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NCERT Solutions for 11th Class Maths: Chapter 11-Conic Sections

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NCERT Solutions for 11th Class Maths: Chapter 11-Conic Sections

Class 11: Maths Chapter 11 solutions. Complete Class 11 Maths Chapter 11 Notes.

NCERT Solutions for 11th Class Maths: Chapter 11-Conic Sections

NCERT 11th Maths Chapter 11, class 11 Maths Chapter 11 solutions

Exercise 11.1

Question 1:

Find the equation of the circle with centre (0, 2) and radius 2

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = (0, 2)$ and radius $(r) = 2$.

Therefore, the equation of the circle is

$(x - 0)^2 + (y - 2)^2 = 2^2$

$x^2 + y^2 + 4 - 4y = 4$

$x^2 + y^2 - 4y = 0$

Question 2:

Find the equation of the circle with centre (–2, 3) and radius 4

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = (-2, 3)$ and radius $(r) = 4$.

Therefore, the equation of the circle is

$(x + 2)^2 + (y - 3)^2 = (4)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$

$x^2 + y^2 + 4x - 6y - 3 = 0$

Question 3:

Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = \left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $(r) = \frac{1}{12}$.

Therefore, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}=\frac{1}{144}$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}-\frac{1}{144}=0$$
$$144x^2-144x+36+144y^2-72y+9-1=0$$
$$144x^2-144x+144y^2-72y+44=0$$
$$36x^2-36x+36y^2-18y+11=0$$
$$36x^2+36y^2-36x-18y+11=0$$

Question 4:

Find the equation of the circle with centre (1, 1) and radius $\sqrt{2}$

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = (1, 1)$ and radius $(r) = \sqrt{2}$.

Therefore, the equation of the circle is

$$(x-1)^2 + (y-1)^2 = \left(\sqrt{2}\right)^2$$
$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$
$$x^2 + y^2 - 2x - 2y = 0$$

Question 5:

Find the equation of the circle with centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = (1, 1)$ and radius $(r) = \sqrt{2}$.

Therefore, the equation of the circle is

$$(x-1)^2 + (y-1)^2 = \left(\sqrt{2}\right)^2$$
$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$
$$x^2 + y^2 - 2x - 2y = 0$$

Question 6:

Find the centre and radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$

Ans:

The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$.

$(x + 5)^2 + (y - 3)^2 = 36$

$\{x - (-5)\}^2 + (y - 3)^2 = 6^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = -5$, $k = 3$, and $r = 6$.

Thus, the centre of the given circle is (–5, 3), while its radius is 6.

Question 7:

Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

Ans:

The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$.

$x^2 + y^2 - 4x - 8y - 45 = 0$

$(x^2 - 4x) + (y^2 - 8y) = 45$

$\{x^2 - 2(x)(2) + 2^2\} + \{y^2 - 2(y)(4) + 4^2\} - 4 - 16 = 45$

$(x - 2)^2 + (y - 4)^2 = 65$

$(x - 2)^2 + (y - 4)^2 = \left(\sqrt{65}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 2$, $k = 4$, and $r = \sqrt{65}$.

Thus, the centre of the given circle is (2, 4), while its radius is $\sqrt{65}$.

Question 8:

Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$

Ans:

The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$.

$x^2 + y^2 - 8x + 10y - 12 = 0$

$(x^2 - 8x) + (y^2 + 10y) = 12$

$\{x^2 - 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\} - 16 - 25 = 12$

$(x - 4)^2 + (y + 5)^2 = 53$

$$\Rightarrow (x-4)^2 + \{y-(-5)\}^2 = (\sqrt{53})^2,$$

which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 4$, $k = -5$, and $r = \sqrt{53}$.

Thus, the centre of the given circle is $(4, -5)$, while its radius is $\sqrt{53}$.

Question 9:

Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

Ans:

The equation of the given circle is $2x^2 + 2y^2 - x = 0$.

$$2x^2+2y^2-x=0$$
$$\Rightarrow\left(2x^2-x\right)+2y^2=0$$
$$\Rightarrow 2\left[\left(x^2-\frac{x}{2}\right)+y^2\right]=0$$
$$\Rightarrow\left\{x^2-2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2\right\}+y^2-\left(\frac{1}{4}\right)^2=0$$

$\Rightarrow\left(x-\frac{1}{4}\right)^2+\left(y-0\right)^2=\left(\frac{1}{4}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = \frac{1}{4}$, $k = 0$, and $r=\frac{1}{4}$.

Thus, the centre of the given circle is $\left(\frac{1}{4},0\right)$, while its radius is $\frac{1}{4}$.

Question 10:

Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

Ans:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (4, 1) and (6, 5),

$(4 - h)^2 + (1 - k)^2 = r^2$ … (1)

$(6 - h)^2 + (5 - k)^2 = r^2$ … (2)

Since the centre (*h*, k) of the circle lies on line $4x + y = 16$,

$4h + k = 16$ … (3)

From equations (1) and (2), we obtain

$(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$

$16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 25 - 10k + k^2$

$16 - 8h + 1 - 2k = 36 - 12h + 25 - 10k$

$4h + 8k = 44$

$h + 2k = 11$ … (4)

On solving equations (3) and (4), we obtain $h = 3$ and $k = 4$.

On substituting the values of *h* and *k* in equation (1), we obtain

$(4 - 3)^2 + (1 - 4)^2 = r^2$

$(1)^2 + (-3)^2 = r^2$

$1 + 9 = r^2$

$r^2 = 10$

$r = \sqrt{10}$

Thus, the equation of the required circle is

$(x - 3)^2 + (y - 4)^2 = \left(\sqrt{10}\right)^2$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$x^2 + y^2 - 6x - 8y + 15 = 0$

Question 11:

Find the equation of the circle passing through the points (2, 3) and (-1, 1) and whose centre is on the line x – 3y – 11 = 0.

Solution:

The equation of the circle is,

$(x - h)2 + (y - k)2 = r2$ ….(i)

Since the circle passes through point (2, 3)

∴ $(2 - h)2 + (3 - k)2 = r2$

⇒ $4 + h2 - 4h + 9 + k2 - 6k = r2$

⇒ $h2+ k2 - 4h - 6k + 13 = r2$ ….(ii)

Also, the circle passes through point (-1, 1)

∴ $(-1 - h)2 + (1 - k)2 = r2$

⇒ $1 + h2 + 2h + 1 + k2 - 2k = r2$

⇒ $h2 + k2 + 2h - 2k + 2 = r2$ ….(iii)

From (ii) and (iii), we have

$h2 + k2 - 4h - 6k + 13 = h2 + k2 + 2h - 2k + 2$

⇒ $-6h - 4k = -11$ ⇒ $6h + 4k = 11$ …(iv)

Since the centre (h, k) of the circle lies on the line x – 3y-11 = 0.

∴ $h - 3k - 11 = 0$ ⇒ $h -3k = 11$ …(v)

Solving (iv) and (v), we get

h = 72 and k = −52

Putting these values of h and k in (ii), we get

$(72)2+(-52)2-4\times72-6\times-52+13=r2$

⇒ 494+254−14+15+13 ⇒ r2=652

Thus required equation of circle is

⇒ (x−72)2+(y+52)2=652

⇒ x2+494−7x+y2+254+5y=652

⇒ 4×2 + 49 – 28x + 4y2 + 25 + 20y = 130

⇒ 4×2 + 4y2 – 28x + 20y – 56 = 0

⇒ 4(x2 + y2 – 7x + 5y -14) = 0

⇒ x2 + y2 – 7x + 5y -14 = 0.

Question 12:

Find the equation of the circle with radius 5 whose centre lies on *x*-axis and passes through the point (2, 3).

Ans:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the radius of the circle is 5 and its centre lies on the x-axis, $k = 0$ and $r = 5$.

Now, the equation of the circle becomes $(x - h)^2 + y^2 = 25$.

It is given that the circle passes through point (2, 3).

$$\therefore (2-h)^2 + 3^2 = 25$$
$$\Rightarrow (2-h)^2 = 25 - 9$$
$$\Rightarrow (2-h)^2 = 16$$
$$\Rightarrow 2 - h = \pm\sqrt{16} = \pm 4$$
$$\text{If } 2 - h = 4, \text{ then } h = -2.$$
$$\text{If } 2 - h = -4, \text{ then } h = 6.$$

When $h = -2$, the equation of the circle becomes

$(x + 2)^2 + y^2 = 25$

$x^2 + 4x + 4 + y^2 = 25$

$x^2 + y^2 + 4x - 21 = 0$

When $h = 6$, the equation of the circle becomes

$(x - 6)^2 + y^2 = 25$

$x^2 - 12x + 36 + y^2 = 25$

$x^2 + y^2 - 12x + 11 = 0$

Question 13:

Find the equation of the circle passing through (0, 0) and making intercepts a and b on the coordinate axes.

Ans:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the centre of the circle passes through (0, 0),

$(0 - h)^2 + (0 - k)^2 = r^2$

$h^2 + k^2 = r^2$

The equation of the circle now becomes $(x - h)^2 + (y - k)^2 = h^2 + k^2$.

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points $(a, 0)$ and $(0, b)$. Therefore,

$(a - h)^2 + (0 - k)^2 = h^2 + k^2$ … (1)

$(0 - h)^2 + (b - k)^2 = h^2 + k^2$ … (2)

From equation (1), we obtain

$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$

$a^2 - 2ah = 0$

$a(a - 2h) = 0$

$a = 0$ or $(a - 2h) = 0$

However, $a \neq 0$; hence, $(a - 2h) = 0$ □ $h = \frac{a}{2}$.

From equation (2), we obtain

$h^2 + b^2 - 2bk + k^2 = h^2 + k^2$

$b^2 - 2bk = 0$

$b(b - 2k) = 0$

$b = 0$ or $(b - 2k) = 0$

However, $b \neq 0$; hence, $(b - 2k) = 0$ □ $k = \frac{b}{2}$.

Thus, the equation of the required circle is

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2$$
$$\Rightarrow\left(\frac{2x-a}{2}\right)^2+\left(\frac{2y-b}{2}\right)^2=\frac{a^2+b^2}{4}$$
$$\Rightarrow 4x^2-4ax+a^2+4y^2-4by+b^2=a^2+b^2$$
$$\Rightarrow 4x^2+4y^2-4ax-4by=0$$
$$\Rightarrow x^2+y^2-ax-by=0$$

Question 14:

Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Ans:

The centre of the circle is given as $(h, k) = (2, 2)$.

Since the circle passes through point (4, 5), the radius (r) of the circle is the distance between the points (2, 2) and (4, 5).

$$\therefore r=\sqrt{(2-4)^2+(2-5)^2}=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$
$$x^2-4x+4+y^2-4y+4=13$$
$$x^2+y^2-4x-4y-5=0$$

Question 15:

Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Ans:

The equation of the given circle is $x^2 + y^2 = 25$.

$x^2 + y^2 = 25$

$(x - 0)^2 + (y - 0)^2 = 5^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 0$, $k = 0$, and $r = 5$.

Centre = (0, 0) and radius = 5

Distance between point (–2.5, 3.5) and centre (0, 0)

$$= \sqrt{(-2.5-0)^2 + (3.5-0)^2}$$
$$= \sqrt{6.25 + 12.25}$$
$$= \sqrt{18.5}$$
$$= 4.3 \text{ (approx.)} < 5$$

Since the distance between point (–2.5, 3.5) and centre (0, 0) of the circle is less than the radius of the circle, point (–2.5, 3.5) lies inside the circle.

Exercise 11.2

NCERT 11th Maths Chapter 11

Question 1:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 12x$

Ans:

The given equation is $y^2 = 12x$.

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$4a = 12$ □ $a = 3$

Coordinates of the focus = $(a, 0) = (3, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of direcctrix, $x = -a$ i.e., $x = -3$ i.e., $x + 3 = 0$

Length of latus rectum = $4a = 4 \times 3 = 12$

Question 2:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = 6y$

Ans:

The given equation is $x^2 = 6y$.

Here, the coefficient of y is positive. Hence, the parabola opens upwards.

On comparing this equation with $x^2 = 4ay$, we obtain

$$4a = 6 \Rightarrow a = \frac{3}{2}$$

Coordinates of the focus = $(0, a)$ = $\left(0, \frac{3}{2}\right)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = -a$ i.e., $y = -\frac{3}{2}$

Length of latus rectum = $4a$ = 6

Question 3:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = -8x$

Ans:

The given equation is $y^2 = -8x$.

Here, the coefficient of x is negative. Hence, the parabola opens towards the left.

On comparing this equation with $y^2 = -4ax$, we obtain

$-4a = -8$ □ $a = 2$

□Coordinates of the focus = $(-a, 0) = (-2, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = a$ i.e., $x = 2$

Length of latus rectum = $4a = 8$

Question 4:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -16y$

Ans:

The given equation is $x^2 = -16y$.

Here, the coefficient of y is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we obtain

$-4a = -16$ □ $a = 4$

□Coordinates of the focus = $(0, -a) = (0, -4)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = a$ i.e., $y = 4$

Length of latus rectum = $4a = 16$

Question 5:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 10x$

Ans:

The given equation is $y^2 = 10x$.

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$$4a = 10 \Rightarrow a = \frac{5}{2}$$

Coordinates of the focus = $(a, 0) = \left(\frac{5}{2}, 0\right)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = -a$, i.e., $x = -\frac{5}{2}$

Length of latus rectum = $4a$ = 10

Question 6:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -9y$

Ans:

The given equation is $x^2 = -9y$.

Here, the coefficient of y is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we obtain

$$-4a = -9 \Rightarrow b = \frac{9}{4}$$

Coordinates of the focus $= (0, -a) = \left(0, -\frac{9}{4}\right)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = a$, i.e., $y = \frac{9}{4}$

Length of latus rectum = 4a = 9

Question 7:

Find the equation of the parabola that satisfies the following conditions: Focus (6, 0); directrix $x = -6$

Ans:

Focus (6, 0); directrix, $x = -6$

Since the focus lies on the x-axis, the x-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $y^2 = 4ax$ or

$y^2 = -4ax$.

It is also seen that the directrix, $x = -6$ is to the left of the y-axis, while the focus (6, 0) is to the right of the y-axis. Hence, the parabola is of the form $y^2 = 4ax$.

Here, $a = 6$

Thus, the equation of the parabola is $y^2 = 24x$.

Question 8:

Find the equation of the parabola that satisfies the following conditions: Focus (0, –3); directrix $y = 3$

Ans:

Focus = (0, –3); directrix $y = 3$

Since the focus lies on the y-axis, the y-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $x^2 = 4ay$ or

$x^2 = -4ay$.

It is also seen that the directrix, $y = 3$ is above the x-axis, while the focus

(0, –3) is below the x-axis. Hence, the parabola is of the form $x^2 = -4ay$.

Here, $a = 3$

Thus, the equation of the parabola is $x^2 = -12y$.

Question 9:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0); focus (3, 0)

Ans:

Vertex (0, 0); focus (3, 0)

Since the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = 4ax$.

Since the focus is (3, 0), $a = 3$.

Thus, the equation of the parabola is $y^2 = 4 \times 3 \times x$, i.e., $y^2 = 12x$

Question 10:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) focus (–2, 0)

Ans:

Vertex (0, 0) focus (–2, 0)

Since the vertex of the parabola is (0, 0) and the focus lies on the negative x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = -4ax$.

Since the focus is (–2, 0), $a = 2$.

Thus, the equation of the parabola is $y^2 = -4(2)x$, i.e., $y^2 = -8x$

Question 11:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) passing through (2, 3) and axis is along x-axis

Ans:

Since the vertex is (0, 0) and the axis of the parabola is the x-axis, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.

The parabola passes through point (2, 3), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $y^2 = 4ax$, while point

(2, 3) must satisfy the equation $y^2 = 4ax$.

$$\therefore 3^2 = 4a(2) \Rightarrow a = \frac{9}{8}$$

Thus, the equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x$$
$$y^2 = \frac{9}{2}x$$
$$2y^2 = 9x$$

Question 12:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis

Ans:

Since the vertex is (0, 0) and the parabola is symmetric about the y-axis, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

The parabola passes through point (5, 2), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $x^2 = 4ay$, while point

(5, 2) must satisfy the equation $x^2 = 4ay$.

$$\therefore (5)^2 = 4 \times a \times 2 \Rightarrow 25 = 8a \Rightarrow a = \frac{25}{8}$$

Thus, the equation of the parabola is

$$x^2 = 4\left(\frac{25}{8}\right)y$$
$$2x^2 = 25y$$

Exercise 11.3

NCERT 11th Maths Chapter 11

Question 1:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Ans:

The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$.

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain *a* = 6 and *b* = 4.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$$

Therefore,

The coordinates of the foci are $\left(2\sqrt{5},0\right)$ and $\left(-2\sqrt{5},0\right)$.

The coordinates of the vertices are (6, 0) and (–6, 0).

Length of major axis = 2*a* = 12

Length of minor axis = 2*b* = 8

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times16}{6}=\frac{16}{3}$

Question 2:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{4}+\frac{y^2}{25}=1$

Ans:

The given equation is $\frac{x^2}{4}+\frac{y^2}{25}=1$ or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$.

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain *b* = 2 and *a* = 5.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{25-4}=\sqrt{21}$

Therefore,

The coordinates of the foci are $\left(0,\sqrt{21}\right)$ and $\left(0,-\sqrt{21}\right)$.

The coordinates of the vertices are (0, 5) and (0, –5)

Length of major axis = 2*a* = 10

Length of minor axis = 2*b* = 4

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{21}}{5}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{5}=\frac{8}{5}$

Question 3:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Ans:

The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$.

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain *a* = 4 and *b* = 3.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{7},0\right)$.

The coordinates of the vertices are $(\pm 4,0)$.

Length of major axis = 2*a* = 8

Length of minor axis = 2*b* = 6

$\text{Eccentricity, } e=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

Question 4:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{25}+\frac{y^2}{100}=1$

Ans:

The given equation is $\frac{x^2}{25}+\frac{y^2}{100}=1$ or $\frac{x^2}{5^2}+\frac{y^2}{10^2}=1$.

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain b = 5 and a = 10.

$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 5\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±10).

Length of major axis = 2a = 20

Length of minor axis = 2b = 10

Eccentricity, $e=\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 25}{10}=5$

Question 5:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{49}+\frac{y^2}{36}=1$

Ans:

The given equation is $\frac{x^2}{49}+\frac{y^2}{36}=1$ or $\frac{x^2}{7^2}+\frac{y^2}{6^2}=1$.

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain *a* = 7 and *b* = 6.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{49-36}=\sqrt{13}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{13},0\right)$.

The coordinates of the vertices are (± 7, 0).

Length of major axis = 2*a* = 14

Length of minor axis = 2*b* = 12

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{13}}{7}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 36}{7}=\frac{72}{7}$

Question 6:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100}+\frac{y^2}{400}=1$

Ans:

The given equation is $\frac{x^2}{100}+\frac{y^2}{400}=1$ or $\frac{x^2}{10^2}+\frac{y^2}{20^2}=1$.

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain *b* = 10 and *a* = 20.

$$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$$

Therefore,

The coordinates of the foci are $\left(0,\pm10\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±20)

Length of major axis = 2*a* = 40

Length of minor axis = 2*b* = 20

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{10\sqrt{3}}{20}=\frac{\sqrt{3}}{2}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times100}{20}=10$

Question 7:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $36x^2 + 4y^2 = 144$

Ans:

The given equation is $36x^2 + 4y^2 = 144$.

It can be written as

$$36x^2 + 4y^2 = 144$$

$$\text{Or, } \frac{x^2}{4} + \frac{y^2}{36} = 1$$

$$\text{Or, } \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 2$ and $a = 6$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$$

Therefore,

The coordinates of the foci are $\left(0, \pm 4\sqrt{2}\right)$.

The coordinates of the vertices are $(0, \pm 6)$.

Length of major axis = $2a = 12$

Length of minor axis = $2b = 4$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$

Question 8:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $16x^2 + y^2 = 16$

Ans:

The given equation is $16x^2 + y^2 = 16$.

It can be written as

$$16x^2 + y^2 = 16$$

$$\text{Or, } \frac{x^2}{1} + \frac{y^2}{16} = 1$$

$$\text{Or, } \frac{x^2}{1^2} + \frac{y^2}{4^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 1$ and $a = 4$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16-1} = \sqrt{15}$$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{15}\right)$.

The coordinates of the vertices are $(0, \pm 4)$.

Length of major axis $= 2a = 8$

Length of minor axis $= 2b = 2$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{15}}{4}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 1}{4} = \frac{1}{2}$$

Question 9:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Ans:

The given equation is $4x^2 + 9y^2 = 36$.

It can be written as

$$4x^2 + 9y^2 = 36$$

$$\text{Or, } \frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\text{Or, } \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $a = 3$ and $b = 2$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{9-4} = \sqrt{5}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{5}, 0\right)$.

The coordinates of the vertices are $(\pm 3, 0)$.

Length of major axis $= 2a = 6$

Length of minor axis $= 2b = 4$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{5}}{3}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$$

Question 10:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±5, 0), foci (±4, 0)

Ans:

Vertices (±5, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, a = 5 and c = 4.

It is known that $a^2=b^2+c^2$.

$$\therefore 5^2=b^2+4^2$$
$$\Rightarrow 25=b^2+16$$
$$\Rightarrow b^2=25-16$$
$$\Rightarrow b=\sqrt{9}=3$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Question 11:

Find the equation for the ellipse that satisfies the given conditions: Vertices (0, ±13), foci (0, ±5)

Ans:

Vertices (0, ±13), foci (0, ±5)

Here, the vertices are on the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, a = 13 and c = 5.

It is known that $a^2 = b^2 + c^2$.

$$\therefore 13^2 = b^2 + 5^2$$
$$\Rightarrow 169 = b^2 + 25$$
$$\Rightarrow b^2 = 169 - 25$$
$$\Rightarrow b = \sqrt{144} = 12$$

Thus, the equation of the ellipse is $\frac{x^2}{12^2}+\frac{y^2}{13^2}=1$ or $\frac{x^2}{144}+\frac{y^2}{169}=1$.

Question 12:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±6, 0), foci (±4, 0)

Ans:

Vertices (±6, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, *a* = 6, *c* = 4.

It is known that $a^2=b^2+c^2$.

$$\therefore 6^2=b^2+4^2$$
$$\Rightarrow 36=b^2+16$$
$$\Rightarrow b^2=36-16$$
$$\Rightarrow b=\sqrt{20}$$

Thus, the equation of the ellipse is $\frac{x^2}{6^2}+\frac{y^2}{\left(\sqrt{20}\right)^2}=1$ or $\frac{x^2}{36}+\frac{y^2}{20}=1$.

Question 13:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Ans:

Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Here, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = 3 and *b* = 2.

Thus, the equation of the ellipse is $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ i.e., $\frac{x^2}{9}+\frac{y^2}{4}=1$.

Question 14:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Ans:

Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Here, the major axis is along the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = $\sqrt{5}$ and *b* = 1.

Thus, the equation of the ellipse is $\frac{x^2}{1^2}+\frac{y^2}{\left(\sqrt{5}\right)^2}=1$ or $\frac{x^2}{1}+\frac{y^2}{5}=1$.

Question 15:

Find the equation for the ellipse that satisfies the given conditions: Length of major axis 26, foci (±5, 0)

Ans:

Length of major axis = 26; foci = (±5, 0).

Since the foci are on the *x*-axis, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, 2*a* = 26 ⇒ *a* = 13 and *c* = 5.

It is known that $a^2=b^2+c^2$.

$\therefore 13^2=b^2+5^2$
$\Rightarrow 169=b^2+25$
$\Rightarrow b^2=169-25$
$\Rightarrow b=\sqrt{144}=12$

Thus, the equation of the ellipse is $\frac{x^2}{13^2}+\frac{y^2}{12^2}=1$ or $\frac{x^2}{169}+\frac{y^2}{144}=1$.

Question 16:

Find the equation for the ellipse that satisfies the given conditions: Length of minor axis 16, foci (0, ±6)

Ans:

Length of minor axis = 16; foci = (0, ±6).

Since the foci are on the y-axis, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, $2b = 16$ □ $b = 8$ and $c = 6$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore a^2 = 8^2 + 6^2 = 64 + 36 = 100$$
$$\Rightarrow a = \sqrt{100} = 10$$

Thus, the equation of the ellipse is $\frac{x^2}{8^2}+\frac{y^2}{10^2}=1$ or $\frac{x^2}{64}+\frac{y^2}{100}=1$.

Question 17:

Find the equation for the ellipse that satisfies the given conditions: Foci (±3, 0), $a = 4$

Ans:

Foci (±3, 0), a = 4

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, c = 3 and a = 4.

It is known that $a^2=b^2+c^2$.

$$\therefore 4^2=b^2+3^2$$
$$\Rightarrow 16=b^2+9$$
$$\Rightarrow b^2=16-9=7$$

Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{7}=1$.

Question 18:

Find the equation for the ellipse that satisfies the given conditions: b = 3, c = 4, centre at the origin; foci on the x axis.

Ans:

It is given that b = 3, c = 4, centre at the origin; foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, b = 3, c = 4.

It is known that $a^2=b^2+c^2$.

$$\therefore a^2=3^2+4^2=9+16=25$$
$$\Rightarrow a=5$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Question 19:

Find the equation for the ellipse that satisfies the given conditions: Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Ans:

Since the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (3, 2) and (1, 6). Hence,

$$\frac{9}{b^2}+\frac{4}{a^2}=1 \qquad ...(2)$$

$$\frac{1}{b^2}+\frac{36}{a^2}=1 \qquad ...(3)$$

On solving equations (2) and (3), we obtain b^2 = 10 and a^2 = 40.

Thus, the equation of the ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$ or $4x^2+y^2=40$.

Question 20:

Find the equation for the ellipse that satisfies the given conditions: Major axis on the x-axis and passes through the points (4, 3) and (6, 2).

Ans:

Since the major axis is on the *x*-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (4, 3) and (6, 2). Hence,

$$\frac{16}{a^2}+\frac{9}{b^2}=1 \qquad ...(2)$$

$$\frac{36}{a^2}+\frac{4}{b^2}=1 \qquad ...(3)$$

On solving equations (2) and (3), we obtain $a^2 = 52$ and $b^2 = 13$.

Thus, the equation of the ellipse is $\frac{x^2}{52}+\frac{y^2}{13}=1$ or $x^2+4y^2=52$.

Exercise 11.4

Question 1:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$

Ans:

The given equation is $\frac{x^2}{16}-\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}-\frac{y^2}{3^2}=1$.

On comparing this equation with the standard equation of hyperbola i.e., $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we obtain a = 4 and b = 3.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 4^2 + 3^2 = 25$$
$$\Rightarrow c = 5$$

Therefore,

The coordinates of the foci are (±5, 0).

The coordinates of the vertices are (±4, 0).

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{5}{4}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

Question 2:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $\frac{y^2}{9}-\frac{x^2}{27}=1$

Ans:

The given equation is $\frac{y^2}{9}-\frac{x^2}{27}=1$ or $\frac{y^2}{3^2}-\frac{x^2}{\left(\sqrt{27}\right)^2}=1$.

On comparing this equation with the standard equation of hyperbola i.e., $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, we obtain a = 3 and $b=\sqrt{27}$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 3^2+\left(\sqrt{27}\right)^2 = 9+27=36$$
$$\Rightarrow c=6$$

Therefore,

The coordinates of the foci are (0, ±6).

The coordinates of the vertices are (0, ±3).

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{6}{3}=2$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 27}{3}=18$

Question 3:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $9y^2 - 4x^2 = 36$

Ans:

The given equation is $9y^2 - 4x^2 = 36$.

It can be written as

$9y^2 - 4x^2 = 36$

$$\text{Or, } \frac{y^2}{4} - \frac{x^2}{9} = 1$$

$$\text{Or, } \frac{y^2}{2^2} - \frac{x^2}{3^2} = 1 \quad \ldots(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = 2$ and $b = 3$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 4 + 9 = 13$$

$$\Rightarrow c = \sqrt{13}$$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{13}\right)$.

The coordinates of the vertices are $\left(0, \pm 2\right)$.

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{13}}{2}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{2} = 9$

Question 4:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 576$

Ans:

The given equation is $16x^2 - 9y^2 = 576$.

It can be written as

$16x^2 - 9y^2 = 576$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$$
$$\Rightarrow \frac{x^2}{6^2} - \frac{y^2}{8^2} = 1 \qquad ...(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we obtain $a = 6$ and $b = 8$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 36 + 64 = 100$$
$$\Rightarrow c = 10$$

Therefore,

The coordinates of the foci are $(\pm 10, 0)$.

The coordinates of the vertices are $(\pm 6, 0)$.

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 64}{6} = \frac{64}{3}$

Question 5:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $5y^2 - 9x^2 = 36$

Ans:

The given equation is $5y^2 - 9x^2 = 36$.

$$\Rightarrow \frac{y^2}{\left(\frac{36}{5}\right)} - \frac{x^2}{4} = 1$$

$$\Rightarrow \frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} - \frac{x^2}{2^2} = 1 \qquad ...(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = \frac{6}{\sqrt{5}}$ and $b = 2$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = \frac{36}{5} + 4 = \frac{56}{5}$$

$$\Rightarrow c = \sqrt{\frac{56}{5}} = \frac{2\sqrt{14}}{\sqrt{5}}$$

Therefore, the coordinates of the foci are $\left(0, \pm\frac{2\sqrt{14}}{\sqrt{5}}\right)$.

The coordinates of the vertices are $\left(0, \pm\frac{6}{\sqrt{5}}\right)$.

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\left(\frac{2\sqrt{14}}{\sqrt{5}}\right)}{\left(\frac{6}{\sqrt{5}}\right)} = \frac{\sqrt{14}}{3}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 4}{\left(\frac{6}{\sqrt{5}}\right)} = \frac{4\sqrt{5}}{3}$

Question 6:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $49y^2 - 16x^2 = 784$

Ans:

The given equation is $49y^2 - 16x^2 = 784$.

It can be written as
$49y^2 - 16x^2 = 784$

$$\text{Or, } \frac{y^2}{16} - \frac{x^2}{49} = 1$$

$$\text{Or, } \frac{y^2}{4^2} - \frac{x^2}{7^2} = 1 \qquad \ldots(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = 4$ and $b = 7$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 16 + 49 = 65$$

$$\Rightarrow c = \sqrt{65}$$

Therefore,

The coordinates of the foci are $\left(0,\ \pm\sqrt{65}\right)$.

The coordinates of the vertices are $(0, \pm 4)$.

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{65}}{4}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 49}{4} = \frac{49}{2}$$

Question 7:

Find the equation of the hyperbola satisfying the give conditions: Vertices (±2, 0), foci (±3, 0)

Ans:

Vertices (±2, 0), foci (±3, 0)

Here, the vertices are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the vertices are (±2, 0), $a = 2$.

Since the foci are (±3, 0), $c = 3$.

We know that $a^2 + b^2 = c^2$.

$$\therefore 2^2 + b^2 = 3^2$$
$$b^2 = 9 - 4 = 5$$

Thus, the equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Question 8.

Vertices (0, ±5), foci (0, ±8)

Solution:

Vertices are (0, ±5) which lie on x-axis. So the equation of hyperbola in standard form is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Now vertices are (0, ±5) $\Rightarrow a = 5$

Foci are (0, ±8) $\Rightarrow ae = 8$

Now, $ae = 8 \Rightarrow e = \frac{8}{a} \Rightarrow e = \frac{8}{5}$

We know that

$$b = a\sqrt{e^2 - 1} \Rightarrow b = 5\sqrt{\frac{64}{25} - 1} = 5\frac{\sqrt{39}}{5} = \sqrt{39}$$

Thus required equation of parabola is

$$\frac{y^2}{(5)^2} - \frac{x^2}{(\sqrt{39})^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{39} = 1.$$

Question 9.

Vertices (0, ±3), foci (0, ±5)

Solution:

Vertices are (0, ±3) which lie on x-axis. So the equation of hyperbola in standard form is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Now vertices are $(0, \pm 3) \Rightarrow a = 3$

Co-ordinates of Foci are $(0, \pm 5) \Rightarrow ae = 5$

Now, $ae = 5 \Rightarrow e = \frac{5}{a} \Rightarrow e = \frac{5}{3}$

We know that

$$b = a\sqrt{e^2 - 1} \Rightarrow b = 3\sqrt{\frac{25}{9} - 1} = 3\sqrt{\frac{16}{9}} = 4$$

Thus required equation of hyperbola is

$$\frac{y^2}{(3)^2} - \frac{x^2}{(4)^2} = 1 \Rightarrow \frac{y^2}{9} - \frac{x^2}{16} = 1.$$

Question 10:

Find the equation of the hyperbola satisfying the give conditions: Foci $(\pm 5, 0)$, the transverse axis is of length 8.

Ans:

Foci (±5, 0), the transverse axis is of length 8.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the foci are (±5, 0), c = 5.

Since the length of the transverse axis is 8, $2a = 8$ □ $a = 4$.

We know that $a^2 + b^2 = c^2$.

$4^2 + b^2 = 5^2$

$b^2 = 25 - 16 = 9$

Thus, the equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Question 11:

Find the equation of the hyperbola satisfying the give conditions: Foci (0, ±13), the conjugate axis is of length 24.

Ans:

Foci (0, ±13), the conjugate axis is of length 24.

Here, the foci are on the *y*-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

Since the foci are (0, ±13), *c* = 13.

Since the length of the conjugate axis is 24, 2*b* = 24 □ *b* = 12.

We know that $a^2 + b^2 = c^2$.

$a^2 + 12^2 = 13^2$

$a^2 = 169 - 144 = 25$

Thus, the equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{144} = 1$.

Question 12:

Find the equation of the hyperbola satisfying the give conditions: Foci $\left(\pm 3\sqrt{5},\ 0\right)$, the latus rectum is of length 8.

Ans:

Foci $\left(\pm 3\sqrt{5},\ 0\right)$, the latus rectum is of length 8.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Since the foci are $\left(\pm 3\sqrt{5},\ 0\right)$, $c = \pm 3\sqrt{5}$.

Length of latus rectum = 8

$\Rightarrow \frac{2b^2}{a}=8$

$\Rightarrow b^2 = 4a$

We know that $a^2 + b^2 = c^2$.

$a^2 + 4a = 45$

$a^2 + 4a - 45 = 0$

$a^2 + 9a - 5a - 45 = 0$

$(a + 9)(a - 5) = 0$

$a = -9, 5$

Since a is non-negative, $a = 5$.

$b^2 = 4a = 4 \times 5 = 20$

Thus, the equation of the hyperbola is $\frac{x^2}{25}-\frac{y^2}{20}=1$.

Question 13:

Find the equation of the hyperbola satisfying the give conditions: Foci (±4, 0), the latus rectum is of length 12

Ans:

Foci (±4, 0), the latus rectum is of length 12.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Since the foci are (±4, 0), c = 4.

Length of latus rectum = 12

$$\Rightarrow \frac{2b^2}{a}=12$$
$$\Rightarrow b^2=6a$$

We know that $a^2 + b^2 = c^2$.

$a^2 + 6a = 16$

$a^2 + 6a - 16 = 0$

$a^2 + 8a - 2a - 16 = 0$

$(a + 8)(a - 2) = 0$

$a = -8, 2$

Since a is non-negative, a = 2.

$b^2 = 6a = 6 \times 2 = 12$

Thus, the equation of the hyperbola is $\frac{x^2}{4}-\frac{y^2}{12}=1$.

Question 14:

Find the equation of the hyperbola satisfying the give conditions: Vertices (±7, 0), $e=\frac{4}{3}$

Ans:

Vertices (±7, 0), $e=\frac{4}{3}$

Here, the vertices are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Since the vertices are (±7, 0), $a = 7$.

It is given that $e=\frac{4}{3}$

$\therefore \frac{c}{a}=\frac{4}{3} \qquad \left[e=\frac{c}{a}\right]$

$\Rightarrow \frac{c}{7}=\frac{4}{3}$

$\Rightarrow c=\frac{28}{3}$

We know that $a^2 + b^2 = c^2$.

$\therefore 7^2+b^2=\left(\frac{28}{3}\right)^2$

$\Rightarrow b^2=\frac{784}{9}-49$

$\Rightarrow b^2=\frac{784-441}{9}=\frac{343}{9}$

Thus, the equation of the hyperbola is $\frac{x^2}{49}-\frac{9y^2}{343}=1$.

Question 15:

Find the equation of the hyperbola satisfying the give conditions: Foci $\left(0,\ \pm\sqrt{10}\right)$, passing through (2, 3)

Ans:

Foci $\left(0, \pm\sqrt{10}\right)$, passing through (2, 3)

Here, the foci are on the *y*-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.

Since the foci are $\left(0, \pm\sqrt{10}\right)$, $c=\sqrt{10}$.

We know that $a^2 + b^2 = c^2$.

$a^2 + b^2 = 10$

$b^2 = 10 - a^2$ ... (1)

Since the hyperbola passes through point (2, 3),

$$\frac{9}{a^2}-\frac{4}{b^2}=1 \qquad ...(2)$$

From equations (1) and (2), we obtain

$$\frac{9}{a^2}-\frac{4}{\left(10-a^2\right)}=1$$
$$\Rightarrow 9\left(10-a^2\right)-4a^2=a^2\left(10-a^2\right)$$
$$\Rightarrow 90-9a^2-4a^2=10a^2-a^4$$
$$\Rightarrow a^4-23a^2+90=0$$
$$\Rightarrow a^4-18a^2-5a^2+90=0$$
$$\Rightarrow a^2\left(a^2-18\right)-5\left(a^2-18\right)=0$$
$$\Rightarrow \left(a^2-18\right)\left(a^2-5\right)=0$$
$$\Rightarrow a^2=18 \text{ or } 5$$

In hyperbola, $c > a$, i.e., $c^2 > a^2$

$a^2 = 5$

$b^2 = 10 - a^2 = 10 - 5 = 5$

Thus, the equation of the hyperbola is $\frac{y^2}{5} - \frac{x^2}{5} = 1$.

NCERT 11th Maths Chapter 11, class 11 Maths Chapter 11 solutions

Chapterwise NCERT Solutions for Class 11 Maths :

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