NCERT Solutions for Maths: Chapter 13 - Surface Areas and Volumes

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NCERT Solutions for Maths: Chapter 13 - Surface Areas and Volumes

Class 10: Maths Chapter 13 solutions. Complete Class 10 Maths Chapter 13 Notes.

NCERT Solutions for Maths: Chapter 13 - Surface Areas and Volumes

NCERT 10th Maths Chapter 13, class 10 Maths Chapter 13 solutions

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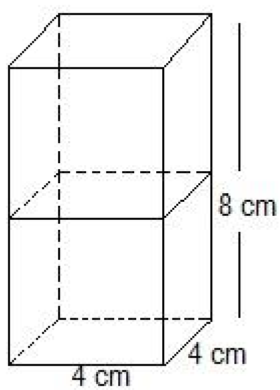
Exercise 13.1

Unless stated otherwise, take π = 22/7.

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Answer



Volume of each cube(a^3) = 64 cm^3

⇒ $a^3 = 64\ cm^3$

⇒ a = 4 cm

Side of the cube = 4 cm

Length of the resulting cuboid = 4 cm

Breadth of the resulting cuboid = 4 cm

Height of the resulting cuboid = 8 cm

∴ Surface area of the cuboid = 2(lb + bh + lh)

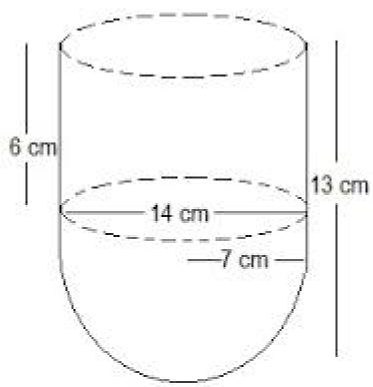
= 2(8×4 + 4×4 + 4×8) cm^2

= 2(32 + 16 + 32) cm^2

$= (2 \times 80)\ cm^2 = 160\ cm^2$

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Answer



Diameter of the hemisphere = 14 cm

Radius of the hemisphere(r) = 7 cm

Height of the cylinder(h) = 13 - 7 = 6 cm

Also, radius of the hollow hemisphere = 7 cm

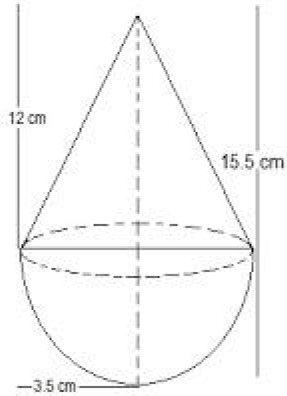
Inner surface area of the vessel = CSA of the cylindrical part + CSA of hemispherical part

$= (2\pi rh+2\pi r^2)\ cm^2= 2\pi r(h+r)\ cm^2$

$= 2 \times 22/7 \times 7\ (6+7)\ cm^2= 572\ cm^2$

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer



Radius of the cone and the hemisphere(r) = 3.5 = 7/2 cm

Total height of the toy = 15.5 cm

Height of the cone(h) = 15.5 - 3.5 = 12 cm

Slant height of the cone(l) = $\sqrt{h^2 + r^2}$

$\Rightarrow$ l = $\sqrt{12^2 + (3.5)^2}$

$\Rightarrow$ l = $\sqrt{12^2 + (7/2)^2}$

$\Rightarrow$ l = $\sqrt{144 + 49/4} = \sqrt{(576 + 49)/4} = \sqrt{625/4}$

$\Rightarrow$ l = 25/2

Curved Surface Area of cone = πrl = 22/7 × 7/2 × 25/2

= 275/2 cm^2

Curved Surface Area of hemisphere = $2\pi r^2$

= 2 × 22/7 × $(7/2)^2$

= 77 cm^2

Total surface area of the toy = CSA of cone + CSA of hemisphere

= (275/2 + 77) cm^2

= (275+154)/2 cm^2

= 429/2 cm^2 = 214.5cm^2

The total surface area of the toy is 214.5cm^2

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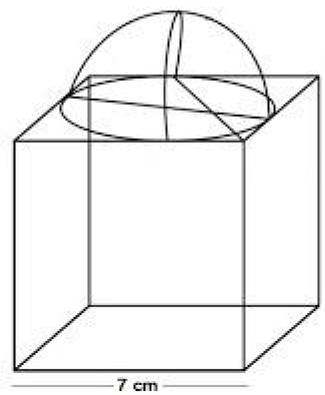
4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Answer



Each side of cube = 7 cm

∴ Radius of the hemisphere = 7/2 cm

Total surface area of solid = Surface area of cubical block + CSA of hemisphere - Area of base of hemisphere

TSA of solid = $6\times(\text{side})^2 + 2\pi r^2 - \pi r^2$

= $6\times(\text{side})^2 + \pi r^2$

= $6\times(7)^2 + (22/7 \times 7/2 \times 7/2)$

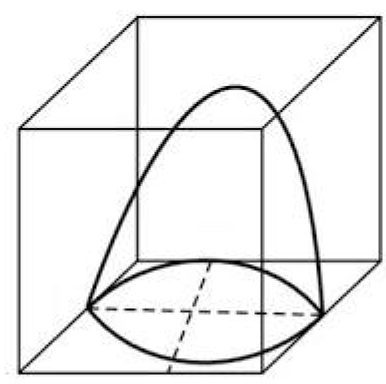
= (6×49) + (77/2)

= 294 + 38.5 = 332.5 cm^2

The surface area of the solid is 332.5 cm^2

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer



Diameter of hemisphere = Edge of cube = l

Radius of hemisphere = l/2

Total surface area of solid = Surface area of cube + CSA of hemisphere - Area of base of hemisphere

TSA of remaining solid = $6\ (edge)^2 + 2\pi r^2 - \pi r^2$

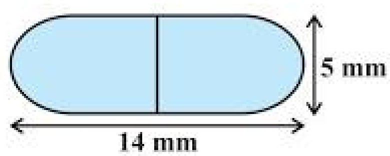
$= 6l^2 + \pi r^2$

$= 6l^2 + \pi(l/2)^2$

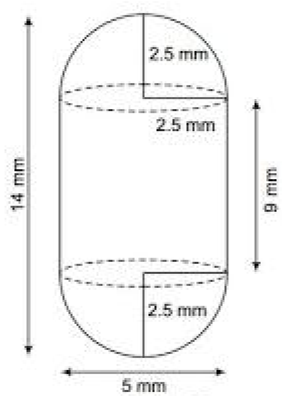
$= 6l^2 + \pi l^2/4$

$= l^2/4(24 + \pi)$ sq units

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Answer



Two hemisphere and one cylinder are given in the figure.

Diameter of the capsule = 5 mm

∴ Radius = 5/2 = 2.5 mm

Length of the capsule = 14 mm

∴ Length of the cylinder = 14 - (2.5 + 2.5) = 9mm

Surface area of a hemisphere = $2\pi r^2$ = 2 × 22/7 × 2.5 × 2.5

= 275/7 mm^2

Surface area of the cylinder = $2\pi rh$

= 2 × 22/7 × 2.5 x 9

= 22/7 × 45

990/7 mm^2

∴ Required surface area of medicine capsule

= 2 × Surface area of hemisphere + Surface area of cylinder

= (2 × 275/7) × 990/7

= 550/7 + 990/7

= 1540/7 = 220 mm^2

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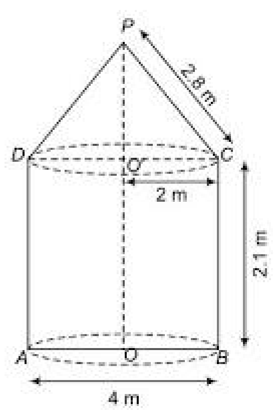
7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Answer

Tent is combination of cylinder and cone.



Diameter = 4 m

Slant height of the cone (l) = 2.8 m

Radius of the cone (r) = Radius of cylinder = 4/2 = 2 m

Height of the cylinder (h) = 2.1 m

∴ Required surface area of tent = Surface area of cone+Surface area of cylinder

= πrl + 2πrh

= πr(l+2h)

= 22/7 × 2 (2.8 + 2×2.1)

= 44/7 (2.8 + 4.2)

= 44/7 × 7 = 44 m^2

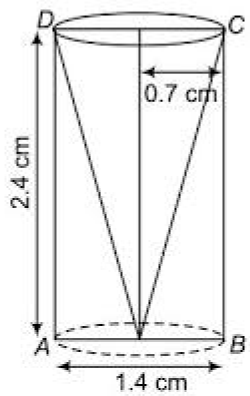
Cost of the canvas of the tent at the rate of ₹500 per m^2

= Surface area × cost per m^2

= 44 × 500 = ₹22000

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Answer



Diameter of cylinder = diameter of conical cavity = 1.4 cm

∴ Radius of cylinder = Radius of conical cavity = 1.4/2 = 0.7

Height of cylinder = Height of conical cavity = 2.4 cm

$$\therefore \text{Slant height of the conical cavity } (l) = \sqrt{h^2 + r^2} = \sqrt{(2.4)^2 + (0.7)^2} = \sqrt{5.76 + 0.49} = \sqrt{6.25} = 2.5 \text{ cm}$$

TSA of remaining solid = Surface area of conical cavity+TSA of cylinder

$= \pi rl + (2\pi rh + \pi r^2)$

$= \pi r (l + 2h + r)$

$= 22/7 \times 0.7 (2.5 + 4.8 + 0.7)$

$= 2.2 \times 8 = 17.6 \text{ cm}^2$

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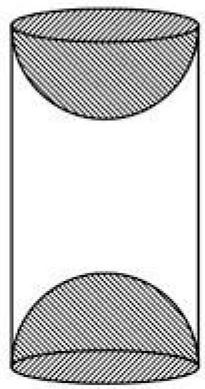
9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.

Answer



Given,

Height of the cylinder, h = 10cm and radius of base of cylinder = Radius of hemisphere (r) = 3.5 cm

Now, required total surface area of the article = 2 × Surface area of hemisphere + Lateral surface area of cylinder

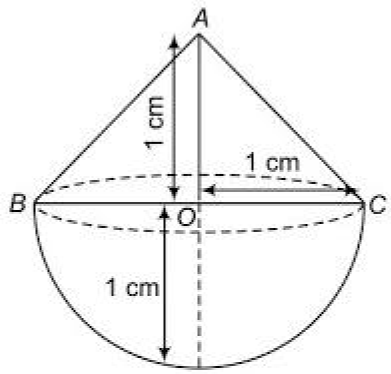
$$= 2 \times (2\pi r^2) + 2\pi r h$$
$$= 2\pi r(2r + h) = 2 \times \frac{22}{7} \times 3.5 \times (2 \times 3.5 + 10)$$
$$= \frac{22}{7} \times 7 \times (7 + 10) = 22 \times 17 = 374 \text{ cm}^2$$

Exercise 13.2

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Answer

Given, solid is a combination of a cone and a hemisphere.



Also, we have radius of the cone (r) = Radius of the hemisphere = 1cm and height of the cone (h) = 1cm

∴ Required volume of the solid = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi (1)^2 (1) + \frac{2}{3}\pi (1)^3$$
$$= \frac{\pi}{3} + \frac{2}{3}\pi$$
$$= \frac{(\pi + 2\pi)}{3}$$
$$= \frac{3\pi}{3} = \pi \text{ cm}^3$$

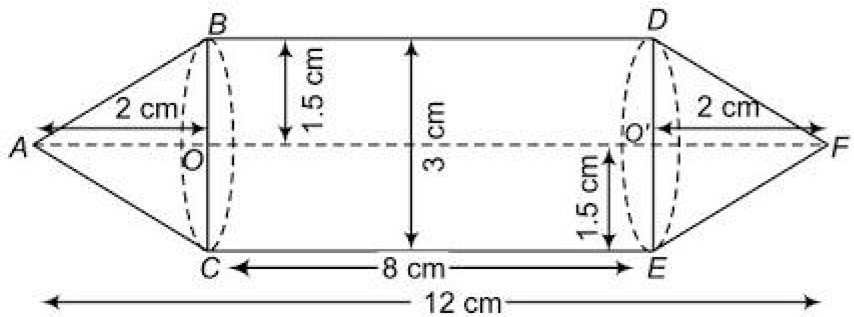
2. Rachel, an engineering, student was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel mode. (Assume the outer and inner dimensions of the model to be nearly the same.)

Answer

Given, model is a combination of a cylinder and two cones. Also, we have, diameter of the model,BC = ED = 3 cm.

$\therefore \quad r = \frac{3}{2} = 1.5$ cm

Height of cone, $h_1 = 2$ cm and length of the model, $AF = 12$ cm



$$\therefore \quad OO' = AF - (AO + O'F)$$
$$= 12 - (2 + 2)$$
$$= 8 \text{ cm}$$

$\therefore$ Height of cylinder, $h_2 = 8$ cm

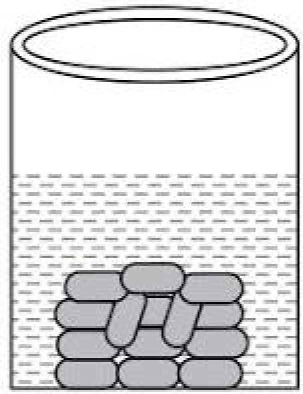
Now, volume of the air inside the model

$$= \text{Volume of air inside (cone + cylinder + cone)}$$
$$= \left(\frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2 + \frac{1}{3}\pi r^2 h_1\right)$$
$$= \frac{1}{3}\pi r^2 (h_1 + 3h_2 + h_1) \quad \ldots\text{(i)}$$
$$= \frac{1}{3} \times \frac{22}{7} \times 1.5 \times 1.5\,(2 + 3 \times 8 + 2)$$
$$= \frac{22}{7} \times \frac{2.25}{3} \times (2 + 24 + 2)$$
$$= \frac{22}{7} \times 0.75 \times 28$$
$$= 22 \times 3$$
$$= 66 \text{ cm}^3$$

3. A gulab jamun, contains sugar syrup upto about 30% of its volume. Find approximately how much syrup would be found in 45 gulab

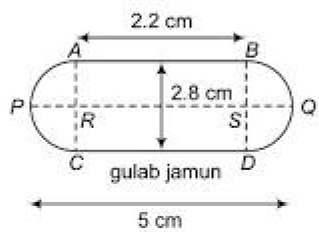
jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see figure).

Answer



Let r be the radius of the hemisphere and cylinder both. h1 be the height of the hemisphere which is equal to its radius and h2 be the height of the cylinder.

Given, length = 5 cm, diameter = 2.8 cm



∴ $r = h_1 = \frac{2.8}{2} = 1.4$ cm

and $h_2 = PQ - (PR + SQ)$

$= 5 - (1.4 + 1.4)$

$= 5 - 2.8 = 2.2$ cm

∴ Volume of one gluab jamun = 2 × (Volume of hemisphere) + Volume of cylinder

$= 2 \times \left\{\frac{2}{3}\pi r^3\right\} + \pi r^2 h_2$

$= \frac{4}{3}\pi r^3 + \pi r^2 h_2$

$= \pi r^2 \left\{\frac{4\pi}{3} + h_2\right\}$

$= \frac{22}{7} \times 1.4 \times 1.4 \left\{\frac{4}{3} \times 1.4 + 2.2\right\}$

$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10} \left\{\frac{4}{3} \times \frac{14}{10} + \frac{22}{10}\right\}$

$= 22 \times \frac{1}{5} \times \frac{7}{5} \left\{\frac{28}{15} + \frac{11}{5}\right\}$

$= \frac{154}{25} \times \frac{61}{15} = \frac{9394}{375}$ cm^3

∴ Volume of 45 gulab jamuns $= 45 \times \frac{9394}{375} = \frac{1}{75} \times 84546$

$= 1127.28$ cm^3

∵ One gulab jamun, contains sugar syrup upto about 30% of its volume

i.e., $\left(\frac{9394}{375}\right) \times 30\%$

Hence, quantity of syrup found in 45 gulab jamuns

$= 1127.28 \times \frac{30}{100}$

$= 1127.28 \times \frac{3}{10} = 338.184$

≈ 338 cm^3

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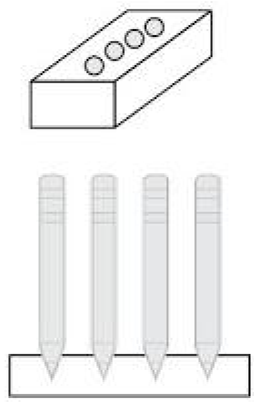
4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is

0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see figure).

Answer



Given, length of cuboid (l) = 15 cm

Breadth of cuboid (b) = 10cm

and height of cuboid (h) = 3.5 cm

∴ Volume of cuboid = l×b×h = 15 ×10 × 3.5 = 525^3

Also, given that

Radius of conical depression $r = 0.5\text{cm}$

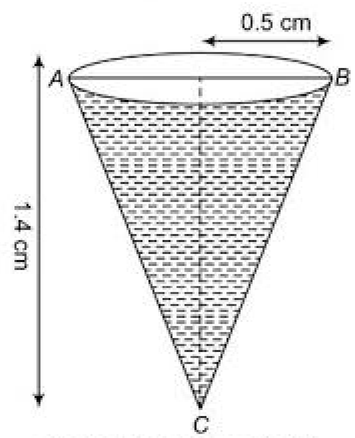
and height of conical depression, $h = 1.4$

$\therefore$ Volume of a conical depression $= \frac{1}{3}\pi \times r^2 \times h$

$$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$$

$$= \frac{22}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{10}$$

$$= \frac{11}{30}\text{ cm}^3$$



Conical depression (cone)

$\therefore$ Volume of 4 conical depressions $= 4 \times$ Volume of a conical depression

$$= 4 \times \frac{11}{30}$$

$$= \frac{22}{15}\text{ cm}^3$$

Hence, the volume of wood in the entire wood

$$= \text{Volume of cuboid} - \text{Volume of 4 conical depressions}$$

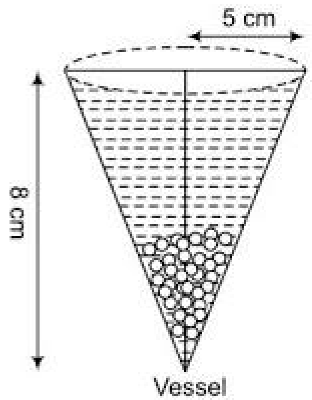
$$= 525 - \frac{22}{15}$$

$$= 525 - 1.46$$

$$= 523.54\text{ cm}^3$$

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel,one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer



Height of the vessel (h) = 8 cm

and radius of the vessel (r) = 5 cm

∴ Volume of water filled in a vessel

$$= \frac{1}{3} \times \pi \times r \times r \times h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8 = \frac{4400}{21} \text{ cm}^3$$

Also, given that,

the radius of a lead shots (sphere) = 0.5 cm

∴ Volume of a lead shots (sphere) $= \frac{4}{3} \times \pi \times r \times r \times r$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{11}{21} \text{ cm}^3$$

Let 'n' be the number of lead shots dropped in the vessel, then according to question.

$n \times$ Volume of a lead shot (sphere)

$$= \frac{1}{4} \times \text{Volume of water filled in a vessel}$$

(∵ Each lead shot flow out the water from the vessel equal to its volume)

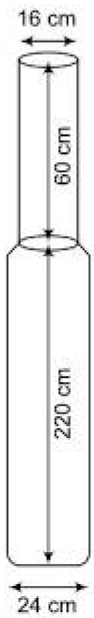
$$\Rightarrow \quad n \times \frac{11}{21} = \frac{4400}{21} \times \frac{1}{4}$$

$$\Rightarrow \quad n \times 11 = 1100$$

$$\Rightarrow \quad n = 100$$

∴ Required number of lead shots (n) = 100.

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use π = 3.14)



Pole

Answer

Height of the first cylinder, $h_1 = 220$ cm

Radius of the first cylinder, $r_1 = \frac{24}{2} = 12$ cm

Height of the second cylinder, $h_2 = 60$ cm

and radius of the second cylinder, $r_2 = 8$ cm

$\therefore$ Volume of iron pole = Volume of first cylinder + Volume of second cylinder

$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$
$$= \pi (r_1^2 h_1 + r_2^2 h_2)$$
$$= 3.14 \{144 \times 220 + 64 \times 60\}$$
$$= 3.14 (31680 + 3840) = 3.14 \times 35520 \text{ cm}^3$$

Also, given that,

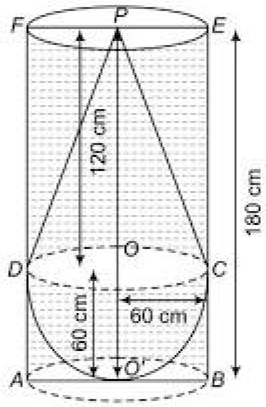
1 cm^3 of iron has approximately mass = 8 g = $\frac{8}{1000}$ kg

$\therefore$ (3.14×35520) cm^3 of iron has approximately mass

$$= 3.14 \times 35520 \times \frac{8}{1000}$$
$$= 3.14 \times 284.160$$
$$= 892.26 \text{ kg}$$

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Answer



Height of the cylinder $(h) = 180 \text{ cm} = 1.8 \text{ m}$ $\quad (\because 1\text{m} = 100 \text{ cm} \Rightarrow 1 \text{ cm} = \frac{1}{100} \text{ m})$

and radius of the cylinder $(r) = 60 \text{ cm} = 0.6 \text{ m}$

$\therefore$ Volume of water filled in a right circular cylinder

$$= \pi r^2 h$$
$$= \frac{22}{7} \times 0.6 \times 0.6 \times 1.8$$
$$= \frac{14.256}{7} \text{ m}^3$$

Also, given solid is a combination of a cone and a hemisphere.

Height of the cone $(h_1) = 120 \text{ cm} = 1.2 \text{ m}$

Radius of the cone $(r_1) = 60 \text{ cm} = 0.6 \text{ m}$

and height/radius of the hemisphere $(r_2) = 60 \text{ cm} = 0.6 \text{ m}$

$\therefore$ Volume of the solid = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3} \times \pi r_1^2 h_1 + \frac{2}{3} \pi r_2^3$$
$$= \frac{1}{3} \times \frac{22}{7} \times (0.6)^2 \times (1.2) + \frac{2}{3} \times \frac{22}{7} \times (0.6)^3$$
$$= \frac{22}{21} \times (0.6)^2 \{1.2 + 2 \times 0.6\}$$
$$= \frac{22}{21} \times 0.36 \, (1.2 + 1.2)$$
$$= \frac{22}{21} \times 0.36 \times 2.4 = \frac{19.008}{21} \text{ m}^3 = \frac{6.336}{7} \text{ m}^3$$

Hence, the volume of water left in the cylinder

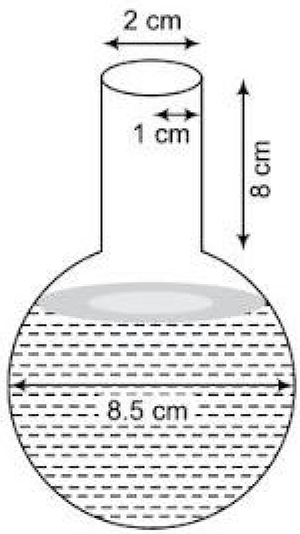
= Volume of water filled in a right circular cylinder − Volume of the solid

$$= \frac{14.256}{7} - \frac{6.336}{7} = \frac{7.92}{7} = 1.131428 \text{ m}^3 \approx 1.131 \text{ m}^3$$

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements and π = 3.14.

Answer

She is not correct.



Given, spherical glass vessel is a combination of sphere as its base and a cylinder as its neck.

Also, we have height of the cylinder (h_1) = 8 cm

Radius of the cylinder (r_1) $= \frac{2}{2} = 1\text{cm}$

and radius of the sphere (r_2) $= \frac{8.5}{2}\text{cm}$

∴ The volume of water filled in a spherical glass vessel

$$= \text{Volume of cylinder} + \text{Volume of sphere}$$

$$= \pi r_1^2 h_1 + \frac{4}{3}\pi r_2^3$$

$$= 3.14 \times 1 \times 1 \times 8 + \frac{4}{3} \times 3.14 \times \frac{8.5}{2} \times \frac{8.5}{2} \times \frac{8.5}{2}$$

$$= 25.12 + \frac{1928.3525}{6}$$

$$= 25.12 + 321.39$$

$$= 346.51$$

So, the correct answer is 346.51 cm^3.

Exercise 13.3

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Answer

Given, the radius of the sphere (r) = 4.2 cm

Radius of the cylinder (r1) = 6 cm

Then, the volume of the sphere $= \frac{4}{3} \times \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2$

$= 4 \times 22 \times 1.4 \times 0.6 \times 4.2$

$= 310.464 \text{ cm}^3$

Let h be the height of the cylinder, then

its volume $= \pi r_1^2 h = \frac{22}{7} \times 6 \times 6 \times h$

Now, according to the question,

Volume of the sphere = Volume of the cylinder

$\Rightarrow \quad 310.464 = \frac{22}{7} \times 6 \times 6 \times h$

$\Rightarrow \quad h = \frac{310.464 \times 7}{22 \times 6 \times 6} = \frac{2173.248}{792}$

$\Rightarrow \quad h = 2.744 \text{ cm}$

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Answer

Let r1, r2 and r3 be the radius of given three spheres and R be the radius of a single solid sphere.

Given, r1 = 6 cm , r2 = 8 cm and r3 =10 cm

Volume of first metallic sphere $(V_1) = \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi(6)^3 = \frac{864}{3}\pi \text{ cm}^3$

Volume of second metallic sphere $(V_2) = \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi (8)^3 = \frac{2048}{3}\pi \text{ cm}^3$

Volume of third metallic sphere $(V_3) = \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi(10)^3 = \frac{4000}{3}\pi \text{ cm}^3$

and the volume of a single solid sphere $(V) = \frac{4}{3}\pi R^3$

Now, according to question,

Volume of three metallic sphere = Volume of a single solid sphere

$\Rightarrow \quad V_1 + V_2 + V_3 = V$

$\Rightarrow \quad \frac{864}{3}\pi + \frac{2048}{3}\pi + \frac{4000}{3}\pi = \frac{4}{3}\pi R^3$

$\Rightarrow \quad \frac{6912}{3} = \frac{4}{3}R^3$

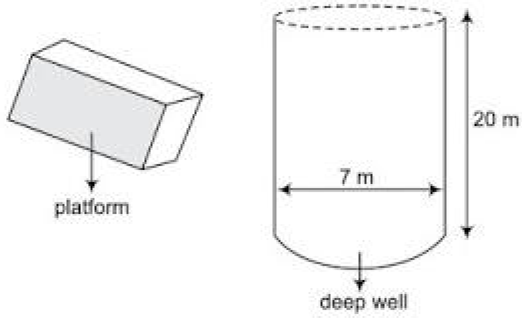
$\Rightarrow \quad R^3 = 1728 = (12)^3$

$\Rightarrow \quad R = 12 \text{ cm}$

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Answer

Given, the height of deep well which form a cylinder (h1) = 20 m



Radius of deep well $(r_1) = \frac{7}{2}$ m

Length of the platform, $l = 22$ m

and breadth of the platform $b = 14$ m

Let height of the platform $= h_2$

Now, according to question,

Volume of deep well (cylinder) = Volume of platform (cuboid)

$$\Rightarrow \pi r_1^2 h_1 = l \times b \times h_2$$

$$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 14 \times h_2$$

$$\Rightarrow \frac{11 \times 7}{2} \times 20 = 22 \times 14 \times h_2$$

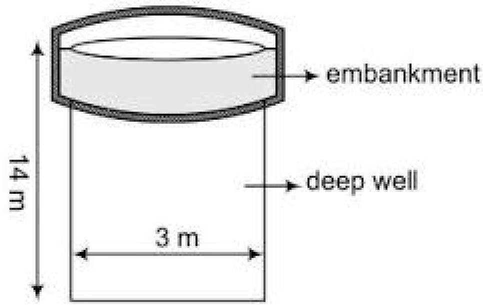
$$\Rightarrow h_2 = \frac{11 \times 7 \times 20}{2 \times 22 \times 14} = \frac{5}{2}$$

$\therefore$ Height of the platform $(h_2) = 2.5$ m.

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4m to form an embankment. Find the height of the embankment.

Answer

Given, the height of deep well which form a cylinder (h1) = 14 m



and radius of deep well $(r_1) = \frac{3}{2}$ m

Let the width and height of the embankment form a circular ring are d and h_2.

Here; $d = 4$ cm (Given)

Now, the volume of deep well (cylinder) $= \pi r_1^2 h_1 = \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 14 = 99\,\text{m}^3$

Here, we observe that the embankment form a hollow cylinder. So, for finding the volume of embankment, we apply the concept of hollow cylinder.

$\therefore$ Volume of hollow cylinder (embankment) $= \pi(r_1 + d)^2 \cdot h_2 - \pi r_1^2 h_2$

$$= \pi\left(\frac{3}{2} + 4\right)^2 \times h_2 - \pi\left(\frac{3}{2}\right)^2 h_2$$

$$= \pi\left[\left(\frac{11}{2}\right)^2 h_2 - \frac{9}{2} h_2\right]$$

Now, according to question,

Volume of deep well (cylinder) = Volume of embankment (hollow cylinder)

$$\Rightarrow \quad 99 = \pi\left[\left(\frac{11}{2}\right)^2 \times h_2 - \frac{9}{4} h_2\right]$$

$$\Rightarrow \quad \frac{7 \times 99}{22} = \frac{121}{4} \times h_2 - \frac{9}{4} h_2 = \frac{112 h_2}{4}$$

$$\Rightarrow \quad h_2 = \frac{7 \times 99 \times 4}{22 \times 112} = \frac{9}{8} = 1.125\text{ m}$$

$\therefore$ Height of the embankment is 1.125 cm.

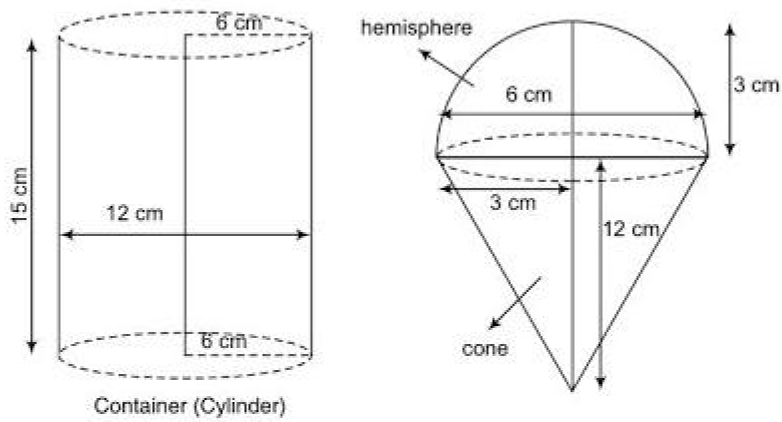
NCERT 10th Maths Chapter 13, class 10 Maths Chapter 13 solutions

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical

shape on the top. Find the number of such cones which can be filled with ice cream.

Answer

Let the height and radius of ice cream container (cylinder) be h1 and r1.



Given, $h_1 = 15$ cm and $r_1 = 6$ cm

$\therefore$ The volume of ice cream container (cylinder) $= \pi r_1^2 h_1 = \frac{22}{7} \times 6 \times 6 \times 15$

$$= \frac{11880}{7} \text{ cm}^3$$

Now, the cone full with ice-cream having top surface just like a hemisphere.

Also, we have,

Height of the cone, $h_2 = 12$ cm

Diameter of the cone, $r_1 = 6$ cm

$\therefore$ Radius of the cone $r_2 = 3$ cm $=$ Radius of the hemisphere

$\because$ The volume of cone full with ice cream

$$= \text{Volume of cone} + \text{Volume of hemisphere (upper surface level)}$$

$$= \frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3 = \frac{1}{3}\pi(r_2^2 h_2 + 2r_2^3)$$

$$= \frac{1}{3} \times \frac{22}{7}[(3)^2 \times 12 + 2\,(3)^3]$$

$$= \frac{22}{3 \times 7} \times 9 \times (12 + 6)$$

$$= \frac{22 \times 3 \times 18}{7} = \frac{1188}{7} \text{ cm}^3$$

Let 'n' be number of cones full with ice cream, then

The volume of ice cream container $= n \times$ volume of one cone full with ice cream

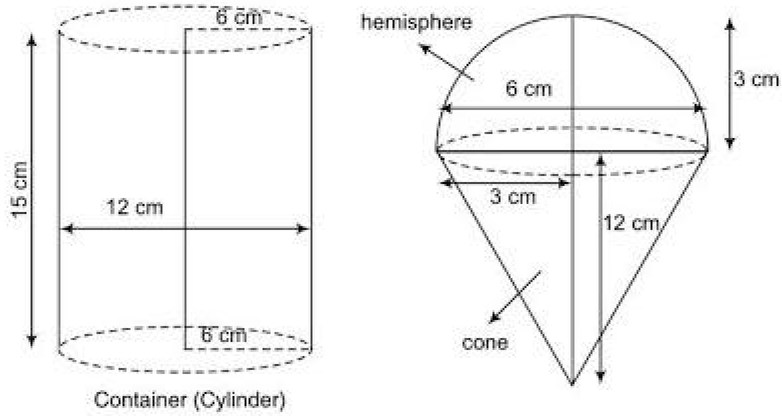
$$\Rightarrow \quad \frac{11880}{7} = n \times \frac{1188}{7} \Rightarrow n = 10$$

Hence, the number of such cones is 10.

6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 10 3 5 . . cm cm cm × ?

Answer

We know that, every coin has a shape of cylinder. Let radius and height of the coin are r1 and h1 respectively.



Given, $h_1 = 15$ cm and $r_1 = 6$ cm

$\therefore$ The volume of ice cream container (cylinder) $= \pi r_1^2 h_1 = \frac{22}{7} \times 6 \times 6 \times 15$

$$= \frac{11880}{7} \text{ cm}^3$$

Now, the cone full with ice-cream having top surface just like a hemisphere.

Also, we have,

Height of the cone, $h_2 = 12$ cm

Diameter of the cone, $r_1 = 6$ cm

$\therefore$ Radius of the cone $r_2 = 3$ cm = Radius of the hemisphere

$\because$ The volume of cone full with ice cream

$$= \text{Volume of cone} + \text{Volume of hemisphere (upper surface level)}$$

$$= \frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3 = \frac{1}{3}\pi(r_2^2 h_2 + 2r_2^3)$$

$$= \frac{1}{3} \times \frac{22}{7}[(3)^2 \times 12 + 2(3)^3]$$

$$= \frac{22}{3 \times 7} \times 9 \times (12 + 6)$$

$$= \frac{22 \times 3 \times 18}{7} = \frac{1188}{7} \text{ cm}^3$$

Let 'n' be number of cones full with ice cream, then

The volume of ice cream container $= n \times$ volume of one cone full with ice cream

$$\Rightarrow \quad \frac{11880}{7} = n \times \frac{1188}{7} \Rightarrow n = 10$$

Hence, the number of such cones is 10.

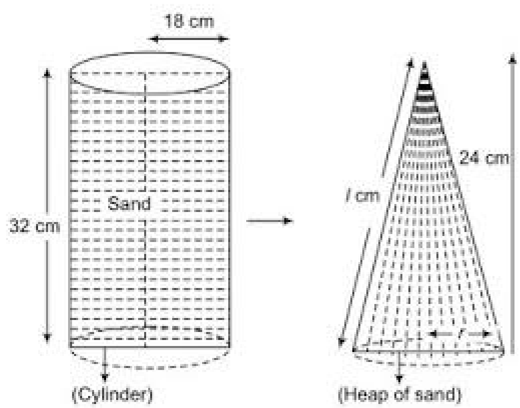
NCERT 10th Maths Chapter 13, class 10 Maths Chapter 13 solutions

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Answer

Let the radius and slant height of the heap of sand are r and l.

Given, the height of the heap of sand h = 24 cm.



$\therefore$ Volume of the heap of sand $= \frac{1}{3}\pi r^2 (24) = 8\pi r^2$

Also, given the height and radius of cylindrical bucket are 32 cm and 18 cm, respectively.

$\therefore$ Volume of cylindrical bucket $= \pi\,(\text{Radius})^2 \times (\text{Height}) = \pi (18)^2 \times 32$

Now, according to question;

Volume of the heap of sand = Volume of the cylindrical bucket

$\Rightarrow \quad 8\pi r^2 = \pi \times 18 \times 18 \times 32$

$\Rightarrow \quad r^2 = 18 \times 18 \times 4 \quad \Rightarrow r = 36 \text{ cm}$

Now, the slant height of the conical heap of sand

$$= \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (36)^2}$$

$$= \sqrt{576 + 1296} = \sqrt{1872}$$

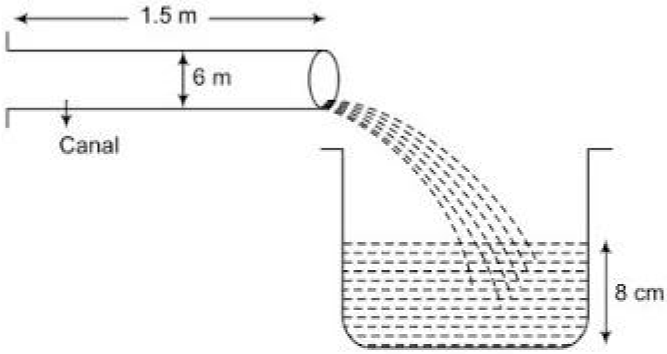
$$l = \sqrt{144 \times 13} = 12\sqrt{13} \text{ cm}$$

8. Water in a canal, 6 m wide and 1.5 m deep is flowing with a speed of 10 kmh^{-1}. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Answer

Given, speed of flow of water (l) = 10 kmh-1 = 10 × 1000 mh^{-1}

Area of canal = 6 × 1.5 = 9m^2



Volume of water flowing in 1 h = $10 \times 1000 \times 9 m^3$

$\therefore$ Volume of water flowing in 1/2 h $= \frac{10 \times 1000 \times 9}{2} = 45000 m^2$

$\therefore$ Required area for covering 8 cm height of standing water

$$= \frac{45000}{8} \times 100 = 562500 m^2 = \frac{562500}{10000} \text{ hectares} = 56.25 \text{ hectares}$$

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 kmh^{-1}, in how much time will the tank be filled?

Answer

Given, speed of flow of water = 3 kmh^{-1} = 3×1000 mh^{-1}

$\therefore$ Length of water in 1 h = 3000 m

Now, area of face of pipe which form a circle $= \pi \left(\frac{20}{2}\right)^2$ $\quad (\because A = \pi r^2)$

$$= 100\pi \text{ cm}^2 = \frac{\pi}{100} m^2$$

Volume of cylindrical tank $= \pi \times \left(\frac{10}{2}\right)^2 \times 2 = 50\pi m^3$ $\quad (\because V = \pi r^2 h)$

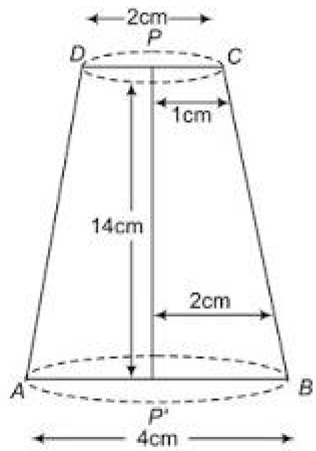
$\therefore$ Required time $= \frac{\text{Volume of cylindrical tank}}{\text{Area of face of pipe} \times \text{Length of water}}$

$$= \frac{50\pi}{\frac{\pi}{100} \times 3000} h = \frac{50 \times 60 \times 100}{3000} \text{ min} = 100 \text{ min}$$

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Answer

Let the height of the frustum of a cone be h.



Drinking glass
(frustum of cone)

and radius of both ends of the frustum are r_1 and r_2, respectively.

Given, $h = 14$ cm, $r_1 = 1$ cm and $r_2 = 2$ cm

$\therefore$ The volume (capacity) of drinking glass (frustum of a cone)

$$= \frac{\pi}{3} h (r_1^2 + r_2^2 + r_1 r_2)$$

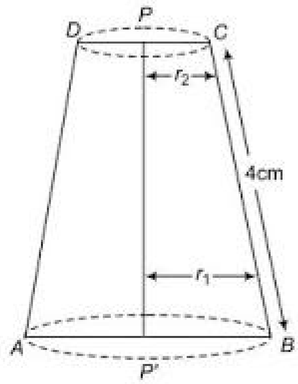
$$= \frac{1}{3} \times \frac{22}{7} \times 14 \{(1)^2 + (2)^2 + (1)(2)\}$$

$$= \frac{44}{3}(1 + 4 + 2) = \frac{44 \times 7}{3} = \frac{308}{3} = 102\frac{2}{3} \text{ cm}^3$$

2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Answer

Let the slant height of the frustum be l and radius of the both ends of the frustum be r1 and r2.



Given that, $l = 4$ cm

Perimeter of one end of frustum which is circular in shape $= 2\pi r_1 = 18$

$\Rightarrow$ $r_1 = \frac{9}{\pi}$ cm

and perimeter of other end of frustum which is also circular in shape $= 2\pi r_2 = 6$

$\Rightarrow$ $r_2 = \frac{3}{\pi}$ cm

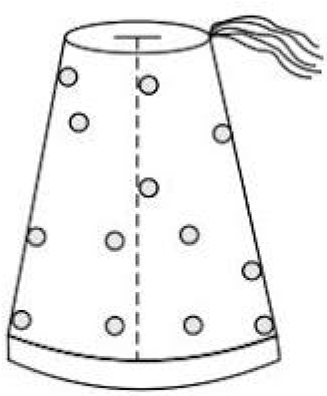
$\therefore$ Required curved surface area of the frustum $= \pi(r_1 + r_2) l$

$$= \pi \left\{\frac{9}{\pi} + \frac{3}{\pi}\right\} \times 4$$

$$= 12 \times 4$$

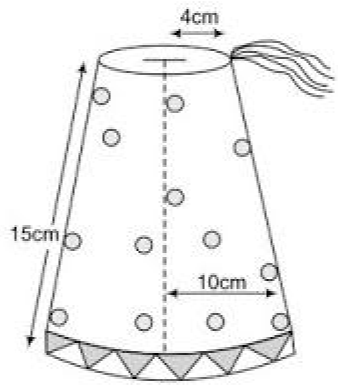
$$= 48 \text{ cm}^2$$

3. A fez, the cap used by the turks, is shaped like the frustum of a cone (see figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.



Answer

Let the slant height of fez be l and the radius of upper end which is closed be r1 and the other end which is open be r2.



'fez'
(frustum of a cone)

Given, $l = 15$ cm, $r_1 = 4$ cm and $r_2 = 10$ cm

$\therefore$ The area of upper end of fez which is circular and closed $= \pi r_1^2$

$$= \pi (4)^2 = 16\pi \text{ cm}^2$$

Hence, total curved surface area of the fez = Lateral surface area of frustum + Area of upper close end

$$= \pi (r_1 + r_2) l + \pi r_1^2$$

$$= \pi\{(4 + 10) 15 + 16\}$$

$$= \frac{22}{7}(210 + 16) = \frac{22 \times 226}{7} = \frac{4972}{7}$$

$$= 710\frac{2}{7} \text{ cm}^2$$

NCERT 10th Maths Chapter 13, class 10 Maths Chapter 13 solutions

4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of ` 20 per litre. Also find the cost of metal sheet used to make the container. If it costs ₹8 per 100 cm^2. (Take π = 3.14)

Answer

Let h be the height of the container, which is in the form of a frustum of a cone whose lower end is closed and upper end is opened. Also, let the radius of its lower end be r1 and upper end be r2.

IndCareer

Given, $h = 16$ cm, $r_1 = 8$ cm and $r_2 = 20$ cm

$\therefore$ Volume of the container $= \dfrac{\pi}{3} \times h\,(r_1^2 + r_2^2 + r_1 r_2)$

$$= \frac{3.14}{3} \times 16\,\{(8)^2 + (20)^2 + 8 \times 20\}$$

$$= \frac{3.14 \times 16 \times (64 + 400 + 160)}{3}$$

$$= \frac{3.14 \times 16 \times 624}{3}\ \text{cm}^3$$

$$= \frac{3.14 \times 16 \times 624}{3 \times 1000}\ \text{L} \qquad \left(\because 1\ \text{cm}^3 = \frac{1}{1000}\ \text{L}\right)$$

$\because$ The cost of 1 L milk $=$ ₹ 20

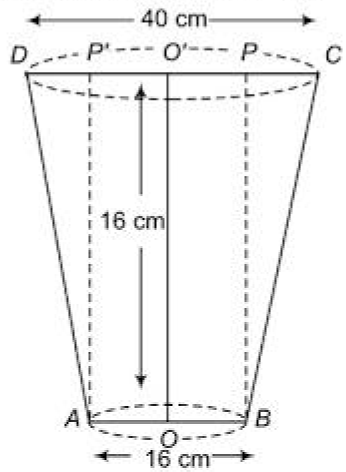
$\therefore$ The cost of $\left(\dfrac{3.14 \times 16 \times 624}{3 \times 1000}\right)$ L milk $=$ ₹ $\dfrac{3.14 \times 16 \times 624 \times 20}{3 \times 1000}$

$$= \frac{626995.2}{3000}$$

$$= 208.99$$

$$= ₹\ 209$$

Now, the area of its lower end (which is closed) $= \pi r_1^2 = 64\pi$ cm^2



From figure, $PC = DP' = \dfrac{CD - PP'}{2} = \dfrac{40 - 16}{2}$ $\quad (\because PP' = AB)$

$$= \frac{24}{2} = 12\ \text{cm}$$

$\therefore$ In right angled BPC,

$$BC^2 = PC^2 + BP^2 \qquad \text{(By Pythagoras theorem)}$$

$\Rightarrow$ $BC^2 = (12)^2 + (16)^2$

$$= 144 + 256 = 400 = (20)^2$$

$\Rightarrow$ $BC = 20$ cm

which is the slant height (l) of the container.

Now, the curved surface area of frustum $= \pi(r_1 + r_2)\,l + \pi r_1^2$

$$= 3.14(8 + 20) \times 20 + 3.14 \times 64$$

$$= 3.14(28 \times 20 + 64) = 3.14 \times 624$$

$$= 1959.36\ \text{cm}^2$$

areas-and-volumes/

$\because$ The cost of metal sheet per 100 cm^2 $=$ ₹ 8

The cost of metal sheet per 1959.36 cm^2 $=$ ₹ $\dfrac{8 \times 1959.36}{100}$

$$= ₹\ 156.7488 \approx ₹\ 156.75$$

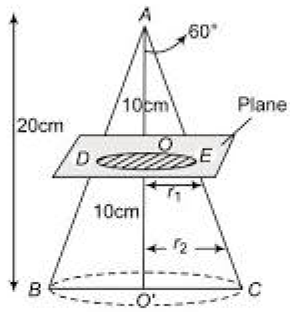
IndCareer

5. A metallic right circular cone 20 cm high and whose vertical angle is 60º is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter 1/16 cm, find the length of the wire.

Answer

Let r1 and r2 be the radii of the frustum of upper and lower ends cut by a plane. Given, height of the cone = 20 cm.

∴ Height of the frustum = 10 cm



Now, In ΔAOE,

$$\cot 60° = \frac{OE}{AO} = \frac{r_1}{10} \quad [\because \theta = 60° \text{ (given)}]$$

$$\Rightarrow \quad \frac{r_1}{10} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad r_1 = \frac{10}{\sqrt{3}} \text{ cm}$$

Again, in $\Delta AO'C$,

$$\cot 60° = \frac{O'C}{AO'} = \frac{r_2}{20} \quad [\because \theta = 60° \text{ (given)}]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{r_2}{20} \Rightarrow r_2 = \frac{20}{\sqrt{3}} \text{ cm}$$

$$\therefore \quad \text{Volume of the frustum} = \frac{\pi}{3} h(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{\pi}{3} \times 10 \left(\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right)$$

$$= \frac{7000\pi}{9} \text{ cm}^3$$

Let the length of the wire be h.

Given, the diameter of drawn wire from frustum is $\frac{1}{16}$ cm

$$\therefore \quad \text{Radius of wire} = \frac{1}{32} \text{ cm}$$

So, $\quad$ volume of wire (cylinder) $= \pi\left(\frac{1}{32}\right)^2 \times h$

Now, according to the question,

Volume of the frustum = Volume of the wire

$$\Rightarrow \quad \frac{7000\pi}{9} = \frac{\pi h}{32 \times 32}$$

$$\Rightarrow \quad h = \frac{7000 \times 32 \times 32}{9}$$

$$\Rightarrow \quad h = \frac{7000 \times 32 \times 32}{9 \times 100}$$

$$\Rightarrow \quad h = 7964.44 \text{ m}$$

Exercise 13.5 (Optional)

NCERT 10th Maths Chapter 13, class 10 Maths Chapter 13 solutions

1. A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm and diameter 10 cm, so as to cover the curved surface

of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Answer

Since, the diameter of the wire is 3 mm.

When a wire is one round wound about a cylinder, it covers a 3 mm of length of the cylinder.

Given, length of the cylinder = 12 cm = 120 mm

∴ Number of rounds to cover 120 mm = 120/3 = 40

Given, diameter of a cylinder is d = 10 cm

∴ Radius, r = 10/2 = 5 cm

∴ Length of wire required to complete one round = 2πr = 2π(5) = 10π cm

∴ Length of the wire in covering the whole surface

= Length of the wire in completing 40 rounds

= 10π × 40 = 400π cm

= 400 × 3.14 = 1256 cm

Now, radius of copper wire = 3/2 mm = 3/20 cm

$$\therefore \quad \text{Volume of wire} = \pi\left(\frac{3}{20}\right)^2 (400\,\pi) = 9\pi^2\ \text{cm}^3$$

$$\therefore \quad \text{Mass of wire} = 9\pi^2 \times \text{density}$$

$$= 9\,(3.14)^2 \times 8.88$$

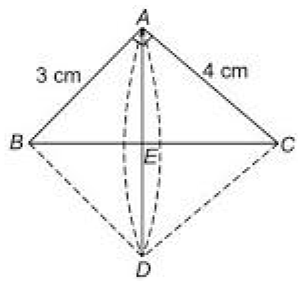
$$= 88.74 \times 8.88 = 788\ \text{g (approx)}$$

2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the

volume and surface area of the double cone so formed. (Choose value of π as found appropriate).

Answer

Here, ABC is a right angled triangle at A and BC is the hypotenuse.



$\therefore \quad BC = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ cm

As Δ ABC revolves about the hypotenuse BC. It formes two cones ABD and ACD.

Since, ΔAEB and ΔABC are similar.

$$\therefore \quad \frac{AE}{CA} = \frac{AB}{BC} \Rightarrow \frac{AE}{4} = \frac{3}{5}$$

$$\Rightarrow \quad AE = \frac{12}{5} = 2.4 \text{ cm}$$

So, radius of the base of cone = AE = 2.4

Now, in right angled ΔABE,

$$BE = \sqrt{AB^2 - AE^2} = \sqrt{9 - (2.4)^2} = \sqrt{3.24}$$

$$= 1.8 \text{ cm}$$

$$\therefore \quad CE = BC - BE = 5 - 1.8 = 3.2 \text{ cm}$$

Now, volume of the cone $ABD = \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 1.8$

$$= \frac{22}{21} \times 10.368 = 10.86 \text{ cm}^3$$

and volume of the cone $ACD = \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 3.2$

$$= \frac{405.504}{21} = 19.31 \text{ cm}^3$$

∴ Required volume of double cone = 10.86 + 19.31 = 30.17 cm^3

Now, surface area of cone $ABD = \pi r l = \frac{22}{7} \times 2.4 \times 3$

$$= \frac{158.4}{7} = 22.63 \text{ cm}^2$$

and surface area of cone $ACD = \frac{22}{7} \times 2.4 \times 4$

$$= \frac{211.2}{7} = 30.17 \text{ cm}^2$$

∴ Required surface area of double cone = 22.63 + 30.17 = 52.8 cm^2

3. A cistern, internally measuring 150 cm × 120 cm× 110 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. how many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Answer

Given, internally dimensions of cistern = 150 cm×120 cm×110 cm

∴ Volume of cistern = 150×120×110

= 1980000 cm^3

Volume of water = 129600 cm^3

∴ Volume of cistern to be filled = 1980000 - 129600 = 1850400 cm^3

Let required number of bricks = n

Then water absorbed by n bricks $= n\left(\frac{1096.875}{17}\right)$ cm^3

Now, $1850400 + n\left(\frac{1096.875}{17}\right) = n\,(1096.875)$

$\Rightarrow \quad n \times 1096.875 \times \frac{16}{17} = 1850400$

$\Rightarrow \quad n = \frac{1850400 \times 17}{1096.875 \times 16}$

$= 1792.4102 \approx 1792$ (approx)

Hence, 1792 brickes can be put in cistern.

NCERT 10th Maths Chapter 13, class 10 Maths Chapter 13 solutions

4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 97280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Answer

Given, area of the valley = 97280 km^2

and rainfall $= 10\text{ cm} = \dfrac{10}{100 \times 1000}\text{ km}$

$\therefore$ Volume of rainfall $= \dfrac{97280 \times 10}{100 \times 1000} = 9.728\text{ km}^3$ ...(i)

Also, given length of the river = 1072 km

breadth of the river $= 75\text{ m} = \dfrac{75}{1000}\text{ km}$

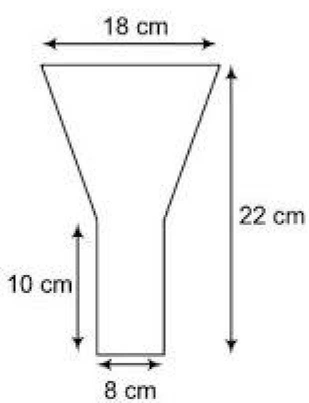
and height (deep) of the river $= 3\text{ m} = \dfrac{3}{1000}\text{ km}$

$\therefore$ Volume of one river $= 1072 \times \dfrac{75}{1000} \times \dfrac{3}{1000} = 0.2412\text{ km}^3$

So, volume of three rivers $= 3 \times 0.2412 = 0.7236\text{ km}^3$ ...(ii)

From Eqs. (i) and (ii), it is clear that total rainfall is not approximately equivalent to normal water of three rivers.

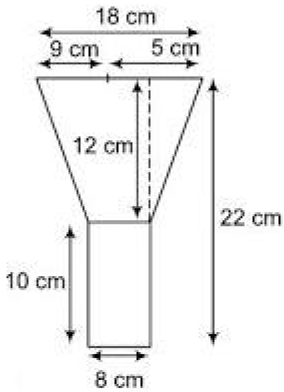
5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel.



Answer

Given, oil funnel is a combination of a cylinder and a frustum of a cone.

Also, given height of cylindrical portion h = 10 cm



Radius (r_1) of upper circular end of frustum part $= \frac{18}{2} = 9$ cm

Radius (r_2) of lower circular end of frustum part = radius of circular end of cylindrical part $= \frac{8}{2} = 4$ cm

Height (h_1) of frustum part $= 22 - 10 = 12$ cm

Height (h_2) of cylindrical part = 10 cm slant height (l) of frustum part

$= \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(9-4)^2 + (12)^2} = 13$ cm

Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

$= \pi(r_1 + r_2)l + 2\pi r_2 h_2$

$= \frac{22}{7} \times (9+4) \times 13 + 2 \times \frac{22}{7} \times 4 \times 10$

$= \frac{22}{7}[169 + 80] = \frac{22 \times 249}{7}$

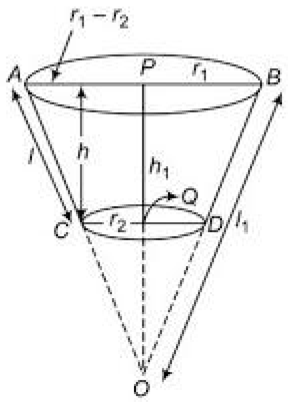
$= 782\frac{4}{7}$ cm^2

NCERT 10th Maths Chapter 13, class 10 Maths Chapter 13 solutions

6. Derive the formula for the curved surface area and total surface area of the frustum of a cone. Using the symbols as explained.

Answer

Leth be the height, l be the slant height and r_1 and r_2 be the radii of the bases (r_1>r_2) of the frustum of a cone. We complete the conical part OCD.



The frustum of the right circular cone can be viewed as the difference of the two right circular cones OAB and OCD. Let slant height of the cone OAB be l_1 and its height be h_1 i.e.,OB = OA = l_1 and OP = h_1

The frustum of the right circular cone can be viewed as the difference of the two right circular cones OAB and OCD. Let slant height of the cone OAB be l_1 and its height be h_1 i.e., OB = OA = l_1 and OP = h_1

Then in ΔACE,

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

Slant height of the cone $OCD = l_1 - l$

$\because$ $\Delta OQD \sim \Delta OPB$ (AA similarity criterion)

$$\frac{OD}{OB} = \frac{DQ}{PB} \Rightarrow \frac{l_1 - l}{l_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad 1 - \frac{l}{l_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{l}{l_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

$$\Rightarrow \quad l_1 = \frac{lr_1}{r_1 - r_2} \quad \ldots(i)$$

$$\therefore \quad l_1 - l = \frac{lr_1}{r_1 - r_2} - l = \frac{lr_2}{r_1 - r_2} \quad \ldots(ii)$$

Hence, curved surface area of the frustum of cone

= curved surface area of the cone OAB

− curved surface area of the cone OCD

$$= \pi r_1 l_1 - \pi r_2 (l_1 - l)$$

$$= \pi r_1 \frac{lr_1}{r_1 - r_2} - \pi r_2 \frac{lr_2}{r_1 - r_2} \quad \text{[From Eqs. (i) and (ii)]}$$

$$= \pi l \left[\frac{r_1^2 - r_2^2}{r_1 - r_2}\right] = \pi l (r_1 + r_2)$$

Therefore, curved surface area of the frustum of cone $= \pi l (r_1 + r_2)$

where, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

If A_1 and A_2 are the surface areas ($A_1 > A_2$) of two circular bases, then

$$A_1 = \pi r_1^2 \text{ and } A_2 = \pi r_2^2$$

So, the total surface area of the frustum of cone

$$= \pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$$

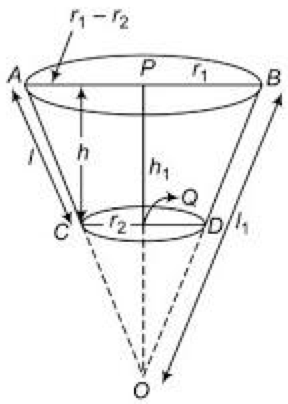
where, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

7. Derive the formula for the volume of the frustum of a cone given to you in the section 13.5 using the symbols as explained.

Answer

Leth be the height, l be the slant height and r_1 and r_2 be the radii of the bases ($r_1 > r_2$) of the frustum of a cone. We complete the conical part OCD.

The frustum of the right circular cone can be viewed as the difference of the two right circular cones OAB and OCD. Let the height of the cone OAB be h_1 and its slant height be l_1.



i.e., OP = h_1 and OA = OB = l_1

Then, height of the cone OCD = h_1 −1

ΔOQD ~ ΔOPB (AA similarity criterion)

$$\therefore \quad \frac{OQ}{OP} = \frac{QD}{PB}$$

$$\Rightarrow \quad \frac{h_1 - h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1} \Rightarrow h_1 = \frac{hr_1}{r_1 - r_2} \quad \text{...(i)}$$

Now, height of the cone $OCD = h_1 - h = \frac{hr_1}{r_1 - r_2} - h$ [From Eq. (i)]

$$= \frac{hr_2}{r_1 - r_2} \quad \text{...(ii)}$$

$\therefore$ Volume of the frustum of cone = Volume of the cone OAB – Volume of the cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$= \frac{\pi}{3}\left[r_1^2 \frac{hr_1}{r_1 - r_2} - r_2^2 \frac{hr_2}{r_1 - r_2}\right]$$

[From Eqs. (i) and (ii)]

$$= \frac{\pi h}{3}\left[\frac{r_1^3 - r_2^3}{r_1 - r_2}\right] = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

Therefore, volume of the frustum of cone is $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$.

If A_1 and A_2 are the surface areas ($A_1 > A_2$) of the two circular bases, then

$$A_1 = \pi r_1^2 \text{ and } A_2 = \pi r_2^2$$

Then, volume of the frustum of the cone $= \frac{h}{3}[\pi r_1^2 + \pi r_2^2 + \sqrt{\pi r_1^2}\sqrt{\pi r^2}]$

$$= \frac{h}{3}(A_1 + A_2 + \sqrt{A_1 A_2})$$

NCERT 10th Maths Chapter 13, class 10 Maths Chapter 13 solutions

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- Chapter 3 Pair of Linear Equations in Two Variables
- Chapter 4 Quadratic Equations
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