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Time: $\mathbf{2 1}^{1 ⁄ 2}$ Hours
MATHEMATICS AND STATISTICS
(New Pattern)

## Subject Code

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INSTRUCTIONS: 1) All questions are compulsory.
2) The question paper consists of $\mathbf{3 0}$ questions divided into five Sections - A, B, C, D and E.
3) Section - A contains $\mathbf{7}$ questions of $\mathbf{1}$ mark each, which are multiple choice type questions. Section - B contains 7 questions of 2 marks each, Section - C contains 7 questions of 3 marks each, Section - D contains 7 questions of 4 marks each and Section - E contains 2 questions of 5 marks each.
4) There is no overall choice in the paper. However internal choice is provided in 2 questions of 3 marks each, 2 questions of 4 marks each and 2 questions of 5 marks. In questions with choices only one of the choices is to be attempted.
5) Use of calculators is not permitted.
6) Log tables will be supplied on request.
7) Graphs should be drawn on the answer paper only.

## SECTION -A

Question numbers 1 to $\mathbf{7}$ carry 1 mark each. In each question, four options are provided, out of which one is correct. Select and write the correct option :

1. If any two rows or columns of a determinant are identical, then value of the determinant is $\qquad$

- 1
-     - 1
- 0
- 2

2. If $\left|\begin{array}{cc}x+1 & 8 \\ 3 & 4\end{array}\right|=8$, then $x$ is

- $x=0$
- $x=4$
- $x=7$
- $x=3$

3. If a function $f: R \rightarrow R$ is defined as $f(x)=2 x+3$, then $(f \circ f)(1)$ is $\qquad$

- 1
- 5
- 10

4. $\int \cot x d x=$ $\qquad$

- $\log |\sec x|+c$
- $\log |\sin x|+c$
- $\log |\sec x+\tan x|+c$
- $\log \left|\tan \frac{x}{2}\right|+c$

5. The interest on the present value of a bill of exchange is known as

- Banker's discount
- True discount
- Banker's gain
- Discounted value

6. Gauri invested ₹ 2,000 for 4 months and Seema invested ₹ 3,000 for 8 months in a partnership deed. Then the adjusted capital ratio is $\qquad$

- 1:3
- 3 : 1
- 2 : 3
- 3 : 2

7. The future value of an ordinary annuity of ₹ 1,000 a year, for 3 years at $5 \%$ per annum, compounded annually (Given $S_{\overline{3} 10.05}=3.1525, S_{\overline{3} 10.025}=3.0756$ ) is $\qquad$

- 3152.5
- 3.1525
- 3075.6
- 307.56


## SECTION - B

Question numbers 8 to 14 carry 2 marks each :
8. If $A^{\prime}=\left[\begin{array}{rr}3 & 4 \\ -1 & 2 \\ 0 & 1\end{array}\right]$ and $B=\left[\begin{array}{rrr}-1 & 2 & 1 \\ 1 & 2 & 3\end{array}\right]$, where $A^{\prime}$ is transpose of $A$. Find $(3 A+B)^{\prime}$.
9. If $f(x)=3^{4 x}$, find the derivative of $f(x)$ with respect to $x$, using first principle.
10. If $\int_{1}^{2}\left[8 x^{3}-2 x+6 K\right] d x=3$. Find value of $K$.
11. Evaluate $\int e^{x}\left[1-\cot x+\cot ^{2} x\right] d x$.
12. A company manufactures fans and sewing machines. The company has only $₹ 5,760$ to invest and can manufacture atmost 200 items. The cost of manufacturing a fan is ₹ 360 and a sewing machine is ₹ 240 . The profit that the company gets on a sale of a fan is ₹ 22 and a sewing machine is ₹ 18 . Write the objective function and the constraints to maximize the profit.
13. The probability that a certain person will buy a shirt is 0.2 , the probability that he will buy a trouser is 0.3 and the probability that he will buy a shirt given that he buys a trouser is 0.4 . Find the probability that he will buy both a shirt and a trouser. Also find the probability that he will buy a trouser given that he buys a shirt.
14. Define "Present Worth" of a firm and "Profit sharing ratio" in a partnership deed.
SECTION - C

Question numbers 15 to 21 carry 3 marks each :
15. On the set $R$ of real numbers binary operation $*$ is defined by $a * b=(a-b)^{2}$. Determine, whether* is commutative and associative.
16. If $y=2^{x+y}$, show that $\frac{d y}{d x}=\frac{y \log 2}{1-y \log 2}$.
17. If $x=\frac{t^{2}-1}{2 t}, y=\frac{t+2}{4 t}$, find $\frac{d y}{d x}$ at $t=2$.
18. Evaluate $\int_{1}^{2} x^{2} \log x d x$.

## OR

Evaluate $\int_{0}^{1} x^{2} \sqrt{1-x} d x$
19. Form a differential equation by eliminating the arbitrary constants $a$ and $b$ from $y=(a+b x) e^{5 x}$.
20. Solve the differential equation $\left(2 x^{2}+1\right) d y+x \sin y d x=0$. Hence find particular solution when $x=y=\frac{\pi}{2}$.
21. Gurmeet, Ram and Rahim are partners in business and decide to share the profits and losses in the ratio 5:6:7. Ram took retirement from the partnership. The new profit sharing ratio of Gurmeet and Rahim is $20: 34$. Find the gaining ratio. Also find the share of Gurmeet from the profit of ₹ $1,08,000$ after retirement of Ram.
OR

Priti, Niki and Nishu enter into a partnership investing ₹ 60,000 , ₹ 36,000 and $₹ 30,000$ respectively. It was agreed that after allowing $\frac{1}{8}^{\text {th }}$ of the profits to Nishu as a manager, the remaining profits will be divided among the partners in the ratio of their capitals. At the end of the year, Nishu received ₹ 8,000. What was the total profit of the business and how much did Priti and Niki receive ?

## SECTION - D

Question numbers 22 to 28 carry 4 marks each :
22. Solve the following system of equations by using the matrix method:

$$
\begin{aligned}
& x+2 y+3 z=20 \\
& 2 x-y+z=5 \\
& 3 x+2 y-z=8
\end{aligned}
$$

23. By using properties of determinants, show that $\left|\begin{array}{ccc}1 & y & y^{2} \\ y^{2} & 1 & y \\ y & y^{2} & 1\end{array}\right|=(1-y)^{2}\left(1+y+y^{2}\right)^{2}$.
24. Discuss the continuity of the function :

$$
\begin{aligned}
f(x) & =\frac{x^{2}-4 \sin x}{x} \quad-1<x<0 \\
& =\frac{x-24}{x+3}+4 \quad x=0 \\
& =\frac{\log (1+8 x)}{-2 x} \quad 0<x<1 \\
\text { at } x & =0
\end{aligned}
$$

25. Evaluate $\int \frac{1 d x}{x \sqrt{6(\log x)^{2}+7 \log x+2}}$.

OR
Evaluate $\int \frac{14 \sin x+8 \cos x}{2 \sin x+3 \cos x} d x$.
26. Solve the following linear programming problem graphically :

Maximize $Z=50 x+15 y$
Subject to the constraints :

$$
\begin{gathered}
5 x+y \leq 100 \\
x+y \leq 60 \\
x, y \geq 0
\end{gathered}
$$

27. There are 6 boys and 4 girls in Room $A$ and 8 boys and 2 girls in Room B. $A$ student is selected at random from one of the two rooms for the post of class representative. What is the probability that the student is from Room A, given that the class representative is a boy.

OR
Past experience shows that $80 \%$ of the surgeries performed by a doctor are successful. If he performs 2 surgeries in a day. Find the probability distribution of the number of successful surgeries performed by the doctor.
28. A bill was drawn on $20^{\text {th }}$ July, 2016 at 3 months after sight and was accepted on presentation on $1^{\text {st }}$ August 2016. It was discounted on $23^{\text {rd }}$ August 2016 at $5 \%$ p.a. interest for $₹ 7,920$. Find the face value of the bill.

## SECTION-E

Question numbers 29 to $\mathbf{3 0}$ carry 5 marks each :
29. If the average cost function of an article manufactured by a company is given by $A C=\frac{1}{300} x^{2}-5 x+\frac{100}{x}$. Find the output at which the Marginal Cost is minimum. OR

A manufacturer finds that his product can be assembled at a total cost $C(x)=₹(200+30 x)$, where $x$ is the number of units manufactured. Assume that the price at which he can sell each is given by $p=₹\left(150-\frac{x}{3}\right)$. What level of production will maximize the total profit ? What is the price at this level of production?

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30. Anupam Sehgal buys a house worth ₹ $2,50,000$. The contract is that Mr. Sehgal will pay ₹ $1,00,000$ immediately and the balance in 15 equal annual instalments at the end of each year with $15 \%$ p.a. compound interest. Find the amount of each instalment. (Use log table).

OR
Rajan wants to accumulate ₹ $5,00,000$ to send his son abroad for further studies after 4 years. How much should he save at the beginning of each year to accumulate this amount for 4 years, if money is worth $6 \%$ compounded annually. (Use log table).

