NCERT Solutions for 12th Class Maths: Chapter 1-Relations and Functions

Class 12: Maths Chapter 1 solutions. Complete Class 12 Maths Chapter 1 Notes.

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Exercise 1.1 (NCERT 12th Class Maths Chapter 1)

https://www.indcareer.com/schools/ncert-solutions-for-12th-class-maths-chapter-1-relations-and-functions/

1. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y) : 3x – y = 0}

(ii) Relation R in the set N of natural numbers defined as R = {(x, y) : y = x + 5 and x < 4}

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as R = {(x, y) : y is divisible by x}

(iv) Relation R in the set Z of all integers defined as R = {(x, y) : x – y is an integer}

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) R = {(x, y) : x and y work at the same place}

(b) R = {(x, y) : x and y live in the same locality}

(c) R = {(x, y) : x is exactly 7 cm taller than y}

(d) R = {(x, y) : x is wife of y}

(e) R = {(x, y) : x is father of y}

Answer

(i) A = {1,2,3,.....,13,14}

R = {x.y) : 3 x – y ≠} = {(x, y) : y = 3 x} = {(1,3), (2, 6), (3, 9), (4, 12)}

(a) R is not reflexive as (x, x) ∉ R [∵ 3 x – x ≠ 0]

(b) R is not symmetric as (x,y) ∈ R does not imply (y, x) ∈ R

[∴ (1, 3) ∈ R does not imply (3. 1) ∈ R]

(c) R is not transitive as (1.3) ∈ R , (3, 9) ∈ R but (1.9) ∉ R

(ii) Relation R in the set N is given by

R = {(x, y) : y = x + 5 and x < 4}

∴ R = {(1,6), (2, 7). (3, 8)}

(a) R is not reflexive as (x, x) ∉ R

(b) R is not symmetric as (x, y) ∈ R ⇏ (v, x) ∈ R

(c) R is not transitive as (x,y) ∈ R, (y, z) ∈ R ⇏ (x, z) ∈ R

(iii) A = {1, 2, 3, 4, 5, 6}

R = {(x, y) : y is divisible by x}

(a) R is reflexive as (x, x) ∈ R ∀ x ∈ A [∴ x divides x ∀ x ∈ A]

(b) R is not symmetric as (1, 6) ∈ R but (6, 1) ∉ R.

(c) Let (x, y), (y, z) ∈ A

∴ y is divisible by x and z is divisible by y

∴ z is divisible by x

∴ (r, y) ∈ R (y, z) ∈ R ⇒ (x, z) ∈ R

∴ R is transitive

(iv) Relation R is in the set Z given by R = {(x,y) : x – y is an integer}

(a) R is reflexive as (x, x) ∈ R [∴ x – x = 0 is an integer]

(b) R is symmetric as (x,y) ∈ R ⇒ (y, x) ∈ A

[∵ x – y is an integer ⇒ y – x is an integer]

(c) R is transitive as (x, y), (y, z) ∈ R ⇒ (x, z) ∈ R

[∵ if x – y, y – z are integers, then (x – y) + (y – z) = x – z is also in integer]

(v) A is the set of human beings in a town at a particular time R is relation in A.

(a) R = {(x, y) : x and y work at the same time}

R is reflexive as (x, x) ∈ R

R is symmetric as (x, y) ∈ R ∈ (y, x) ∈ R

[∵ x and y work at the same time ⇒ y and x work at the same time]

R is transitive as (x, y), (y, z) ∈ R ⇒ (x, z) ∈ R

[∴ if x and y, y and z work at the same time, then x and z also work at the same time]

(b) R = {(x,y) : x and y live in the same locality}

R is reflexive as (x, x) ∈ R

R is symmetric as (x, y) ∈ R ⇒ (y, x) ∈ R

[∴ x and y live in the same locality ⇒ y and x live in the same locality]

R is transitive as (x, y), (y, z) ∈ R ⇒ (x, z) ∈ R

[∵ if x and y, y and z live in the same locality. then x and z also live in the same locality]

(c) R = {(x,y) : x is exactly 7 cm taller than y}

Since (x, x) ∉ R as x cannot be 7 cm taller than x.

∴ R is not reflexive.

(x, y) ∈ R ⇒ (y.x) ∈ R as if x is taller than y, then y cannot be taller than x.

∴ R is not symmetric.

Again, (x,y), (y,z) ∈ R ⇏ (x, z) ∈ R

[∵ if x is taller than y by 7 cm and y is taller than z by 7 cm, then x is taller than z by 14 cm]

∴ R is not transitive.

(d) R = {(x,y) : x is wife of y}

R is not reflexive as (x,y) ∉ R [∴ x cannot be wife of x]

Also, (x, y) ∈ R ⇏ (y, x) ∈ R

[∵ if x is wife of y, then y cannot be wife of x]

∴ R is not symmetric.

Also, this does not imply that x is the wife of z.

∴ (x, z) ∉ R

∴ R is not transitive.

(e) R = {(x,y) : x is father of y}

R is not reflexive as (x, x) ∉ R [∵ x cannot be father of x]

Also, (x,y) ∈ R ⇐ (y, x) ∈ R [∵ if x is father of y. then y cannot be father of x]

∴ R is not symmetric.

Let (x, y) ∈ R and (y, z) ∈ R.

⇒ x is the father of y and y is the father of z.

⇒ x is not the father of z.

∴ x is the grandfather of z.

∴ (x, z) ∉ R

∴R is not transitive.

2. Show that the relation R in the set R of real numbers, defined as R = {(a, b) : a ≤ b^2} is neither reflexive nor symmetric nor transitive.

Answer

R = {(a, b) : a ≤ b^2}

(i) Since (a, a) ∉ R

[Take a = 1/3 then 1/3 > $(1/3)^2$]

∴ R is not reflexive.

(ii) Also, (a, b) ∈ R ⇏ (b, a) ∈ R

[Take a = 2 ,b = 6, then 2 ≤ 62 but (6)2 < 2 is not true]

∴ R is not symmetric.

(iii) Now (a, b), (b, c) ∈ R ∉ (a, c) ∈ R

[Take a = 1, b = -2, c = -3 ∴ a ≤ b^2, b ≤ c^2 but a ≤ c^2 is not true]

∴ R is not transitive.

3. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

Answer

Let,

A = {1, 2, 3, 4, 5, 6}

R = {(a, b) : b = a + 1} = {(a, a + 1)}

= {(1, 2), (2, 3), (3, 4), (4,5), (5,6)}

(i) R is not reflexive as (a, a) ∉ R ∀ a ∈ A

(ii) (a,b) ∈ R ⇏ (b,a) ∈ R

[∵ (a, b) ∈ R ⇒ b = a + 1 ⇒ a = b –1]

∴ R is not symmetric.

(iii) (a, b) ∈ R, (b, c) ∈ R ⇏ (a, c) ∈ R

[∵ $(a, b), (b, c) \in R \Rightarrow b = a + 1, c = b + 1 \Rightarrow c = a + 2$]

∴ R is not transitive.

4. Show that the relation R in R defined as R = {(a, b) : a ≤ b}, is reflexive and transitive but not symmetric.

Answer

$R = \{(a, b) : a \leq b\}$

(i) Since $(a, a) \in R\ \forall\ a \in R$

[∵ $a \leq a\ \forall\ a \in R$]

∴ R is reflexive.

(ii) $(a, b) \in R \nRightarrow (b, a) \in R$

[∵ if $a \leq b$. then $b \leq a$ is not true]

∴ R is not symmetric.

(iii) Let $(a, b), (b, c) \in R$

∴ $a \leq b, b \leq c$

∴ $a \leq c \Rightarrow (a, c) \in R$

∴ $(a, b), (b. c) \in R \Rightarrow (a, c) \in R$

∴ R is transitive.

5. Check whether the relation R in R defined by R = {(a, b) : a ≤ b^3 } is reflexive, symmetric or transitive.

Answer

$R = \{(a, b) : a \leq b^3\}$

(i) Since $(a, a) \notin R$ as $a \leq a^3$ is not always true.

[Take a = 1/3. then $a \leq a^3$ is not true]

∴ R is not reflexive.

(ii) Also $(a, b) \in R \nRightarrow (b, a) \in R$

[Take a = 1, b = 4, ∴ $1 \leq 4^3$ but $4 \nleq (l)^3$]

∴ R is not symmetric.

(iii) Now $(a, b) \in R, (b, c) \in R \nRightarrow (a, c) \in R$

[Take a = 100, b = 5, c = 3, ∴ $100 \leq 5^3$, $5 \leq 3^3$ but $100 \geq 3^3$] R is not symmetric.

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6. Show that the relation R in the set{1, 2, 3} given by R={(1, 2), (2, 1)}is symmetric but neither reflexive nor transitive.

Answer

A = {1,2, 3}

R = {(1.2), (2, 1)}

Since, $(a, a) \in R\ \forall\ a \in A$ R is not reflexive.

Now $(1, 2) \in R \Rightarrow (2, 1) \in R$ and $(2, 1) \in R \Rightarrow (1,2) \in R$

∴ $(a, b) \in R \Rightarrow (b, a) \in R\ \forall\ (a, b) \in R$

∴ R is symmetric

Again, $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$

∴ R is not transitive.

7. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.

Answer

A is the set of all books in a library of a college.

R = {(x,y) : x and y have same number of pages}

Since, (x, x) ∈ R as x and x have the same number of pages ∀ x ∈ A.

∴ R is reflexive.

Also (x, y) ∈ R

⇒ x and y have the same number of pages

⇒ y and x have the same number of pages

⇒ (y, x) ∈ R

∴ R is symmetric.

Now, (x, y) ∈ R and (y, z) ∈ R.

⇒ x and y have the same number of pages and y and z have the same number of pages

⇒ x and z have he same number of pages

⇒ (x, z) ∈ R

∴ R is transitive.

8. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by

R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

Answer

A = {1, 2, 3, 4, 5}

R = {(a, b) : | a – b | is even}

Now | a – a | = 0 is an even number,

∴ (a, a) ∈ R ∀ a ∈ A

⇒ R is reflexive.

Again (a, b) ∈ R

⇒ | a – b | is even

⇒ | – (b – a) | is even

⇒ | b – a | is even

⇒ (b, a) ∈ R

∴ R is symmetric.

Let (a, b) ∈ R and (b, c) ∈ R

⇒ | a – b | is even and | b – c | is even

⇒ a – b is even and b – c is even

⇒ (a – b) + (b – c) is even

⇒ a – c is even ⇒ | a – c | is even

⇒ (a, c) ∈ R

∴ R is transitive.

∴ R is an equivalence relation.

∵ | 1 – 3 | = 2, | 3 – 5 | = 2 and | 1 – 5 | = 4 are even,

All the elements of {1, 3, 5} are related to each other.

∵ | 2 – 4 | = 2 is even,

All the elements of {2, 4} are related to each other.

Now | 1 – 2 | = 1, | 1 – 4 | = 3, | 3 – 2 | = 1, | 3 – 4 | = 1, | 5 – 2 | = 3 and | 5 – 4 | = 1 are all odd

No element of the set {1, 3, 5} is related to any element of (2, 4}.

9. Show that each of the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12}, given by

(i) R = {(a, b) : |a – b| is a multiple of 4}

(ii) R = {(a, b) : a = b} is an equivalence relation.

Find the set of all elements related to 1 in each case.

Answer

A= {x ∈ Z : 0 ≤ x ≤ 12}

= {0, 1, 2, 3, 4, 5, 6, 7, 8,9, 10, 11, 12 }

(i) R = {(a, b) : | a – b | is a multiple of 4|

As | a – a | = 0 is divisible by 4

∴ (a, a) ∈ R ∀ a ∈ A.

∴ R is reflexive.

Next, let (a, b) ∈ R

⇒ | a – b | is divisible by 4

⇒ | – (b – a) | is divisible by 4

⇒ | b – a | is divisible by 4 ⇒ (b, a) ∈ R

∴ R is symmetric.

Again. (a, b) ∈ R and (b, c) ∈ R

⇒ |a – A| is a multiple of 4 and |b – c| is a multiple of 4

⇒ a – b is a multiple of 4 and b – c is a multiple of 4

⇒ (a – b) + (b – c) is a multiple of 4

⇒ a – c is a multiple of 4

$\Rightarrow$ |a – c| is a multiple of 4

$\Rightarrow$ (a, c) $\in$ R

$\therefore$ R is transitive.

$\therefore$ R is an equivalence relation.

Set of elements which are related to | = {a $\in$ A : (a, 1) $\in$ R}

= {a $\in$ A : |a – 1| is a multiple of 4]

= {1, 5, 9}

[$\because$ | 1 – 1 | = 0, | 5 – 1 | = 4 and I 9 – 1 | = 8 are multiples of 4] (ii) R = {(a, b) : a = b} $\therefore$ a = a $\forall$ a $\in$ A,

$\therefore$ R is reflexive.

Again, (a, b) $\in$ R $\Rightarrow$ a = b $\Rightarrow$ b = a $\Rightarrow$ (b,a) $\in$ R $\therefore$ R is symmetric.

Next. (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ a = b and b = c

$\Rightarrow$ a = c $\Rightarrow$ (a, c) $\in$ R

$\therefore$ R is transitive.

$\therefore$ R is an equivalence relation.

Set of elements of A which are related to I = {a $\in$ A : (a,1) $\in$ R }

= {a $\in$ A : a = 1} = {1}

10. Give an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

Answer

(i) Let A = {1, 2}.

Then A x A = {(1,1), (1,2), (2,1), (2,2) }.

Let R = {(1,2), (2,1)} .

Then R ⊆ A x A and hence R is a relation on the set A.

R is symmetric since (a, b) ∈ R ⇒ (b. a) ∈ R.

R is not reflexive since I ∈ A but (1,1) ∉ R.

R is not transitive since (1, 2) ∈ R, (2,1) ∈ R but (1,1) ∉ R.

(ii) Let A = {1,2,3}

Then A x A = {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)}.

Let R = {(1,1), (2,2), (1,2), (2,1), (1,3), (2,3)}.

Then R is transitive since (a, b) ∈ R, (b, c) ∈ R ⇒ (a, c) ∈ R.

R is not reflexive since 3 G A but (3.3) ∉ R.

R is not symmetric since (1,3) ∈R but (3,1) ∉ R.

(iii) Let A = {1,2 3}

Then A x A = {(1, 1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) }.

Let R = {(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)}.

R is a relation on A as R ⊆ A x A.

R is reflexive as (a, a) ∈ R ∀ a ∈ A.

Also. R is symmetric since (a. b) ∈ R implies that (b, a) ∈R.

But R is not transitive since (1,2) ∈R arid (2,3) ∈R but (1,3) ∉ R.

(iv) Let A = {1,2,3}.

Then A x A = {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)}.

Let R = {(1,1), (2,2), (3,3), (1,3)}.

Then R is a relation on A as R ⊆ A x A.

R is reflexive since (a, a) ∈R ∀ a ∈ A.

R is not symmetric as (1,3) ∈R and (3,1) ∉ R. R is transitive since (a, b) ∈R and (b, c) ∈R implies that (a, c) ∈R.

(v) Let A = {1,2,3}

Then A x A = {(1,1), (1,2), (1, 3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)}

Let R = {(1,1), (1,2), (2,1), (2,2)}.

R is not reflexive as 3∈ A and (3,3) ∉ R.

R is symmetric as (a, b) ∈ R ⇒ (b, a) ∈R.

R is transitive since (a, b) ∈ R and (b, c) ∈R implies that (a, c) ∈ R.

11. Show that the relation R in the set A of points in a plane given by R = {(P, Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

Answer

A is the set of points in a plane.

R = {(P. Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}

= {(P, Q) : | OP | = | OQ | where O is origin}

Since, | OP | = | OP |, (P, P) ∈ R ∀ P ∈ A.

∴ R is reflexive.

Also (P. Q) ∈ R

⇒ | OP | = | OQ |

⇒ | OQ | = | OP |

⇒ (Q.P) ∈ R ⇒ R is symmetric.

Next let (P, Q) ∈ R and (Q, T) ∈ R ⇒ | OP | = | OQ | and | OQ | = | OT |

⇒ | OP | = | OT |

⇒ (P,T) ∈ R

∴ R is transitive

∴ R is an equivalence relation

Set of points related to P ≠ O

= {Q ∈ A : (Q,P) ∈ R} = {Q ∈ A : | OQ | = | OP |}

= {Q ∈ A :Q lies on a circle through P with centre O}

12. Show that the relation R defined in the set A of all triangles as R = {(T_1 , T_2) : T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Answer

R = {(T_1, T_2) : T_1 is similar to T_2}

Since every triangle is similar to itself

∴ R is reflexive.

Also (T_1 T_2) ∈ R ⇒ T_1 is similar to T_2 ⇒ T_2 is similar to T_1

∴ (T_2,T_1) ⇒ R

$\therefore$ $(T_1,T_2) \in R \Rightarrow (T_2,T_1) \in R \Rightarrow$ R is symmetric.

Again $(T_1, T_2), (T_2, T_3) \in R$

$\Rightarrow$ T_1 is similar to T_2 and T_2 is similar to T_3.$\therefore$ T_1 is similar to $T_3 \Rightarrow (T_1,T_3) \in R$

$\therefore$ $(T_1, T_2), (T_2,T_3) \in R \Rightarrow (T_1, T_3) \in R$

$\therefore$ R is transitive.

$\therefore$ R is reflexive, symmetric and transitive

$\therefore$ R is an equivalence relation.

Now T_1, T_2, T_3 are triangles with sides 3, 4, 5 ; 5, 12, 13 and 6, 8, 10.

Since, 3/6 = 4/8 = 5/10

$\therefore$ T_1 is similar to T_3 i.e. T_3 is related to T_1.

13. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2) : P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Answer

A is the set of all polygons

R = {(P_1, P_2) : P_1 and P_2 have same number of sides }

Since P_1 and P_2 have the same number of sides

$\therefore$ $(P_1, P_2) \in R\ \forall\ P \in A$.

$\therefore$ R is reflexive.

Let $(P_1, P_2) \in R$

$\Rightarrow$ P_1 and P_2 have the same number of sides $\Rightarrow$ P_2 and P_1 have the same number of sides $\Rightarrow (P_2, P_1) \in R$

$\therefore$ $(P_1, P_2) \in R \Rightarrow (P_2, P_1) \in R$

∴ R is symmetric.

Let $(P_1, P_2) \in R$ and $(P_2, P_3) \in R$.

⇒ P_1 and P_2 have the same number of sides and P_2 and P_3 have same number of sides

⇒ P_1 and P_3 have the same number of sides

⇒ $(P_1, P_3) \in R$

∴ $(P_1, P_2), (P_2, P_3) \in R \in (P_1, P_3) \in R$

∴ R is transitive.

∴ R is an equivalence relation.

Now T is a triangle.

Let P be any element of A.

Now P ∈ A is related to T iff P and T have the same number of sides P is a triangle required set is the set of all triangles in A.

14. Let L be the set of all lines in XY plane and R be the relation in L defined as R = $\{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

Answer

R is reflexive because, in (L_1, L_1), L_1 and L_1 are parallel.

R is symmetric because, if L_1 and L_2 are parallel, then L_2 and L_1 are also parallel.

R is transitive because, if L_1 and L_2 are parallel; L_2 and L_3 are parallel, then L_1 and L_3 are also parallel.

Set of all lines parallel to y = 2x + 4 are all lines of the equation y = 2x + k , where k is any constant . (Two parallel lines will have the ratio of their x coefficients equal to that of y coefficients.).

15. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

Answer

(A) R is reflexive and symmetric but not transitive

(B) R is reflexive and transitive but not symmetric

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

R is reflexive because we have (1,1) (2,2) (3,3) and (4,4) in R.

R is not symmetric because (2,1), (3,1) doesn't lie in R.

R is transitive because (1,3) (3,2) (1,2) lie in R.

Thus option B is the correct answer

16. Let R be the relation in the set N given by R = {(a, b) : a = b – 2, b > 6}.

Choose the correct answer.

(A) (2, 4) ∈ R

(B) (3, 8) ∈ R

(C) (6, 8) ∈ R

(D) (8, 7) ∈ R

Answer

The correct option will C.

Option A is not possible, since b should be greater than 6.

Option B is not possible, since 3 = 8 - 2 is not possible.

Option C is possible , since 6 = 8 - 2

Option D is not possible, since 8 = 7 − 2 is not possible.

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Exercise 1.2 (NCERT 12th Class Maths Chapter 1)

1. Show that the function $f : R* \to R*$ defined by $f(x) = 1/x$ is one-one and onto, where R∗ is the set of all non-zero real numbers. Is the result true, if the domain R∗ is replaced by N with co-domain being same as R∗?

Answer

It is given that f: $R_* \rightarrow R_*$ is defined by $f(x) = \frac{1}{x}$

For one – one:

Let $x, y \in R_*$ such that $f(x) = f(y)$

$\Rightarrow \frac{1}{x} = \frac{1}{y}$

$\Rightarrow x = y$

$\therefore f$ is one – one.

For onto:

It is clear that for $y \in R_*$, there exists $x = \frac{1}{y} \in R_*$ [as $y \neq 0$] such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$$

$\therefore f$ is onto.

Thus, the given function f is one – one and onto.

Now, consider function g: $\mathbf{N} \rightarrow R_*$ defined by $g(x) = \frac{1}{x}$

We have,

$g(x_1) = g(x_2) \quad \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \quad \Rightarrow x_1 = x_2$

$\therefore g$ is one – one.

Further, it is clear that g is not onto as for $1.2 \in R_*$ there does not exit any x in $\mathbf{N}$ such that $g(x) = \frac{1}{1.2}$.

Hence, function g is one-one but not onto.

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2. Check the injectivity and surjectivity of the following functions:

(i) f : N → N given by f(x) = x^2

(ii) f : Z → Z given by f(x) = x^2

(iii) $f : R \rightarrow R$ given by $f(x) = x^2$

(iv) $f : N \rightarrow N$ given by $f(x) = x^3$

(v) $f : Z \rightarrow Z$ given by $f(x) = x^3$

Answer

(i) $f: \mathbf{N} \rightarrow \mathbf{N}$ is given by $f(x) = x^2$

It is seen that for $x, y \in \mathbf{N}$, $f(x) = f(y) \Rightarrow x^2 = y^2 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{N}$. But, there does not exist any x in $\mathbf{N}$ such that $f(x) = x^2 = 2$.

$\therefore f$ is not surjective.

Hence, function f is injective but not surjective.

(ii) $f: \mathbf{Z} \rightarrow \mathbf{Z}$ is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore f$ is not injective.

Now, $-2 \in \mathbf{Z}$. But, there does not exist any element $x \in \mathbf{Z}$ such that

$f(x) = -2 \quad \text{or} \quad x^2 = -2$.

$\therefore f$ is not surjective.

Hence, function f is neither injective nor surjective.

(iii) $f: \mathbf{R} \rightarrow \mathbf{R}$ is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore f$ is not injective.

Now, $-2 \in \mathbf{R}$. But, there does not exist any element $x \in \mathbf{R}$ such that

$f(x) = -2 \quad \text{or} \quad x^2 = -2$.

$\therefore f$ is not surjective.

Hence, function f is neither injective nor surjective.

(iv) $f: \mathbf{N} \rightarrow \mathbf{N}$ given by $f(x) = x^3$

It is seen that for $x, y \in \mathbf{N}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{N}$. But, there does not exist any element $x \in \mathbf{N}$ such that

$f(x) = 2 \quad \text{or} \quad x^3 = 2$.

$\therefore f$ is not surjective

Hence, function f is injective but not surjective.

(v) $f: \mathbf{Z} \rightarrow \mathbf{Z}$ is given by $f(x) = x^3$

It is seen that for $x, y \in \mathbf{Z}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{Z}$. But, there does not exist any element $x \in \mathbf{Z}$ such that

$f(x) = 2 \quad \text{or} \quad x^3 = 2$.

3. Prove that the Greatest Integer Function f : R → R, given by f(x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

Answer

f : R → R is given by, f(x) = [x]

It is seen that $f(1.2) = [1.2] = 1$

$f(1.9)= [1.9] = 1$

∴ $f(1.2) = f(1.9)$, but $1.2 \neq 1.9$

∴ f is not one-one.

Now, consider $0.7 \in$ R.

It is known that $f(x) = [x]$ is always an integer.

Thus, there does not exist any element $x \in$ R such that $f(x) = 0.7$.

∴ f is not onto.

Hence, the greatest integer function is neither one-one nor onto.

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4. Show that the Modulus Function f : R → R, given by f(x) = |x|, is neither one-ne nor onto, where |x| is x, if x is positive or 0 and |x| is – x, if x is negative.

Answer

$$f: \mathbf{R} \to \mathbf{R} \text{ is given by } f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

It is clear that $f(-1) = |-1| = 1$

and $f(1) = |1| = 1$

∴ $f(-1) = f(1)$, but $-1 \neq 1$

Now, consider -1 ∈ R.

∴ *f* is not one-one.

It is known that $f(x) = |x|$ is always non-negative.

Thus, there does not exist any element x in domain R such that $f(x) = |x| = -1$.

∴ *f* is not onto.

Hence, the modulus function is neither one-one nor onto.

5. Show that the Signum Function *f* : R → R, given by

$$f(x)=\begin{cases}1, \text{if } x>0\\ 0, \text{if } x=0\\ -1, \text{if } x<0\end{cases}$$

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Answer

$$f: \mathbf{R} \rightarrow \mathbf{R} \text{ is given by } f(x)=\begin{cases}1, & \text{if } x>0\\ 0, & \text{if } x=0\\ -1, & \text{if } x<0\end{cases}$$

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It is seen that $f(1) = f(2) = 1$, but $1 \neq 2$.

∴ *f* is not one-one.

Now, as $f(x)$ takes only 3 values (1, 0, or -1) for the element -2 in co-domain R, there does not exist any x in domain R such that $f(x) = -2$.

∴ *f* is not onto.

Hence, the Signum function is neither one-one nor onto.

6. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let *f* = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

Answer

It is given that A= (1, 2, 3), B= (4, 5, 6, 7).

A → B is defined as *f* = {(1, 4), (2, 5), (3, 6)}

f(1)= 4,

f(2) = 5,

f(3) = 6

It is seen that the images of distinct elements of A under *f* are distinct. Hence, function *f* is one-one.

7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) *f* : R → R defined by *f*(x) = 3 - 4x

(ii) *f* : R → R defined by *f*(x) = $1+x^2$

Answer

(i) *f* : R → 12 is defined as *f*(x) = 3 - 4x.

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 3-4x_1 = 3-4x_2$

$\Rightarrow -4x_1 = -4x_2$

$\Rightarrow x_1 = x_2$

∴ f is one-one.

For any real number (y) in **R**, there exists $\frac{3-y}{4}$ in **R** such that

$$f\left(\frac{3-y}{4}\right) = 3-4\left(\frac{3-y}{4}\right) = y$$

∴ f is onto.

Hence, f is bijective.

(ii) f: R → R is defined as f(x) = 1 + x2

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

⇒ $1+ x_1^2 = 1 + x_2^2$

⇒ $x_1^2 = x_2^2$

⇒ $x_1 = \pm x_2$

∴ $f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$

8. Let A and B be sets. Show that f : A×B → B×A such that f(a, b) = (b, a) is bijective function.

Answer

f: A × B → B × A is defined as f(a, b) = (b, a).

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that $f(a_1, b_1) = f(a_2, b_2)$

⇒ $(b_1, a_1) = (b_2, a_2)$

⇒ $b_1 = b_2$ and $a_1 = a_2$

⇒ $(a_1, b_1) = (a_2, b_2)$

∴ f is one – one.

Now, let (b, a) ∈ B × A be any element.

Then, there exists (a, b) ∈ A × B such that f(a, b) = (b, a). [By definition of f]

∴ f is onto.

Hence, f is bijective.

9.

Let $f : \mathbf{N} \rightarrow \mathbf{N}$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2} \quad, \text{ if } n \text{ is even} \end{cases}$ for all $n \in \mathbf{N}$.

State whether the function f is bijective. Justify your answer.

Answer

$f\!: \mathbf{N} \rightarrow \mathbf{N}$ is defined as $f(n) = \begin{cases} \frac{n+1}{2}, \text{if n is odd} \\ \frac{n}{2}, \ \text{if n is even} \end{cases}$ for all $n \in \mathbf{N}$

It can be observed that:

$f(1) = \frac{1+1}{2} = 1$ and $f(2) = \frac{2}{2} = 1$ [By definition of f(n)]

$f(1) = f(2)$, where $1 \neq 2$

$\therefore f$ is not one-one.

Consider a natural number (n) in co-domain $\mathbf{N}$.

Case **I:** n is odd

$\therefore n = 2r + 1$ for some $r \in \mathbf{N}$. Then, there exists $4r + 1 \in \mathbf{N}$ such that

$$f(4r+1) = \frac{4r+1+1}{2} = 2r+1$$

Case **II:** n is even

$\therefore n = 2r$ for some $r \in \mathbf{N}$. Then, there exists $4r \in \mathbf{N}$ such that

$f(4r) = \frac{4r}{2} = 2r$.

$\therefore f$ is onto.

Hence, f is not a bijective function.

10. Let A = R – {3} and B = R – {1}. Consider the function f : A → B defined by f(x) = (x-2)/(x-3). Is f one-one and onto? Justify your answer.

Answer

$A = \mathbf{R} - \{3\}$, $B = \mathbf{R} - \{1\}$ and $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x, y \in A$ such that $f(x) = f(y)$

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$

$\Rightarrow xy - 3x - 2y + 6 = xy - 2x - 3y + 6$

$\Rightarrow -3x - 2y = -2x - 3y \quad \Rightarrow x = y$

$\therefore f$ is one-one.

Let $y \in B = \mathbf{R} - \{1\}$. Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now, $f(x) = y$

$\Rightarrow \frac{x-2}{x-3} = y$

$\Rightarrow x - 2 = xy - 3y \quad \Rightarrow x(1-y) = -3y + 2$

$\Rightarrow x = \frac{2-3y}{1-y} \in A \qquad [y \neq 1]$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$

$\therefore f$ is onto.

Hence, function f is one – one and onto.

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11. Let f : R → R be defined as f(x) = x^4. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto.

Answer

f: R → R is defined as $f(x) = x^4$

Let x, y ∈ R such that f(x) = f(y).

⇒ $x^4 = y^4$

⇒ x = ± y

∴ f(x) = f(y) does not imply that x = y.

For example f(1) = f(–1) = 1

∴ f is not one-one.

Consider an element 2 in co-domain R. It is clear that there does not exist any x in domain R such that f(x) = 2.

∴ f is not onto.

Hence, function f is neither one – one nor onto.

The correct answer is D.

12. Let f : R → R be defined as f(x) = 3x. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto.

Answer

f: R → R is defined as f(x) = 3x.

Let x, y ∈ R such that f(x) = f(y).

⇒ 3x = 3y

⇒ x = y

∴f is one-one.

Also, for any real number (y) in co-domain R, there exists y/3 in R such that f(y/3) = 3(y/3) = y

∴ f is onto.

Hence, function f is one – one and onto.

The correct answer is A.

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Exercise 1.3 (NCERT 12th Class Maths Chapter 1)

1. Let f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof

Answer

The functions f: {1, 3, 4} → {1, 2, 5} and g: {1, 2, 5} → {1, 3} are defined as f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}.

gof(1) = g[f(1)] = g(2) = 3 [as f(1) = 2 and g(2) = 3]

gof(3) = g[f(3)] = g(5) = 1 [as f(3) = 5 and g(5) = 1]

gof(4) = g[f(4)] = g(1) = 3 [as f(4) = 1 and g(1) = 3]

∴ gof = {(1, 3), (3, 1), (4, 3)}

2. Let f, g and h be functions from R to R. Show that

(f + g) oh = foh + goh

(f . g) oh = (foh) . (goh)

Answer

To prove: (f + g)oh = foh + goh

LHS = [(f + g)oh](x)

= (f + g)[h(x)]

= f [h(x)] + g[h(x)]

= (foh)(x) + (goh)(x)

= {(foh)(x) + (goh)}(x) = RHS

∴ {(f + g)oh}(x) = {(foh)(x) + (goh)}(x) for all x ∈R

Hence, (f + g)oh = foh + goh

To Prove: (f.g)oh = (foh).(goh)

LHS = [(f.g)oh](x)

= (f.g)[h(x)]

= f[h(x)] . g[h(x)]

= (foh)(x) . (goh)(x)

= {(foh).(goh)}(x) = RHS

∴ [(f.g)oh](x) = {(foh).(goh)}(x) for all x ∈R

Hence, (f.g)oh = (foh).(goh)

3. Find gof and fog, if

(i) f(x) = | x | and g(x) = | 5x – 2 |

(ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$.

Answer

(i). $f(x) = |x|$ and $g(x) = |5x-2|$

$\therefore gof(x) = g(f(x)) = g(|x|) = |5|x|-2|$

$fog(x) = f(g(x)) = f(|5x-2|) = ||5x-2|| = |5x-2|$

(ii). $f(x) = 8x^3$ and $g(x) = x^{1/3}$

$\therefore gof(x) = g(f(x)) = g(8x^3) = (8x^3)^{1/3} = 2x$

$fog(x) = f(g(x)) = f(x^{1/3}) = 8(x^{1/3})^3 = 8x$

4. If f(x) = (4x+3)/(6x-4), x ≠ 2/3, show that fof(x) = x, for all x ≠ 2/3. What is the inverse of f ?

Answer

It is given that $f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$

$$(fof)(x) = f(f(x)) = f\left(\frac{4x+3}{6x-4}\right) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34}$$

$$= x$$

$\therefore fof(x) = x$, for all $x \neq \frac{2}{3}$.

$\Rightarrow fof = I_x$

Hence, the given function f is invertible and the inverse of f is f itself.

5. State with reason whether following functions have inverse

(i) f : {1, 2, 3, 4} → {10} with

f = {(1, 10), (2, 10), (3, 10), (4, 10)}

(ii) g : {5, 6, 7, 8} → {1, 2, 3, 4} with

$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii) $h : \{2, 3, 4, 5\} \to \{7, 9, 11, 13\}$ with

$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

Answer

(i) $f: \{1, 2, 3, 4\} \to \{10\}$ defined as $f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

From the given definition of f, we can see that f is a many one function as $f(1) = f(2) = f(3) = f(4) = 10$

∴f is not one – one.

Hence, function f does not have an inverse.

(ii) $g: \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ defined as $g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

From the given definition of g, it is seen that g is a many one function as $g(5) = g(7) = 4$.

∴ g is not one – one.

Hence, function g does not have an inverse.

(iii) $h: \{2, 3, 4, 5\} \to \{7, 9, 11, 13\}$ defined as $h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

It is seen that all distinct elements of the set {2, 3, 4, 5} have distinct images under h.

∴ Function h is one – one.

Also, h is onto since for every element y of the set {7, 9, 11, 13}, there exists an element x in the set {2, 3, 4, 5}, such that $h(x) = y$.

Thus, h is a one – one and onto function.

Hence, h has an inverse.

6. Show that $f : [-1, 1] \to R$, given by $f(x) = x/(x+2)$ is one-one. Find the inverse of the function $f : [-1, 1] \to$ Range f.

[Hint: For $y \in$ Range f, $y = f(x) = x/(x+2)$, for some x in $[-1, 1]$, i.e., $x = 2y/(2y-1)$]

Answer

f : [–1, 1] → R, given by f (x) = x/(x+2)

For one-one,

Let $f(x) = f(y)$

$\Rightarrow \frac{x}{x+2} = \frac{y}{y+2}$

$\Rightarrow xy + 2x = xy + 2y$

$\Rightarrow 2x = 2y$

$\Rightarrow x = y$

$\therefore f$ is a one – one function.

It is clear that f: $[-1, 1] \to$ Range f is onto.

$\therefore f$: $[-1, 1] \to$ Range f is one – one and onto and therefore, the inverse of the function f: $[-1, 1] \to$ Range f exists.

Let g: Range $f \to [-1, 1]$ be the inverse of f.

Let y be an arbitrary element of range f.

Since f: $[-1, 1] \to$ Range f is onto, we have

$y = f(x)$ for some $x \in [-1, 1]$

$\Rightarrow y = \frac{x}{x+2}$

$\Rightarrow xy + 2y = x$

$\Rightarrow x(1-y) = 2y$

$\Rightarrow x = \frac{2y}{1-y}, \quad y \neq 1$

Now, let us define g: Range $f \to [-1, 1]$ as

$g(y) = \frac{2y}{1-y}, \quad y \neq 1$

Now,

$$(gof)(x) = g(f(x)) = g\left(\frac{x}{x+2}\right) = \frac{2\left(\frac{x}{x+2}\right)}{1-\left(\frac{x}{x+2}\right)} = \frac{2x}{x+2-x} = \frac{2x}{2} = x$$

and

$$(fog)(y) = f(g(y)) = f\left(\frac{2y}{1-y}\right) = \frac{\frac{2y}{1-y}}{\frac{2y}{1-y}+2} = \frac{2y}{2y+2-2y} = \frac{2y}{2} = y$$

$\therefore gof = x = I_{[-1,1]}$ and $fog = y = I_{\text{Range } f}$

$\therefore f^{-1} = g$

$\Rightarrow f^{-1}(y) = \frac{2y}{1-y}, \quad y \neq 1$

7. Consider f : R → R given by f(x) = 4x + 3. Show that f is invertible. Find the inverse of f.

Answer

f: R → R is given by, f(x) = 4x + 3

For one – one

Let f(x) = f(y)

⇒ 4x + 3 = 4y + 3

⇒ 4x = 4y

⇒ x = y

∴ f is a one – one function.

For onto

For y ∈ R, let y = 4x + 3.

⇒ x = y-3/4 ∈ R

Therefore, for any y ∈ R, there exists x = y-3/4 ∈ R, such that

$f(x) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y.$

$\therefore f$ is onto.

Thus, f is one – one and onto and therefore, f^{-1} exists.

Let us define g: $\mathbf{R} \to \mathbf{R}$ by $g(x) = \frac{y-3}{4}$

Now,

$$(gof)(x) = g(f(x)) = g(4x+3) = \frac{(4x+3)-3}{4} = \frac{4x}{4} = x$$

and

$$(fog)(y) = f(g(y)) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y-3+3 = y$$

$\therefore gof = fog = I_R$

Hence, f is invertible and the inverse of f is given by $f^{-1}(y) = g(y) = \frac{y-3}{4}$. NCERT Solutions for 12th Class Maths: Chapter 1-Relations and Functions

8. Consider f : R+ → [4, ∞) given by f(x) = x^2 + 4. Show that f is invertible with the inverse f^{-1} of f given by f^{-1}(y) = √(y-4) , where R+ is the set of all non-negative real numbers.

Answer

f: R+ → [4, ∞) is given as f(x) = $x^2 + 4$.

For one – one

Let f(x) = f(y)

⇒ $x^2 + 4 = y^2 + 4$

⇒ $x^2 = y^2$

⇒ x = y [as x = y ∈ R+]

∴ f is a one–one function.

For onto

For $y \in [4, \infty)$, let $y = x^2 + 4$

$\Rightarrow x^2 = y - 4 \geq 0$ [as $y \geq 4$]

$\Rightarrow x = \sqrt{y-4} \geq 0$

Therefore, for any $y \in [4, \infty)$, there exists $x = \sqrt{y-4} \in R_+$, such that

$f(x) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = y-4+4 = y$

$\therefore f$ is onto.

Thus, f is one – one and onto and therefore, f^{-1} exists.

Let us define g: $[4, \infty) \rightarrow R_+$ by $g(y) = \sqrt{y-4}$

Now,

$$(gof)(x) = g(f(x)) = g(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$$

and

$$(fog)(y) = f(g(y)) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = y-4+4 = y$$

$\therefore gof = fog = I_R$

Hence, f is invertible and the inverse of f is given by $f^{-1}(y) = g(y) = \sqrt{y-4}$ NCERT Solutions for 12th Class Maths: Chapter 1-Relations and Functions

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9. Consider f : R+ → [– 5, ∞) given by f (x) = 9x^2 + 6x - 5. Show that f is invertible with

$$f^{-1}(y) = \left(\frac{(\sqrt{y+6})-1}{3}\right)$$ NCERT Solutions for 12th Class Maths: Chapter 1-Relations and Functions que 9

Answer

f: R+ → [−5, ∞) is given as f(x) = 9x^2 + 6x − 5.

Let y be an arbitrary element of [−5, ∞).

Let y = 9x^2 + 6x – 5

⇒ y = (3x+1)^2 -1 - 5 = (3x+1)^2 -6

⇒ y + 6 = (3x + 1)^2

⇒ 3x + 1 = √(y + 6)

$\Rightarrow x = \frac{(\sqrt{y+6})-1}{3}$

$\therefore f$ is onto, thereby range $f = [-5, \infty)$.

Let us define g: $[-5, \infty) \rightarrow R_+$ as $g(y) = \frac{(\sqrt{y+6})-1}{3}$

Now,

$$(gof)(x) = g(f(x)) = g(9x^2 + 6x\text{-}5) = g((3x+1)^2\text{-}6)$$
$$= \sqrt{(3x+1)^2\text{-}6+6}\text{-}1$$
$$= \frac{3x+1\text{-}1}{3} = \frac{3x}{3} = x$$

and

$$(fog)(y) = f(g(y)) = f\left(\frac{\sqrt{y+6}\text{-}1}{3}\right) = \left[3\left(\frac{\sqrt{y+6}\text{-}1}{3}\right)+1\right]^2 \text{-}6$$
$$= \left(\sqrt{y+6}\right)^2\text{-}6 = y + 6\text{-}6 = y$$

$\therefore gof = x = I_R$ and $fog = y = I_{\text{Range } f}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \left(\frac{(\sqrt{y+6})-1}{3}\right)$$

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10. Let f : X → Y be an invertible function. Show that f has unique inverse.

(Hint: suppose g_1 and g_2 are two inverses of f. Then for all y ∈ Y, $fog_1(y) = I_Y(y) = fog_2(y)$. Use one-one ness of f).

Answer

Let f: X → Y be an invertible function.

Also, suppose f has two inverses (say g_1 and g_2)

Then, for all y∈Y, we have

$fog_1(y) = I_Y(y) = fog_2(y)$

$\Rightarrow f(g_1(y)) = f(g_2(y))$

$\Rightarrow g_1(y) = g_2(y)$ [as f is invertible ⇒ f is one–one]

⇒ $g_1 = g_2$ [as g is one–one]

Hence, f has a unique inverse.

11. Consider f:{1, 2, 3} → {a, b, c} given by f(1) = a,f(2) = b and f(3) = c. Find f^{-1} and show that $(f^{-1})^{-1} = f$.

Answer

Function f: {1, 2, 3} → {a, b, c} is given by f(1) = a, f(2) = b, and f(3) = c

If we define g: {a, b, c} → {1, 2, 3} as g(a) = 1, g(b) = 2, g(c) = 3.

We have,

(fog)(a) = f(g(a)) = f(1) = a

(fog)(b) = f(g(b)) = f(2) = b

(fog)(c) = f(g(c)) = f(3) = c

and

(gof)(1) = g(f(1)) = f(a) = 1

(gof)(2) = g(f(2)) = f(b) = 2

(gof)(3) = g(f(3)) = f(c) = 3

∴ gof = I_X and fog = I_Y, where X = {1, 2, 3} and Y= {a, b, c}.

Thus, the inverse of f exists and f^{-1} = g.

∴ f^{-1}:{a, b, c} → {1, 2, 3} is given by $f^{-1}(a) = 1$, $f^{-1}(b) = 2$, $f^{-1}(c) = 3$

Let us now find the inverse of f^{-1} i.e., find the inverse of g.

If we define h: {1, 2, 3} → {a, b, c} as h(1) = a, h(2) = b, h(3) = c

We have

(goh)(1) = g(h(1)) = g(a) = 1

$(goh)(2) = g(h(2)) = g(b) = 2$

$(goh)(3) = g(h(3)) = g(c) = 3$

and

$(hog)(a) = h(g(a)) = h(1) = a$

$(hog)(b) = h(g(b)) = h(2) = b$

$(hog)(c) = h(g(c)) = h(3) = c$

∴ $goh = I_X$ and $hog = I_Y$, where $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$.

Thus, the inverse of g exists and $g^{-1} = h \Rightarrow (f^{-1})^{-1} = h$.

It can be noted that $h = f$.

Hence, $(f^{-1})^{-1} = f$.

12. Let f: X → Y be an invertible function. Show that the inverse of f^{-1} is f, i.e., $(f^{-1})^{-1}$ = f

Answer

Let f: X → Y be an invertible function.

Then, there exists a function g: Y → X such that $gof = I_X$ and $fog = I_Y$.

Here, $f^{-1} = g$.

Now, $gof = I_X$ and $fog = I_Y$

$\Rightarrow f^{-1}of = I_X$ and $fof^{-1} = I_Y$

Hence, f^{-1}: Y → X is invertible and f is the inverse of f^{-1}

i.e., $(f^{-1})^{-1} = f$.

13. 13. If f: R → R be given by $f(x) = ((3 - x^3))^{1/3}$, then fof(x) is

(a) $x)^{1/3}$

(b) x^3

(c) x

(d) $(3 - x^3)$

Answer

f: $\mathbf{R} \rightarrow \mathbf{R}$ be given as $f(x) = (3-x^3)^{\frac{1}{3}}$

$$\therefore fof(x) = f(f(x)) = f\left((3-x^3)^{\frac{1}{3}}\right) = \left[3-\left((3-x^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}}$$

$$= [3-(3-x^3)]^{\frac{1}{3}} = (x^3)^{\frac{1}{3}} = x$$

$\therefore fof(x) = x$ NCERT Solutions for 12th Class Maths: Chapter 1-Relations and Functions que 13

The correct option is C.

14. Let $f: \mathbf{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbf{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. The inverse of f is the map g : Range $f \rightarrow \mathbf{R} - \left\{-\frac{4}{3}\right\}$ given by

(A) $g(y) = \frac{3y}{3-4y}$ (B) $g(y) = \frac{4y}{4-3y}$

(C) $g(y) = \frac{4y}{3-4y}$ (D) $g(y) = \frac{3y}{4-3y}$ NCERT Solutions for 12th Class Maths: Chapter 1-Relations and Functions que 13

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Exercise 1.4

1. Determine whether or not each of the definition of ∗ given below gives a binary operation. In the event that ∗ is not a binary operation, give justification for this.

(i) On Z^+, define ∗ by a ∗ b = a – b

(ii) On Z^+, define ∗ by a ∗ b = ab

(iii) On R, define ∗ by a ∗ b = ab^2

(iv) On Z$^+$, define ∗ by a ∗ b = |a – b|

(v) On Z$^+$, define ∗ by a ∗ b = a

Answer

(i) On Z+, * is defined by a * b = a − b.

It is not a binary operation as the image of (1, 2) under * is 1 * 2 = 1 − 2 = −1 ∉ Z+

(ii) On Z+, * is defined by a * b = ab.

It is seen that for each a, b ∈ Z+, there is a unique element ab in Z+.

This means that * carries each pair (a, b) to a unique element a * b = ab in Z+.

Therefore, * is a binary operation.

(iii) On R, * is defined by a * b = ab^2.

It is seen that for each a, b ∈ R, there is a unique element ab^2 in R.

This means that * carries each pair (a, b) to a unique element a * b = ab^2 in R.

Therefore, * is a binary operation.

(iv) On Z+,* is defined by a * b = |a − b|.

It is seen that for each a, b ∈ Z+, there is a unique element |a − b| in Z+.

This means that * carries each pair (a, b) to a unique element a * b = |a − b| in Z+.

Therefore,* is a binary operation.

(v) On Z+,* is defined by a * b = a.

It is seen that for each a, b ∈ Z+, there is a unique element a in Z+.

This means that * carries each pair (a, b) to a unique element a * b = a in Z+.

Therefore, * is a binary operation.

2. For each operation ∗ defined below, determine whether ∗ is binary, commutative or associative.

(i) On Z, define a ∗ b = a – b

(ii) On Q, define a ∗ b = ab + 1

(iii) On Q, define a ∗ b = ab/2

(iv) On Z+ , define a ∗ b = 2^{ab}

(v) On Z + , define a ∗ b = a^b

(vi) On R –{– 1}, define a ∗ b = a/b+1

Answer

(i) On Z, * is defined by a * b = a − b.

It can be observed that 1 * 2 = 1 − 2 = −1 and 2 * 1 = 2 − 1 = 1.

∴1 * 2 ≠ 2 * 1, where 1, 2 ∈ Z

Hence, the operation * is not commut

Also, we have

(1 * 2) * 3 = (1 − 2) * 3 = −1 * 3 = −1 − 3 = −4

1 * (2 * 3) = 1 * (2 − 3) = 1 * −1 = 1 − (−1) = 2

∴ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1, 2, 3 ∈ Z

Hence, the operation * is not associative.

(ii) On Q, * is defined by a * b = ab + 1.

It is known that: ab = ba for all a, b ∈ Q

⇒ ab + 1 = ba + 1 for all a, b ∈ Q

⇒ a * b = a * b for all a, b ∈ Q

Therefore, the operation * is commutative.

It can be observed that

(1 * 2) * 3 = (1 × 2 + 1) * 3 = 3 * 3 = 3 × 3 + 1 = 10

1 * (2 * 3) = 1 * (2 × 3 + 1) = 1 * 7 = 1 × 7 + 1 = 8

∴ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1, 2, 3 ∈ Q

Therefore, the operation * is not associative.

(iii) On Q, * is defined by a * b =ab/2

It is known that: ab = ba for all a, b ∈ Q

⇒ab/2 = ba/2

for all a, b ∈ Q

⇒ a * b = b * a for all a, b ∈ Q

Therefore, the operation * is commutative.

For all a, b, c ∈ Q, we have

$$a * (b * c) = a * \left(\frac{bc}{2}\right) = \frac{a\left(\frac{bc}{2}\right)}{2} = \frac{abc}{4}$$

and

$$(a * b) * c = \left(\frac{ab}{2}\right) * c = \frac{\left(\frac{ab}{2}\right)c}{2} = \frac{abc}{4}$$

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∴ (a*b)*c = a*(b*c), where a, b, c ∈ Q

Therefore, the operation * is associative.

(iv) On Z+, * is defined by a * b = 2^{ab}.

It is known that: ab = ba for all a, b ∈ Z+

$\Rightarrow 2^{ab} = 2^{ba}$ for all a, b $\in$ Z+

$\Rightarrow$ a * b = b * a for all a, b $\in$ Z+

Therefore, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = 2^{1\times2} * 3 = 4 * 3 = 2^{4\times3} = 2^{12}$ and

$1 * (2 * 3) = 1 * 2^{2\times3} = 1 * 2^6 = 1 * 64 = 2^{1\times64} = 2^{64}$

∴ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1, 2, 3 $\in$ Z+

Therefore, the operation * is not associative.

(v) On Z+, * is defined by a * b = a^b.

It can be observed that

$1*2 = 1^2 = 1$ and $2*1 = 2^1 = 2$

∴ 1 * 2 ≠ 2 * 1, where 1, 2 $\in$ Z+

Therefore, the operation * is not commutative.

It can also be observed that

$(2 * 3) * 4 = 2^3 * 4 = 8 * 4 = 8^4 = 2^{12}$ and

$2 * (3 * 4) = 2 * 3^4 = 2 * 81 = 2^{81}$

∴ (2 * 3) * 4 ≠ 2 * (3 * 4), where 2, 3, 4 $\in$ Z+

Therefore, the operation * is not associative.

(vi) On R, * −{−1} is defined by a ∗ b = a/b+1

It can be observed that

1 ∗ 2 = 1/2+1 = 1/3 and 2 ∗ 1 = 2/1+1 = 2/2 = 1

∴1 * 2 ≠ 2*1, where 1, 2 $\in$ R − {−1}

Therefore, the operation * is not commutative.

It can also be observed that

$$(1*2)*3=\frac{1}{2+1}*3=\frac{1}{3}*3=\frac{\frac{1}{3}}{3+1}=\frac{1}{12}$$

and

$$1*(2*3)=1*\frac{2}{3+1}=1*\frac{2}{4}=1*\frac{1}{2}=\frac{1}{\frac{1}{2}+1}=\frac{1}{\frac{3}{2}}=\frac{2}{3}$$

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∴ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1, 2, 3 ∈ R −{−1}

Therefore, the operation * is not associative.

3. Consider the binary operation ∧ on the set {1, 2, 3, 4, 5} defined by a ∧ b = min {a, b}. Write the operation table of the operation ∧.

Answer

The binary operation ∧ on the set {1, 2, 3, 4, 5} is defined as a ∧ b = min {a, b} for all a, b ∈ {1, 2, 3, 4, 5}.

Thus, the operation table for the given operation ∧ can be given as:

∧	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

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4. Consider a binary operation ∗ on the set {1, 2, 3, 4, 5} given by the following multiplication table (Table 1.2).

(i) Compute (2 ∗ 3) ∗ 4 and 2 ∗ (3 ∗ 4)

(ii) Is ∗ commutative?

(iii) Compute (2 ∗ 3) ∗ (4 ∗ 5). (Hint: use the following table)

Table 1.2

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

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Answer

(i) (2 * 3) * 4 = 1 * 4 = 1

2 * (3 * 4) = 2 * 1 = 1

(ii) For every a, b ∈ {1, 2, 3, 4, 5}, we have a * b = b * a.

Therefore, the operation * is commutative.

(iii) (2 * 3) = 1 and (4 * 5) = 1

∴ (2 * 3) * (4 * 5) = 1 * 1 = 1

5. Let ∗′ be the binary operation on the set {1, 2, 3, 4, 5} defined by a ∗′ b = H.C.F. of a and b. Is the operation ∗′ same as the operation ∗ defined in Exercise 4 above? Justify your answer.

Answer

The binary operation *′ on the set {1, 2, 3 4, 5} is defined as a *′ b = H.C.F of a and b.

The operation table for the operation *′ can be given as:

*′	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

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We observe that the operation tables for the operations * and *′ are the same.

Thus, the operation *′ is same as the operation*.

6. Let ∗ be the binary operation on N given by a ∗ b = L.C.M. of a and b. Find

(i) 5 ∗ 7, 20 ∗ 16

(ii) Is ∗ commutative?

(iii) Is ∗ associative?

(iv) Find the identity of ∗ in N

(v) Which elements of N are invertible for the operation ∗?

Answer

The binary operation * on N is defined as a * b = L.C.M. of a and b.

(i) 5 * 7 = L.C.M. of 5 and 7 = 35

20 * 16 = L.C.M of 20 and 16 = 80

(ii) It is known that

L.C.M of a and b = L.C.M of b and a for all a, b ∈ N.

∴ a * b = b * a

Thus, the operation * is commutative.

(iii) For a, b, c ∈ N, we have

(a * b) * c = (L.C.M of a and b) * c = LCM of a, b, and c

a * (b * c) = a * (LCM of b and c) = L.C.M of a, b, and c

∴ (a * b) * c = a * (b * c)

Thus, the operation * is associative.

(iv) It is known that:

L.C.M. of a and 1 = a = L.C.M. 1 and a for all a ∈ N

⇒ a * 1 = a = 1 * a for all a ∈ N

Thus, 1 is the identity of * in N.

(v) An element a in N is invertible with respect to the operation * if there exists

an element b in N, such that a * b = e = b * a.

Here, e = 1

This means that

L.C.M of a and b = 1 = L.C.M of b and a

This case is possible only when a and b are equal to 1.

Thus, 1 is the only invertible element of N with respect to the operation *.

7. Is ∗ defined on the set {1, 2, 3, 4, 5} by a ∗ b = L.C.M. of a and b a binary operation? Justify your answer.

Answer

The operation * on the set A = {1, 2, 3, 4, 5} is defined as a * b = L.C.M. of a and b.

Then, the operation table for the given operation * can be given as:

*	1	2	3	4	5
1	1	2	3	4	5
2	2	2	6	4	10
3	3	6	3	12	15
4	4	4	12	4	20
5	5	10	15	20	5

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It can be observed from the obtained table that

$3 * 2 = 2 * 3 = 6 \notin A$,

$5 * 2 = 2 * 5 = 10 \notin A$,

$3 * 4 = 4 * 3 = 12 \notin A$,

$3 * 5 = 5 * 3 = 15 \notin A$,

$4 * 5 = 5 * 4 = 20 \notin A$

Hence, the given operation * is not a binary operation.

8. Let ∗ be the binary operation on N defined by a ∗ b = H.C.F. of a and b.

Is ∗ commutative? Is ∗ associative? Does there exist identity for this binary operation on N?

Answer

The binary operation * on N is defined as: a * b = H.C.F. of a and b

It is known that

H.C.F. of a and b = H.C.F. of b and a for all a, b ∈ N.

∴ a * b = b * a

Thus, the operation * is commutative.

For a, b, c ∈ N, we have

(a * b)* c = (H.C.F. of a and b) * c = H.C.F. of a, b and c

a *(b * c) = a *(H.C.F. of b and c) = H.C.F. of a, b, and c

∴ (a * b) * c = a * (b * c)

Thus, the operation * is associative.

Now, an element e ∈ N will be the identity for the operation * if a * e = a = e* a for all a ∈ N.

But this relation is not true for any a ∈ N.

Thus, the operation * does not have any identity in N.

9. Let ∗ be a binary operation on the set Q of rational numbers as follows:

(i) a ∗ b = a – b

(ii) a ∗ b = $a^2 + b^2$

(iii) a ∗ b = a + ab

(iv) a ∗ b = $(a – b)^2$

(v) a ∗ b = ab/4

(vi) a ∗ b = ab^2

Answer

(i) On Q, the operation * is defined as a * b = a − b. It can be observed that:

$$\frac{1}{2}*\frac{1}{3}=\frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}$$

and

$$\frac{1}{3}*\frac{1}{2}=\frac{1}{3}-\frac{1}{2}=\frac{2-3}{6}=\frac{-1}{6}$$

$\therefore \frac{1}{2}*\frac{1}{3}\neq\frac{1}{3}*\frac{1}{2}, where\ \frac{1}{2},\frac{1}{3}\in \boldsymbol{Q}$

Thus, the operation * is not commutative.

It can also be observed that

$$\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}=\left(\frac{1}{2}-\frac{1}{3}\right)*\frac{1}{4}=\left(\frac{3-2}{6}\right)*\frac{1}{4}=\frac{1}{6}*\frac{1}{4}=\frac{1}{6}-\frac{1}{4}=\frac{2-3}{12}=\frac{-1}{12}$$

and

$$\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)=\frac{1}{2}*\left(\frac{1}{3}-\frac{1}{4}\right)=\frac{1}{2}*\left(\frac{4-3}{12}\right)=\frac{1}{2}*\frac{1}{12}=\frac{1}{2}-\frac{1}{12}=\frac{6-1}{12}=\frac{5}{12}$$

$\therefore \left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}\neq\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right), where\ \frac{1}{2},\frac{1}{3},\frac{1}{4}\in \boldsymbol{Q}$

Thus, the operation * is not associative.

(ii) On Q, the operation * is defined as a * b = $a^2 + b^2$.

For a, b ∈ Q, we have

a * b = $a^2 + b^2 = b^2 + a^2$ = b * a

∴ a * b = b * a

Thus, the operation * is commutative.

It can be observed that

(1 * 2) * 3 = $(1^2 + 2^2)$ * 3 = (1 + 4) * 3 = 5 * 3 = $5^2 + 3^2 = 3^4$ and

1 * (2 * 3) = 1 * $(2^2 + 3^2)$ = 1 * (4 + 9) = 1 * 13 = $1^2 + 13^2$ =170

∴ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1, 2, 3 ∈ Q

Thus, the operation * is not associative.

(iii) On Q, the operation * is defined as a * b = a + ab.

It can be observed that

1 * 2 = 1 + 1×2 = 1 + 2 = 3

2 * 1 = 2 + 2×1 = 2 + 2 = 4

∴ 1 * 2 ≠ 2 * 1, where 1, 2 ∈ Q

Thus, the operation * is not commutative.

It can also be observed that

(1 * 2) * 3 = (1+ 1×2) * 3 = (1 + 2) * 3 = 3 * 3 = 3 + 3×3 = 3 + 9 = 12 and

1 * (2 * 3) = 1 * (2 + 2×3) = 1 * (2 + 6) = 1 * 8 = 1 + 1×8 =1 + 8 = 9

∴ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1, 2, 3 ∈ Q

Thus, the operation * is not associative.

(iv) On Q, the operation * is defined by a * b = $(a - b)^2$.

For a, b ∈ Q, we have

a * b = $(a - b)^2$

b * a = $(b - a)^2 = [-(a - b)]^2 = (a - b)^2$

∴ a * b = b * a

Thus, the operation * is commutative.

It can be observed that

(1 * 2) * 3 = $(1 - 2)^2$ * 3 = $(-1)^2$ * 3 = 1 * 3 = $(1 - 3)^2$ = (-2) = 4 and

1 * (2 * 3) = 1 * $(2 - 3)^2$ = 1 * $(-1)^2$ = 1 * 1 = $(1 - 1)^2$ = 0

∴ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1, 2, 3 ∈ Q

Thus, the operation * is not associative.

(v) On Q, the operation * is defined as a*b = ab/4.

For a, b ∈ Q, we have a*b = ab/4 = ba/4 = b*a

∴ a * b = b * a

Thus, the operation * is commutative.

For a, b, c ∈ Q, we have

$$(a*b)*c = \left(\frac{ab}{4}\right)*c = \frac{\left(\frac{ab}{4}\right).c}{4} = \frac{abc}{16}$$

and

$$a*(b*c) = a*\left(\frac{bc}{4}\right) = \frac{a.\left(\frac{bc}{4}\right)}{4} = \frac{abc}{16}$$

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∴ (a * b) * c = a * (b * c), where a, b, c ∈ Q

Thus, the operation * is associative.

(vi) On Q, the operation * is defined as a * b = ab^2

It can be observed that

$$\frac{1}{2}*\frac{1}{3} = \frac{1}{2}.\left(\frac{1}{3}\right)^2 = \frac{1}{2}.\frac{1}{9} = \frac{1}{18}$$

and

$$\frac{1}{3}*\frac{1}{2} = \frac{1}{3}.\left(\frac{1}{2}\right)^2 = \frac{1}{3}.\frac{1}{4} = \frac{1}{12}$$

$\therefore \frac{1}{2}*\frac{1}{3} \neq \frac{1}{3}*\frac{1}{2}$, where $\frac{1}{2}$ and $\frac{1}{3} \in \mathbf{Q}$

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∴ 1/2 * 1/3 ≠ 1/3 * 1/2, where 1/2 and 1/3 ∈ Q

Thus, the operation * is not commutative.

It can also be observed that

$$\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}=\left[\frac{1}{2}\left(\frac{1}{3}\right)^2\right]*\frac{1}{4}=\frac{1}{18}*\frac{1}{4}=\frac{1}{18}\cdot\left(\frac{1}{4}\right)^2=\frac{1}{18\times 16}=\frac{1}{288}$$

and

$$\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)=\frac{1}{2}*\left[\frac{1}{3}\left(\frac{1}{4}\right)^2\right]=\frac{1}{2}*\frac{1}{48}=\frac{1}{2}\left(\frac{1}{48}\right)^2=\frac{1}{2\times 2304}=\frac{1}{4608}$$

$$\therefore\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}\neq\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right), where\ \frac{1}{2},\frac{1}{3},\frac{1}{4}\in \boldsymbol{Q}$$

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Thus, the operation * is not associative.

Hence, the operations defined in (ii), (iv), (v) are commutative and the operation defined in (v) is associative.

10. Find which of the operations given above has identity.

Answer

An element e ∈ Q will be the identity element for the operation *

if a * e = a = e * a, for all a ∈ Q.

However, there is no such element e ∈ Q with respect to each of the six operations satisfying the above condition.

Thus, none of the six operations has identity.

11. Let A = N × N and ∗ be the binary operation on A defined by

(a, b) ∗ (c, d) = (a + c, b + d)

11. Show that ∗ is commutative and associative. Find the identity element for ∗ on A, if any.

Answer

A = N × N and * is a binary operation on A and is defined by

(a, b) * (c, d) = (a + c, b + d)

Let (a, b), (c, d) ∈ A

Then, a, b, c, d $\in$ N

We have:

(a, b) * (c, d) = (a + c, b + d)

(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d)

[Addition is commutative in the set of natural numbers]

$\therefore$ (a, b) * (c, d) = (c, d) * (a, b)

Therefore, the operation * is commutative.

Now, let (a, b), (c, d), (e, f) $\in$ A

Then, a, b, c, d, e, f $\in$ N

We have

[(a, b) $\ast$ (c, d)] $\ast$ (e, f) = (a + c, b + d) $\ast$ (e, f) = (a + c + e, b + d + f)

and

(a, b) $\ast$ [(c, d) $\ast$ (e, f)] = (a, b) $\ast$ (c + e, d + f) = (a + c + e, b + d + f)

$\therefore$ [(a, b) $\ast$ (c, d)] $\ast$ (e, f) = (a, b) $\ast$ [(c, d) $\ast$ (e, f)]

Therefore, the operation * is associative.

Let an element e = (e_1, e_2) $\in$ A will be an identity element for the operation *

if a * e = a = e * a for all a = (a_1, a_2) $\in$ A

i.e., $(a_1 + e_1, a_2 + e_2) = (a_1, a_2) = (e_1 + a_1, e_2 + a_2)$

Which is not true for any element in A.

Therefore, the operation * does not have any identity element.

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12. State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation ∗ on a set N, a ∗ a = a ∀ a ∈ N.

(ii) If ∗ is a commutative binary operation on N, then a ∗ (b ∗ c) = (c ∗ b) ∗ a

Answer

(i) Define an operation * on N as a * b = a + b ∀ a, b ∈ N

Then, in particular, for b = a = 3, we have

3 * 3 = 3 + 3 = 6 ≠ 3

Therefore, statement (i) is false.

(ii) R.H.S. = (c * b) * a

= (b * c) * a [* is commutative]

= a * (b * c) [Again, as * is commutative]

= L.H.S.

∴ a * (b * c) = (c * b) * a

Therefore, statement (ii) is true.

13. Consider a binary operation ∗ on N defined as a ∗ b = $a^3 + b^3$. Choose the correct answer.

(A) Is ∗ both associative and commutative?

(B) Is ∗ commutative but not associative?

(C) Is ∗ associative but not commutative?

(D) Is ∗ neither commutative nor associative?

Answer

On N, the operation * is defined as a * b = a3 + b3.

For, a, b, ∈ N, we have

$a * b = a^3 + b^3 = b^3 + a^3 = b * a$ [Addition is commutative in N]

Therefore, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = (1^3 + 2^3) * 3 = (1 + 8) * 3 = 9 * 3 = 9^3 + 3^3 = 729 + 27 = 756$

and $1* (2 * 3) = 1 * (2^3 + 3^3) = 1* (8 + 27) = 1*35 = 1^3 + 35^3 = 1 + 42875 = 42876$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in N$

Therefore, the operation * is not associative.

Hence, the operation * is commutative, but not associative.

Thus, the correct answer is B.

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