NCERT Solutions for Maths: Chapter 10 - Circles

NCERT Solutions for 10th Class Maths: Chapter 10 - Circles

Class 10: Mathematics Chapter 10 solutions. Complete Class 10 Mathematics Chapter 10 Notes.

NCERT Solutions for 10th Class Maths: Chapter 10 - Circles

NCERT 10th Mathematics Chapter 10, class 10 Mathematics Chapter 10 solutions

Page No: 209

Exercise: 10.1

1. How many tangents can a circle have?

Answer

A circle can have infinite tangents.

2. Fill in the blanks :

(i) A tangent to a circle intersects it in .... point(s).

(ii) A line intersecting a circle in two points is called a ....

(iii) A circle can have .... parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ....

Answer

(i) one

(ii) secant

(iii) two

(iv) point of contact

3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

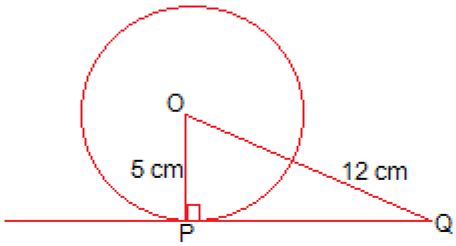
(A) 12 cm

(B) 13 cm

(C) 8.5 cm

(D) $\sqrt{119}$ cm

Answer



The line drawn from the centre of the circle to the tangent is perpendicular to the tangent.

∴ OP ⊥ PQ

By Pythagoras theorem in ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

⇒ $(12)^2 = 5^2 + PQ^2$

⇒$PQ^2 = 144 - 25$

⇒$PQ^2 = 119$

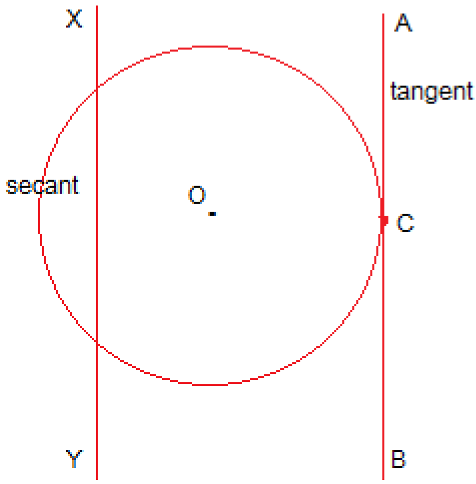
⇒$PQ = \sqrt{119}$ cm

(D) is the correct option.

4. Draw a circle and two lines parallel to a given line such that one is a tangent and the

other, a secant to the circle.

Answer



AB and XY are two parallel lines where AB is the tangent to the circle at point C while XY is the secant to the circle.

Exercise: 10.2

In Q.1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm

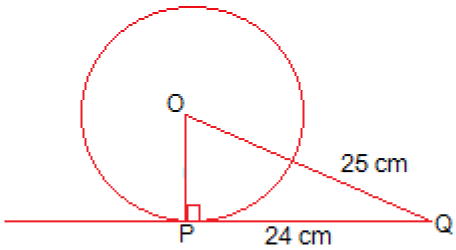
(B) 12 cm

(C) 15 cm

(D) 24.5 cm

Answer

The line drawn from the centre of the circle to the tangent is perpendicular to the tangent.



∴ OP ⊥ PQ

also, ΔOPQ is right angled.

OQ = 25 cm and PQ = 24 cm (Given)

By Pythagoras theorem in ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

⇒ $(25)^2 = OP^2 + (24)^2$

⇒ $OP^2 = 625 - 576$

⇒ $OP^2 = 49$

⇒ OP = 7 cm

The radius of the circle is option (A) 7 cm.

NCERT 10th Mathematics Chapter 10, class 10 Mathematics Chapter 10 solutions

2. In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to

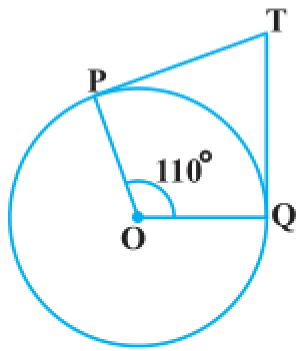
(A) 60°

(B) 70°

(C) 80°

(D) 90°

Answer



OP and OQ are radii of the circle to the tangents TP and TQ respectively.

∴ OP ⊥ TP and,

∴ OQ ⊥ TQ

∠OPT = ∠OQT = 90°

In quadrilateral POQT,

Sum of all interior angles = 360°

∠PTQ + ∠OPT + ∠POQ + ∠OQT = 360°

⇒ ∠PTQ + 90° + 110° + 90° = 360°

⇒ ∠PTQ = 70°

∠PTQ is equal to option (B) 70°.

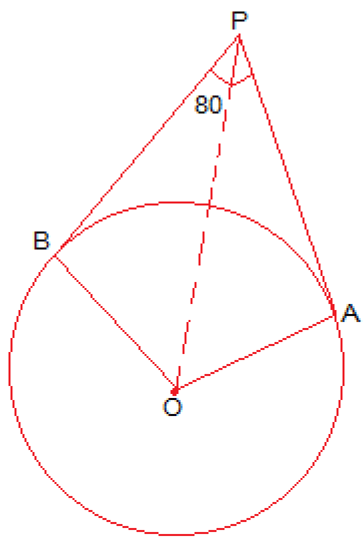
3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠ POA is equal to

(A) 50°

(B) 60°

(C) 70°

(D) 80°



OA and OB are radii of the circle to the tangents PA and PB respectively.

∴ OA ⊥ PA and,

∴ OB ⊥ PB

∠OBP = ∠OAP = 90°

In quadrilateral AOBP,

Sum of all interior angles = 360°

∠AOB + ∠OBP + ∠OAP + ∠APB = 360°

⇒ ∠AOB + 90° + 90° + 80° = 360°

⇒ ∠AOB = 100°

Now,

In ΔOPB and ΔOPA,

AP = BP (Tangents from a point are equal)

OA = OB (Radii of the circle)

OP = OP (Common side)

∴ $\Delta OPB \cong \Delta OPA$ (by SSS congruence condition)

Thus $\angle POB = \angle POA$

$\angle AOB = \angle POB + \angle POA$

$\Rightarrow 2 \angle POA = \angle AOB$

$\Rightarrow \angle POA = 100°/2 = 50°$

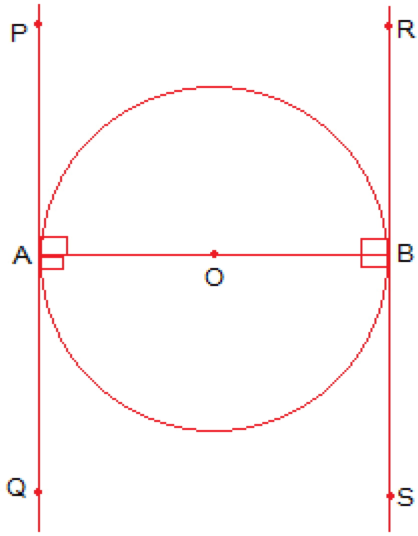
$\angle POA$ is equal to option (A) 50°

NCERT 10th Mathematics Chapter 10, class 10 Mathematics Chapter 10 solutions

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer

Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.



Radii of the circle to the tangents will be perpendicular to it.

∴ OB ⊥ RS and,

∴ OA ⊥ PQ

∠OBR = ∠OBS = ∠OAP = ∠OAQ = 90º

From the figure,

∠OBR = ∠OAQ (Alternate interior angles)

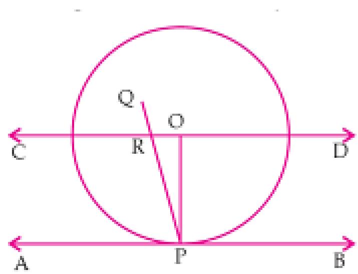
∠OBS = ∠OAP (Alternate interior angles)

Since alternate interior angles are equal, lines PQ and RS will be parallel.

Hence Proved that the tangents drawn at the ends of a diameter of a circle are parallel.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer



Let AB be the tangent to the circle at point P with centre O.

We have to prove that PQ passes through the point O.

Suppose that PQ doesn't passes through point O. Join OP.

Through O, draw a straight line CD parallel to the tangent AB.

PQ intersect CD at R and also intersect AB at P.

AS, CD // AB PQ is the line of intersection,

∠ORP = ∠RPA (Alternate interior angles)

but also,

∠RPA = 90° (PQ ⊥ AB)

⇒ ∠ORP = 90°

∠ROP + ∠OPA = 180° (Co-interior angles)

⇒∠ROP + 90° = 180°

⇒∠ROP = 90°

Thus, the ΔORP has 2 right angles i.e. ∠ORP and ∠ROP which is not possible.

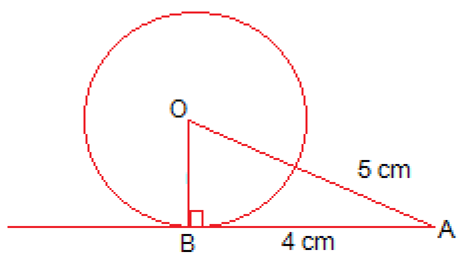
Hence, our supposition is wrong.

∴ PQ passes through the point O.

6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer

AB is a tangent drawn on this circle from point A.



∴ OB ⊥ AB

OA = 5cm and AB = 4 cm (Given)

In ΔABO,

By Pythagoras theorem in ΔABO,

$OA^2 = AB^2 + BO^2$

⇒ $5^2 = 4^2 + BO^2$

⇒ $BO^2 = 25 - 16$

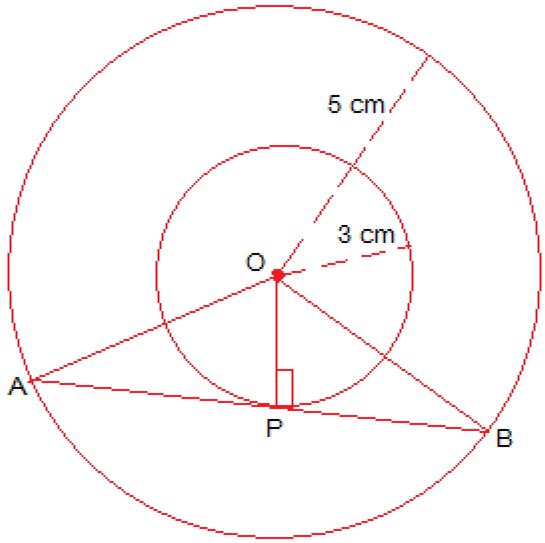
⇒ $BO^2 = 9$

⇒ BO = 3

∴ The radius of the circle is 3 cm.

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer



Let the two concentric circles with centre O.

AB be the chord of the larger circle which touches the smaller circle at point P.

∴ AB is tangent to the smaller circle to the point P.

⇒ OP ⊥ AB

By Pythagoras theorem in ΔOPA,

$OA^2 = AP^2 + OP^2$

⇒ $5^2 = AP^2 + 3^2$

⇒ $AP^2 = 25 - 9$

⇒ AP = 4

In ΔOPB,

Since OP ⊥ AB,

AP = PB (Perpendicular from the center of the circle bisects the chord)

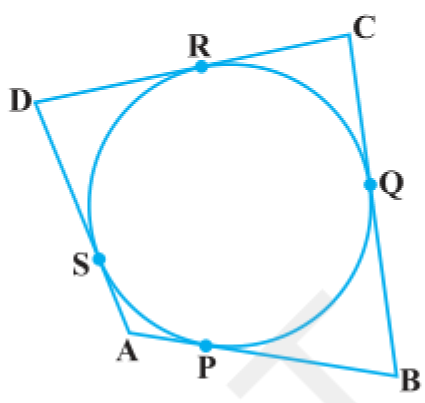
AB = 2AP = 2 × 4 = 8 cm

∴ The length of the chord of the larger circle is 8 cm.

NCERT 10th Mathematics Chapter 10, class 10 Mathematics Chapter 10 solutions

8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that AB + CD = AD + BC

Answer



From the figure we observe that,

DR = DS (Tangents on the circle from point D) … (i)

AP = AS (Tangents on the circle from point A) … (ii)

BP = BQ (Tangents on the circle from point B) … (iii)

CR = CQ (Tangents on the circle from point C) … (iv)

Adding all these equations,

DR + AP + BP + CR = DS + AS + BQ + CQ

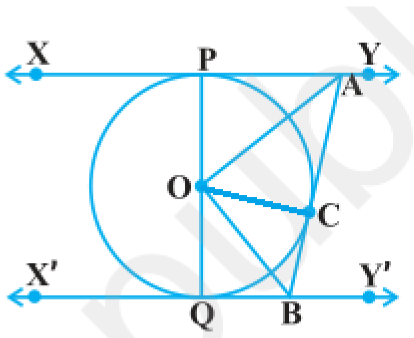
⇒ (BP + AP) + (DR + CR) = (DS + AS) + (CQ + BQ)

⇒ CD + AB = AD + BC

9. In Fig. 10.13, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that ∠ AOB = 90°.

Answer

We joined O and C



A/q,

In ΔOPA and ΔOCA,

OP = OC (Radii of the same circle)

AP = AC (Tangents from point A)

AO = AO (Common side)

∴ ΔOPA ≅ ΔOCA (SSS congruence criterion)

⇒ ∠POA = ∠COA … (i)

Similarly,

ΔOQB ≅ ΔOCB

∠QOB = ∠COB … (ii)

Since POQ is a diameter of the circle, it is a straight line.

∴ ∠POA + ∠COA + ∠COB + ∠QOB = 180 º

From equations (i) and (ii),

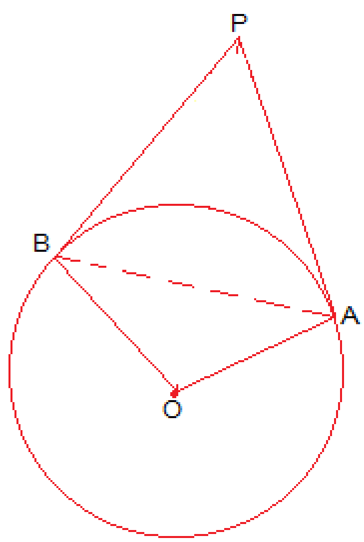
2∠COA + 2∠COB = 180º

⇒ ∠COA + ∠COB = 90º

⇒ ∠AOB = 90°

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer



Consider a circle with centre O. Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends ∠AOB at center O of the circle.

It can be observed that

OA ⊥ PA

∴ ∠OAP = 90°

Similarly, OB ⊥ PB

∴ ∠OBP = 90°

In quadrilateral OAPB,

Sum of all interior angles = 360º

∠OAP +∠APB +∠PBO +∠BOA = 360º

⇒ 90º + ∠APB + 90º + ∠BOA = 360º

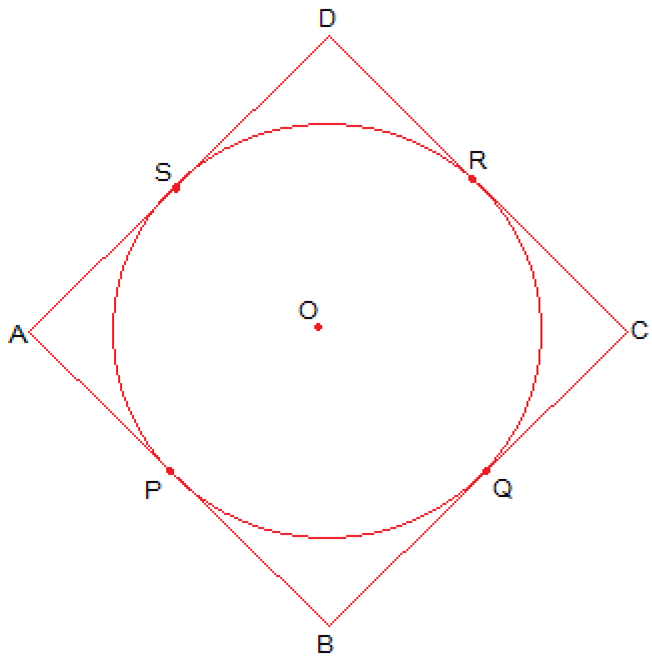
⇒ ∠APB + ∠BOA = 180º

∴ The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

NCERT 10th Mathematics Chapter 10, class 10 Mathematics Chapter 10 solutions

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Answer



ABCD is a parallelogram,

∴ AB = CD ... (i)

∴ BC = AD ... (ii)

From the figure, we observe that,

DR = DS (Tangents to the circle at D)

CR = CQ (Tangents to the circle at C)

BP = BQ (Tangents to the circle at B)

AP = AS (Tangents to the circle at A)

Adding all these,

DR + CR + BP + AP = DS + CQ + BQ + AS

⇒ (DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

⇒ CD + AB = AD + BC ... (iii)

Putting the value of (i) and (ii) in equation (iii) we get,

2AB = 2BC

⇒ AB = BC ... (iv)

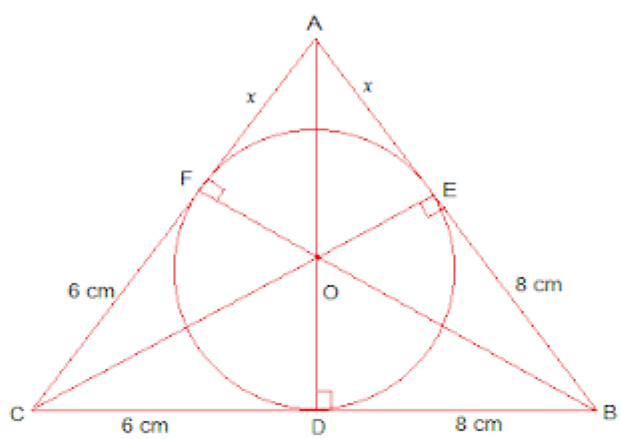
By Comparing equations (i), (ii), and (iv) we get,

AB = BC = CD = DA

∴ ABCD is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

Answer



In ΔABC,

Length of two tangents drawn from the same point to the circle are equal,

∴ CF = CD = 6cm

∴ BE = BD = 8cm

∴ AE = AF = x

We observed that,

AB = AE + EB = $x + 8$

BC = BD + DC = 8 + 6 = 14

CA = CF + FA = 6 + x

Now semi perimeter of triangle (s) is,

⇒ 2s = AB + BC + CA

= $x + 8 + 14 + 6 + x$

= $28 + 2x$

⇒s = $14 + x$

Area of ΔABC = √s (s - a)(s - b)(s - c)

= $\sqrt{(14 + x)\,(14 + x - 14)(14 + x - x - 6)(14 + x - x - 8)}$

= $\sqrt{(14 + x)\,(x)(8)(6)}$

= $\sqrt{(14 + x)\,48x}$ ... (i)

also, Area of ΔABC = 2×area of (ΔAOF + ΔCOD + ΔDOB)

= 2×[(1/2×OF×AF) + (1/2×CD×OD) + (1/2×DB×OD)]

= 2×1/2 ($4x$ + 24 + 32) = 56 + $4x$ ... (ii)

Equating equation (i) and (ii) we get,

$\sqrt{(14 + x)\,48x} = 56 + 4x$

Squaring both sides,

$48x\ (14 + x) = (56 + 4x)^2$

$\Rightarrow 48x = [4(14 + x)]^2/(14 + x)$

$\Rightarrow 48x = 16\ (14 + x)$

$\Rightarrow 48x = 224 + 16x$

$\Rightarrow 32x = 224$

$\Rightarrow x = 7$ cm

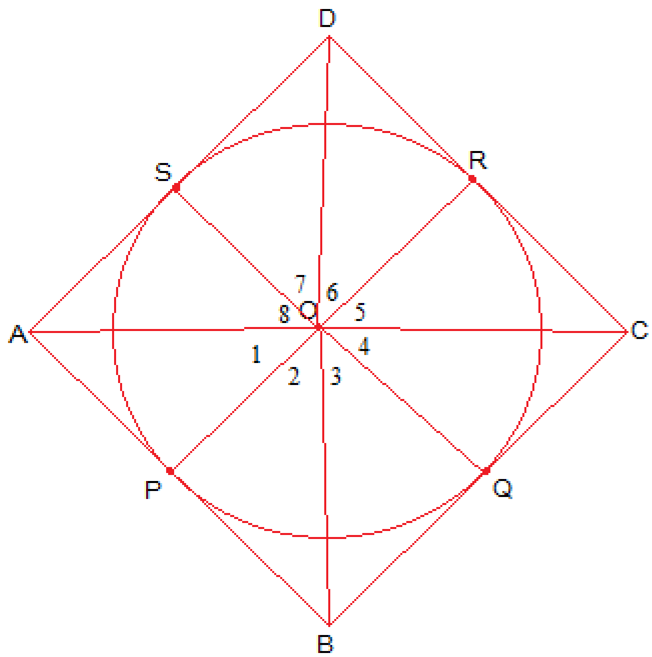
Hence, AB = $x + 8 = 7 + 8 = 15$ cm

CA = $6 + x = 6 + 7 = 13$ cm

NCERT 10th Mathematics Chapter 10, class 10 Mathematics Chapter 10 solutions

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer



Let ABCD be a quadrilateral circumscribing a circle with O such that it touches the circle at point P, Q, R, S. Join the vertices of the quadrilateral ABCD to the center of the circle.

In ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS (SSS congruence condition)

∴ ∠POA = ∠AOS

⇒∠1 = ∠8

Similarly we get,

∠2 = ∠3

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

Adding all these angles,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$\Rightarrow (\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$

$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$

$\Rightarrow 2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$

$\Rightarrow \angle AOB + \angle COD = 180^\circ$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^\circ$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

NCERT 10th Maths Chapter 10

Chapterwise NCERT Solutions for Class 10 Maths:

- Chapter 1 Real Numbers
- Chapter 2 Polynomials
- Chapter 3 Pair of Linear Equations in Two Variables
- Chapter 4 Quadratic Equations
- Chapter 5 Arithmetic Progressions
- Chapter 6 Triangles
- Chapter 7 Coordinate Geometry
- Chapter 8 Introduction to Trigonometry
- Chapter 9 Applications of Trigonometry
- Chapter 10 Circle
- Chapter 11 Constructions
- Chapter 12 Areas related to Circles
- Chapter 13 Surface Areas and Volumes
- Chapter 14 Statistics

- Chapter 15 Probability

About NCERT

The National Council of Educational Research and Training is an autonomous organization of the Government of India which was established in 1961 as a literary, scientific, and charitable Society under the Societies Registration Act. The major objectives of NCERT and its constituent units are to: undertake, promote and coordinate research in areas related to school education; prepare and publish model textbooks, supplementary material, newsletters, journals and develop educational kits, multimedia digital materials, etc.
Organise pre-service and in-service training of teachers; develop and disseminate innovative educational techniques and practices;collaborate and network with state educational departments, universities, NGOs and other educational institutions; act as a clearing house for ideas and information in matters related to school education; and act as a nodal agency for achieving the goals of Universalisation of Elementary Education. In addition to research, development, training, extension, publication and dissemination activities, NCERT is an implementation agency for bilateral cultural exchange programmes with other countries in the field of school education.Its headquarters are located at Sri Aurobindo Marg in New Delhi. Visit the Official NCERT website to learn more.