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NCERT Solutions for 12th Class Maths: Chapter 11-Three Dimensional Geometry

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Class 12: Maths Chapter 11 solutions. Complete Class 12 Maths Chapter 11 Notes.

Ex 11.1 Class 12 Maths Question 1.

If a line makes angles 90°, 135°, 45° with the and z axes respectively, find its direction cosines.

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Solution:

Direction angles are 90°, 135°, 45°

Direction cosines are

$$\ell = \cos 90^\circ = 0, m = \cos\ 135^\circ = -\frac{1}{\sqrt{2}},$$

$$n = \cos 45^\circ = \frac{1}{\sqrt{2}}\ ,$$

Hence, D, C's $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Ex 11.1 Class 12 Maths Question 2.

Find the direction cosines of a line which makes equal angles with coordinate axes.

Solution:

Let direction angle be α each

∴ Direction cosines are cos α, cos α, cos α

But $l^2 + m^2 + n^2 = 1$

∴ $\cos^2 a + \cos^2 a + \cos^2 a = 1$

$$3\cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

Thus, the direction cosines of the line equally inclined to the coordiante axes are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$.

Ex 11.1 Class 12 Maths Question 3.

If a line has the direction ratios – 18,12, -4 then what are its direction cosines?

Solution:

Now given direction ratios of a line are -8,12,-4

∴ a = -18,b = 12,c = -4

Direction cosines are

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{-9}{11},$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{6}{11}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{-2}{11}$$

Hence, direction cosines are $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

Ex 11.1 Class 12 Maths Question 4.

Show that the points (2,3,4) (-1,-2,1), (5,8,7) are collinear.

Solution:

Let the points be A(2,3,4), B (-1, -2,1), C (5,8,7).

Let direction ratios of AB be

$a_1 = x_2 - x_1 = -3, \ b_1 = y_2 - y_1 = -5$
$c_1 = z_2 - z_1 = -3$

Similarly again let direction of BC be

$a_2 = 6, \ b_2 = 10, c_2 = 6$

Now $\frac{a_1}{a_2} = \frac{-1}{2}, \frac{b_1}{b_2} = \frac{-1}{2}, \frac{c_1}{c_2} = \frac{-1}{2}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow$ AB || BC but one point B is common to both,
therefore A, B, C are collinear.

Ex 11.1 Class 12 Maths Question 5.

Find the direction cosines of the sides of the triangle whose vertices are (3,5, -4), (-1,1,2) and (-5,-5,-2).

Solution:

The vertices of triangle ABC are A (3, 5, -4), B (-1,1,2), C (-5, -5, -2)

(i) Direction ratios of AB are (-4,-4,6)

Direction cosines are

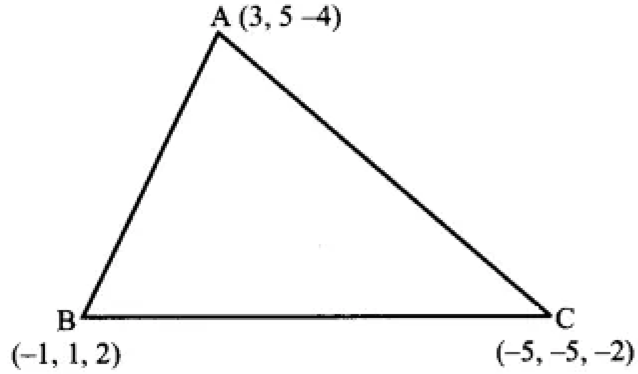
$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\therefore\ a^2+b^2+c^2=(-4)^2+(-4)^2+6^2$$
$$=16+16+36=68$$

$$\therefore\ \sqrt{a^2+b^2+c^2}=2\sqrt{17}$$

Direction cosines of AB are

$$\frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}} \text{ or } \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{17}}$$



(ii) Points B and C are (–1, 1, 2) and (–5, –5, –2) respectively

Direction ratios of BC are –2, –3, –2

$$\therefore\ \sqrt{a^2+b^2+c^2}=\sqrt{4+9+4}=\sqrt{17}$$

$\Rightarrow$ Direction cosines of BC are

$$\frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$$

(iii) Points C and A are (–5, –5, –2) and (3, 5, –4) respectively Direction ratios of CA are 4, 5, –1

$$\therefore \sqrt{a^2+b^2+c^2} = \sqrt{4^2+5^2+1^2} = \sqrt{42}$$

$\therefore$ Direction cosines of CA are

$$\frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$$

Thus direction cosines of AB, BC and CA are

$$\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$$

and $\frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$.

Ex 11.2 Class 12 Maths Question 1.

Show that the three lines with direction cosines:

$$\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}, \frac{4}{13}, \frac{12}{13}, \frac{3}{13}, \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$$

are mutually perpendicular.

Solution:

Let the lines be L_1,L_2 and L_3.

∴ For lines L1 and L2

$l_1l_2 + m_1m_2 + n_1n_2$

$= \frac{12}{13} \times \frac{4}{13} + \left(\frac{-3}{13}\right) \times \frac{12}{13} + \left(\frac{-4}{13}\right) \times \frac{3}{13} = 0$

$\therefore \quad L_1 \perp L_2$

Similarly, Again for lines L_2 and L_3

$l_1l_2 + m_1m_2 + n_1n_2 = 0 \therefore L_2 \perp L_3$

Similarly, Again for lines L_3 and L_1

$l_1l_2 + m_1m_2 + n_1n_2 = 0 \therefore L_1 \perp L_3$

Hence the three lines are mutually perpendicular.

Ex 11.2 Class 12 Maths Question 2.

Show that the line through the points (1,-1,2) (3,4, -2) is perpendicular to the line through the points (0,3,2) and (3,5,6).

Solution:

Let A, B be the points (1, -1, 2), (3, 4, -2) respectively Direction ratios of AB are 2,5, -4

Let C, D be the points (0, 3, 2) and (3, 5, 6) respectively Direction ratios of CD are 3, 2,4 AB is Perpendicular to CD if

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

i.e. $2 \times 3 + 5 \times 2 + (-4) \times 4 = 6 + 10 - 16 = 0$

which is true $\Rightarrow AB \perp CD$.

Ex 11.2 Class 12 Maths Question 3.

Show that the line through the points (4,7,8) (2,3,4) is parallel to the line through the points (-1,-2,1) and (1,2,5).

Solution:

Let the points be A(4,7,8), B (2,3,4), C (-1,-2,1) andD(1,2,5).

Now direction ratios of AB are

$a_1 = x_2 - x_1 = -2, b_1 = y_2 - y_1 = -4$
$c_1 = z_2 - z_1 = -4$
Similarly, Again direction ratios of CD are
$a_2 = 2, b_2 = 4, c_2 = 4$

Now $\frac{a_1}{a_2} = -1, \frac{b_1}{b_2} = -1, \frac{c_1}{c_2} = -1$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence AB is parallel to CD.

Ex 11.2 Class 12 Maths Question 4.

Find the equation of the line which passes through the point (1,2,3) and is parallel to the vector

$$3\hat{i} + 2\hat{j} - 2\hat{k}$$

Solution:

Equation of the line passing through the point

$\vec{a}$ and parallel to vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$
$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 2\hat{k}$
$\therefore$ Equation of the required line is
$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k}), \lambda \in R$

Ex 11.2 Class 12 Maths Question 5.

Find the equation of the line in vector and in cartesian form that passes through the point with position vector

$$2\hat{i} - \hat{j} + 4\hat{k}$$

and is in the direction

$\hat{i}+2\hat{j}-\hat{k}$

.

Solution:

The vector equation of a line passing through a point with position vector

$\vec{a}$

and parallel to the

vector $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$

$\vec{a}=2\hat{i}-\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$

Ex 11.2 Class 12 Maths Question 6.

Find the cartesian equation of the line which passes through the point (-2,4, -5) and parallel to the line is given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Solution:

The cartesian equation of the line passing through the point (-2,4, -5) and parallel to the

line is given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is

$\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

So, the vector equation of the required line is

$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ ...(i)

Where λ is a parameter. In cartesian form,

Putting $r = x\hat{i} + y\hat{j} + z\hat{k}$ in (i), we get

$x\hat{i} + y\hat{i} + z\hat{k} = (2+\lambda)\hat{i} + (-1+2\lambda)\hat{j} + (4-\lambda)\hat{k}$

Equating coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$.

$\Rightarrow \quad x - 2 = \lambda, \frac{y+1}{2} = \lambda, \frac{z-4}{-1} = \lambda$

Eliminating λ, we have, $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$

Hence, the cartesian form of the equation (i) is

$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$

Ex 11.2 Class 12 Maths Question 7.

The cartesian equation of a line is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$$

write its vector form.

Solution:

The cartesian equation of the line is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$$

Clearly (i) passes through the point (5, – 4, 6) and has 3,7,2 as its direction ratios.

=> Line (i) passes through the point A with position vector $\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$ and is in the direction $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$.

So, the vector equation of the required line is

$$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k}).$$

Ex 11.2 Class 12 Maths Question 8.

Find the vector and the cartesian equations of the lines that passes through the origin and (5,-2,3).

Solution:

The line passes through point

$$\therefore \overrightarrow{a} = \overrightarrow{0}$$

Direction ratios of the line passing through the

points (x_1, y_1, z_1) and (x_2, y_2, z_2) are
$x_2 - x_1, y_2 - y_1, z_2 - z_1$

$\therefore$ Direction ratios of the line joining the point A (0, 0, 0) and B (5, –2, 3) are 5, –2, 3

$\therefore \quad \vec{b} = 5\hat{i} - 2\hat{j} + 3\hat{k}$

(i) Equation of line AB in vector form

$$\vec{r} = \vec{a} + \lambda\vec{b} = 0 + \lambda\,(5\hat{i} - 2\hat{j} + 3\hat{k})$$

or $\quad \vec{r} = \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})$

(ii) Equation of the line AB in Cartesian form is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Where a, b, c are in the direction ratios of the line which passes through (x_1, y_1, z_1) Direction ratios are (5, –2, 3)

The line passes trhough (0, 0, 0)

$\therefore$ Required equation of AB

$$\frac{x-0}{5} = \frac{y-0}{-2} = \frac{z-0}{3} \quad \text{or} \quad \frac{x}{5} = \frac{y}{2} = \frac{z}{3}.$$

Ex 11.2 Class 12 Maths Question 9.

Find the vector and cartesian equations of the line that passes through the points (3, -2, -5), (3,-2,6).

Solution:

The PQ passes through the point P(3, -2, -5)

$\therefore \quad \vec{a} = 3\hat{i} - 2\hat{j} - 5\hat{k}$

Direction ratios of PQ where P and Q are (3, –2, –5) and (3, –2 6) are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

i.e. $3 - 3, -2 - (-2), 6 - (-5)$ or $0, 0, 11$

$\therefore \vec{b} = 11\hat{k}$

(i) Equation of line PQ in vector form

$$\vec{r} = \vec{a} + \lambda\vec{b} = (3\hat{i} - 2\hat{j} - 5\hat{k}) + \lambda 11\hat{k}$$

or $\vec{r} = (3\hat{i} - 2\hat{j} - 5\hat{k}) + 11\lambda\hat{k}$

(ii) Equation of PQ in cartesian form

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

P (x_1, y_1, z_1) is (3, –2, –5) and direction ratios a, b, c are 0, 0, 11

$\therefore$ Equation of PQ is

$$\frac{x - 3}{0} = \frac{y + 2}{0} = \frac{z + 5}{11}.$$

Ex 11.2 Class 12 Maths Question 10.

Find the angle between the following pair of lines

(i)

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$$
$$and \quad \vec{r} = 7\hat{i} - 6\hat{j} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

(ii)

$$\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j} - 2\hat{k})$$
$$\vec{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$$

Solution:

(i) Let θ be the angle between the given lines.The given lines are parallel to the vectors

$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$ and $\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore$ The angle θ between them is given by

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|} = \frac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{|3\hat{i}+2\hat{j}+6\hat{k}||\hat{i}+2\hat{j}+2\hat{k}|}$$

$$= \frac{19}{21} \quad \Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right).$$

(ii) Let θ be the angle between the given lines. The given lines are parallel to the vector.

$\vec{b}_1 = \hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b}_2 = 3\hat{i} - 5\hat{j} - 4\hat{k}$ respectively.

$\therefore$ The angle θ between them is given by

$$\cos\theta = \frac{\vec{b}_1 \vec{b}_2}{|\vec{b}_1||\vec{b}_2|} = \frac{(1)(3)+(-1)(-5)+(-2)(-4)}{\sqrt{1+1+4}\sqrt{9+25+16}}$$

$$= \frac{8\sqrt{3}}{15} \Rightarrow \theta = \cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$$

Ex 11.2 Class 12 Maths Question 11.

Find the angle between the following pair of lines

(i)

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} and \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

(ii)

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} and \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$$

Solution:

Given

(i)

$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} and \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

(ii)

$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} and \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

(i) Let $\vec{b}_1$ and $\vec{b}_2$ be the vectors parallel to

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{7}$$

$$\therefore \quad \vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\hat{k}, \ \vec{b}_2 = -\hat{i} + 8\hat{j} - 4\hat{k}$$

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|} = \frac{(2\hat{i}+5\hat{j}-3\hat{k})\cdot(-\hat{i}+8\hat{j}+4\hat{k})}{|2\hat{i}+5\hat{k}-3\hat{k}|\,|-\hat{i}+8\hat{j}+4\hat{k}|}$$

$$= \frac{26}{\sqrt{38}\sqrt{81}} = \frac{26}{9\sqrt{38}} \therefore \quad \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right).$$

(ii) The given equations are

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} \text{ and } \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$$

$$\therefore \vec{b}_1 = 2\hat{i} + 2\hat{j} + \hat{k} \text{ and } \vec{b}_2 = 4\hat{i} + \hat{j} + 8\hat{k}$$

Let θ be the angle between the two lines,then

$$\cos\theta = \frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1||\vec{b}_2|} = \frac{2(4)+2(1)+1(8)}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$$

$$= \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Ex 11.2 Class 12 Maths Question 12.

Find the values of p so that the lines

$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} and \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

are at right angles

Solution:

The given equation are not in the standard form

The equation of given lines is

$$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2} \quad ...(i)$$

and $$\frac{x-1}{3p/-7} = \frac{y-5}{1} = \frac{z-6}{-5} \quad ...(ii)$$

The direction ratios of the given lines are -3, $\frac{2P}{7}, 2$ and $\frac{-3P}{7}, 1, -5$.

The lines are perpendicualr to each other

$$\therefore (-3)\left(\frac{-3P}{7}\right)+\left(\frac{2P}{7}\right)(1)+2(-5)=0 \Rightarrow P=\frac{70}{11}$$

Ex 11.2 Class 12 Maths Question 13.

Show that the lines

$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1} and \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

are perpendicular to each other

Solution:

Given lines

$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$

…(i)

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$$

…(ii)

Let $\vec{p}_1$ and $\vec{p}_2$ be the vectors parallel to (i) and (ii) respectively.

$\therefore \quad \vec{p}_1=7\hat{i}-5\hat{j}+\hat{k}$ and $\vec{p}_2=\hat{i}+2\hat{j}+3\hat{k}$

Consider $\vec{p}_1 \cdot \vec{p}_2=7(1)+(-5)(2)+1(3)=0$

Hence the two given lines are perpendicular.

Ex 11.2 Class 12 Maths Question 14.

Find the shortest distance between the lines

$$\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$$

and

$$\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$$

Solution:

The shortest distance between the lines

$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

Comparing, $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$

$\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$ and

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = (-2-1)i - (2-2)\hat{j} + (1+2)\hat{k}$$

$$= -3\hat{i} - o\hat{j} + 3\hat{k}$$

$\therefore \quad (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (-3\hat{i} + 3\hat{k})$

$= (1)(-3) + (-3)(0) + (-2)(3) = -3 - 6 = -9$

and $|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$\therefore \quad d = \left| \frac{-9}{3\sqrt{2}} \right| = \left| \frac{-3}{\sqrt{2}} \right| = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}.$

Ex 11.2 Class 12 Maths Question 15.

Find the shortest distance between the lines

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} and \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Solution:

Shortest distance between the lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and}$$

$$\frac{x-x_2}{a_2}=\frac{y=y_2}{b_2}=\frac{z-z_2}{c_2} \text{ is:}$$

$$\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}=\frac{N}{D}.$$

The given lines are $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

$$N=\begin{vmatrix} 4 & 6 & 0 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$\therefore\ N=4(-6+2)+6(1-7)+8(-14+6)=-116$$

$$\therefore\ D=\sqrt{(-6+2)^2+(1-7)^2+(-14+6)^2}=\sqrt{116}$$

$$\therefore\ S.D.=\left|\frac{-116}{\sqrt{116}}\right|=\sqrt{116}=2\sqrt{29}.$$

Ex 11.2 Class 12 Maths Question 16.

Find the distance between die lines whose vector equations are:

$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$$

and

$$\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$$

Solution:

Comparing the given equations with

$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ respectively, we have $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -9\hat{i} + 3\hat{j} + 9\hat{k}$

$|\vec{b}_1 \times \vec{b}_2| = 3\sqrt{19}$ $\therefore (\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 9$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{9}{3\sqrt{19}} = \frac{3}{\sqrt{19}}$$

Ex 11.2 Class 12 Maths Question 17.

Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$$

and

$$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$$

Solution:

Comparing these equation with

$\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$
and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$

$\therefore \quad \vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \ \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$ and

$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}, \ \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$ & $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{4+16+9} = \sqrt{29}$ and

$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k}) = 8$

The shortest distance between two lines is

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \left| \frac{8}{\sqrt{29}} \right| = \frac{8}{\sqrt{29}}.$$

Ex 11.3 Class 12 Maths Question 1.

In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

(a) z = 2

(b) x+y+z = 1

(c) 2x + 3y – z = 5

(d) 5y+8 = 0

Solution:

(a) Direction ratios of the normal to the plane are 0,0,1

=> a = 0, b = 0, c = 1

$$\sqrt{a^2+b^2+c^2}=\sqrt{0^2+0^2+1^2}=1$$

$$\therefore \quad \frac{0x}{1}+\frac{0y}{1}+\frac{z}{1}=\frac{2}{2}$$

Comparing with $lx+my+nz=p$; $p=\frac{2}{1}=2$

(b) $x+y+z=1$

Direction ratios of the normal to the plane are: $a=1,\ b=1, c=1$

Now $\sqrt{a^2+b^2+c^2}=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

$$\therefore \quad \frac{x}{\sqrt{3}}+\frac{y}{\sqrt{3}}+\frac{z}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

Comparing with $lx+my+nz=p$; $p=\frac{1}{\sqrt{3}}$

(c) $2x+3y-z=5$; Now $a=2; b=3; c=-1$

$$\therefore \quad \sqrt{a^2+b^2+c^2}=\sqrt{2^2+3^2+(-1)^2}=\sqrt{14}$$

$$\therefore \quad \frac{2x}{\sqrt{14}}+\frac{3y}{\sqrt{14}}-\frac{z}{\sqrt{14}}=\frac{5}{\sqrt{14}}$$

Comparing with $lx+my+nz=p$; $p=\frac{5}{\sqrt{14}}$

(d) $5y+8=0 \Rightarrow \quad 5y=-8 \quad \Rightarrow -5y=8$

Now $a=0; b=-5; c=0$

$$\therefore \quad \sqrt{a^2+b^2+c^2}=\sqrt{0^2+(-5)^2+0^2}=5$$

$$\therefore \quad \frac{0x}{5}+\frac{-5y}{5}+\frac{0z}{5}=\frac{8}{5}$$

Comparing with $lx+my+nz=p$; $\therefore$ $p=\frac{8}{5}$

Ex 11.3 Class 12 Maths Question 2.

Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector

$$3\hat{i} + 5\hat{j} - 6\hat{k}$$

Solution:

Let $\vec{n} - 3\hat{i} + 5\hat{j} - 6\hat{k}$; $|\vec{n}| = \sqrt{3^2 + 5^2 + (-6)^2} = \sqrt{70}$,

$$\frac{n}{|\vec{n}|} = \frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}$$

The required equation of the plane is $\vec{r} \cdot \hat{n} = d$

i.e. $\vec{r} \cdot \left(\frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k} \right) = 7$

or $\vec{r} \cdot (3\hat{i} + 5\hat{j} - 6\hat{k}) = 7\sqrt{70}$

Ex 11.3 Class 12 Maths Question 3.

Find the Cartesian equation of the following planes.

(a)

$$\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$$

(b)

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$$

(c)

$$\vec{r} \cdot [(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}) = 15$$

Solution:

(a) $\vec{r}$ is the position vector of any arbitrary point P (x, y, z) on the plane.

$$\therefore \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \Rightarrow \vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 2$$

$$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}).\ (\hat{i} + \hat{j} - \hat{k}) = 2$$

$$\Rightarrow \quad (x)(1) + (y)(1) + (z)(-1) = 2$$

$$\Rightarrow \quad x + y - z = 2$$

which is the required cartesian equation.

(b) $\vec{r}$ is the position vector of any arbitrary point P (x, y, z) on the plane.

$$\therefore \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \ \therefore\ \vec{r}.(2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$$

$$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}).\ (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$$

$$\Rightarrow \quad 2x + 3y - 4z = 1$$

which is the required cartesian equation.

(c) $\vec{r} \cdot [(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}] = 15$

The vector equation of the plane is

$$\vec{r} \cdot [(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}] = 15$$

$$(x\hat{i} + y\hat{i} + z\hat{k}) \cdot [(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}] = 15$$

$$(s - 2t)\,x + (3 - t)\,y + (2s + t)\,z = 15$$

Which is the required cartesian equation.

Ex 11.3 Class 12 Maths Question 4.

In the following cases find the coordinates of the foot of perpendicular drawn from the origin

(a) 2x + 3y + 4z – 12 = 0

(b) 3y + 4z – 6 = 0

(c) x + y + z = 1

(d) 5y + 8 = 0

Solution:

(a) Let N (x1, y1, z1) be the foot of the perpendicular from the origin to the plane 2x+3y+4z-12 = 0

∴ Direction ratios of the normal are 2, 3, 4.

Also the direction ratios of ON are (x1,y1,z1)

$\Rightarrow \quad \frac{x_1}{2} = \frac{y_1}{3} = \frac{z_1}{4} = k \quad \text{(say)}$

$\therefore \quad x_1 = 2k, y_1 = 3k, z = 4k \quad \text{...(i)}$

The point (x_1, y_1, z_1) lies on the plane

$\therefore \; 2(2k) + 3(3k) + 4(4k) - 12 = 0$

$(4 + 9 + 16)k = 12, \quad \therefore \; k = \frac{12}{29}$

Putting value of k in (i)

$x_1 = 2 \cdot \frac{12}{29} = \frac{24}{29}, \quad y_1 = 3k = 3 \cdot \frac{12}{29} = \frac{36}{29}$

$\therefore \quad z_1 = 4k = 4 \cdot \frac{12}{29} = \frac{48}{29}$

Hence, the foot of the normal from the origin to the given plane is $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$.

(b) $3y + 4z - 6 = 0$

Direction ratios of the normal to the plane are: $a = 0, b = 3, c = 4$

$\therefore$ Equation of the line through (0, 0, 0) are

$\frac{x-0}{0} = \frac{y-0}{3} = \frac{z-0}{4} \Rightarrow \frac{x}{0} = \frac{y}{3} = \frac{z}{4} = \lambda \text{ (let)}$

$\therefore \quad x = 0, y = 3\lambda, z = 4\lambda$

Let foot of the perpendicular from the origin be $(0, 3\lambda, 4\lambda)$ which lies on the plane

$3y + 4z - 6 = 0 \Rightarrow 3(3\lambda) + 4(4\lambda) - 6 = 0$

$\Rightarrow \quad \lambda = \frac{6}{25} \quad \therefore x = 0, y = \frac{18}{25}, z = \frac{24}{25}$

Hence the foot of the perpendicular is

$\left(0, \frac{18}{25}, \frac{24}{25}\right)$

(c) Direction ratios of the normal to the plane x + y + z = 1 a = 1, b = 1, c = 1
Now equation of the line through (0, 0, 0)
are : $\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-0}{1} \Rightarrow x=y=z=\lambda$ (let)
$\therefore$ $x=\lambda, y=\lambda, z=\lambda$
Let foot of the perpendicular from the origin be $(\lambda, \lambda, \lambda,)$ which lies on the plane $x+y+z=1$
$\Rightarrow$ $\lambda+\lambda+\lambda=1 \Rightarrow \lambda=\frac{1}{3} \therefore x=\frac{1}{3}, y=\frac{1}{3}, z=\frac{1}{3}$
Hence the foot of the perpendicular is $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(d) $5y+8=0$
Direction ratios of the normal to the plane are : a = 0, b = 5, c = 0
Now equation of the line through (0, 0, 0) are:
$\frac{x-0}{0}=\frac{y-0}{5}=\frac{z-0}{0} \Rightarrow \frac{x}{0}=\frac{y}{5}=\frac{z}{0}=\lambda$ (let)
$\therefore$ $x=0, y=5\lambda, z=0.$
Let foot of the perpendicular be $(0, 5\lambda, 0)$ which lies on: $5y+8=0$
$\Rightarrow 5(5\lambda)+8=0 \Rightarrow \lambda=\frac{-8}{25}$
$\therefore$ $x=0, y=5\times\left(\frac{-8}{25}\right)=\frac{-8}{5}, z=0$
Hence the foot of the perpendicular is $\left(0, \frac{-8}{5}, 0\right)$

Ex 11.3 Class 12 Maths Question 5.

Find the vector and cartesian equation of the planes

(a) that passes through the point (1,0, -2) and the normal to the plane is

$$\hat{i} + \hat{j} - \hat{k}$$

(b) that passes through the point (1,4,6) and the normal vector to the plane is

$$\hat{i} - 2\hat{j} + \hat{k}$$

Solution:

(a) Normal to the plane is i + j – k and passes through (1,0,-2)

$\therefore \quad \vec{b} = i + j - k$ and $\vec{a} = i - 2k$

Equation of the plane in vector form is

$\vec{r} = \vec{a} + \lambda\vec{b} \Rightarrow \vec{r} = (\hat{i} - 2\hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$

For cartesian form : Direction ratio of the normal are 1, 1, –1.

Now equation of the plane in cartesian form is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\Rightarrow \quad x + y - z - 3 = 0$

(b) The plane passes through (1, 4, 6) is

$\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

Normal to the plane is $\hat{n} = \hat{i} - 2\hat{j} + \hat{k}$

Equation of the plane passing through (1, 4, 6) are with normal $\hat{i} - 2\hat{j} + \hat{k}$ is

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

or $\quad [\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$

$\Rightarrow \quad \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) + 1 = 0,$

Cartesian form : Equation of the plane passing through (x_1, y_1, z_1) with direction ratio of normal a, b, c is

$a(x - x_1) + b(y - y_1) + (c(z - z_1) = 0$

the plane passes through (1, 4, 6)

$\therefore \quad x_1 = 1, y_1 = 4, z_1 = 6$

Direction ratio of normal $\hat{i} - 2\hat{j} + \hat{k}$ are 1, –2, 1

$\therefore \quad a = 1, \; b = -2, \; c = 0$

Putting these values in (i)

$1 \cdot (x - 1) - 2(y - 4) + 1 \cdot (z - 6) = 0 \Rightarrow x - 2y + z + 1 = 0$

Ex 11.3 Class 12 Maths Question 6.

Find the equations of the planes that passes through three points

(a) (1,1,-1) (6,4,-5), (-4, -2,3)

(b) (1,1,0), (1,2,1), (-2,2,-1)

Solution:

(a) The plane passes through the points (1,1,-1) (6,4,-5), (-4,-2,3)

Let the equation of the plane passing through(1,1,-1)be

$$a(x-1)+b(y-1)+c(z+1)=0 \quad ...(i)$$

(6, 4, –5) lies on it

$$\therefore \; a(6-1)+b(4-1)+c(-5+1)=0$$

or $5a+3b-4c=0 \quad ...(ii)$

(–4, –2, 3) lies on the plane

$$\therefore \; a(-4-1)+b(-2-1)+c(3+1)=0$$

or $5a+3b-4c=0 \quad ...(iii)$

from (ii) & (iii)

$$\frac{a}{-12+12}=\frac{b}{-20+20}=\frac{c}{15-15} \Rightarrow \frac{a}{0}=\frac{b}{0}=\frac{c}{0}$$

No unique plane can be drawn.

(b) (b) Let the points be A(1, 1, 0), B(1, 2, 1), C (–2, 2, –1).

Now equation of the plane is

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix}=0$$

$$\Rightarrow (x-1)(-2)-(y-1)\times 3+z(0+3)=0$$

$$\Rightarrow 2x+3y-3z-5=0.$$

Ex 11.3 Class 12 Maths Question 7.

Find the intercepts cut off by the plane 2x+y-z = 5.

Solution:

Equation of the plane is 2x + y- z = 5 x y z

Dividing by 5:

$$\Rightarrow \frac{x}{\frac{5}{2}}+\frac{y}{5}-\frac{z}{-5}=1$$

∴ The intercepts on the axes OX, OY, OZ are

$$\frac{5}{2}$$

, 5, -5 respectively

Ex 11.3 Class 12 Maths Question 8.

Find the equation of the plane with intercept 3 on the y- axis and parallel to ZOX plane.

Solution:

Any plane parallel to ZOX plane is y=b where b is the intercept on y-axis.

∴ b = 3.

Hence equation of the required plane is y = 3.

Ex 11.3 Class 12 Maths Question 9.

Find the equation of the plane through the intersection of the planes 3x – y + 2z – 4 = 0 and x + y + z – 2 = 0 and the point (2,2,1).

Solution:

Given planes are:

3x – y + 2z – 4 = 0 and x + y + z – 2 = 0

Any plane through their intersection is

3x – y + 2z – 4 + λ(x + y + z – 2) = 0

point (2,2,1) lies on it,

∴3 x 2 – 2 + 2 x 1 – 4 +λ(2+2+1-2)=0

=>λ = $\frac{-2}{3}$

Now required equation is 7x – 5y + 4z – 8 = 0

Ex 11.3 Class 12 Maths Question 10.

Find the vector equation of the plane passing through the intersection of the planes

$$\vec{r} \cdot \left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 7, \vec{r} \cdot \left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) = 9$$

and through the point (2,1,3).

Solution:

Equation of the plane passing through the line of intersection of the planes

$$\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7+\lambda[\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9]=0 \quad ...(i)$$

or $$\vec{r}\cdot[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}]-7-9\lambda=0 \quad ...(ii)$$

It passes through the point $(2\hat{i}+\hat{j}+3\hat{k})$

$$\Rightarrow \quad (2\hat{i}+\hat{j}+3\hat{k})\cdot[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}]-7-9\lambda=0$$

$$\Rightarrow \quad (4+4\lambda)+(2+5\lambda)+(-9+9\lambda)-7-9\lambda=0$$

$$\Rightarrow \quad \lambda=\frac{10}{9}$$; Putting value of λ in (i)

$$\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7+\frac{10}{9}[2\hat{i}+5\hat{j}+3\hat{k}-9]=0$$

$$\Rightarrow \quad \vec{r}\cdot(38\hat{i}+68\hat{j}+3\hat{k})=153$$

Ex 11.3 Class 12 Maths Question 11.

Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z = 0.

Solution:

Given planes are

x + y + z – 1 = 0 …(i)

2x + 3y + 4z – 5 = 0 …(ii)

x – y + z = 0 ….(iii)

Any plane through the intersection of (i) and (ii) is
$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-1-5\lambda=0$ ...... (iv)

Direction ratio of the normal of (iii) are 1, –1, 1
Also direction ratios of normal of (iv) are
$1+2\lambda, 1+3\lambda, 1+4\lambda$

Two planes are perpendicular if their normals are perpendicular.

$$\Rightarrow \quad 1+2\lambda-1-3\lambda+1+4\lambda=0 \Rightarrow \lambda=\frac{-1}{3}$$

Now equation of the required plane is : $x-z+2=0$

Ex 11.3 Class 12 Maths Question 12.

Find the angle between the planes whose vector equations are

$$\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5, \vec{r}\cdot\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=3$$

Solution:

The angle θ between the given planes is

$$\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|} \text{ where; } n_1=2\hat{i}+2\hat{j}-3\hat{k} \text{ \&}$$

$n_2=3\hat{i}+3\hat{j}-5\hat{k}$,

$$\cos\theta=\frac{(2\hat{i}-2\hat{j}-3\hat{k})\cdot(3\hat{i}-3\hat{j}+5\hat{k})}{\sqrt{4+4+9}\cdot\sqrt{9+9+25}}$$

$$=\frac{-15}{\sqrt{17}\sqrt{43}} \quad \therefore \theta=\cos^{-1}\left(\frac{15}{\sqrt{17}\sqrt{43}}\right).$$

Ex 11.3 Class 12 Maths Question 13.

In the following determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angle between them.

(a) 7x + 5y + 6z + 30 = 0 and 3x – y – 10z + 4 = 0

(b) 2x + y + 3z – 2 = 0 and x – 2y + 5 = 0

(c) 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z – 1 = 0

(d) 2x – y + 3z – 1 = 0 and 2x – y + 3z + 3 = 0

(e) 4x + 8y + z – 8 = 0 and y + z – 4 = 0.

Solution:

(a) Direction ratios of the normal of the planes 7x + 5y + 6z + 30 = 0 are 7,5,6

Direction ratios of the normal of the plane 3x – y – 10z + 4 = 0 are 3,-1,-10

The plane 7x + 5y + 6z + 30 = 0 …(i)

3x – y – 10z + y = 0 …(ii)

are perpendicualr to each other if
$a_1 \cdot a_2 + b_1b_2 + c_1c_2 = 0$
or $7 \cdot 3 + 5 \cdot (-1) + 6(-10) = 21 - 5 - 60 \neq 0$
Planes (i) and (ii) are not perpendicualr
Direction ratio of normal of the plane (i) and (ii) are not proportional as

$$\frac{7}{3} \neq \frac{5}{-1} \neq \frac{6}{4}$$

These planes are not parallel. Angle between the planes is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}} = \frac{-2}{5}$$

$$\therefore \quad \theta = \cos^{-1}\left(-\frac{2}{5}\right).$$

(b) $2x + y + 3z - 2 = 0$ and $x - 2y + 5 = 0$
Now $\vec{p}_1 = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{p}_2 = \hat{i} - 2\hat{j}$
$\vec{p}_1 \cdot \vec{p}_2 = 0 \;\therefore\; \vec{p}_1 \perp \vec{p}_2$
hence planes are perpendicular.

(c) $2x-2y+4z+5=0$ and $3x-3y+6z-1=0$

Now $\vec{p}_1=2\hat{i}-2\hat{j}+4\hat{k}$ and $\vec{p}_2=3\hat{i}-3\hat{j}+6\hat{k}$

Now $\frac{a_1}{a_2}=\frac{2}{3},\frac{b_1}{b_2}=\frac{2}{3},\frac{c_1}{c_2}=\frac{2}{3}$

$\therefore \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow$ Normals are parallel.

Hence, planes are parallel.

(d) $2x-y+3z-1=0$ and $2x-y+3z+3=0$

Now

$\vec{p}_1=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{p}_2=2\hat{i}-\hat{j}+3\hat{k}$

Now $\frac{a_1}{a_2}=\frac{2}{2}=1,\ \frac{b_1}{b_2}=\frac{-1}{-1}=1,\ \frac{c_1}{c_2}=\frac{3}{3}=1$

$\therefore \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\therefore$ Normals are parallels. Hence planes are parallels.

(e) $4x+8y+z-8=0$ and $y+z-4=0$

$\vec{p}_1=4\hat{i}+8\hat{j}+\hat{k}$ and $\vec{p}_2=\hat{j}+\hat{k}$

$\vec{p}_1\cdot\vec{p}_2=4(0)+8(1)+1(1)=9$

$|\vec{p}_1|=\sqrt{4^2+8^2+1^2}=\sqrt{81}$ and

$|\vec{p}_2|=\sqrt{1^2+1^2}=\sqrt{2}$

Angle between planes is the angle between their normals. Let θ be the angle between the planes.

$\therefore \quad \cos\theta=\frac{\vec{p}_1\cdot\vec{p}_2}{|\vec{p}_1||\vec{p}_2|}=\frac{1}{\sqrt{2}} \Rightarrow \theta=45^\circ.$

Ex 11.3 Class 12 Maths Question 14.

In the following cases, find the distance of each of the given points from the corresponding given plane.

Point Plane

(a) (0, 0,0) 3x – 4y + 12z = 3

(b) (3,-2,1) 2x – y + 2z + 3 = 0.

(c) (2,3,-5) x + 2y – 2z = 9

(d) (-6,0,0) 2x – 3y + 6z – 2 = 0

Solution:

(a) Given plane: 3x – 4y + 12z – 3 = 0

Given point (0, 0, 0) $\therefore d = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

$$d = \left|\frac{3(0) - 4(0) + 12(0) - 3}{\sqrt{3^2 + (-4)^2 + 12^2}}\right| = \frac{3}{13}$$

(b) The point is (3, –2, 1), the plane $2x - y + 2z + 3 = 0$

$$\therefore \quad d = \left|\frac{2\cdot 3 - (-2) + 2\cdot 1 + 3}{\sqrt{2^2 + (-1)^2 + 2^2}}\right| = \frac{13}{8}$$

(c) Given plane : $x + 2y - 2z - 9 = 0$ and point (2, 3, –5)

$$d = \frac{|2 + 2\times 3 - 2(-5) - 9|}{\sqrt{1^2 + 2^2 + (-2)^2}} = 3.$$

(d) Given point (–6 , 0, 0) and plane $2x - 3y + 6z - 2 = 0$

$$d = \frac{|2(-6) - 3(0) + 6(0) - 2|}{\sqrt{2^2 + (-3)^2 + 6^2}} = 2.$$

Chapterwise NCERT Solutions for Class 12 Maths :

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