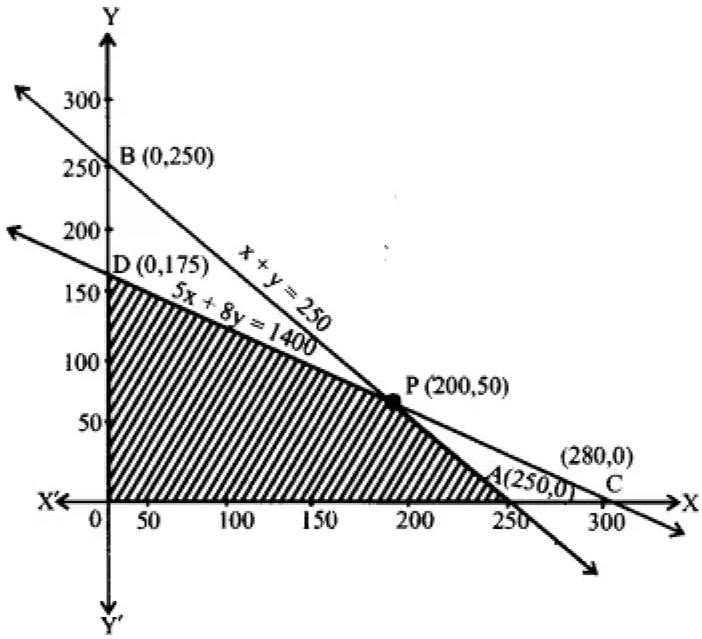
profit on 1 desktop computer is Rs 4500 and on 1 portable is Rs 5000, total profit,

$Z = 4500x + 5000y$

objective function to maximise $Z = 4500x + 5000y$ and constraints are $x + y \leq 250$, $5x + 8y \leq 1400$ $x, y \geq 0$

(i) The line $x + y = 250$ passes through $A(250, 0)$, $B(0, 250)$ putting $x = 0$, $y = 0$ in $x + y \leq 250$, $0 \leq 25$, which is true.

$\Rightarrow$ $x + y \leq 250$ lies on or below AB.



(ii) The line $5x + 8y = 1400$ passes through $C(280, 0), D(0, 175)$

put $x = 0, y = 0$ in $5x + 8y \leq 14000, 0 \leq 1400$ which is true.

$\Rightarrow$ $5x + 8y \leq 1400$ lies on and below CD.

(iii) $x \geq 0$, lies on and to the right to axis.

(iv) $y \geq 0$, lies on and above x-axis.

The shaded area OAPD represent the feasible region where P is the point of intersection of

$5x + 8y = 1400$ ...(i)

$x + y = 250$ ...(ii)

Multiplying (ii) by 8 and subtracting (i) from (ii)

$3x = 2000 - 1400 = 600 \Rightarrow x = 200$

$\therefore$ $y = 250 - x = 250 - 200 = 50$

The point P is (200,50) values of Z at A, P, D, O

At A (250, 0), $Z = 4500x + 5000y = 4500 \times 250 + 0 = 1125000$

At P (200, 50), $Z = 4500 \times 200 + 5000 \times 50 = 900000 + 250000 = 1150000$

At D (0, 175), $Z = 0 + 5000 \times 175 = 875000$

$\Rightarrow$ Maximum profit of ₹ 1150000 is obtained when he stocks 200 desktop computer and 50 portable.

Ex 12.2 Class 12 Maths Question 9.

A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods Ft and F2 are available. Food F1 costs Rs 4 per unit food and F2 costs Rs 6 per unit.One unit of food F2 contains 3 units of vitamin A and 4 units of minerals. One unit of food F2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Solution:

Let there be x units of food F1 and y units of food F2.

Food	Quantity	Vitamin A	Minerals	Costs
F_1	x units	3 units	4 units	₹ 4
F_2	y units	6 units	3 units	₹ 6
	80 units	100 units		

vitamin A contained in food F_1 and $F_2 = 3x + 6y$
least units of vitamin A = 80 units
$\Rightarrow 3x + 6y \geq 80$ ...(i)
Minerals contained in food F_1 and $F_2 = 4x + 3y$
least quantity mineral required = 100 units
$\therefore \quad 4x + 3y \geq 100$
L. P. P is to minimize $Z = 4x + 6y$
Subject to Constraints are $3x + 6y \geq 80$, $4x + 3y \geq 100$
and $x, y \geq 0$

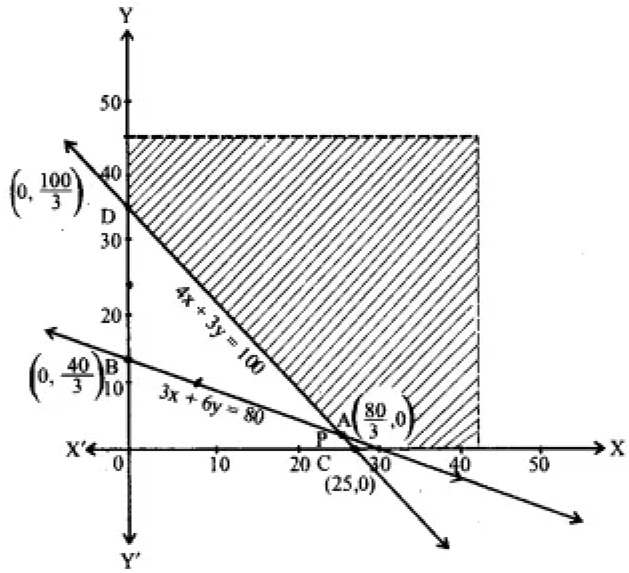
(i) The line $3x + 6y = 80$ passes through

$A\left(\frac{80}{3}, 0\right)$, $B\left(0, \frac{40}{3}\right)$ putting

$x = 0, y = 0,$

$0 \geq 80$ which is not true.

$\Rightarrow$ $3x + 6y \geq 80$ lies on and above the line AB.



(ii) The line $4x + 3y = 100$ passes through

C (25,0), D $\left(0, \frac{100}{3}\right)$ putting $x = 0, y = 0$ in

$4x + 3y \geq 100$, $0 \geq 100$ which is not true.

$\Rightarrow$ $4x + 3y \geq 100$ lies on and above CD.

(iii) $x \geq 0$ lies on and to the right of y-axis.

(iv) $y \geq 0$ lies on and above x-axis.

Shaded area YDPAX is the feasible region which is unbounded where P is the point of intersection of AB and CD *i.e.*,

$3x + 6y = 80$ ... (i)

$4x + 3y = 100$ ... (ii)

Mulitplying equ. (ii) by 2 and subtracting (i) from it

$5x = 200 - 80 = 120 \quad \Rightarrow x = 24$

form (i) $72 + 6y = 80,\ 6y = 8$ or $y = \frac{4}{3}$

Point P is $\left(24, \frac{4}{3}\right)$, values of $z = 4x + 6y$,

At D$\left(0, \frac{100}{3}\right)$, $z = 0 + 6 \times \frac{100}{3} = 200$

At P $\left(24, \frac{4}{3}\right)$, $z = 4 \times 24 + 6 \times \frac{4}{3}$

$= 96 + 8 = 104$ min^m

At A $\left(\frac{80}{3}, 0\right)$, $z = 4 \times \frac{80}{3} + 0 = \frac{320}{3} = 106\frac{2}{3}$

Minimum value of Z in 104. Feasible region is unbounded. Consider the inequality

$4x + 6y < 104$ or $2x + 3y < 52$.

The line $2x + 3y = 52$ passes through (26, 0), $\left(0, 17\frac{1}{3}\right)$. No point of feasible region and $4x + 6y < 104$ is common to them.

$\Rightarrow$ Minimum cost of diet is ₹ 104.

Ex 12.2 Class 12 Maths Question 10.

There are two types of fertilisers F1, and F2. F1 consists of 10% nitrogen and 6% phosphoric acid and F2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a fanner finds that she needs atleast 14 kg of nitrogen and 14 kg of phsophoric acid for her crop. If F2 costs Rs 6/kg and F2 costs Rs 5/ kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost What is the minimum cost?

Solution:

Let x kg of festiliser and y kg of fertilised F2. be required.

Fertiliser	Nitrogen	Phosphoric Acid	Quantity	Cost
F_1	10%	6%	x kg	₹ 6
F_2	5%	10%	y kg	₹ 5
Total	14 kg	14 kg		

Quantity of nitrogen in F_1 and F_2 is

10% of $x + 5\%$ of $y = \frac{x}{10} + \frac{y}{20}$,

least quantity of nitrogen = 14 kg

$\Rightarrow \quad \frac{x}{10} + \frac{y}{20} \geq 14 \quad \text{or} \quad 2x + y \leq 280$,

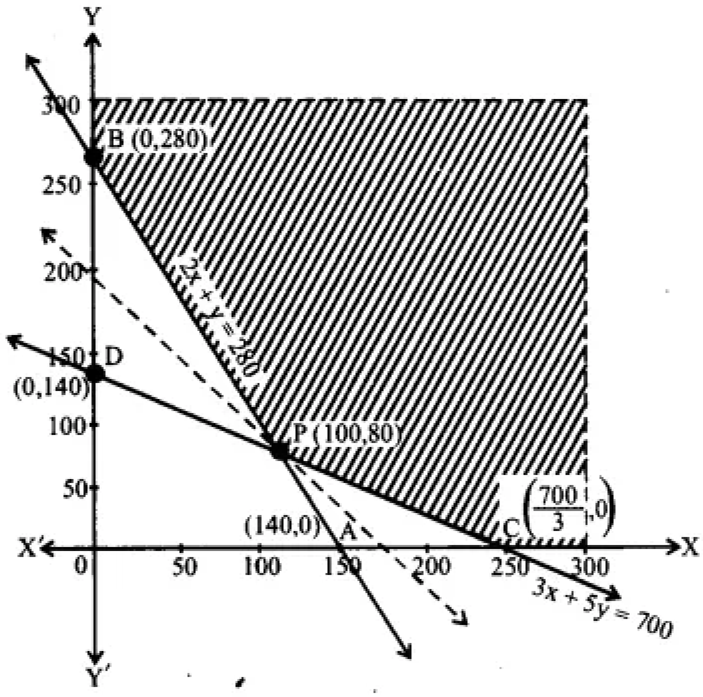
Quantity phosphoric acid in fertilisers F_1 and F_2

$= 6\%$ of $x + 10\%$ of $y = \frac{6x}{100} + \frac{y}{10}$,

least quantity of phosphoric Acid =14 kg

$\Rightarrow \frac{6x}{100} + \frac{y}{10} \geq 14$

or $\quad 6x + 10y \geq 1400 \quad$ or $3x + 5y \geq 700$



∴ we have to minimize the cost $Z = 6x + 5y$ and constraints are :

$2x + y \geq 280,\ 3x + 5y \geq 700\ x,\ y \geq 0$

(i) The line $2x + y = 280$ passes through $A(140, 0)$, $B(0, 280)$, putting $x = 0, y = 0$ in $2x + y \geq 280$, $0 \geq 280$ which is not true.

⇒ $2x + y \geq 280$ lies and above AB.

(ii) The line $3x + 5y = 700$ passes through

$C\left(\frac{700}{3}, 0\right)$, D (0,140), putting x = 0,

y = 0, in $3x + 5y \geq 700$, $0 \geq 70$ which is not true

$\Rightarrow$ It lies on and above CD.

(iii) $x \geq 0$ lies on and to the right of y-axis.

(iv) $y \geq 0$ lies on and above the x-axis.

The shaded area YBPCX is the feasible region where P is the intersection of AB and CD *i.e.*,

$2x + y = 280$ ... (i)

$3x + 5y = 700$ ...(ii)

Multiplying (i) by 5 and subtracting (ii) from it

$7x = 700 \quad \therefore \quad x = 100$

from (i) $y = 80$

$\therefore$ The point P is (100, 80).

At B (0, 280) $\quad Z = 6x + 5y = 0 + 5 \times 280 = 1400$

At P (100 ,80) $\quad Z = 600 + 400 = 1000 \text{ min}^m$

At $C\left(\frac{700}{3}, 0\right)$ $\quad Z = 1400$

$\therefore$ Z is minimum at P (100, 80), where Z = 1000

But the feasible region is unbounded consider the inequality $6x + 5y < 1000$

$6x + 5y = 1000$ passes through $\left(\frac{500}{3}, 0\right)$ and (0, 200)

Putting x = 0, y = 0 in $6x + 5y < 1000$, $0 < 1000$ which is true.

$\therefore$ $6x + 5y < 1000$ lies below the line $6x + 5y = 1000$.

There is no point in common between feasible region and $6x + 5y < 1000$. Hence, minimum cost of Z is ₹ 1000 when quantites of fertiliser F_1 and F_2 are 100 kg and 80 kg respectively.

Ex 12.2 Class 12 Maths Question 11.

The corner points of the feasible region determined by the following system of linear inequalities:

$2x+y\leq10, x+3y\leq15$, $x,y\geq0$ are (0,0),(5,0), (3,4) and (0,5).

Let $Z=px+qy$, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is

(a) $p=q$

(b) $p=2q$

(c) $p=3q$

(d) $q=3p$

Solution:

Maximum value of $Z = px + qy$ occurs at (3,4) and (0,5),

At (3,4), $Z = px + qy = 3p + 4q$

At (0,5), $Z = 0 + q . 5 = 5q$

Both are the maximum values

=> $3p + 4q = 5q$ or $q = 3p$

Option (d) is correct.

Chapterwise NCERT Solutions for Class 12 Maths :

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