2019 III 13 **1000** **Seat No.** | | | | | |

Time : 2½ Hours

MATHEMATICS & STATISTICS

Subject Code

H	6	0	6

Total No. of Questions : 30 **(Printed Pages : 8)** **Maximum Marks : 80**

INSTRUCTIONS **:**

(*i*) *All* questions are compulsory.

(*ii*) The question paper consists of **30** questions divided into five sections A, B, C, D and E.

(*iii*) Section A contains **7** questions of **1** mark each, which are multiple choice type questions, Section B contains **7** questions of 2 marks each, Section C contains **7** questions of 3 marks each, Section D contains **7** questions of **4** marks each and Section E contains **2** questions of **5** marks each.

(*iv*) There is no overall choice in the paper. However internal choice is provided in **2** questions of **3** marks, **2** questions of **4** marks each and **2** questions of **5** marks. In questions with choices, only one of the choices is to be attempted.

(*v*) Use of calculators is not permitted.

(*vi*) Logarithmic tables will be supplied on request.

(*vii*) Graphs should be drawn on the answer paper only.

Section A

Question Nos. **1** to **7** carry **1** mark each. In each question, *four* options are provided, out of which *one* is correct. Write the correct option.

1. If $\begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} + 2X = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$, then matrix X = ...

- $\begin{bmatrix} -2 & 2 \\ 2 & 0 \end{bmatrix}$
- $\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$
- $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$
- $\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$

2. If $y = \log(\sec x)$, then $\frac{dy}{dx}$ = ...

- $\sec x \tan x$
- $\sec x$
- $\tan x$
- $\sec x + \tan x$

3. Which statement among the following statements is *false* ?

- Banker's gain is interest on true discount
- Banker's discount is interest on face value
- True discount is interest on present value
- Banker's gain is interest on Banker's discount

4. The total time from the beginning of the first payment period to the end of the last payment period of an annuity is called :

- Payment interval
- Interval of deferment
- Term
- Time period

5. In a partnership deed on admission of a new partner, the premium brought by the new partner is shared among the old partners in ....

- Sacrificing ratio
- Gaining ratio
- New profit sharing ratio
- Old profit sharing ratio

6. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \text{............} + C$

- $\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right|$
- $\frac{1}{2a} \log \left| \frac{x+a}{x-a} \right|$
- $\log (x + \sqrt{x^2 - a^2})$
- $\log (x - \sqrt{x^2 - a^2})$

7. $\int_0^{\pi/4} \sec^2 x \, dx = \text{.............}$

- 0
- $\frac{1}{2}$
- -1
- 1

Section B

Question Nos. **8** to **14** carry **2** marks each.

8. If $f : \mathbf{R}^+ \to (1, \infty)$ such that $f(x) = x^2 + 1$, then show that the function 'f' is bijective.

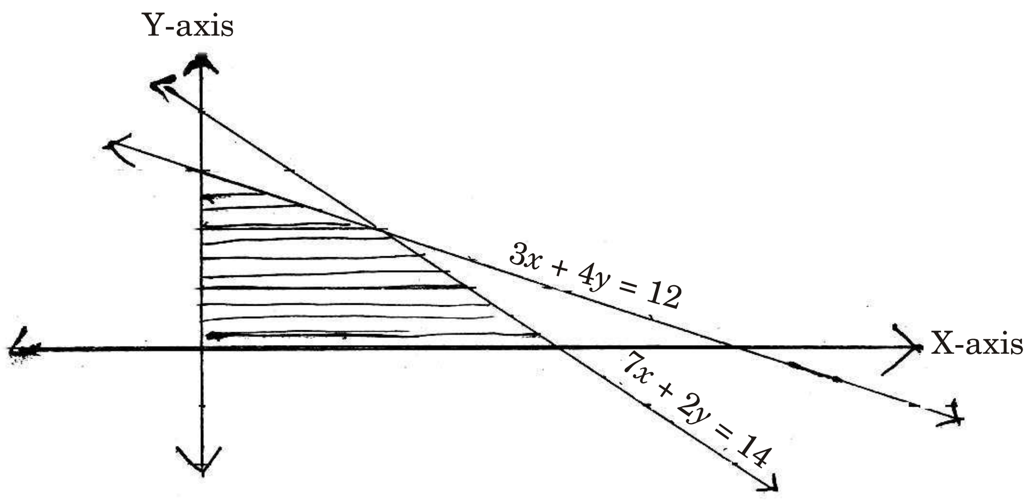
9. If $a * b = \dfrac{ab}{2}$ and $(x * 5) \times 3 = 15$, then find x.

10. If $A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$, then show that $A^3 = A$.

11. If the point (1, 1), (0, 2) and (3, y) are collinear points, then find y using determinant.

12. Total cost function of a commodity is given by $C(x) = 3x^2 + 7x + 12$. Find x so that marginal cost is equal to average cost.

13. Write the linear constraints of a linear programming problem if the feasible region of the problem is given by the shaded region in the following figure :



14. If A and B are two independent events such that $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{4}$, then find $P(A' \cup B')$.

Section C

Question Nos. **15** to **21** carry **3** marks each.

15. Find x, y and z if :

$$\left\{3\begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} - 4\begin{bmatrix} 2 & -2 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

16. If $e^x + e^y = e^{x+y}$, then show that :

$$\frac{dy}{dx} = \frac{-e^y}{e^x}.$$

17. A monopolist's demand function for one of its products is $p(x) = ax + b$, where x is the number of units produced and p is the price per unit. If on selling 10 units, the price is Rs. 1,200 per unit and on selling 7 units, the price per unit is Rs. 1,500, then find total revenue function and marginal revenue function of the product.

Or

Profit function of a commodity is given by $P(x) = 600x - 2000 - 3x^2$, where x is number of items. Then find x so that profit is maximum.

18. Find :

$$\int \frac{x+1}{x^2 - 2x - 3} dx.$$

19. Evaluate :

$$\int_0^{\pi} x \sin x \, dx.$$

20. Solve the differential equation :

$$y(x + 1)\, dx + x(y + 1)\, dy = 0\,.$$

Also find the particular solution given that $y = 1$ when $x = 1$.

Or

Solve the differential equation :

$$x^2 \frac{dy}{dx} = x^2 - y^2 + xy.$$

21. Form the differential equation by eliminating the arbitrary constants A and B from the equation :

$$y = \mathrm{A}x^2 + \mathrm{B}x.$$

Section D

Question Nos. **22** to **28** carry **4** marks each.

22. By using properties of determinants prove that :

$$\begin{vmatrix} 2a+b & a & a \\ a & 2a+b & a \\ a & a & 2a+b \end{vmatrix} = (4a + b)\,(a + b)^2.$$

23. If f is a function such that :

$$\begin{aligned} f(x) &= 4x^4 - 3x^3, \ 0 < x \leq 1 \\ &= Ax + B, \ 1 < x < 2 \\ &= 5x^2 - 8x, \ 2 \leq x \leq 3 \end{aligned}$$

and f is continuous on its domain, then find A and B.

24. If $x = t^2 \log t$ and $y = \frac{\log t}{t}$, then find $\frac{dy}{dx}$ when $t = 1$.

25. There are three boxes containing bulbs. In Box I out of 10 bulbs 4 are defective. In Box II out of 9 bulbs 3 are defective. In Box III out of 8 bulbs 4 are defective. One of the boxes is selected at random and a bulb is selected from the box at random. If the selected bulb is defective, then find the probability that it is from box II.

Or

A die is tossed 3 times. If getting a number greater than 2 is a success then find the probability distribution of number of successes. Also find mean of the probability distribution.

26. Solve the following linear programming problem graphically.

Minimize :

$$Z = 8x + 9y$$

Subject to the constraints :

$$x + y \leq 8, \ 4x + 3y \geq 7,$$
$$x + 6y \geq 7, \ x \geq 0, \text{ and } y \geq 0.$$

27. A bill of Rs. 60,000 drawn on 2nd January, 2017 for 9 months and discounted on 12th May 2017 for Rs. 58,680. Find the rate of interest per annum.

28. Find :

$$\int \frac{\sin x \, dx}{\sqrt{\cos^2 x - 5 \cos x + 7}}.$$

Or

Find :

$$\int \frac{1}{2 \sin^2 x - 3 \cos^2 x - 1} \, dx.$$

Section E

Question Nos. **29** and **30** carry **5** marks each.

29. A loan of Rs. 5,05,700 to purchase a car is re-paid in 18 half yearly equal instalments, at 8% per annum compounded half yearly. The first instalment being paid at the end of first half year. Find the amount of each instalment.

Or

Find the amount of deferred annuity of Rs. 9,000 payable at the end of every quarter year, the first instalment being made at the end of 2 years and last instalment at the end of 10 years, if money is worth 12% per annum compounded quarterly.

30. X and Y started a business in partnership. X invested $\frac{2}{3}$ of the capital and rest by Y. After 9 months, X withdrew half of his investment. After n months, they divided their profit in the ratio 3 : 2. Then find n.

Or

A, B and C are partners of a firm contributing Rs. 5,00,000, Rs. 3,00,000 and Rs. 2,00,000 respectively. The partnership deed provided an interest of 5% on the capital investment. B and C gets Rs. 30,000 and Rs. 20,000 per month respectively, as salary for their services to the firm. At the end of the year the firm gets a profit of Rs. 12,00,000. What will each receive if the remaining profit is shared in ratio of capitals.