NCERT Solutions for 12th Class Maths: Chapter 10-Vector Algebra

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Class 12: Maths Chapter 10 solutions. Complete Class 12 Maths Chapter 10 Notes.

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Ex 10.1 Class 12 Maths Question 1.

Represent graphically a displacement of 40km, 30° east of north.

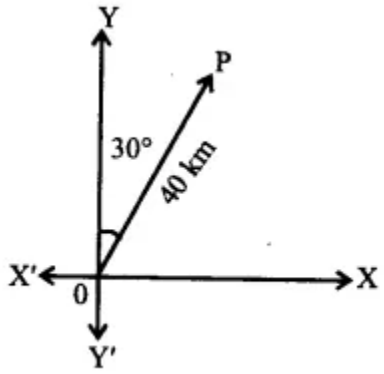
Solution:

A line segment of 2 cm is drawn on the right of OY making an angle of 30° with it. OP = 40 km,

scale 1cm = 20 km. Vector

$\overrightarrow{OP}$

represents displacement of 40 km 30° east of north.



Ex 10.1 Class 12 Maths Question 2.

Classify the following measures as scalars and vectors.

(i) 10 kg

(ii) 2 metres north- west

(iii) 40°

(iv) 40 watt

(v) 10^{-19} coulomb

(vi) 20 m/sec².

Solution:

(i) Mass-scalar

(ii) Directed distance-vector

(iii) Temperature-scalar

(iv) Rate of electricity-scalar

(v) Electric charge-vector

(vi) Acceleration-vector

Ex 10.1 Class 12 Maths Question 3.

Classify the following as scalar and vector quantities

(i) time period

(ii) distance

(iii) force

(iv) velocity

(v)work.

Solution:

Scalar Quantity: (i) time period (ii) distance (v) work.

Vector Quantity: (iii) force (iv) velocity

Ex 10.1 Class 12 Maths Question 4.

In a square, identify the following vectors

(i) Co-initial

(ii) Equal

(iii) collinear but not equal

Solution:
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(i) Co initial vectors are

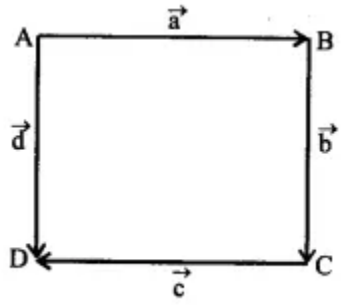
$\overrightarrow{a}, \overrightarrow{d}$

(ii) Equal Vectors are

$\overrightarrow{b}, \overrightarrow{d}$

(iii) Collinear but not equal vectors are

$\overrightarrow{a}, \overrightarrow{c}$



Ex 10.1 Class 12 Maths Question 5.

Answer the following as true or false:

(i)

$\overrightarrow{a}, -\overrightarrow{a}$

are collinear.

(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

Solution:

(i) True

(ii) False

(iii) False

(iv) False.

Ex 10.2 Class 12 Maths Question 1.

Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k} \ \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Solution:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k} \, |\vec{a}| = \sqrt{1^2 + 1^2 + 1^2}$$

$$\left[\because |x\hat{i}+y\hat{j}+z\hat{k}| = \sqrt{x^2+y^2+z^2}\right].$$

$$\therefore \quad |\vec{a}| = \sqrt{3}$$

(ii) $\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$

$$\therefore \quad |\vec{b}| = \sqrt{2^2+(-7)^2+3^2} = \sqrt{62}$$

(iii) $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$,

$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2} = 1$$

Ex 10.2 Class 12 Maths Question 2.

Write two different vectors having same magnitude.

Solution:

$$|\vec{a}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

and $|b| = \sqrt{3^2+2^2+1^2} = \sqrt{14}$

$\vec{a} \neq \vec{b}$ but $|\vec{a}| = |\vec{b}|$

$\overrightarrow{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{b} = 3\hat{i} + 2\hat{j} + \hat{k}$

Such possible answers are infinite

Ex 10.2 Class 12 Maths Question 3.

Write two different vectors having same direction.

Solution:

Let the two vectors be

$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \ \vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ and

d'c's of $\vec{b}$ are $\left(\frac{3}{\sqrt{27}}, \frac{3}{\sqrt{27}}, \frac{3}{\sqrt{27}}\right)$ *i.e.*,

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Hence vectors

$\vec{a}, \vec{b}$

have the same direction but different magnitude

Ex 10.2 Class 12 Maths Question 4.

Find the values of x and y so that the vectors

$\vec{2i} + \vec{3j} \quad and \quad \hat{x}i + \hat{y}j$

are equal.

Solution:

We are given

$\vec{2i} + \vec{3j} \quad and \quad \hat{x}i + \hat{y}j$

If vectors are equal, then their respective components are equal. Hence x = 2, y = 3.

Ex 10.2 Class 12 Maths Question 5.

Find the scalar and vector components of the vector with initial point (2,1) and terminal point (-5,7).

Solution:

LetA(2, 1) be the initial point and B(-5,7) be the terminal point

$$\overrightarrow{AB} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} = -7\hat{i} + 6\hat{j}$$

∴The vector components are

$$-7\hat{i} and 6\hat{j}$$

and scalar components are – 7 and 6.

Ex 10.2 Class 12 Maths Question 6.

Find the sum of three vectors:

$$\overrightarrow{a} = \hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = -2\hat{i} + 4\hat{j} + 5\hat{k} \quad and \quad \overrightarrow{c} = \hat{i} - 6\hat{j} - 7\hat{k},$$

Solution:

$$\overrightarrow{a} = \hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = -2\hat{i} + 4\hat{j} + 5\hat{k} \quad and \quad \overrightarrow{c} = \hat{i} - 6\hat{j} - 7\hat{k},$$

$$\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = 0\hat{i} - 4\hat{j} - \hat{k} = -4\hat{i} - \hat{k}$$

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Ex 10.2 Class 12 Maths Question 7.

Find the unit vector in the direction of the vector

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$

Solution:

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \quad |\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

Unit vector in the direction of vector $\vec{a}$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k}) = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

Ex 10.2 Class 12 Maths Question 8.

Find the unit vector in the direction of vector

$$\overrightarrow{PQ}$$

, where P and Q are the points (1,2,3) and (4,5,6) respectively.

Solution:

The points P and Q are (1, 2, 3) and (4, 5, 6) respectively

$$\overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k}$$

$= 3i + 3\hat{j} + 3\hat{k}$,

$|\overrightarrow{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{27} = 3\sqrt{3}$

$\therefore$ Unit vector in the direction of $\overrightarrow{PQ}$

$$\hat{PQ} = \frac{1}{|\overrightarrow{PQ}|}\overrightarrow{PQ} = \frac{1}{3\sqrt{3}}(3\hat{i} + 3\hat{j} + 3\hat{k})$$

$$= \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Ex 10.2 Class 12 Maths Question 9.

For given vectors

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \quad and \quad \vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

find the unit vector in the direction of the vector

$$\vec{a} + \vec{b}$$

Solution:

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \quad and \quad \vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

$\therefore \ \vec{a} + \vec{b} = (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} - \hat{k}) = \hat{i} + \hat{k}$

Now $|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Hence uint vector along

$$\vec{a} + \vec{b} = \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}.$$

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Ex 10.2 Class 12 Maths Question 10.

Find a vector in the direction of

$$5\hat{i} - \hat{j} + 2\hat{k}$$

which has magnitude 8 units.

Solution:

The given vector is

$$\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$$

$$|\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25+1+4} = \sqrt{30}$$

$\therefore$ Unit vector in the direction of vector $\vec{a}$

$$= \frac{1}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$$

$\therefore$ Vector of magnitude 8 in the direction of vector $\vec{a}$

$$= 8 \times \text{unit vector } \vec{a} = 8 \cdot \frac{1}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$$

$$= = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

Ex 10.2 Class 12 Maths Question 11.

Show that the vector

$$2\hat{i} - 3\hat{j} + 4\hat{k} \quad and \quad -4\hat{i} + 6\hat{j} - 8\hat{k}$$

are collinear.

Solution:

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$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k} \quad and \quad \vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

$= -2(2\hat{i} - 3\hat{j} + 4\hat{k})$

vector

$\vec{a} \quad and \quad \vec{b}$

have the same direction they are collinear.

Ex 10.2 Class 12 Maths Question 12.

Find the direction cosines of the vector

$\hat{i} + 2\hat{j} + 3\hat{k}$

Solution:

let

$\vec{p} = \hat{i} + 2\hat{j} + 3\hat{k}$

Now a = 1,b = 2,c = 3

$\therefore$ Direction cosines of $\vec{p}$ are:

$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{1}{\sqrt{14}}, \text{m} = \frac{2}{\sqrt{14}}, \text{n} = \frac{3}{\sqrt{14}}$

Hence direction cosines are $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

Ex 10.2 Class 12 Maths Question 13.

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Find the direction cosines of the vector joining the points A (1,2, -3) and B(-1, -2,1), directed from A to B.

Solution:

Vector joining the points A and B is

$$(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

A is $(1,-2,-3) \Rightarrow x_1 = 1, y_1 = 2, z_1 = -3$,
B is $(-1,-2,1) \Rightarrow x_2 = -1, y_2 = -2, z_2 = 1$
$\therefore$ vector joining the points A and B

$$= \overline{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + [1-(-3)]\hat{k}$$

$$= -2\hat{i} - 4\hat{j} + 4\hat{k}$$

Direction cosines of vectors $x\hat{i} + y\hat{j} + z\hat{k}$ are

$$\left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

Hence, the direction cosines of

$$\overline{AB} \text{ are } \left(-\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$$

Ex 10.2 Class 12 Maths Question 14.

Show that the vector

$$\hat{i} + \hat{j} + \hat{k}$$

are equally inclined to the axes OX, OY, OZ.

Solution:

Let

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$\hat{i}+\hat{j}+\hat{k}=\vec{a}$

, Direction cosines of vector

$x\hat{i}+y\hat{j}+z\hat{k}$

are

$$\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}},$$

$\therefore$ The D.C's of vector $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

which shows that the vector a is equally inclined to the axes OX, OY, OZ.

Ex 10.2 Class 12 Maths Question 15.

Find the position vector of a point R which divides the line joining the points whose positive vector are

$P(\hat{i}+2\hat{j}-\hat{k}) \quad and \quad Q(-\hat{i}+\hat{j}+\hat{k})$

in the ratio 2:1

(i) internally

(ii) externally.

Solution:

(i) The point R which divides the line joining the point

$P(\vec{a}) \quad and \quad Q(\vec{b})$

in the ratio m : n

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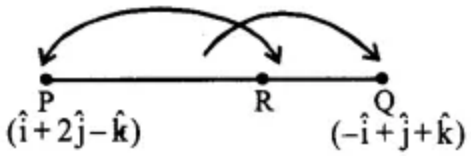


$$\vec{R}=\frac{m\vec{b}+n\vec{a}}{m+n}$$

$\Rightarrow$ $m=2, n=1,$

$\vec{a}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}+\hat{k}$

$$\therefore \quad \vec{R} \text{ is } \frac{2\times(-\hat{i}+\hat{j}+\hat{k})+1\times(\hat{i}+2\hat{j}-\hat{k})}{2+1}$$

$$=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}=-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(ii) Similarly the point 'R' which divides PQ externally in the ratio m : n is given by

$$\vec{R}=\frac{m\vec{b}-n\vec{a}}{m-n}=\frac{2(-\hat{i}+\hat{j}+\hat{k})-1\cdot(\hat{i}+2\hat{j}-\hat{k})}{2-1}$$

$$=3\hat{i}+0\cdot\hat{j}+3\hat{k}=-3\hat{i}+3\hat{k}$$

Ex 10.2 Class 12 Maths Question 16.

Find position vector of the mid point of the vector joining the points P (2,3,4) and Q (4,1, -2).

Solution:

Let

$$\overrightarrow{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k} \quad and \quad \overrightarrow{OQ} = 4\hat{i} + \hat{j} - 2\hat{k}$$

Let mid-point of PQ be R $\therefore \overline{OR} = \frac{\overline{OP} + \overline{OQ}}{2}$

$\overline{OR} = \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2}$. Hence

$\overline{OR} = 3\hat{i} + 2\hat{j} + \hat{k}$.

Ex 10.2 Class 12 Maths Question 17.

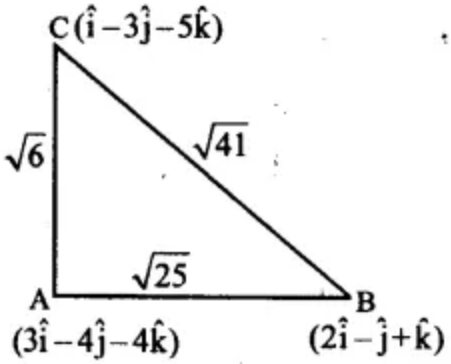
Show that the points A, B and C with position vector

$$\overrightarrow{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \overrightarrow{b} = 2\hat{i} - \hat{j} + \hat{k} and \quad \overrightarrow{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

respectively form the vertices of a right angled triangle.

Solution:

$$\overrightarrow{AB} = \overrightarrow{b} - \overrightarrow{a} = -\hat{i} + 3\hat{j} + 5\hat{k}$$



$\therefore \quad |\overrightarrow{AB}|^2 = 35,\ \overrightarrow{BC} = \vec{c}-\vec{b} = \hat{i}-2\hat{j}-6\hat{k}$

$\therefore \quad |\overrightarrow{BC}|^2 = 41,\ \overrightarrow{CA} = \vec{a}-\vec{c} = 2\hat{i}-\hat{j}+\hat{k}$

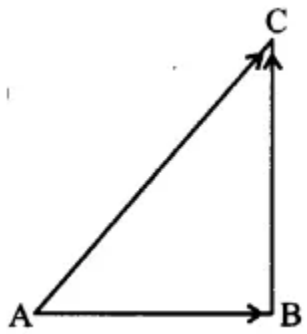
$\therefore \quad |\overrightarrow{CA}|^2 = 6$

Now $|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = |\overrightarrow{BC}|^2$

Hence ΔABC is a right angled triangle

Ex 10.2 Class 12 Maths Question 18.

In triangle ABC (fig.), which of the following is not



(a)

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$$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{0}$$

(b)

$$\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\overrightarrow{0}$$

(c)

$$\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\overrightarrow{0}$$

(d)

$$\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\overrightarrow{0}$$

Solution:

We know that

$$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{0}\ \overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\overrightarrow{0}$$

Hence option (c) is not correct

Ex 10.2 Class 12 Maths Question 19.

If

$$\overrightarrow{a},\overrightarrow{b}$$

are two collinear vectors then which of the following are incorrect:

(a)

$\vec{b} = \lambda \vec{a}$

, for some scalar λ.

(b)

$\vec{a} = \pm \vec{b}$

(c) the respective components of

$\vec{a}, \vec{b}$

are proportional.

(d) both the vectors

$\vec{a}, \vec{b}$

have same direction, but different magnitudes.

Solution:

Options (d) is incorrect since both the vectors

$\vec{a}, \vec{b}$

, being collinear, are not necessarily in the same direction. They may have opposite directions. Their magnitudes may be different.

Ex 10.3 Class 12 Maths Question 1.

Find the angle between two vectors

$\vec{a}, \vec{b}$

with magnitudes √3 and 2 respectively, and such that

$$\vec{a} \cdot \vec{b} = \sqrt{6}$$

Solution:

Angle θ between two vectors

$$\vec{a}, \vec{b}$$

,

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3}\,(2)} = \frac{\sqrt{3}\times\sqrt{2}}{\sqrt{3}\times 2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{4} \qquad \therefore\ \theta = \frac{\pi}{4}$$

Ex 10.3 Class 12 Maths Question 2.

Find the angle between the vectors

$$\hat{i} - 2\hat{j} + 3\hat{k} \quad and \quad 3\hat{i} - 2\hat{j} + \hat{k}$$

Solution:

Let

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k} \quad and \quad \vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$

Let θ be the angle between

$\vec{a}, \vec{b}$

,

$$\therefore \quad \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\Rightarrow \cos\theta = \frac{10}{\sqrt{14}\sqrt{14}} = \frac{10}{14} = \frac{5}{7}$$

$$\text{Hence} \quad \theta = \cos^{-1}\left(\frac{5}{7}\right).$$

Ex 10.3 Class 12 Maths Question 3.

Find the projection of the vector

$\vec{i} - \vec{j}$

, on the line represented by the vector

$\vec{i} + \vec{j}$

,

Solution:

let

$\vec{a} = \hat{i} - \hat{j} \quad and \quad \vec{b} = \hat{i} + \hat{j}$

Then, projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$ ..... (i)

From (i), we have,

Projection of $\vec{a}$ and $\vec{b} = \frac{0}{\sqrt{2}} = 0.$

Ex 10.3 Class 12 Maths Question 4.

Find the projection of the vector

$$\hat{i} + 3\hat{j} + 7\hat{k}$$

on the vector

$$7\hat{i} - \hat{j} + 8\hat{k}$$

Solution:

let

$$\overrightarrow{a} = \hat{i} + 3\hat{j} + 7\hat{k} \quad and \quad \overrightarrow{b} = 7\hat{i} - \hat{j} + 8\hat{k}$$

then

projection $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ ... (i)

Now, $\vec{a}\cdot\vec{b} = (\hat{i}+3\hat{j}+7\hat{k})\cdot(7\hat{i}-\hat{j}+8\hat{k}) = 60$ and

$|\vec{b}| = \sqrt{7^2 + (-1)^2 + 8^2} = \sqrt{49+1+64} = \sqrt{114}$

from (i), we have projection of $\vec{a}$ on $\vec{b} = \frac{60}{\sqrt{114}}$

Ex 10.3 Class 12 Maths Question 5.

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Show that each of the given three vectors is a unit vector

$$\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right), \frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right), \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

Also show that they are mutually perpendicular to each other.

Solution:

$$Let \quad \vec{a}=\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right), \vec{b}=\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right), \vec{c}=\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

Now, $|\vec{a}|=1$, $|\vec{b}|=1$, $|\vec{c}|=1$

$\therefore$ $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors.

Now, $\vec{a}\cdot\vec{b}=0$, $\vec{b}\cdot\vec{c}=0$, $\vec{c}\cdot\vec{a}=0$

$\Rightarrow$ $\vec{a}\perp\vec{b}$, $\vec{b}\perp\vec{c}$ and $\vec{c}\perp\vec{a}$

Hence, $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular unit vectors.

Ex 10.3 Class 12 Maths Question 6.

$$Find\,|\vec{a}|\,and\,\left|\vec{b}\right|\,if\,\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=8 \quad and\,|\vec{a}|=8\left|\vec{b}\right|$$

Solution:

Given

$$\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=8$$

$\vec{a}\cdot\left(\vec{a}-\vec{b}\right)+\vec{b}\cdot\left(\vec{a}-\vec{b}\right)=8,$

$\vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}=8$

$\Rightarrow \quad |\vec{a}|^2-|\vec{b}|^2=8 \quad [\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}]$

$\Rightarrow \quad 64\,|\vec{b}|^2-|\vec{b}|^2=8 \quad [\because |\vec{a}|=8\,|\vec{b}|]$

$\therefore \quad |\vec{b}|^2=\frac{8}{63},$

But $|\vec{a}|=8\,|\vec{b}|=8\times\sqrt{\frac{8}{63}}=\frac{8\sqrt{8}}{\sqrt{63}}$

Thus $|\vec{a}|=\frac{8\sqrt{8}}{\sqrt{63}}, |\vec{b}|=\sqrt{\frac{8}{63}}$

Ex 10.3 Class 12 Maths Question 7.

Evaluate the product :

$$\left(3\vec{a}-5\vec{b}\right)\cdot\left(2\vec{a}+7\vec{b}\right)$$

Solution:

$$\left(3\vec{a}-5\vec{b}\right)\cdot\left(2\vec{a}+7\vec{b}\right)$$

$$=6\vec{a}.\vec{a}-10\vec{b}\,\vec{a}+21\vec{a}.\vec{b}-35\vec{b}.\vec{b}$$

$$=6|\vec{a}|^2-11\vec{a}\,\vec{b}-35\left|\vec{b}\right|^2$$

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Ex 10.3 Class 12 Maths Question 8.

Find the magnitude of two vectors

$\vec{a}, \vec{b}$

having the same magnitude and such that the angle between them is 60° and their scalar product is

$\frac{1}{2}$

Solution:

We know that

$$\vec{a} . \vec{b} = |\vec{a}| \left|\vec{b}\right| cos\theta$$

$$\vec{a} \cdot \vec{b} = \frac{1}{2}, |\vec{a}| = |\vec{b}| \text{ and } \theta = 60°$$

$$\therefore \quad \frac{1}{2} = |\vec{a}||\vec{b}| \cos 60°$$

$$\Rightarrow \quad \frac{1}{2} = |\vec{a}|^2 \left(\frac{1}{2}\right) \Rightarrow |\vec{a}|^2 = 1 \left[\because |\vec{a}| = |\vec{b}|\right]$$

$$\Rightarrow \quad |\vec{a}| = 1, \ |\vec{b}| = |\vec{a}| = 1$$

Ex 10.3 Class 12 Maths Question 9.

Find

$|\vec{x}|$

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, if for a unit vector

$$\vec{a}, (\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$$

Solution:

Given

$$\vec{a}, (\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$$

$$(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = \vec{x}(\vec{x} + \vec{a}) - \vec{a}(\vec{x} + \vec{a})$$

$$= |\vec{x}|^2 + \vec{x} \cdot \vec{a} - \vec{x} \cdot \vec{a} - |\vec{a}|^2 \quad [\because \vec{a} \cdot \vec{x} = \vec{x} \cdot \vec{a}]$$

Now, $|\vec{a}| = 1, (\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 12$

$\therefore \quad |\vec{x}|^2 - |\vec{a}|^2 = 12$

or $\quad |\vec{x}|^2 = 12 + 1 \quad [\because |\vec{a}| = 1] \quad |\vec{x}| = \sqrt{13}$

Ex 10.3 Class 12 Maths Question 10.

If

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k} and \vec{c} = 3\hat{i} + \hat{j}$$

such that

$$\vec{a} + \lambda \vec{b} \perp \vec{c}$$

, then find the value of λ.

Solution:

Given

$\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \overrightarrow{b} = -\hat{i} + 2\hat{j} + \hat{k} and \overrightarrow{c} = 3\hat{i} + \hat{j}$

$\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}, \vec{b} = -\hat{i}+2\hat{j}+\hat{k}$

$\therefore \quad \vec{a} + \lambda\vec{b} = (2\hat{i}+2\hat{j}+3\hat{k}) + \lambda(-\hat{i}+2\hat{j}+\hat{k})$

$= (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k},$

$\vec{c} = 3\hat{i} + \hat{j}, \quad \vec{a} + \lambda\vec{b}$ is pendicular to $\vec{c}$

$\Rightarrow \quad (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$

$3(2-\lambda)+(2+2\lambda)=0$

$\Rightarrow \quad 6-3\lambda+2+2\lambda=0 \quad \text{or} \quad \lambda=8$

Ex 10.3 Class 12 Maths Question 11.

Show that

$$|\overrightarrow{a}|\overrightarrow{b} + \left|\overrightarrow{b}\right| a \quad \perp \quad |\overrightarrow{a}| \cdot \overrightarrow{b} - \left|\overrightarrow{b}\right| a$$

for any two non-zero vectors

$\overrightarrow{a}, \overrightarrow{b}$

Solution:

$\overrightarrow{a}, \overrightarrow{b}$

are any two non zero vectors

Consider : $[|\vec{a}|\vec{b}+|\vec{b}|\vec{a}].[|\vec{a}|\vec{b}-|\vec{b}|\vec{a}]^2$

$= [|\vec{a}|\vec{b}]^2 - [|\vec{b}|\ \vec{a}]^2$

$= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2 = 0$

Hence $|\vec{a}|\vec{b}+|\vec{b}|\vec{a} \perp |\vec{a}|\vec{b}-|\vec{b}|\vec{a}.$

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Ex 10.3 Class 12 Maths Question 12.

If

$$\vec{a} \cdot \vec{a} = 0 \quad and \quad \vec{a} \cdot \vec{b} = 0$$

, then what can be concluded about the vector

$$\vec{b}$$

?

Solution:

$$\vec{a}\,\vec{a} = 0 \quad and \quad \vec{a}.\vec{b} = 0$$

=>

$$\vec{b}$$

= 0

Hence b is any vector.

Ex 10.3 Class 12 Maths Question 13.

If

$$\vec{a}, \vec{b}, \vec{c}$$

are the unit vector such that

$$\vec{a} + \vec{b} + \vec{c} = 0$$

, then find the value of

$$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$$

Solution:

We have

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow \quad \vec{a}\cdot(\vec{a} + \vec{b} + \vec{c}) = \vec{a}\cdot\vec{0}$$
$$\Rightarrow \quad \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = 0$$
$$\Rightarrow \quad |\vec{a}|^2 + \vec{a}\cdot\vec{b} + \vec{c}\cdot\vec{a} = 0$$
$$\Rightarrow \quad 1 + \vec{a}\cdot\vec{b} + \vec{c}\cdot\vec{a} = 0 \qquad \text{...(i)}$$

Similarly, by taking dot product with $\vec{b}$ and $\vec{c}$, we get

$$\vec{a}\cdot\vec{b} + 1 + \vec{b}\cdot\vec{c} = 0 \qquad \text{...(ii)}$$

$$\vec{c}\cdot\vec{a} + \vec{b}\cdot\vec{c} + 1 \qquad \text{...(iii)}$$

Adding (i), (ii) and (iii), we have

$$2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) + 3 = 0$$
$$\therefore \quad \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}$$

Ex 10.3 Class 12 Maths Question 14.

If either vector

$$\vec{a} = 0 \quad or \quad \vec{b} = 0$$

then

$$\vec{a}.\vec{b} = 0$$

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. But the converse need not be true. Justify your answer with an example.

Solution:

Given:

$$\vec{a} = 0 \quad or \quad \vec{b} = 0$$

To prove:

$$\vec{a} . \vec{b} = 0$$

Proof: L.H.S. $= \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

If $\vec{a}$ or $|\vec{a}| = 0$, then L.H.S. $= 0$

Again if $\vec{b}$ or $|\vec{b}| = 0$, then L.H.S. $= 0$

Converse:

Given: $\vec{a}.\vec{b} = 0$

To prove : $\vec{a} = 0$ or $\vec{b} = 0$ or none

Proof: $\vec{a}.\vec{b} = 0 \Rightarrow |\vec{a}||\vec{b}|\cos\theta = 0$

Clearly if $|\vec{a}| = 0$ then $\vec{a}.\vec{b} = 0$

or $|\vec{b}| = 0$ then $\vec{a}.\vec{b} = 0$

or if $|\vec{a}| \neq 0, \vec{b} \neq 0,$

then if $\theta = \frac{\pi}{2}$ Converse in not true.

Ex 10.3 Class 12 Maths Question 15.

If the vertices A,B,C of a triangle ABC are (1,2,3) (-1,0,0), (0,1,2) respectively, then find ∠ABC.

Solution:

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Let O be the origin then.

$$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}, \overrightarrow{OB} = -\hat{i} \text{ and}$$

$$\overrightarrow{OC} = \hat{j} + 2\hat{k}$$

$$\therefore \quad \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\text{Now } \cos \angle ABC = \frac{\overrightarrow{BC} \cdot \overrightarrow{BA}}{\left|\overrightarrow{BC}\right| \left|\overrightarrow{BA}\right|}$$

$$= \frac{(\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 2\hat{j} + 3\hat{k})}{|\hat{i} + \hat{j} + 2\hat{k}||2\hat{i} + 2\hat{j} + 3\hat{k}|}$$

$$= \frac{10}{\sqrt{102}},$$

$$\Rightarrow \quad \angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

$\frac{1}{2}$

Ex 10.3 Class 12 Maths Question 16.

Show that the points A (1,2,7), B (2,6,3) and C (3,10, -1) are collinear.

Solution:

The position vectors of points A, B, C are

$\hat{i} + 2\hat{j} + 7\hat{k}, 2\hat{i} + 6\hat{j} + 3\hat{k}, 3\hat{i} + 10\hat{j} - \hat{k}$

$\overline{AB} = \overline{OB} - \overline{OA} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overline{BC} = \overline{OC} - \overline{OB} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overline{AC} = \overline{OC} - \overline{OB} = 2(\hat{i} + 4\hat{j} - 4\hat{k})$

$|\overline{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$,

$|\overline{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$

$|\overline{AC}| = \sqrt{2^2 + 8^2 + (-8)^2} = \sqrt{4 + 64 + 64} = 2\sqrt{33}$

Thus AB + BC = AC. Hence A, B, C are colinear.

Ex 10.3 Class 12 Maths Question 17.

Show that the vectors

$$2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$$

and

$$\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$$

from the vertices of a right angled triangle.

Solution:

The position vectors of the points A, B and C are

$$2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$$

and

$\left(3\hat{i}-4\hat{j}-4\hat{k}\right)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\left|\overrightarrow{AB}\right|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 41$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 2\hat{i} - \hat{j} + \hat{k}$

$\left|\overrightarrow{BC}\right|^2 = BC^2 = 2^2 + (-1)^2 + 12 = 4 + 1 + 1 = 6$

$\left|\overrightarrow{AC}\right| = \overrightarrow{OC} - \overrightarrow{OA} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\left|\overrightarrow{AC}\right|^2 = AC^2 = 1^2 + (-3)^2 + (-5)^2$

$= 1 + 9 + 25 = 35,$

Now, $BC^2 + AC^2 = 6 + 35 = 41 = AB^2$

$\Rightarrow$ $BC^2 + AC^2 = AB^2$

$\Rightarrow$ Triangle ABC is a right angled triangle.

Ex 10.3 Class 12 Maths Question 18.

If

$\vec{a}$

is a non-zero vector of magnitude ‘a’ and λ is a non- zero scalar, then λ

$\vec{a}$

is unit vector if

(a) λ = 1

(b) λ = – 1

(c) a = |λ|

(d) a =

$\frac{1}{|\lambda|}$

Solution:

$|\vec{a}| = a$

Given :

$\lambda \vec{a}$

is a unit vectors

$\Rightarrow \quad |\lambda \vec{a}| = 1$

$\Rightarrow \quad |\lambda||\vec{a}| = 1$

$\Rightarrow \quad |\lambda| a = 1$

$\Rightarrow \quad a = \frac{1}{|\lambda|}$

Hence option (d), is correct.

Ex 10.4 Class 12 Maths Question 1.

Find

$\left|\vec{a} \times \vec{b}\right|, if \quad \vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \quad and \quad \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

Solution:

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Given

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \quad and \quad \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} = 19\hat{j} + 19\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-19)^2 + (-19)^2} = 19\sqrt{2}.$$

Ex 10.4 Class 12 Maths Question 2.

Find a unit vector perpendicular to each of the vector

$$\vec{a} + \vec{b} \quad and \quad \vec{a} - \vec{b}$$

, where

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \quad and \quad \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Solution:

we have

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \quad and \quad \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$\therefore \quad \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$

$\therefore \quad \vec{a} - \vec{b} = (2\hat{i} + 4\hat{k})$

$$\text{Now } (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$= 16\hat{i} - 16\hat{j} - 8\hat{k}$

$\therefore$ A unit vector perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ is given by

$$\hat{n} = \pm \frac{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})}{|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|}$$

$$= \pm \frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k}$$

Ex 10.4 Class 12 Maths Question 3.

If a unit vector $\vec{a}$ makes angle $\frac{\pi}{3} with \quad \hat{i}, \frac{\pi}{4} with \quad \hat{j}$ and an acute angle θ with $\vec{k}$,then find θ and hence the components of $\vec{a}$

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.

Solution:

$$Let \quad \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} such \quad that \quad |\vec{a}| = 1$$

$$\Rightarrow \quad \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) \cdot \hat{i} = |\bar{a}|\,|\hat{i}| \cos\frac{\pi}{3}$$

$$\Rightarrow \quad (a_1)(1) = (1)(1)\frac{1}{2} \Rightarrow a_1 = \frac{1}{2}$$

$$\text{Also } \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) \hat{j} = |\bar{a}||\hat{j}| \cos\frac{\pi}{4}$$

$$\Rightarrow \quad a_2 = \frac{1}{\sqrt{2}} \quad \text{As } |\bar{a}| = 1 \Rightarrow a_1^2 + a_2^2 + a_3^2 = 1$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + a_3^2 = 1$$

$$\Rightarrow \quad a_3 = \frac{1}{2} \qquad \therefore \quad \bar{a} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}$$

$$\text{and } \left(\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}\right) \cdot \hat{k} = \left|\frac{1}{4} + \frac{1}{2} + \frac{1}{4}\right| |1| \cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} \text{ i.e., } \theta = \frac{\pi}{3}$$

$$\therefore \quad \text{components of } \bar{a} \text{ are } \frac{1}{2}\hat{i}, \frac{1}{\sqrt{2}}\hat{j}, \frac{1}{2}\hat{k}$$

Ex 10.4 Class 12 Maths Question 4.

Show that

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$$\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$$

Solution:

LHS =

$$\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)$$

$$=\vec{a}\times\vec{a}-\vec{b}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{b}$$

$$=0+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}-0=2\left(\vec{a}\times\vec{b}\right)=\text{R.H.S.}$$

Ex 10.4 Class 12 Maths Question 5.

Find λ and μ if

$$\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=0$$

Solution:

$$\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=0$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0$$

$\Rightarrow (6\mu-27\lambda)\,\hat{i}-(2\mu-27)$

$\hat{j}+(2\lambda-6)\,\hat{k}=0$ comparing,

we have $6\mu-27\lambda=0$ ...(i)

$\Rightarrow 2\mu-27=0 \Rightarrow \mu=\frac{27}{2}$ ...(ii)

and $2\lambda-6=0 \Rightarrow \lambda=\frac{6}{2}=3$ ...(iii)

Putting for λ, μ in (i) $6\left(\frac{27}{2}\right)-27(3)=81-81=0$

Hence, $\lambda=3, \mu=\frac{27}{2}$

Ex 10.4 Class 12 Maths Question 6.

Given that

$$\vec{a}.\vec{b}=0 \quad and \quad \vec{a}\times\vec{b}=0$$

. What can you conclude about the vectors

$$\vec{a}, \vec{b}$$

?

Solution:

$\vec{a} . \vec{b} = 0 \quad and \quad \vec{a} \times \vec{b} = 0$

$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta = 0$ and $|\vec{a} \times \vec{b}| = 0$

$\Rightarrow \quad |\vec{a}||\vec{b}|\sin\theta = 0$

$\Rightarrow \quad (\vec{a} = 0 or \vec{b} = \vec{0} or \vec{a} \perp \vec{b})$ and

$(\vec{a} = \vec{0} or \vec{b} = \vec{0} or \vec{a} \| \vec{b})$

$\Rightarrow \quad \vec{a} = \vec{0}$ or $\vec{b} = \vec{o}$

[$\because \quad \vec{a} \perp \vec{b}$ and $\vec{a} \| \vec{b}$ can never hold simultaneously]

Ex 10.4 Class 12 Maths Question 7.

Let the vectors

$\vec{a}, \vec{b}, \vec{c}$

are given

$a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

. Then show that

$= \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Solution:

Given

$\vec{a}, \vec{b}, \vec{c}$

are given

$$a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\vec{b} + \vec{c} = \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) + \left(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\right)$$

$$= (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$$

$$\text{L.H.S.} = \vec{a} \times \left(\vec{b} + \vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$$

$$= [(a_2 b_3 - a_3 b_2) + (a_2 c_3 - a_3 c_2)]$$

$$\hat{i} - [(a_1 b_3 - a_3 b_1) + (a_1 c_3 - c_3 c_1)\ \hat{j}\ ]$$

$$+ [a_1 b_2 - a_2 b_1) + (a_1 c_2 - a_2 c_1)\hat{k}\ ] \qquad \text{...(ii)}$$

$$\text{R.H.S.} = \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= [(a_2 b_3 - a_3 b_2)\ \hat{i} - (a_1 b_3 - a_3 b_1)$$

$$\hat{j} + (a_1 b_2 - a_2 b_1)\ \hat{k}\ ] + [(a_2 c_3 - a_3 c_2)$$

$$\hat{i} - (a_1 c_3 - a_3 c_1)\ \hat{j} + (a_1 c_2 - a_2 c_1)\hat{k}\ ] \qquad \text{...(ii)}$$

from (i) and (ii), we conclude that

$$\vec{a} \times \left(\vec{b} + \vec{c}\right) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

Ex 10.4 Class 12 Maths Question 8.

If either

$$\vec{a} = 0 \quad or \quad \vec{b} = 0 \quad then \quad \hat{a} \times \hat{b} = 0$$

.Is the

converse true? Justify your answer with an example.

Solution:

$$\vec{a} = 0 \Rightarrow |\vec{a}| = 0$$

$\therefore \quad \vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \hat{n}$

Where θ is the angle between $\vec{a}$ and $\vec{b}$

$= 0 \times |\vec{b}| \sin\theta = 0$

Similarly, when $\vec{b} = \vec{0} \Rightarrow |\vec{b}| = 0$, $\vec{a} \times \vec{b} = 0$

Converse let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = pa_1\hat{i} + pa_2\hat{j} + pa_3\hat{k}$ *i.e.*, $\vec{a}$ and $\vec{b}$ are

parallel so $\theta = 0$, $|\vec{a}| \neq 0, |\vec{b}| \neq 0$

But $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta, \hat{n} = 0$

Thus $\vec{a} \times \vec{b} = 0$, even when $\vec{a} \neq 0, \vec{b} \neq 0$

Ex 10.4 Class 12 Maths Question 9.

Find the area of the triangle with vertices A (1,1,2), B (2,3,5) and C (1,5,5).

Solution:

A (1,1,2), B (2,3,5) and C (1,5,5).

$\overrightarrow{OA} = \hat{i} + \hat{j} + 2\hat{k}$, $\overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

and $\overrightarrow{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 4\hat{j} + 3\hat{k}$

Now $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$

Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{61}$ sq. units.

Ex 10.4 Class 12 Maths Question 10.

Find the area of the parallelogram whose adjacent sides are determined by the vectors

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}, \ \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

Solution:

We have

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}, \ \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

Area of the parallelogram whose sides are vectors $\vec{a}$ and $\vec{b}$

$$= |\vec{a} \times \vec{b}| = |20\hat{i} + 5\hat{j} - 5\hat{k}|$$

$$= \sqrt{(20)^2 + 5^2 + (-5)^2} = 15\sqrt{2}$$

Ex 10.4 Class 12 Maths Question 11.

Let the vectors

$\vec{a}, \vec{b}$

such that

$|\vec{a}| = 3, \left|\vec{b}\right| = \frac{\sqrt{2}}{3}$

then

$\vec{a} \times \vec{b}$

is a unit vector if the angle between

$\vec{a}, \vec{b}$

is

(a)

$\frac{\pi}{6}$

(b)

$\frac{\pi}{4}$

(c)

$\frac{\pi}{3}$

(d)

$\frac{\pi}{2}$

Solution:

Given

$$\left|\vec{a} \times \vec{b}\right| = 1 \, |\vec{a}| = 3, \left|\vec{b}\right| = \frac{\sqrt{2}}{3}$$

Now $|(\vec{a} \times \vec{b}) = |\vec{a}||\vec{b}| \sin\theta$

$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

Hence correct, option is (b).

Ex 10.4 Class 12 Maths Question 12.

https://www.indcareer.com/schools/ncert-solutions-for-12th-class-maths-chapter-10-vector-algebra/

Area of a rectangles having vertices

$$A\left(-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right), B\left(\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right),$$
$$C\left(\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}\right), D\left(-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}\right),$$

(a)

$\frac{1}{2}$

sq units

(b) 1sq.units

(c) 2sq.units

(d) 4sq.units

Solution:

$$\overrightarrow{OA} = \left(-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}\right) \overrightarrow{OB} = \left(\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}\right)$$

$\overrightarrow{OC} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ $\overrightarrow{OD} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 2\hat{i}$

$\overrightarrow{AC} = \overrightarrow{OC}' - \overrightarrow{OA} = 2\hat{i} - \hat{j}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 2 & -1 & 0 \end{vmatrix} = \hat{k}(-2) = -2\hat{k}$$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-2)^2} = 2$ sq. units

Hence area of rectangle = 2 sq. units.

Thus option (c) is correct.

Chapterwise NCERT Solutions for Class 12 Maths :

- Chapter 1 – Relations and Functions
- Chapter 2 – Inverse Trigonometric Functions.
- Chapter 3 – Matrices
- Chapter 4 – Determinants.
- Chapter 5 – Continuity and Differentiability.0.0
- Chapter 6 – Application of Derivatives.
- Chapter 7 – Integrals.
- Chapter 8 – Application of Integrals.
- Chapter 9: Differential Equations
- Chapter 10: Vector Algebra
- Chapter 11: Three Dimensional Geometry
- Chapter 12: Linear Programming
- Chapter 13: Probability

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