

NCERT Solutions for 11th Class Maths: Chapter 3-Trigonometric Functions

Class 11: Maths Chapter 3 solutions. Complete Class 11 Maths Chapter 3 Notes.

NCERT Solutions for 11th Class Maths: Chapter 3-Trigonometric Functions

NCERT 11th Maths Chapter 3, class 11 Maths Chapter 3 solutions

Exercise 3.1

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Question 1:

Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) - 47° 30' (iii) 240° (iv) 520°

Ans:



(i) 25°

We know that $180^\circ = \pi$ radian

 $\therefore 25^{\circ} = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$ (ii) - 47° 30' - 47° 30' = -47 $\frac{1}{2}$ degree [1° = 60'] -95

$$=\frac{-95}{2}$$
 degree

Since $180^\circ = \pi$ radian

$$\frac{-95}{2} \operatorname{deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \operatorname{radian} = \left(\frac{-19}{36 \times 2}\right) \pi \operatorname{radian} = \frac{-19}{72} \pi \operatorname{radian}$$
$$\therefore -47^{\circ} \ 30' = \frac{-19}{72} \pi \operatorname{radian}$$
(iii) 240°

We know that $180^\circ = \pi$ radian

$$\therefore 240^\circ = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

We know that $180^\circ = \pi$ radian

$$\therefore 520^\circ = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

Question 2:





Find the degree measures corresponding to the following radian measures

$$\left(\text{Use } \pi = \frac{22}{7}\right).$$

(i) $\frac{11}{16}$ (ii) $- 4$ (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Ans:

(i)
$$\frac{11}{16}$$

We know that π radian = 180°

$$\therefore \frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree} = \frac{45 \times 11}{\pi \times 4} \text{ deg ree}$$
$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree} = \frac{315}{8} \text{ deg ree}$$
$$= 39\frac{3}{8} \text{ deg ree}$$
$$= 39^{\circ} + \frac{3 \times 60}{8} \text{ min utes} \qquad [1^{\circ} = 60']$$
$$= 39^{\circ} + 22' + \frac{1}{2} \text{ min utes}$$
$$= 39^{\circ} 22' 30'' \qquad [1' = 60'']$$



(ii) – 4

We know that π radian = 180°

$$-4 \operatorname{radian} = \frac{180}{\pi} \times (-4) \operatorname{deg ree} = \frac{180 \times 7(-4)}{22} \operatorname{deg ree}$$
$$= \frac{-2520}{11} \operatorname{deg ree} = -229 \frac{1}{11} \operatorname{deg ree}$$
$$= -229^{\circ} + \frac{1 \times 60}{11} \operatorname{min utes} \qquad [1^{\circ} = 60']$$
$$= -229^{\circ} + 5' + \frac{5}{11} \operatorname{min utes}$$
$$= -229^{\circ} 5' 27'' \qquad [1' = 60'']$$

(iii) $\frac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3} \text{ radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ deg ree} = 300^{\circ}$$
(iv) $\frac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6} \text{ radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210^{\circ}$$

Question 3:

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Ans:



Number of revolutions made by the wheel in 1 minute = 360

: Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian, i.e.,

 $12 \,\pi \, radian$

Thus, in one second, the wheel turns an angle of 12π radian.

Question 4:

In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Ans:



Diameter of the circle = 40 cm

:. Radius (r) of the circle =
$$\frac{40}{2}$$
 cm = 20 cm

Let AB be a chord (length = 20 cm) of the circle.

In $\triangle OAB$, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, $\triangle OAB$ is an equilateral triangle.

$$\therefore \theta = 60^\circ = \frac{\pi}{3}$$
 radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$.

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Longrightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm .

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at

the centre, then
$$\theta = \frac{r}{r}$$

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Longrightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm

Question 5:



If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Ans:

Let the radii of the two circles be r_1 and r_2 . Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1 , while let an arc of length l subtend an angle of 75° at the centre of the circle of radius r_2 .

Now, $60^\circ = \frac{\pi}{3}$ radian and $75^\circ = \frac{5\pi}{12}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$ or $l = r\theta$.

$$\therefore l = \frac{r_1 \pi}{3} \text{ and } l = \frac{r_2 5 \pi}{12}$$
$$\Rightarrow \frac{r_1 \pi}{3} = \frac{r_2 5 \pi}{12}$$
$$\Rightarrow r_1 = \frac{r_2 5}{4}$$
$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

Thus, the ratio of the radii is 5:4.

Exercise 3.2

Question 1:

Find the values of other five trigonometric functions if $\cos x = -\frac{1}{2}$, x lies in third quadrant.

Ans:



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$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$

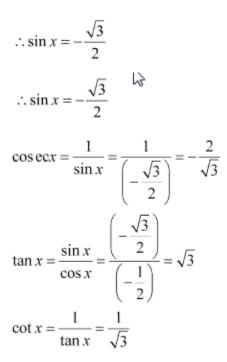
$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$$

Since x lies in the 3^{rd} quadrant, the value of sin x will be negative.





Question 2:

Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, x lies in third quadrant.

Ans:



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$$\cot x = \frac{3}{4}$$
$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$
$$1 + \tan^2 x = \sec^2 x$$
$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$
$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$
$$\Rightarrow \frac{25}{9} = \sec^2 x$$
$$\Rightarrow \sec x = \pm \frac{5}{3}$$

Since x lies in the 3^{rd} quadrant, the value of sec x will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\csc x = \frac{1}{\sin x} = -\frac{5}{4}$$

Question 3:



Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, x lies in fourth quadrant.

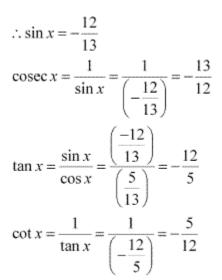
Ans:



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$$\sec x = \frac{13}{5}$$
$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$
$$\sin^2 x + \cos^2 x = 1$$
$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$
$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$
$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$
$$\Rightarrow \sin x = \pm \frac{12}{13}$$

Since x lies in the 4^{th} quadrant, the value of sin x will be negative.



Question 4:



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Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$, x lies in second quadrant.

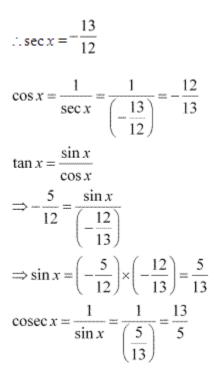
Ans:



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$$\tan x = -\frac{5}{12}$$
$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$
$$1 + \tan^2 x = \sec^2 x$$
$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$
$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$
$$\Rightarrow \frac{169}{144} = \sec^2 x$$
$$\Rightarrow \sec x = \pm \frac{13}{12}$$

Since x lies in the 2^{nd} quadrant, the value of sec x will be negative.





Question 5:

Find the value of the trigonometric function sin 765°

Ans:

It is known that the values of sin x repeat after an interval of 2π or 360° .

$$\therefore \sin 765^\circ = \sin (2 \times 360^\circ + 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

Question 6:

Find the value of the trigonometric function $\tan \frac{19\pi}{3}$

Ans:

It is known that the values of tan x repeat after an interval of π or 180°.

$$\therefore \tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan \left(6\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \tan 60^\circ = \sqrt{3}$$

Question 7:

Find the value of the trigonometric function $\sin\!\left(-\frac{11\pi}{3}\right)$

Ans:



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It is known that the values of sin x repeat after an interval of 2π or 360° .

$$\therefore \sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2 \times 2\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Question 8:

Find the value of the trigonometric function $\cot\left(-\frac{15\pi}{4}\right)$

Ans:

It is known that the values of $\cot x$ repeat after an interval of π or 180°.

$$\therefore \cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right) = \cot\frac{\pi}{4} = 1$$

Exercise 3.3

Question 1:

Prove

$$\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$$

Ans:



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L.H.S. =
$$\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$$

= $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$
= $\frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2}$
= R.H.S.

Question 2:

Prove that
$$2\sin^2\frac{\pi}{6} + \csc^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$$

Ans:

L.H.S. =
$$2\sin^2 \frac{\pi}{6} + \csc^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3}$$

= $2\left(\frac{1}{2}\right)^2 + \csc^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$
= $2 \times \frac{1}{4} + \left(-\csc \frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$
= $\frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$
= $\frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$
= R.H.S.



Question 3:

Prove that
$$\cot^2 \frac{\pi}{6} + \cos ec \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$$

Ans:

L.H.S.
$$= \cot^2 \frac{\pi}{6} + \csc \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6}$$

 $= (\sqrt{3})^2 + \csc \left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$
 $= 3 + \csc \frac{\pi}{6} + 3 \times \frac{1}{3}$
 $= 3 + 2 + 1 = 6$
 $= \text{R.H.S}$

Question 4:

Find the value of:

(i) sin 75°

(ii) tan 15°

Ans:



(i) $\sin 75^\circ = \sin (45^\circ + 30^\circ)$

 $= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

 $[\sin(x+y) = \sin x \cos y + \cos x \sin y]$

$$= \left(\frac{1}{\sqrt{2}}\right) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\tan 15^\circ = \tan (45^\circ - 30^\circ)$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \qquad \left[\tan \left(x - y \right) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \right]$$
$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \left(\frac{1}{\sqrt{3}} \right)} = \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$$
$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\left(\sqrt{3} - 1\right)^2}{\left(\sqrt{3} + 1\right)\left(\sqrt{3} - 1\right)} = \frac{3 + 1 - 2\sqrt{3}}{\left(\sqrt{3}\right)^2 - \left(1\right)^2}$$
$$= \frac{4 - 2\sqrt{3}}{3 - 1} = 2 - \sqrt{3}$$

Question 5:

Prove that:
$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$$

Question 6:



Prove that:
$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

Ans:

It is known that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ and $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

∴L.H.S. =

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \frac{\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)} = \frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)^2} = \text{R.H.S.}$$

Question 7:

Prove that
$$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$$

Ans:



L.H.S. =
$$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$$
$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$
$$= \frac{-\cos^2 x}{-\sin^2 x}$$
$$= \cot^2 x$$
$$= R.H.S.$$

Question 8:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos\left(2\pi+x\right)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot\left(2\pi+x\right)\right]=1$$

Ans:

L.H.S. =
$$\cos\left(\frac{3\pi}{2} + x\right)\cos\left(2\pi + x\right)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot\left(2\pi + x\right)\right]$$

$$= \sin x \cos x \left[\tan x + \cot x \right]$$
$$= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$
$$= \left(\sin x \cos x \right) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right]$$
$$= 1 = \text{R.H.S.}$$



Question 9:

Prove that $\sin (n + 1)x \sin (n + 2)x + \cos (n + 1)x \cos (n + 2)x = \cos x$

Ans:

L.H.S. = sin $(n + 1)x \sin(n + 2)x + \cos(n + 1)x \cos(n + 2)x$

$$= \frac{1}{2} \Big[2\sin(n+1)x\sin(n+2)x + 2\cos(n+1)x\cos(n+2)x \Big]$$

=
$$\frac{1}{2} \Big[\cos\{(n+1)x - (n+2)x\} - \cos\{(n+1)x + (n+2)x\} + \cos\{(n+1)x - (n+2)x\} \Big]$$

=
$$\frac{1}{2} \Big[\cos\{(n+1)x + (n+2)x\} + \cos\{(n+1)x - (n+2)x\} \Big]$$

=
$$\frac{1}{2} \cos\{(n+1)x - (n+2)x\} = \cos(A+B) + \cos(A-B) \Big]$$

=
$$\frac{1}{2} \times 2\cos\{(n+1)x - (n+2)x\} = \cos(-x) = \cos x = R.H.S.$$

Question 10:

Prove that
$$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2}\sin x$$

Ans:



It is known that
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$$
.
 \therefore L.H.S. $= \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$
 $= -2\sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) + \left(\frac{3\pi}{4} - x\right)}{2}\right\}. \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right\}$
 $= -2\sin\left(\frac{3\pi}{4}\right)\sin x$
 $= -2\sin\left(\frac{3\pi}{4}\right)\sin x$
 $= -2\sin\frac{\pi}{4}\sin x$
 $= -2\times\frac{1}{\sqrt{2}}\times\sin x$
 $= -\sqrt{2}\sin x$
 $= R.H.S.$

Question 11:

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Ans:



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It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \ \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

 $\therefore L.H.S. = \sin^2 6x - \sin^2 4x$

$$= (\sin 6x + \sin 4x) (\sin 6x - \sin 4x)$$
$$= \left[2\sin\left(\frac{6x + 4x}{2}\right)\cos\left(\frac{6x - 4x}{2}\right)\right] \left[2\cos\left(\frac{6x + 4x}{2}\right).\sin\left(\frac{6x - 4x}{2}\right)\right]$$

- $= (2 \sin 5x \cos x) (2 \cos 5x \sin x)$
- $= (2 \sin 5x \cos 5x) (2 \sin x \cos x)$
- $= \sin 10x \sin 2x$
- = R.H.S.

Question 12:

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Ans:



It is known that $\cos A + \cos B = 2 \cos \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right), \quad \cos A - \cos B = -2 \sin \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right)$ $\therefore L.H.S. = \cos^{2} 2x - \cos^{2} 6x$ $= (\cos 2x + \cos 6x) (\cos 2x - 6x)$ $= \left[2 \cos \left(\frac{2x+6x}{2}\right) \cos \left(\frac{2x-6x}{2}\right)\right] \left[-2 \sin \left(\frac{2x+6x}{2}\right) \sin \frac{(2x-6x)}{2}\right]$ $= \left[2 \cos 4x \cos (-2x)\right] \left[-2 \sin 4x \sin (-2x)\right]$ $= \left[2 \cos 4x \cos 2x\right] \left[-2 \sin 4x (-\sin 2x)\right]$ $= (2 \sin 4x \cos 4x) (2 \sin 2x \cos 2x)$ $= \sin 8x \sin 4x$ = R.H.S.

Question 13:

Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

Ans:



 $L.H.S = \cot 4x (\sin 5x + \sin 3x)$

$$= \frac{\cos 4x}{\sin 4x} \left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right) \right]$$
$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right]$$
$$= \left(\frac{\cos 4x}{\sin 4x}\right) [2\sin 4x\cos x]$$

 $= 2 \cos 4x \cos x$

 $R.H.S. = \cot x \ (\sin 5x - \sin 3x)$

$$= \frac{\cos x}{\sin x} \left[2\cos\left(\frac{5x+3x}{2}\right) \sin\left(\frac{5x-3x}{2}\right) \right]$$
$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \right]$$
$$= \frac{\cos x}{\sin x} \left[2\cos 4x \sin x \right]$$

 $= 2 \cos 4x \cos x$

L.H.S. = R.H.S.

Question 14:

Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Ans:



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It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore L.H.S. = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$
$$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$
$$= \frac{\sin 4x}{\cos 4x}$$
$$= \tan 4x = \text{R.H.S.}$$

Question 15:

Prove that
$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x - y}{2}$$

Ans:



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It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\therefore L.H.S. = \frac{\sin x - \sin y}{\cos x + \cos y}$$
$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$
$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = R.H.S.$$

Question 16:

Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Ans:



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It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\therefore L.H.S. = \frac{\sin x + \sin 3x}{2}$$

$$\cos x + \cos 3x$$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$
$$=\frac{\sin 2x}{\cos 2x}$$
$$=\tan 2x$$
$$= R.H.S$$

Question 17:

Prove that
$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$$

Ans:



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It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \ \cos^2 A - \sin^2 A = \cos 2A$$

$$\therefore L.H.S. = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x\sin(-x)}{-\cos 2x}$$

$$= -2 \times (-\sin x)$$

$$= 2\sin x = R.H.S.$$

Question 18:

Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Ans:



L.H.S. = $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$= \frac{\left(\cos 4x + \cos 2x\right) + \cos 3x}{\left(\sin 4x + \sin 2x\right) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x + 2x}{2}\right)\cos\left(\frac{4x - 2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x + 2x}{2}\right)\cos\left(\frac{4x - 2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right), \sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)\right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x (2\cos x + 1)}{\sin 3x (2\cos x + 1)}$$

$$= \cot 3x = R.H.S.$$

Question 19:

Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Ans:



L.H.S. = $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$ = $\cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$ = $\cot x \cot 2x - \cot (2x + x) (\cot 2x + \cot x)$ = $\cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right] (\cot 2x + \cot x)$ $\left[\because \cot (A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$ = $\cot x \cot 2x - (\cot 2x \cot x - 1)$ = 1 = R.H.S.

Question 20:

Prove that
$$\tan 4x = \frac{4\tan x \left(1 - \tan^2 x\right)}{1 - 6\tan^2 x + \tan^4 x}$$

Ans:



It is known that $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$.

 \therefore L.H.S. = tan 4x = tan 2(2x)

$$= \frac{2 \tan 2x}{1 - \tan^{2}(2x)}$$

$$= \frac{2\left(\frac{2 \tan x}{1 - \tan^{2} x}\right)}{1 - \left(\frac{2 \tan x}{1 - \tan^{2} x}\right)^{2}}$$

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^{2} x}\right)}{\left[1 - \frac{4 \tan^{2} x}{(1 - \tan^{2} x)^{2}}\right]}$$

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^{2} x}\right)}{\left[\left(\frac{4 \tan x}{1 - \tan^{2} x}\right)^{2} - 4 \tan^{2} x}\right]}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{\left(1-\tan^2 x\right)^2 - 4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$
$$= \frac{4\tan x \left(1-\tan^2 x\right)}{\left(1-\tan^2 x\right)^2 - 4\tan^2 x}$$
$$= \frac{4\tan x \left(1-\tan^2 x\right)}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$
$$= \frac{4\tan x \left(1-\tan^2 x\right)}{1-6\tan^2 x + \tan^4 x} = \text{R.H.S.}$$





Question 21:

Prove that $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

Ans:

L.H.S. = $\cos 4x$ = $\cos 2(2x)$ = $1 - 2 \sin^2 2x [\cos 2A = 1 - 2 \sin^2 A]$ = $1 - 2(2 \sin x \cos x)^2 [\sin 2A = 2 \sin A \cos A]$ = $1 - 8 \sin^2 x \cos^2 x$ = R.H.S.

Question 22:

Prove that: $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Ans:



L.H.S. =
$$\cos 6x$$

= $\cos 3(2x)$
= $4 \cos^3 2x - 3 \cos 2x [\cos 3A = 4 \cos^3 A - 3 \cos A]$
= $4 [(2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1) [\cos 2x = 2 \cos^2 x - 1]$
= $4 [(2 \cos^2 x)^3 - (1)^3 - 3 (2 \cos^2 x)^2 + 3 (2 \cos^2 x)] - 6\cos^2 x + 3$
= $4 [8\cos^6 x - 1 - 12 \cos^4 x + 6 \cos^2 x] - 6 \cos^2 x + 3$
= $32 \cos^6 x - 4 - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3$
= $32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$
= R.H.S.

Exercise 3.4

Question 1:

Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

Ans:



 $\tan x = \sqrt{3}$

It is known that
$$\tan\frac{\pi}{3} = \sqrt{3}$$
 and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now, $\tan x = \tan \frac{\pi}{3}$ $\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in \mathbb{Z}$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in \mathbb{Z}$

Question 2:

Find the principal and general solutions of the equation $\sec x = 2$

Ans:



 $\sec x = 2$

It is known that
$$\sec\frac{\pi}{3} = 2$$
 and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Now, $\sec x = \sec \frac{\pi}{3}$ $\Rightarrow \cos x = \cos \frac{\pi}{3}$ $\left[\sec x = \frac{1}{\cos x}\right]$ $\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in \mathbb{Z}$

Question 3:

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

Ans:



 $\cot x = -\sqrt{3}$

It is known that
$$\cot \frac{\pi}{6} = \sqrt{3}$$

$$\therefore \cot \left(\pi - \frac{\pi}{6} \right) = -\cot \frac{\pi}{6} = -\sqrt{3} \text{ and } \cot \left(2\pi - \frac{\pi}{6} \right) = -\cot \frac{\pi}{6} = -\sqrt{3}$$
i.e., $\cot \frac{5\pi}{6} = -\sqrt{3}$ and $\cot \frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now,
$$\cot x = \cot \frac{5\pi}{6}$$

 $\Rightarrow \tan x = \tan \frac{5\pi}{6}$ $\left[\cot x = \frac{1}{\tan x}\right]$
 $\Rightarrow x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}$, where $n \in \mathbb{Z}$

Question 4:

Find the general solution of the equation $\cos 4x = \cos 2x$

Ans:



 $\cos 4x = \cos 2x$ $\Rightarrow \cos 4x - \cos 2x = 0$ $\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$ $\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$ $\Rightarrow \sin 3x \sin x = 0$ $\Rightarrow \sin 3x \sin x = 0$ $\Rightarrow \sin 3x = 0 \text{ or } \sin x = 0$ $\Rightarrow \sin 3x = n\pi \text{ or } x = n\pi, \text{ where } n \in \mathbb{Z}$ $\Rightarrow x = \frac{n\pi}{3} \text{ or } x = n\pi, \text{ where } n \in \mathbb{Z}$

Question 5:

Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

Ans:



 $\cos 3x + \cos x - \cos 2x = 0$

$$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$
$$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$$
$$\Rightarrow \cos 2x (2\cos x - 1) = 0$$
$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$
$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$\therefore 2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos\frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$
$$\Rightarrow x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Question 6:

Find the general solution of the equation $\ sec^2 \, 2x = 1 - \tan 2x$

Ans:



sec² 2x = 1 - tan 2x $\Rightarrow 1 + tan^{2} 2x = 1 - tan 2x$ $\Rightarrow tan^{2} 2x + tan 2x = 0$ $\Rightarrow tan 2x (tan 2x + 1) = 0$ $\Rightarrow tan 2x = 0 \quad \text{or} \quad tan 2x + 1 = 0$ Now, tan 2x = 0 $\Rightarrow tan 2x = tan 0$ $\Rightarrow 2x = n\pi + 0, \text{ where } n \in Z$ $\Rightarrow x = \frac{n\pi}{2}, \text{ where } n \in Z$ tan 2x + 1 = 0 $\Rightarrow tan 2x = -1 = -tan \frac{\pi}{4} = tan \left(\pi - \frac{\pi}{4}\right) = tan \frac{3\pi}{4}$ $\Rightarrow 2x = n\pi + \frac{3\pi}{4}, \text{ where } n \in Z$ $\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{4}, \text{ where } n \in Z$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8}$, $n \in \mathbb{Z}$.

Question 7:

Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

Ans:



 $\sin x + \sin 3x + \sin 5x = 0$ $\Rightarrow \sin 3x (2 \cos 2x + 1) = 0$ $\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2 \cos 2x + 1 = 0$ Now, $\sin 3x = 0 \Rightarrow 3x = n\pi$, where $n \in Z$ i.e., $x = \frac{n\pi}{3}$, where $n \in Z$ $2 \cos 2x + 1 = 0$ $\Rightarrow \cos 2x = \frac{-1}{2} = -\cos \frac{\pi}{3} = \cos \left(\pi - \frac{\pi}{3} \right)$ $\Rightarrow \cos 2x = \cos \frac{2\pi}{3}$ $\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$, where $n \in Z$ $(\sin x + \sin 5x) + \sin 3x = 0$ $\Rightarrow \left[2\sin \left(\frac{x + 5x}{2} \right) \cos \left(\frac{x - 5x}{2} \right) \right] + \sin 3x = 0$ $\Rightarrow \left[\sin A + \sin B = 2 \sin \left(\frac{A + B}{2} \right) \cos \left(\frac{A - B}{2} \right) \right]$ $\Rightarrow 2 \sin 3x \cos(-2x) + \sin 3x = 0$





Chapterwise NCERT Solutions for Class 11 Maths :

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- <u>Chapter 2-Relations and Functions</u>
- <u>Chapter 3-Trigonometric Functions</u>
- <u>Chapter 4-Principle of Mathematical Induction</u>
- <u>Chapter 5-Complex Numbers and Quadratic Equations</u>
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