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Exercise 9.1

Question 1:

Write the first five terms of the sequences whose nth term is $a_n = n(n+2)$

Ans:

$a_n = n(n+2)$

Substituting n = 1, 2, 3, 4, and 5, we obtain

$$a_1 = 1(1+2) = 3$$
$$a_2 = 2(2+2) = 8$$
$$a_3 = 3(3+2) = 15$$
$$a_4 = 4(4+2) = 24$$
$$a_5 = 5(5+2) = 35$$

Therefore, the required terms are 3, 8, 15, 24, and 35.

Question 2:

Write the first five terms of the sequences whose nth term is $a_n = \frac{n}{n+1}$

Ans:

$a_n = \frac{n}{n+1}$

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = \frac{1}{1+1} = \frac{1}{2},\ a_2 = \frac{2}{2+1} = \frac{2}{3},\ a_3 = \frac{3}{3+1} = \frac{3}{4},\ a_4 = \frac{4}{4+1} = \frac{4}{5},\ a_5 = \frac{5}{5+1} = \frac{5}{6}$$

Therefore, the required terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, and $\frac{5}{6}$.

Question 3:

Write the first five terms of the sequences whose nth term is $a_n = 2^n$

Ans:

$a_n = 2^n$

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = 2^1 = 2$$
$$a_2 = 2^2 = 4$$
$$a_3 = 2^3 = 8$$
$$a_4 = 2^4 = 16$$
$$a_5 = 2^5 = 32$$

Therefore, the required terms are 2, 4, 8, 16, and 32.

Question 4:

Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{2n-3}{6}$

Ans:

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$$
$$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$$
$$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$$
$$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$$
$$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$$

Therefore, the required terms are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6},$ and $\frac{7}{6}$.

Question 5:

Write the first five terms of the sequences whose n^{th} term is $a_n = (-1)^{n-1} 5^{n+1}$

Ans:

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Therefore, the required terms are 25, –125, 625, –3125, and 15625.

Question 6:

Write the first five terms of the sequences whose n^{th} term is $a_n = n\frac{n^2+5}{4}$

Ans:

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = 1 \cdot \frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$$
$$a_2 = 2 \cdot \frac{2^2+5}{4} = 2 \cdot \frac{9}{4} = \frac{9}{2}$$
$$a_3 = 3 \cdot \frac{3^2+5}{4} = 3 \cdot \frac{14}{4} = \frac{21}{2}$$
$$a_4 = 4 \cdot \frac{4^2+5}{4} = 21$$
$$a_5 = 5 \cdot \frac{5^2+5}{4} = 5 \cdot \frac{30}{4} = \frac{75}{2}$$

Therefore, the required terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21,$ and $\frac{75}{2}$.

Question 7:

Find the 17th term in the following sequence whose n^{th} term is $a_n = 4n - 3; a_{17}, a_{24}$

Ans:

Substituting $n = 17$, we obtain

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Substituting $n = 24$, we obtain

$a_{24} = 4(24) - 3 = 96 - 3 = 93$

Question 8:

Find the 7th term in the following sequence whose n^{th} term is $a_n = \frac{n^2}{2n}; a_7$

Ans:

Substituting $n = 7$, we obtain

$$a_7 = \frac{7^2}{2^7} = \frac{49}{128}$$

Question 9:

Find the 9th term in the following sequence whose n^{th} term is $a_n = (-1)^{n-1} n^3; a_9$

Ans:

Substituting $n = 9$, we obtain

$$a_9 = (-1)^{9-1}(9)^3 = (9)^3 = 729$$

Question 10:

Find the 20th term in the following sequence whose n^{th} term is $a_n = \frac{n(n-2)}{n+3}; a_{20}$

Ans:

Substituting $n = 20$, we obtain

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20(18)}{23} = \frac{360}{23}$$

Question 11:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = 3, a_n = 3a_{n-1} + 2$ for all $n > 1$

Ans:

$a_1 = 3, a_n = 3a_{n-1} + 2$ for all $n > 1$

$$\Rightarrow a_2 = 3a_1 + 2 = 3(3) + 2 = 11$$
$$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$$
$$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$$
$$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$$

Hence, the first five terms of the sequence are 3, 11, 35, 107, and 323.

The corresponding series is 3 + 11 + 35 + 107 + 323 + ...

Question 12:

Write the first five terms of the following sequence and obtain the corresponding series:

$$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$$

Ans:

$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

$\Rightarrow a_2 = \frac{a_1}{2} = \frac{-1}{2}$

$a_3 = \frac{a_2}{3} = \frac{-1}{6}$

$a_4 = \frac{a_3}{4} = \frac{-1}{24}$

$a_5 = \frac{a_4}{4} = \frac{-1}{120}$

Hence, the first five terms of the sequence are $-1, \frac{-1}{2}, \frac{-1}{6}, \frac{-1}{24}$, and $\frac{-1}{120}$.

The corresponding series is $(-1)+\left(\frac{-1}{2}\right)+\left(\frac{-1}{6}\right)+\left(\frac{-1}{24}\right)+\left(\frac{-1}{120}\right)+\ldots$

Question 13:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

Ans:

$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

$\Rightarrow a_3 = a_2 - 1 = 2 - 1 = 1$

$a_4 = a_3 - 1 = 1 - 1 = 0$

$a_5 = a_4 - 1 = 0 - 1 = -1$

Hence, the first five terms of the sequence are 2, 2, 1, 0, and −1.

The corresponding series is 2 + 2 + 1 + 0 + (−1) + ...

Question 14:

The Fibonacci sequence is defined by

$1 = a_1 = a_2 \text{ and } a_n = a_{n-1} + a_{n-2}, n > 2$

Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5

Ans:

$$1 = a_1 = a_2$$
$$a_n = a_{n-1} + a_{n-2}, n > 2$$
$$\therefore a_3 = a_2 + a_1 = 1 + 1 = 2$$
$$a_4 = a_3 + a_2 = 2 + 1 = 3$$
$$a_5 = a_4 + a_3 = 3 + 2 = 5$$
$$a_6 = a_5 + a_4 = 5 + 3 = 8$$

$$\therefore \text{For } n = 1, \ \frac{a_n + 1}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$$

$$\text{For } n = 2, \ \frac{a_n + 1}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2$$

$$\text{For } n = 3, \frac{a_n + 1}{a_n} = \frac{a_4}{a_3} = \frac{3}{2}$$

$$\text{For } n = 4, \ \frac{a_n + 1}{a_n} = \frac{a_5}{a_4} = \frac{5}{3}$$

$$\text{For } n = 5, \ \frac{a_n + 1}{a_n} = \frac{a_6}{a_5} = \frac{8}{5}$$

Exercise 9.2

Question 1:

Find the sum of odd integers from 1 to 2001.

Ans:

The odd integers from 1 to 2001 are 1, 3, 5, ...1999, 2001.

This sequence forms an A.P.

Here, first term, a = 1

Common difference, d = 2

$$\text{Here, } a+(n-1)d = 2001$$
$$\Rightarrow 1+(n-1)(2) = 2001$$
$$\Rightarrow 2n-2 = 2000$$
$$\Rightarrow n = 1001$$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$\therefore S_n = \frac{1001}{2}\left[2\times1+(1001-1)\times2\right]$$
$$= \frac{1001}{2}\left[2+1000\times2\right]$$
$$= \frac{1001}{2}\times2002$$
$$= 1001\times1001$$
$$= 1002001$$

Thus, the sum of odd numbers from 1 to 2001 is 1002001.

Question 2:

Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Ans:

The natural numbers lying between 100 and 1000, which are multiples of 5, are 105, 110, ... 995.

Here, $a = 105$ and $d = 5$

$$a + (n-1)d = 995$$
$$\Rightarrow 105 + (n-1)5 = 995$$
$$\Rightarrow (n-1)5 = 995 - 105 = 890$$
$$\Rightarrow n - 1 = 178$$
$$\Rightarrow n = 179$$

$$\therefore S_n = \frac{179}{2}\left[2(105) + (179-1)(5)\right]$$
$$= \frac{179}{2}\left[2(105) + (178)(5)\right]$$
$$= 179\left[105 + (89)5\right]$$
$$= (179)(105 + 445)$$
$$= (179)(550)$$
$$= 98450$$

Question 3:

In an A.P, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Ans:

First term = 2

Let d be the common difference of the A.P.

Therefore, the A.P. is 2, 2 + d, 2 + 2d, 2 + 3d, …

Sum of first five terms = 10 + 10d

Sum of next five terms = 10 + 35d

According to the given condition,

$$10+10d=\frac{1}{4}(10+35d)$$
$$\Rightarrow 40+40d=10+35d$$
$$\Rightarrow 30=-5d$$
$$\Rightarrow d=-6$$

$$\therefore a_{20}=a+(20-1)d=2+(19)(-6)=2-114=-112$$

Thus, the 20th term of the A.P. is –112.

Question 4:

Let the sum of n terms of the given A.P. be –25.

It is known that, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, where n = number of terms, a = first term, and d = common difference

Here, a = –6

$$d = -\frac{11}{2}+6 = \frac{-11+12}{2} = \frac{1}{2}$$

Therefore, we obtain

$$-25 = \frac{n}{2}\left[2\times(-6)+(n-1)\left(\frac{1}{2}\right)\right]$$
$$\Rightarrow -50 = n\left[-12+\frac{n}{2}-\frac{1}{2}\right]$$
$$\Rightarrow -50 = n\left[-\frac{25}{2}+\frac{n}{2}\right]$$
$$\Rightarrow -100 = n(-25+n)$$
$$\Rightarrow n^2-25n+100=0$$
$$\Rightarrow n^2-5n-20n+100=0$$
$$\Rightarrow n(n-5)-20(n-5)=0$$
$$\Rightarrow n = 20 \text{ or } 5$$

Ans:

Let the sum of *n* terms of the given A.P. be –25.

It is known that, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, where *n* = number of terms, *a* = first term, and *d* = common difference

Here, *a* = –6

$$d = -\frac{11}{2} + 6 = \frac{-11+12}{2} = \frac{1}{2}$$

Therefore, we obtain

$$-25 = \frac{n}{2}\left[2\times(-6)+(n-1)\left(\frac{1}{2}\right)\right]$$
$$\Rightarrow -50 = n\left[-12+\frac{n}{2}-\frac{1}{2}\right]$$
$$\Rightarrow -50 = n\left[-\frac{25}{2}+\frac{n}{2}\right]$$
$$\Rightarrow -100 = n(-25+n)$$
$$\Rightarrow n^2 - 25n + 100 = 0$$
$$\Rightarrow n^2 - 5n - 20n + 100 = 0$$
$$\Rightarrow n(n-5) - 20(n-5) = 0$$
$$\Rightarrow n = 20 \text{ or } 5$$

Question 5:

Ans:

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Question 6:

Ans:

Let the sum of n terms of the given A.P. be 116.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Here, $a = 25$ and $d = 22 - 25 = -3$

$$\therefore S_n = \frac{n}{2}\left[2 \times 25 + (n-1)(-3)\right]$$

$$\Rightarrow 116 = \frac{n}{2}[50 - 3n + 3]$$

$$\Rightarrow 232 = n(53 - 3n) = 53n - 3n^2$$

$$\Rightarrow 3n^2 - 53n + 232 = 0$$

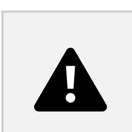
$$\Rightarrow 3n^2 - 24n - 29n + 232 = 0$$

$$\Rightarrow 3n(n-8) - 29(n-8) = 0$$

$$\Rightarrow (n-8)(3n-29) = 0$$

$$\Rightarrow n = 8 \text{ or } n = \frac{29}{3}$$

Question 7:



Ans:

It is given that the k^{th} term of the A.P. is $5k + 1$.

k^{th} term $= a_k = a + (k - 1)d$

$a + (k - 1)d = 5k + 1$

$a + kd - d = 5k + 1$

Comparing the coefficient of k, we obtain $d = 5$

$a - d = 1$

$a - 5 = 1$

$a = 6$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{n}{2}\left[2(6) + (n-1)(5)\right]$$
$$= \frac{n}{2}[12 + 5n - 5]$$
$$= \frac{n}{2}(5n + 7)$$

Question 8:

If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants, find the common difference.

Ans:

Question 9:

Ans:

Question 10:

Ans:

Let a_1 and d be the first term and the common difference of the A.P. respectively.

According to the given information,

$$S_p = \frac{p}{2}\left[2a_1 + (p-1)d\right] = a$$

$$\Rightarrow 2a_1 + (p-1)d = \frac{2a}{p} \qquad ...(1)$$

$$S_q = \frac{q}{2}\left[2a_1 + (q-1)d\right] = b$$

$$\Rightarrow 2a_1 + (q-1)d = \frac{2b}{q} \qquad ...(2)$$

$$S_r = \frac{r}{2}\left[2a_1 + (r-1)d\right] = c$$

$$\Rightarrow 2a_1 + (r-1)d = \frac{2c}{r} \qquad ...(3)$$

Subtracting (2) from (1), we obtain

Question 11:

Ans:

Question 12:

Ans:

Let A_1, A_2, A_3, A_4, and A_5 be five numbers between 8 and 26 such that

8, A_1, A_2, A_3, A_4, A_5, 26 is an A.P.

Here, $a = 8$, $b = 26$, $n = 7$

Therefore, $26 = 8 + (7 - 1)\, d$

$6d = 26 - 8 = 18$

$d = 3$

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2 \times 3 = 8 + 6 = 14$

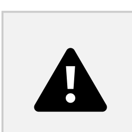
$A_3 = a + 3d = 8 + 3 \times 3 = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 4 \times 3 = 8 + 12 = 20$

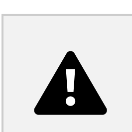
$A_5 = a + 5d = 8 + 5 \times 3 = 8 + 15 = 23$

Thus, the required five numbers between 8 and 26 are 11, 14, 17, 20, and 23.

Question 13:



Ans:



Question 14:

Ans:

Question 15:

Ans:

Question 16:

Ans:

Exercise 9.3

Question 1:

Ans:

Question 2:

Ans:

Common ratio, $r = 2$

Let a be the first term of the G.P.

$a_8 = ar^{8-1} = ar^7$

$ar^7 = 192$

$a(2)^7 = 192$

$a(2)^7 = (2)^6\,(3)$

$$\Rightarrow a = \frac{(2)^6 \times 3}{(2)^7} = \frac{3}{2}$$

$$\therefore a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$$

Question 3:

Ans:

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_5 = a\,r^{5-1} = a\,r^4 = p$ ... (1)

$a_8 = a\,r^{8-1} = a\,r^7 = q$ ... (2)

$a_{11} = a\,r^{11-1} = a\,r^{10} = s$ ... (3)

Dividing equation (2) by (1), we obtain

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \qquad ...(4)$$

Dividing equation (3) by (2), we obtain

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$

$$\Rightarrow r^3 = \frac{s}{q} \qquad ...(5)$$

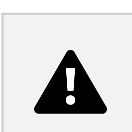
Equating the values of r^3 obtained in (4) and (5), we obtain

$$\frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow q^2 = ps$$

Thus, the given result is proved.

Question 4:



Ans:

Question 5:

Ans:

Question 6:

Ans:

Question 7:

Ans:

Question 8:

Ans:

Question 9:

Ans:

Question 10:

Ans:

Question 11:

Ans:

Question 12:

Ans:

Question 13:

Ans:

Question 14:

Ans:

Question 15:

Ans:

Question 16:

Ans:

$$\Rightarrow a = \frac{-4}{3}$$

$$\text{Also, } -4 = \frac{a\left[1-(-2)^2\right]}{1-(-2)} \text{ for } r = -2$$

$$\Rightarrow -4 = \frac{a(1-4)}{1+2}$$

$$\Rightarrow -4 = \frac{a(-3)}{3}$$

$$\Rightarrow a = 4$$

Thus, the required G.P. is

$$\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots \text{ or } 4, -8, 16, -32, \ldots$$

Question 17:

If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P.

Ans:

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_4 = a\,r^3 = x$ ... (1)

$a_{10} = a\,r^9 = y$ ... (2)

$a_{16} = a\,r^{15} = z$ ... (3)

Dividing (2) by (1), we obtain

$$\frac{y}{x} = \frac{ar^9}{ar^3} \Rightarrow \frac{y}{x} = r^6$$

Dividing (3) by (2), we obtain

$$\frac{z}{y} = \frac{ar^{15}}{ar^9} \Rightarrow \frac{z}{y} = r^6$$

$$\frac{y}{x} = \frac{z}{y}$$

Thus, x, y, z are in G. P.

Question 18:

Find the sum to n terms of the sequence, 8, 88, 888, 8888…

Ans:

The given sequence is 8, 88, 888, 8888…

This sequence is not a G.P. However, it can be changed to G.P. by writing the terms as

S_n = 8 + 88 + 888 + 8888 + …………….. to n terms

$$= \frac{8}{9}[9 + 99 + 999 + 9999 + \text{..........to } n \text{ terms}]$$

$$= \frac{8}{9}\left[(10-1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \text{........to } n \text{ terms}\right]$$

$$= \frac{8}{9}\left[(10 + 10^2 + \text{......} n \text{ terms}) - (1 + 1 + 1 + \text{....} n \text{ terms})\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$$

$$= \frac{80}{81}(10^n - 1) - \frac{8}{9}n$$

Question 19:

Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

Ans:

Required sum = $2\times 128+4\times 32+8\times 8+16\times 2+32\times \frac{1}{2}$

$=64\left[4+2+1+\frac{1}{2}+\frac{1}{2^2}\right]$

Here, 4, 2, 1, $\frac{1}{2},\frac{1}{2^2}$ is a G.P.

First term, a = 4

Common ratio, $r=\frac{1}{2}$

It is known that, $S_n=\frac{a(1-r^n)}{1-r}$

$$\therefore S_5=\frac{4\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}}=\frac{4\left[1-\frac{1}{32}\right]}{\frac{1}{2}}=8\left(\frac{32-1}{32}\right)=\frac{31}{4}$$

Required sum = $64\left(\frac{31}{4}\right)=(16)(31)=496$

Question 20:

Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \ldots ar^{n-1}$ and $A, AR, AR^2, \ldots AR^{n-1}$ form a G.P, and find the common ratio.

Ans:

It has to be proved that the sequence, aA, $arAR$, ar^2AR^2, ...$ar^{n-1}AR^{n-1}$, forms a G.P.

$$\frac{\text{Second term}}{\text{First term}} = \frac{arAR}{aA} = rR$$
$$\frac{\text{Third term}}{\text{Second term}} = \frac{ar^2AR^2}{arAR} = rR$$

Thus, the above sequence forms a G.P. and the common ratio is rR.

Question 21:

Let a be the first term and r be the common ratio of the G.P.

$a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

By the given condition,

$a_3 = a_1 + 9$

$ar^2 = a + 9$ ... (1)

$a_2 = a_4 + 18$

$ar = ar^3 + 18$ ... (2)

From (1) and (2), we obtain

$a(r^2 - 1) = 9$ ... (3)

$ar(1 - r^2) = 18$ ... (4)

Dividing (4) by (3), we obtain

Ans:

Let a be the first term and r be the common ratio of the G.P.

$a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

By the given condition,

$a_3 = a_1 + 9$

$ar^2 = a + 9$ … (1)

$a_2 = a_4 + 18$

$ar = ar^3 + 18$ … (2)

From (1) and (2), we obtain

$a(r^2 - 1) = 9$ … (3)

$ar(1 - r^2) = 18$ … (4)

Dividing (4) by (3), we obtain

$$\frac{ar(1-r^2)}{a(r^2-1)} = \frac{18}{9}$$
$$\Rightarrow -r = 2$$
$$\Rightarrow r = -2$$

Substituting the value of r in (1), we obtain

$4a = a + 9$

$3a = 9$

$a = 3$

Thus, the first four numbers of the G.P. are 3, $3(-2)$, $3(-2)^2$, and $3(-2)^3$ i.e., 3, –6, 12, and –24.

Question 22:

If the p^{th}, q^{th} and r^{th} terms of a G.P. are *a*, *b* and *c*, respectively. Prove that $a^{q-r}b^{r-p}c^{p-q}=1$

Ans:

Let *A* be the first term and *R* be the common ratio of the G.P.

According to the given information,

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

$a^{q-r}\, b^{r-p}\, c^{p-q}$

$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= A^{q-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Thus, the given result is proved.

Question 23:

If the first and the n^{th} term of a G.P. are a ad b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Ans:

The first term of the G.P is a and the last term is b.

Therefore, the G.P. is a, ar, ar^2, ar^3, … ar^{n-1}, where r is the common ratio.

$b = ar^{n-1}$ … (1)

P = Product of n terms

$= (a)\,(ar)\,(ar^2) \ldots (ar^{n-1})$

$= (a \times a \times \ldots a)\,(r \times r^2 \times \ldots r^{n-1})$

$= a^n\, r^{1+2+\ldots(n-1)}$ … (2)

Here, 1, 2, …(n – 1) is an A.P.

$$1 + 2 + \ldots\ldots\ldots + (n-1) = \frac{n-1}{2}\left[2+(n-1-1)\times 1\right] = \frac{n-1}{2}\left[2+n-2\right] = \frac{n(n-1)}{2}$$

$$P = a^n r^{\frac{n(n-1)}{2}}$$
$$\therefore P^2 = a^{2n} r^{n(n-1)}$$
$$= \left[a^2 r^{(n-1)}\right]^n$$
$$= \left[a \times ar^{n-1}\right]^n$$
$$= (ab)^n \qquad \left[\text{Using } (1)\right]$$

Thus, the given result is proved.

Question 24:

Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Ans:

Let a be the first term and r be the common ratio of the G.P.

Sum of first n terms $= \frac{a\left(1-r^{n}\right)}{(1-r)}$

Since there are n terms from $(n+1)^{th}$ to $(2n)^{th}$ term,

Sum of terms from$(n + 1)^{th}$ to $(2n)^{th}$ term $= \frac{a_{n+1}\left(1-r^{n}\right)}{(1-r)}$

$a^{n+1} = ar^{n+1-1} = ar^{n}$

Thus, required ratio = $\frac{a\left(1-r^{n}\right)}{(1-r)} \times \frac{(1-r)}{ar^{n}\left(1-r^{n}\right)} = \frac{1}{r^{n}}$

Thus, the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^{n}}$.

Question 25:

If a, b, c and d are in G.P. show that $\left(a^{2}+b^{2}+c^{2}\right)\left(b^{2}+c^{2}+d^{2}\right)=(ab+bc+cd)^{2}$.

Ans:

a, b, c, d are in G.P.

Therefore,

$bc = ad$ … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

It has to be proved that,

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc - cd)^2$

R.H.S.

$= (ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [Using (1)]

$= [ab + d(a + c)]^2$

$= a^2b^2 + 2abd(a + c) + d^2(a + c)^2$

$= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [Using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$

[Using (2) and (3) and rearranging terms]

$= a^2(b^2 + c^2 + d^2) + b^2 (b^2 + c^2 + d^2) + c^2 (b^2 + c^2 + d^2)$

$= (a^2 + b^2 + c^2) (b^2 + c^2 + d^2)$

= L.H.S.

L.H.S. = R.H.S.

$$\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=\left(ab+bc+cd\right)^2$$

Question 26:

Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Ans:

Let G_1 and G_2 be two numbers between 3 and 81 such that the series, 3, G_1, G_2, 81, forms a G.P.

Let a be the first term and r be the common ratio of the G.P.

$81 = (3)(r)^3$

$r^3 = 27$

$r = 3$ (Taking real roots only)

For $r = 3$,

$G_1 = ar = (3)(3) = 9$

$G_2 = ar^2 = (3)(3)^2 = 27$

Thus, the required two numbers are 9 and 27.

Question 27:

The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Ans:

Let the two numbers be a and b.

G.M. = $\sqrt{ab}$

According to the given condition,

$$a+b=6\sqrt{ab} \quad ...(1)$$
$$\Rightarrow (a+b)^2 = 36(ab)$$

Also,

$$(a-b)^2 = (a+b)^2 - 4ab = 36ab - 4ab = 32ab$$
$$\Rightarrow a-b = \sqrt{32}\sqrt{ab}$$
$$= 4\sqrt{2}\sqrt{ab} \quad ...(2)$$

Adding (1) and (2), we obtain

$$2a = \left(6+4\sqrt{2}\right)\sqrt{ab}$$
$$\Rightarrow a = \left(3+2\sqrt{2}\right)\sqrt{ab}$$

Substituting the value of a in (1), we obtain

$$b = 6\sqrt{ab} - \left(3+2\sqrt{2}\right)\sqrt{ab}$$
$$\Rightarrow b = \left(3-2\sqrt{2}\right)\sqrt{ab}$$
$$\frac{a}{b} = \frac{\left(3+2\sqrt{2}\right)\sqrt{ab}}{\left(3-2\sqrt{2}\right)\sqrt{ab}} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Thus, the required ratio is $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Question 28:

If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Ans:

It is given that *A* and *G* are A.M. and G.M. between two positive numbers. Let these two positive numbers be *a* and *b*.

$$\therefore AM = A = \frac{a+b}{2} \quad ...(1)$$

$$GM = G = \sqrt{ab} \quad ...(2)$$

From (1) and (2), we obtain

$a + b = 2A$ ... (3)

$ab = G^2$ ... (4)

Substituting the value of *a* and *b* from (3) and (4) in the identity $(a-b)^2 = (a+b)^2 - 4ab$, we obtain

$(a-b)^2 = 4A^2 - 4G^2 = 4(A^2 - G^2)$

$(a-b)^2 = 4(A+G)(A-G)$

$$(a-b) = 2\sqrt{(A+G)(A-G)} \quad ...(5)$$

From (3) and (5), we obtain

$$2a = 2A + 2\sqrt{(A+G)(A-G)}$$
$$\Rightarrow a = A + \sqrt{(A+G)(A-G)}$$

Substituting the value of *a* in (3), we obtain

$$b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$$

Thus, the two numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Question 29:

The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour?

Ans:

It is given that the number of bacteria doubles every hour. Therefore, the number of bacteria after every hour will form a G.P.

Here, $a = 30$ and $r = 2$

$a_3 = ar^2 = (30)(2)^2 = 120$

Therefore, the number of bacteria at the end of 2^{nd} hour will be 120.

$a_5 = ar^4 = (30)(2)^4 = 480$

The number of bacteria at the end of 4^{th} hour will be 480.

$a_{n+1} = ar^n = (30)\, 2^n$

Thus, number of bacteria at the end of n^{th} hour will be $30(2)^n$.

Question 30:

If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Ans:

Let the root of the quadratic equation be a and b.

According to the given condition,

$$\text{A.M.} = \frac{a+b}{2} = 8 \Rightarrow a+b=16 \qquad ...(1)$$
$$\text{G.M.} = \sqrt{ab} = 5 \Rightarrow ab = 25 \qquad ...(2)$$

The quadratic equation is given by,

$x^2 - x$ (Sum of roots) + (Product of roots) = 0

$x^2 - x(a+b) + (ab) = 0$

$x^2 - 16x + 25 = 0$ [Using (1) and (2)]

Thus, the required quadratic equation is $x^2 - 16x + 25 = 0$

Exercise 9.4

Question 1:

Find the sum to n terms of the series $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \ldots$

Ans:

The given series is 1 × 2 × 3 + 2 × 3 × 4 + 3 × 4 × 5 + ...

n^{th} term, $a_n = n\,(n+1)\,(n+2)$

$= (n^2 + n)\,(n + 2)$

$= n^3 + 3n^2 + 2n$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$

$$= \sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + 2n + 1 + 2\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{n^2 + n + 4n + 6}{2}\right]$$

$$= \frac{n(n+1)}{4}\left(n^2 + 5n + 6\right)$$

$$= \frac{n(n+1)}{4}\left(n^2 + 2n + 3n + 6\right)$$

$$= \frac{n(n+1)\left[n(n+2) + 3(n+2)\right]}{4}$$

$$= \frac{n(n+1)(n+2)(n+3)}{4}$$

Question 2:

Find the sum to n terms of the series $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots$

Ans:

The given series is $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots$

n^{th} term, $a_n = (2n + 1)\, n^2 = 2n^3 + n^2$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$

$$= \sum_{k=1}^{n} = \left(2k^3 + k^2\right) = 2\sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n^2(n+1)^2}{2} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left[n(n+1) + \frac{2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2 + 3n + 2n + 1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2 + 5n + 1}{3}\right]$$

$$= \frac{n(n+1)(3n^2 + 5n + 1)}{6}$$

Question 3:

Find the sum to n terms of the series $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots$

Ans:

The given series is $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+...$

n^{th} term, $a_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$ (By partial fractions)

$$a_1 = \frac{1}{1} - \frac{1}{2}$$
$$a_2 = \frac{1}{2} - \frac{1}{3}$$
$$a_3 = \frac{1}{3} - \frac{1}{4} ...$$
$$a_n = \frac{1}{n} - \frac{1}{n+1}$$

Adding the above terms column wise, we obtain

$$a_1 + a_2 + ... + a_n = \left[\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...\frac{1}{n}\right] - \left[\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...\frac{1}{n+1}\right]$$
$$\therefore S_n = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$$

Question 4:

Find the sum to n terms of the series $5^2+6^2+7^2+...+20^2$

Ans:

Question 5:

Ans:

The given series is 3 × 8 + 6 × 11 + 9 × 14 + ...

a_n = (n^{th} term of 3, 6, 9 ...) × (n^{th} term of 8, 11, 14, ...)

= (3n) (3n + 5)

= 9n^2 + 15n

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \left(9k^2 + 15k\right)$$
$$= 9\sum_{k=1}^{n} k^2 + 15\sum_{k=1}^{n} k$$
$$= 9 \times \frac{n(n+1)(2n+1)}{6} + 15 \times \frac{n(n+1)}{2}$$
$$= \frac{3n(n+1)(2n+1)}{2} + \frac{15n(n+1)}{2}$$
$$= \frac{3n(n+1)}{2}(2n+1+5)$$
$$= \frac{3n(n+1)}{2}(2n+6)$$
$$= 3n(n+1)(n+3)$$

Question 6:

Find the sum to n terms of the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots$

Ans:

The given series is $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^3) + \ldots$

$a_n = (1^2 + 2^2 + 3^3 + \ldots\ldots + n^2)$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(2n^2+3n+1)}{6} = \frac{2^3+3n^2+n}{6}$$

$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$

$$= \sum_{k=1}^{n}\left(\frac{1}{3}k^3 + \frac{1}{2}k^2 + \frac{1}{6}k\right)$$

$$= \frac{1}{3}\sum_{k=1}^{n}k^3 + \frac{1}{2}\sum_{k=1}^{n}k^2 + \frac{1}{6}\sum_{k=1}^{n}k$$

$$= \frac{1}{3}\frac{n^2(n+1)^2}{(2)^2} + \frac{1}{2}\times\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\times\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{6}\left[\frac{n(n+1)}{2} + \frac{(2n+1)}{2} + \frac{1}{2}\right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n^2+n+2n+1+1}{2}\right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n^2+n+2n+2}{2}\right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n(n+1)+2(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{6}\left[\frac{(n+1)(n+2)}{2}\right]$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

Question 7:

Find the sum to n terms of the series whose n^{th} term is given by $n(n + 1)(n + 4)$.

Ans:

$a_n = n(n + 1)(n + 4) = n(n^2 + 5n + 4) = n^3 + 5n^2 + 4n$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 4\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+20n+10+24}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+23n+34}{6}\right]$$

$$= \frac{n(n+1)(3n^2+23n+34)}{12}$$

Question 8:

Find the sum to n terms of the series whose n^{th} terms is given by $n^2 + 2^n$

Ans:

$a_n = n^2 + 2^n$

$$\therefore S_n = \sum_{k=1}^{n} k^2 + 2^k = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 2^k \qquad (1)$$

Consider $\sum_{k=1}^{n} 2^k = 2^1 + 2^2 + 2^3 + \ldots$

The above series 2, 2^2, 2^3, … is a G.P. with both the first term and common ratio equal to 2.

$$\therefore \sum_{k=1}^{n} 2^k = \frac{(2)\left[(2)^n - 1\right]}{2-1} = 2\left(2^n - 1\right) \qquad (2)$$

Therefore, from (1) and (2), we obtain

$$S_n = \sum_{k=1}^{n} k^2 + 2\left(2^n - 1\right) = \frac{n(n+1)(2n+1)}{6} + 2\left(2^n - 1\right)$$

Question 9:

Find the sum to n terms of the series whose n^{th} terms is given by $(2n-1)^2$

Ans:

$a_n = (2n-1)^2 = 4n^2 - 4n + 1$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n}\left(4k^2 - 4k + 1\right)$$

$$= 4\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$$

$$= \frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n$$

$$= n\left[\frac{2\left(2n^2+3n+1\right)}{3} - 2(n+1) + 1\right]$$

$$= n\left[\frac{4n^2+6n+2-6n-6+3}{3}\right]$$

$$= n\left[\frac{4n^2-1}{3}\right]$$

$$= \frac{n(2n+1)(2n-1)}{3}$$

NCERT 11th Maths Chapter 9, class 11 Maths Chapter 9 solutions

Chapterwise NCERT Solutions for Class 11 Maths :

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