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Senior Secondary School Certificate Examination

July 2017 (Compartment)

Marking Scheme — Mathematics 65/1, 65/2, 65/3

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\lim\limits_{x\to 0}\dfrac{\sin\frac{3x}{2}}{2} = \lim\limits_{x\to 0}\dfrac{3}{2}\cdot\dfrac{\sin\frac{3x}{2}}{\frac{3x}{2}} = \dfrac{3}{2}$ $\qquad \frac{1}{2}$

$\Rightarrow \quad k = \dfrac{3}{2}$ $\qquad \frac{1}{2}$

2. $|A^{-1}| = \dfrac{1}{|A|}$ $\qquad \frac{1}{2}$

$= \dfrac{1}{4}$ $\qquad \frac{1}{2}$

3. $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 225 \Rightarrow |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 225$ $\qquad \frac{1}{2}$

$\Rightarrow \quad (5)^2|\vec{b}|^2 = 225 \Rightarrow |\vec{b}| = 3$ $\qquad \frac{1}{2}$

4. $\displaystyle\int\frac{3x}{3x-1}dx = \int\frac{3x-1+1}{3x-1}dx$ $\qquad \frac{1}{2}$

$= x + \dfrac{1}{3}\log|3x-1| + C$ $\qquad \frac{1}{2}$

SECTION B

5. Getting $\begin{pmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$ $\qquad 1$

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow \quad x = 2, y = 9$ $\qquad 1$

6. $f(x) = \sin 2x - \cos 2x$

$\Rightarrow \quad f'(x) = 2\cos 2x + 2\sin 2x$ $\qquad 1$

$f'\left(\dfrac{\pi}{6}\right) = 2\left[\cos\dfrac{\pi}{3} + \sin\dfrac{\pi}{3}\right] = (1+\sqrt{3})$ $\qquad 1$

7. $6y = x^3 + 2 \Rightarrow 6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ 1

$\frac{dy}{dt} = 2\frac{dx}{dt} \Rightarrow 12 = 3x^2 \Rightarrow x = \pm 2$ $\frac{1}{2}$

$\therefore$ The points are (2, 5/3), (–2, –1) $\frac{1}{2}$

8. $\int \frac{x}{\sqrt{32 - x^2}}dx = -\int 1.dt$ where $32 - x^2 = t^2$ 1

$= -t + C = -\sqrt{32 - x^2} + C$ 1

9. $\log\left(\frac{dy}{dx}\right) = 3x + 4y \Rightarrow \frac{dy}{dx} = e^{3x}.e^{4y}$ $\frac{1}{2}$

$\Rightarrow \int e^{-4y}dy = \int e^{3x}dx$ $\frac{1}{2}$

$\Rightarrow -\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} + C$ 1

10. Given differential equation can be written as

$\frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$ 1

Getting integrating factor = $e^{x - \log x}$ or $\frac{e^x}{x}$ 1

11. For coplanarity of vectors $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ \lambda & 7 & 3 \end{vmatrix} = 0$ 1

Solving to get $\lambda = 0$ 1

12. Let, Number of executive class tickets be x and economy class tickets be y.

$\therefore$ LPP is Maximise Profit $P = 1500x + 1000y$ 1

Subject to: $x + y \leq 250, x \geq 25, y \geq 3x$ 1

SECTION C

13. Let the award for regularily be ₹ x and for hard work be ₹ y.

$\therefore$ $x + y = 6000$ and 1

$x + 3y = 11000$

$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$ or $A.X = B$ 1

$\therefore$ $X = A^{-1}B$ as $|A| = 2 \neq 0$.

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3500 \\ 2500 \end{pmatrix}$ $\therefore$ x = ₹ 3500, y = ₹ 2500 1

Any two values like obedience, respect for elders,... 1

14. Given equation can be written as $\tan^{-1}(1) - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$ $1\frac{1}{2}$

$\Rightarrow \frac{3}{2}\tan^{-1}x = \frac{\pi}{4}$ or $\tan^{-1}x = \frac{\pi}{6}$ $1\frac{1}{2}$

$\Rightarrow x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ 1

15. Let $u = (\cos x)^x \Rightarrow \log u = x.\log\cos x$ $\frac{1}{2}$

$\Rightarrow \frac{du}{dx} = (\cos x)^x.[-x\tan x + \log\cos x]$ $1\frac{1}{2}$

$\therefore$ $y = (\cos x)^x + \sin^{-1}\sqrt{3x} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-3x}}.\frac{\sqrt{3}}{2\sqrt{x}}$ $1\frac{1}{2}$

$\therefore$ $\frac{dy}{dx} = (\cos x)^x[-x\tan x + \log\cos x] + \frac{\sqrt{3}}{2\sqrt{x}}.\frac{1}{\sqrt{1-3x}}$ $\frac{1}{2}$

OR

$$y = (\sec^{-1}x)^2 \quad \Rightarrow \quad \frac{dy}{dx} = 2\sec^{-1}x.\frac{1}{x\sqrt{x^2-1}}$$ 1

$$\therefore \quad x\sqrt{x^2-1}\cdot\frac{dy}{dx} = 2\sec^{-1}x$$ $\frac{1}{2}$

$$\Rightarrow \quad x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}\right) = \frac{2}{x\sqrt{x^2-1}}$$ $1\frac{1}{2}$

$$\Rightarrow \quad x^2(x^2-1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot x(2x^2-1) = 2$$ 1

$$\text{i.e.,} \quad x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3-x)\frac{dy}{dx} = 2$$

16. $f'(x) = 6x^2 - 6x - 36$ $\frac{1}{2}$

$= 6(x^2 - x - 6) = 6(x-3)(x+2)$

$f'(x) = 0 \Rightarrow x = -2, x = 3$ 1

$\therefore$ the intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

$\left.\begin{array}{l}\text{getting } f'(x) \text{ +ve in } (-\infty, -2) \cup (3, \infty) \\ \text{and –ve in } (-2, 3)\end{array}\right\}$ $1\frac{1}{2}$

$\therefore$ f(x) is strictly increasing in $(-\infty, -2) \cup (3, \infty)$, and $\frac{1}{2}$

strictly decreasing in $(-2, 3)$

17. $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$ $1\frac{1}{2}$

$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$ $\frac{1}{2}$

$\Rightarrow \quad I = \frac{-\pi}{2} \int_1^{-1} \frac{dt}{1+t^2} = \frac{\pi}{2} \int_{-1}^{1} \frac{dt}{1+t^2}$, where $\cos x = t$ 1

$I = \frac{\pi}{2} [\tan^{-1} t]_{-1}^{1} = \frac{\pi}{2} \left[\frac{\pi}{4} + \frac{\pi}{4} \right] = \frac{\pi^2}{4}$ 1

18. For $\int \frac{x^2 + x + 1}{(x+1)^2 (x+2)} dx = \int \left[\frac{3}{x+2} - \frac{2}{x+1} + \frac{1}{(x+1)^2} \right] dx$ $2\frac{1}{2}$

$= 3\log|x+2| - 2\log|x+1| - \frac{1}{x+1} + C$ $1\frac{1}{2}$

OR

$I = \int (x-3)\sqrt{3-2x-x^2}\, dx = \int \left[-\frac{1}{2}(-2-2x) - 4 \right] \sqrt{3-2x-x^2}\, dx$ 1

$= -\frac{1}{2} \int (-2-2x)\sqrt{3-2x-x^2}\, dx - 4\int \sqrt{4-(x+1)^2}\, dx$ $\frac{1}{2}+1$

$= -\frac{1}{3}(3-2x-x^2)^{3/2} - 4\left[\frac{(x+1)}{2} \sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2} \right) \right] + C$ $\frac{1}{2}+1$

19. Given differential equation can be written as $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$

$\frac{y}{x} = v \quad \Rightarrow \quad y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1+v^2}{2v} \quad \Rightarrow \quad x\frac{dv}{dx} = \frac{1-v^2}{2v}$

$\Rightarrow \quad \int \frac{2v}{v^2 - 1} dv = -\int \frac{dx}{x}$ 1

$\Rightarrow \quad \log|v^2 - 1| + \log|x| = \log C$ 1

$\Rightarrow \quad x(v^2 - 1) = C$

$\Rightarrow \quad y^2 - x^2 = Cx$ 1

20. Let the vector $\vec{p} = (2\vec{a} + \vec{b} + 2\vec{c})$ makes angles α, β, γ respectively with the vector $\vec{a}, \vec{b}, \vec{c}$

Given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{a} = 0$ 1

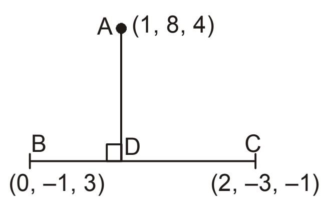
$$\cos\alpha = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|}$$ $\frac{1}{2}$

$$= \frac{2|a|^2}{3|\vec{a}||\vec{a}|} = \frac{2}{3} \quad \Rightarrow \quad \alpha = \cos^{-1}\frac{2}{3}$$ 1

$$\cos\beta = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{b}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{b}|} = \frac{|\vec{b}|^2}{3|\vec{b}||\vec{b}|} = \frac{1}{3} \quad \Rightarrow \quad \beta = \cos^{-1}\frac{1}{3}$$ 1

$$\cos\gamma = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{c}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{c}|} = \frac{2|\vec{c}|^2}{3|\vec{c}||\vec{c}|} = \frac{2}{3} \quad \Rightarrow \quad \gamma = \cos^{-1}\frac{2}{3}$$ $\frac{1}{2}$

21.



Equation of line passing through B and C is

$$\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} \quad \text{or} \quad \frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2}$$ 1

Any point D on BC can be

$[\lambda, -\lambda - 1, -2\lambda + 3]$ for some value of λ. 1

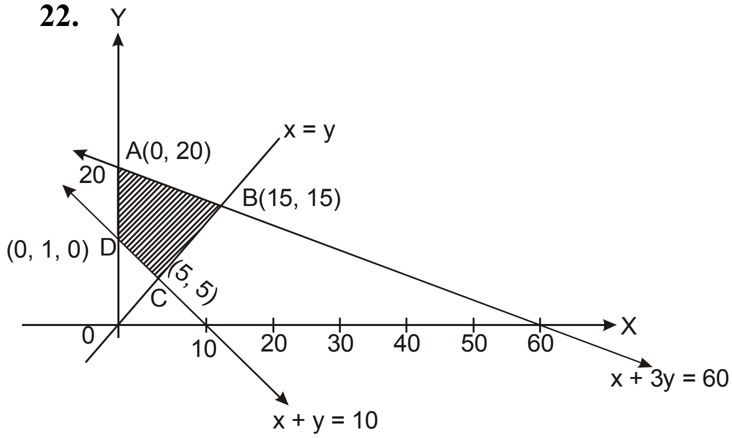
$\therefore$ Direction ratios of AD are $<\lambda - 1, -\lambda - 9, -2\lambda - 1>$ $\frac{1}{2}$

$AD \perp BC \Rightarrow 1(\lambda - 1) - 1(-\lambda - 9) - 2(-2\lambda - 1) = 0$ $\frac{1}{2}$

$\Rightarrow \quad \lambda = -\frac{5}{3}$ $\frac{1}{2}$

$\therefore$ D is $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ $\frac{1}{2}$

22.



Correct graph of three lines $1\frac{1}{2}$

Correct shading 1

Vertices of feasible region are

A(0, 20), B(15, 15), C(5, 5), D(0, 10)

$Z(A) = 180$

$Z(B) = 180$

$Z(C) = 60$ 1

$Z(D) = 90$

$\therefore$ Z = 60 is minimum at x = 5, y = 5 $\frac{1}{2}$

23. Let the events be

E_1: tansferring a red ball from A to B

E_2: transferring a black ball from A to B

A: Getting a red ball from bag B $\frac{1}{2}$

$$P(E_1) = \frac{3}{5},\ P(E_2) = \frac{2}{5}$$ $\frac{1}{2}$

$$P(A/E_1) = \frac{1}{2},\ P(A/E_2) = \frac{1}{3}$$ 1

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$ $\frac{1}{2}$

$$= \frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2}+\frac{2}{5}\cdot\frac{1}{3}} = \frac{9}{13}$$ $1+\frac{1}{2}$

OR

Required probability $= P(A \cup B)$ 1

$= P(A) + P(B) - P(A)\cdot P(B)$ $\frac{1}{2}$

$= P(A)\,[1 - P(B)] + 1 - P(B')$ $\frac{1}{2}$

$= P(A)\,P(B') - P(B') + 1$ 1

$= (1 - P(B')\,(1 - P(A)) = 1 - P(A')\,P(B')$ 1

SECTION D

24. $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$ 1

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow \quad (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$$ 1+1

$\Rightarrow \quad -(a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca) = 0$ 1

$\Rightarrow \quad \frac{-1}{2}(a+b+c)\,[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ 1

$\Rightarrow \quad a - b = 0 = b - c = c - a$ as $a + b + c \neq 0$ 1

$\Rightarrow \quad a = b = c$

25. (i) for any $A, B \in P(X)$, $A*B = A \cap B$ and $B*A = B \cap A$

as $A \cap B = B \cap A \therefore A*B = B*A$ 2

$\Rightarrow$ * is commutative

(ii) for any $A, B, C \in P(X)$

$(A*B)*C = (A \cap B)*C = (A \cap B) \cap C$

and $A*(B*C) = A*(B \cap C) = A \cap (B \cap C)$

Since $(A \cap B) \cap C = A \cap (B \cap C) \Rightarrow$ * is associative 2

(iii) for every $A \in P(X)$, $A*X = A \cap X = A$

$X*A = X \cap A = A$ 1

$\Rightarrow$ X is the identity element

(iv) $X*X = X \cap X = X \Rightarrow$ X is the only invertible element. $\because$ it is true only for X. 1

OR

$$f(x) = \frac{4x}{3x+4}$$

for $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$, $f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

$\therefore \quad 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$ 2

$\therefore \quad$ f is a 1 – 1 function.

for $y = \frac{4}{3}$, there is no x such that $f(x) = \frac{4}{3}$

$\therefore \quad$ f is not invertible 1

But $f : R - \left\{-\frac{4}{3}\right\} \rightarrow$ Range of f is ONTO so invertible. 1

and $f^{-1}(y) = \frac{4y}{4-3y}$ 2

26. Let given volume of cone be, $V = \frac{1}{3}\pi r^2 h$ ...(i) $\frac{1}{2}$

$\therefore \quad$ Surface area (curved) $S = \pi r l = \pi r\sqrt{r^2 + h^2}$ $\frac{1}{2}$

or $\quad A = S^2 = \pi r^2 (r^2 + h^2)$

$$A = S^2 = \pi^2 r^2 \left[r^2 + \left(\frac{3V}{\pi r^2}\right)^2\right] \quad \text{[using (i)]}$$

$$= \pi^2 \left[r^4 + \frac{9V^2}{\pi^2 r^2}\right] \qquad 1\frac{1}{2}$$

$$\frac{dA}{dr} = \pi^2 \left[4r^3 - \frac{18V^2}{\pi^2 r^3}\right] \qquad 1$$

$$\frac{dA}{dr} = 0 \Rightarrow 4\pi^2 r^6 = 18 \cdot \frac{1}{9}\pi^2 r^4 h^2$$

$\Rightarrow$ $2r^2 = h^2$ or $h = \sqrt{2}r$ $1\frac{1}{2}$

$$\frac{d^2A}{dr^2} = \pi^2\left[12r^2 + \frac{54V^2}{\pi^2 r^4}\right] > 0$$ 1

$\Rightarrow$ for least curved surface area, height = $\sqrt{2}$ (radius)

OR

$$x = a\cos\theta + a\theta\sin\theta \Rightarrow \frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta$$

$= a\theta\cos\theta$ 1

$$y = a\sin\theta - a\theta\cos\theta \Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta$$

$= a\theta\sin\theta$ 1

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$ $\frac{1}{2}$

Equation of tangent is

$y - (a\sin\theta - a\theta\cos\theta) = \tan\theta(x - a\cos\theta - a\theta\sin\theta)$ 1

Equation of normal is

$$y - (a\sin\theta - a\theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta)$$ 1

$\Rightarrow$ $y\sin\theta + x\cos\theta = a$ $\frac{1}{2}$

distance of normal from origin $= \dfrac{|-a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a| =$ constant 1

27. Equation of plane through A(2, –2, 1), B(4, 1, 3) and C(–2, –2, 5) is

$$\begin{vmatrix} x-2 & y+2 & z-1 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0 \Rightarrow 3x - 4y + 3z - 17 = 0$$ 2+1

For the given line $3(3) - 4(3) + 3(1) = 0$ 1

$\Rightarrow$ line is parallel to the plane

$\therefore$ Distance, $d = \dfrac{|3(5) - 4(4) + 3(8) - 17|}{\sqrt{9+16+9}} = \dfrac{6}{\sqrt{34}}$ 2

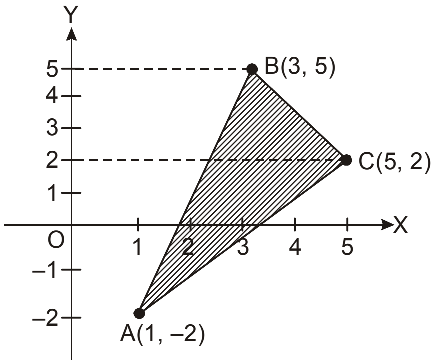
28. $P(\text{Head}) = 4P(\text{Tail}) \Rightarrow P(H) = \dfrac{4}{5}, P(T) = \dfrac{1}{5}$ 1

X (Number of tails)	0	1	2	3	$\frac{1}{2}$
P(X)	$\left(\frac{4}{5}\right)^3 = \frac{64}{125}$	$3\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) = \frac{48}{125}$	$3\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^2 = \frac{12}{125}$	$\left(\frac{1}{5}\right)^3 = \frac{1}{125}$	2
XP(X):	0	$\frac{48}{125}$	$\frac{24}{125}$	$\frac{3}{125}$	$\frac{1}{2}$
$X^2P(X)$:	0	$\frac{48}{125}$	$\frac{48}{125}$	$\frac{9}{125}$	$\frac{1}{2}$

Mean $= \Sigma XP(X) = \dfrac{75}{125} = \dfrac{3}{5}$ $\frac{1}{2}$

Variance: $\Sigma X^2P(X) - [\Sigma XP(X)]^2 = \dfrac{105}{125} - \dfrac{9}{25} = \dfrac{60}{125} = \dfrac{12}{25}$ 1

29. **For Correct Figure** 1



Equation of AB: $x = \frac{1}{7}(2y + 11)$

Equation of BC: $x = \frac{1}{3}(19 - 2y)$ $1\frac{1}{2}$

Equation of AC: $x = y + 3$

Required area $= \int_{-2}^{2} (y+3)dy + \frac{1}{3}\int_{2}^{5}(19-2y)dy - \frac{1}{7}\int_{-2}^{5}(2y+11)dy$ $1\frac{1}{2}$

$$\Rightarrow \quad A = \left.\frac{(y+3)^2}{2}\right]_{-2}^{2} + \frac{1}{3}\left.\frac{(19-2y)^2}{-4}\right]_{2}^{5} - \frac{1}{7}\left.\frac{(2y+11)^2}{4}\right]_{-2}^{5}$$ 1

$$= \frac{1}{2}(25-1) - \frac{1}{12}(81-225) - \frac{1}{28}(441-49) = 10 \text{ sq.units}$$ 1

OR

Here $h = \frac{4}{n}$ or $nh = 4$, $f(x) = 3x^2 + 2x + 1$ $\frac{1}{2}$

$$\int_0^4 (3x^2 + 2x + 1)dx = \lim_{h \to 0} h[f(0) + f(0+h) + f(0+2h) + \ldots + f(0 + \overline{n-1}\,h)]$$ 1

$$= \lim_{h \to 0} h[(1) + (3h^2 + 2h + 1) + (3.2^2 h^2 + 2.2h + 1) + \ldots + (3(n-1)^2 h^2 + 2(n-1)h + 1)]$$ $1\frac{1}{2}$

$$= \lim_{h \to 0} h\left[n + 3h^2 \frac{n(n-1)(2n-1)}{6} + 2h\frac{n(n-1)}{2}\right]$$ 1

$$= \lim_{h \to 0}\left[nh + \frac{(nh)(nh-h)(2nh-h)}{2} + (nh)(nh-h)\right]$$ 1

$= 4 + 64 + 16 = 84$ 1

QUESTION PAPER CODE 65/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2 = 225 \Rightarrow |\vec{a}|^2|\vec{b}|^2\,(\sin^2\theta+\cos^2\theta)=225$ $\frac{1}{2}$

$\Rightarrow \quad (5)^2\,|\vec{b}|^2 = 225 \Rightarrow |\vec{b}| = 3$ $\frac{1}{2}$

2. $\int \frac{3x}{3x-1}dx = \int \frac{3x-1+1}{3x-1}dx$ $\frac{1}{2}$

$= x+\frac{1}{3}\log|3x-1|+C$ $\frac{1}{2}$

3. $\lim\limits_{x\to 0}\frac{\sin\frac{3x}{2}}{2} = \lim\limits_{x\to 0}\frac{3}{2}\cdot\frac{\sin\frac{3x}{2}}{\frac{3x}{2}} = \frac{3}{2}$ $\frac{1}{2}$

$\Rightarrow \quad k = \frac{3}{2}$ $\frac{1}{2}$

4. $|A^{-1}| = \frac{1}{|A|}$ $\frac{1}{2}$

$= \frac{1}{4}$ $\frac{1}{2}$

SECTION B

5. Given differential equation can be written as

$$\frac{dy}{dx}+\left(1-\frac{1}{x}\right)y = \frac{1}{x}$$ 1

Getting integrating factor $= e^{x-\log x}$ or $\frac{e^x}{x}$ 1

6. For coplanarity of vectors $\begin{vmatrix}1 & 3 & 1\\ 2 & -1 & -1\\ \lambda & 7 & 3\end{vmatrix}=0$ 1

Solving to get $\lambda = 0$ 1

7. Let, Number of executive class tickets be x and economy class tickets be y.

$\therefore$ LPP is Maximise Profit P = 1500x + 1000y 1

Subject to: $x + y \le 250,\ x \ge 25,\ y \ge 3x$ 1

8. Getting $\begin{pmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$ 1

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow\quad x = 2,\ y = 9$ 1

9. $f(x) = \sin 2x - \cos 2x$

$\Rightarrow\quad f'(x) = 2\cos 2x + 2\sin 2x$ 1

$f'\left(\frac{\pi}{6}\right) = 2\left[\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right] = (1 + \sqrt{3})$ 1

10. $\frac{y}{y+2}dy = \frac{(x+2)}{x}dx \quad \Rightarrow \quad \int\left(1 - \frac{2}{y+2}\right)dy = \int\left(1 + \frac{2}{x}\right)dx$ $\frac{1}{2}+\frac{1}{2}$

$y - 2\log|y+2| = x + 2\log|x| + C$ 1

11. Put $\sin x = t \Rightarrow \cos x\, dx = dt$ $\frac{1}{2}$

Integral reduces to $\int \frac{dt}{\sqrt{8-t^2}} = \sin^{-1}\left(\frac{t}{\sqrt{8}}\right) + C$ $\frac{1}{2}+\frac{1}{2}$

$= \sin^{-1}\left(\frac{\sin x}{\sqrt{8}}\right) + C$ $\frac{1}{2}$

12. $\frac{dr}{dt} = 5$ cm/min, $\frac{dh}{dt} = -4$ cm/min

$V = \pi r^2 h$ $\frac{1}{2}$

$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2hr\frac{dr}{dt}\right)$ 1

$\left.\frac{dV}{dt}\right)_{r=8,\,h=6} = 224\,\pi\ \text{cm}^3/\text{min}$ $\frac{1}{2}$

$\therefore$ Volume is increasing at the rate of 224π cm^3/min.

SECTION C

13. Let the award for regularily be ₹ x and for hard work be ₹ y.

$\therefore$ $x + y = 6000$ and 1

$x + 3y = 11000$

$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$ or $A.X = B$ 1

$\therefore$ $X = A^{-1}B$ as $|A| = 2 \neq 0$.

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3500 \\ 2500 \end{pmatrix}$ $\therefore$ $x = ₹\ 3500,\ y = ₹\ 2500$ 1

Any two values like obedience, respect for elders,... 1

14. $f'(x) = 6x^2 - 6x - 36$ $\frac{1}{2}$

$= 6(x^2 - x - 6) = 6(x - 3)(x + 2)$

$f'(x) = 0 \Rightarrow x = -2, x = 3$ 1

$\therefore$ the intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

$\left.\begin{array}{l} \text{getting } f'(x) \text{ +ve in } (-\infty, -2) \cup (3, \infty) \\ \text{and –ve in } (-2, 3) \end{array}\right\}$ $1\frac{1}{2}$

$\therefore$ f(x) is strictly increasing in $(-\infty, -2)$ U$(3, \infty)$, and $\frac{1}{2}$

strictly decreasing in $(-2, 3)$

15. For $\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = \int\left[\frac{3}{x+2}-\frac{2}{x+1}+\frac{1}{(x+1)^2}\right]dx$ $2\frac{1}{2}$

$= 3\log|x+2| - 2\log|x+1| - \frac{1}{x+1} + C$ $1\frac{1}{2}$

OR

$I = \int (x-3)\sqrt{3-2x-x^2}\,dx = \int\left[-\frac{1}{2}(-2-2x)-4\right]\sqrt{3-2x-x^2}\,dx$ 1

$= -\frac{1}{2}\int(-2-2x)\sqrt{3-2x-x^2}\,dx - 4\int\sqrt{4-(x+1)^2}\,dx$ $\frac{1}{2}+1$

$= -\frac{1}{3}(3-2x-x^2)^{3/2} - 4\left[\frac{(x+1)}{2}\sqrt{3-2x-x^2}+2\sin^{-1}\left(\frac{x+1}{2}\right)\right]+C$ $\frac{1}{2}+1$

16. Let the vector $\vec{p} = (2\vec{a}+\vec{b}+2\vec{c})$ makes angles α, β, γ respectively with the vector $\vec{a}, \vec{b}, \vec{c}$

Given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{a} = 0$ 1

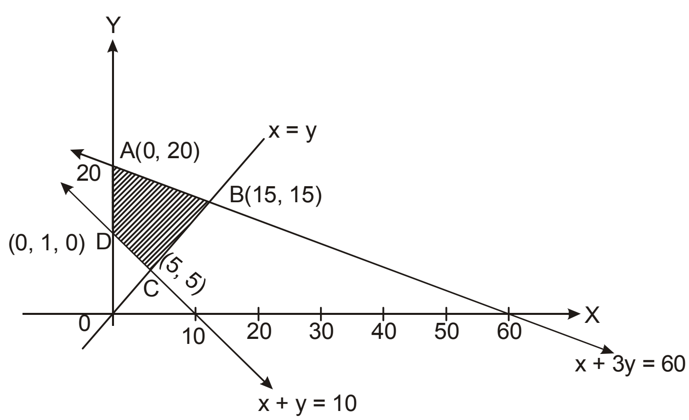
$\cos\alpha = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|}$ $\frac{1}{2}$

$= \frac{2|a|^2}{3|\vec{a}||\vec{a}|} = \frac{2}{3} \Rightarrow \alpha = \cos^{-1}\frac{2}{3}$ 1

$\cos\beta = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{b}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{b}|} = \frac{|\vec{b}|^2}{3|\vec{b}||\vec{b}|} = \frac{1}{3} \Rightarrow \beta = \cos^{-1}\frac{1}{3}$ 1

$\cos\gamma = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{c}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{c}|} = \frac{2|\vec{c}|^2}{3|\vec{c}||\vec{c}|} = \frac{2}{3} \Rightarrow \gamma = \cos^{-1}\frac{2}{3}$ $\frac{1}{2}$

17.



Correct graph of three lines $1\frac{1}{2}$

Correct shading 1

Vertices of feasible region are

A(0, 20), B(15, 15), C(5, 5), D(0, 10)

Z(A) = 180

Z(B) = 180

Z(C) = 60 1

Z(D) = 90

$\therefore$ Z = 60 is minimum at x = 5, y = 5 $\frac{1}{2}$

18. Let the events be

E_1: tansferring a red ball from A to B

E_2: transferring a black ball from A to B

A: Getting a red ball from bag B $\frac{1}{2}$

$$P(E_1) = \frac{3}{5},\ P(E_2) = \frac{2}{5}$$ $\frac{1}{2}$

$$P(A/E_1) = \frac{1}{2},\ P(A/E_2) = \frac{1}{3}$$ 1

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$ $\frac{1}{2}$

$$= \frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2}+\frac{2}{5}\cdot\frac{1}{3}} = \frac{9}{13}$$ $1+\frac{1}{2}$

OR

Required probability $= P(A \cup B)$ 1

$$= P(A) + P(B) - P(A)\cdot P(B)$$ $\frac{1}{2}$

$$= P(A)\,[1 - P(B)] + 1 - P(B')$$ $\frac{1}{2}$

$$= P(A)\,P(B') - P(B') + 1$$ 1

$$= (1 - P(B')\,(1 - P(A)) = 1 - P(A')\,P(B')$$ 1

19. Given equation can be written as $\tan^{-1}(1) - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{3}{2}\tan^{-1}x = \frac{\pi}{4} \quad \text{or} \quad \tan^{-1}x = \frac{\pi}{6}$$ $1\frac{1}{2}$

$$\Rightarrow \quad x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$ 1

20. Let $u = (\cos x)^x \Rightarrow \log u = x.\log\cos x$ $\frac{1}{2}$

$\Rightarrow \quad \frac{du}{dx} = (\cos x)^x.[-x\tan x + \log\cos x]$ $1\frac{1}{2}$

$\therefore \quad y = (\cos x)^x + \sin^{-1}\sqrt{3x} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-3x}}.\frac{\sqrt{3}}{2\sqrt{x}}$ $1\frac{1}{2}$

$\therefore \quad \frac{dy}{dx} = (\cos x)^x[-x\tan x + \log\cos x] + \frac{\sqrt{3}}{2\sqrt{x}}.\frac{1}{\sqrt{1-3x}}$ $\frac{1}{2}$

OR

$y = (\sec^{-1}x)^2 \quad \Rightarrow \quad \frac{dy}{dx} = 2\sec^{-1}x.\frac{1}{x\sqrt{x^2-1}}$ 1

$\therefore \quad x\sqrt{x^2-1}\cdot\frac{dy}{dx} = 2\sec^{-1}x$ $\frac{1}{2}$

$\Rightarrow \quad x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}\right) = \frac{2}{x\sqrt{x^2-1}}$ $1\frac{1}{2}$

$\Rightarrow \quad x^2(x^2-1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot x(2x^2-1) = 2$ 1

i.e., $\quad x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3-x)\frac{dy}{dx} = 2$

21. Let $\quad I = \int_0^\pi \frac{x\tan x\,dx}{\tan x + \sec x} = \int_0^\pi \frac{(\pi-x)\tan(\pi-x)}{\tan(\pi-x)+\sec(\pi-x)}dx$ $\frac{1}{2}$

So, $\quad I = \int_0^\pi \frac{(\pi-x)(-\tan x)}{-\tan x - \sec x}dx \quad \Rightarrow \quad I = \pi\int_0^\pi \frac{\tan x}{\sec x + \tan x}dx - I$ $\frac{1}{2} + \frac{1}{2}$

$\Rightarrow \quad 2I = \pi\int_0^\pi \frac{\tan x}{\sec x + \tan x}dx = \pi\int_0^\pi (\sec x\tan x - \tan^2 x)dx$ $\frac{1}{2}$

$= \pi\int_0^\pi \sec x\tan x\,dx - \pi\int_0^\pi \sec^2 x\,dx + \pi\int_0^\pi dx$ 1

$$\left.\begin{aligned} &= \pi\,|\sec x|_0^\pi - \pi\,|\tan x|_0^\pi + \pi^2 \\ &= \pi^2 - 2\pi \end{aligned}\right\}$$ 1

$\Rightarrow \quad I = \left(\frac{\pi^2}{2} - \pi\right)$

22. Writing the given differential equation in the form

$$\frac{dx}{dy} = \frac{xe^{x/y} + y^2}{ye^{x/y}}$$

$$= \frac{x}{y} + \frac{y}{e^{x/y}} \qquad 1$$

Put $\frac{x}{y} = v$ so, $x = vy$

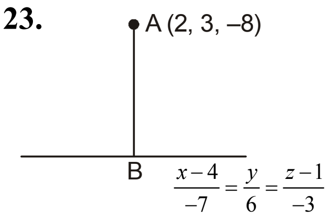
and $\frac{dx}{dy} = v + y\frac{dv}{dy}$ 1

$\therefore \quad v + y\frac{dv}{dy} = v + \frac{y}{e^v} \Rightarrow \frac{dv}{dy} = \frac{1}{e^v}$ 1

$\Rightarrow \quad \int e^v dv = \int dy \quad$ so, $y = e^v + C$ 1

hence, $y = e^{\frac{x}{y}} + C$

23.



Writing line in symmetric form $\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$ $\frac{1}{2}$

$\Rightarrow \quad \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$ gives co-ordinates of B as

$x = -2\lambda + 4,\ y = 6\lambda,\ z = -3\lambda + 1$ for some λ. 1

So, direction ratios of AB are $-2\lambda + 2,\ 6\lambda - 3,\ -3\lambda + 9$ $\frac{1}{2}$

Since AB is perpendicular to the given line

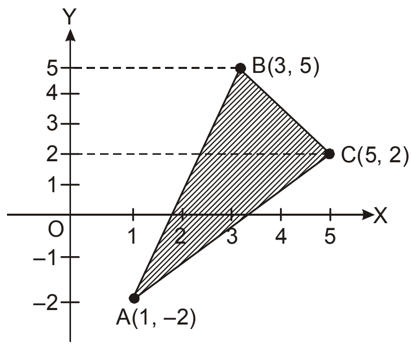
$-2(-2\lambda + 2) + 6(6\lambda - 3) + -3(-3\lambda + 9) = 0$ 1

$\Rightarrow \quad \lambda = 1$

So, foot of perpendicular is B(2, 6, –2) 1

SECTION D

24. **For Correct Figure** 1



Equation of AB: $x = \frac{1}{7}(2y + 11)$

Equation of BC: $x = \frac{1}{3}(19 - 2y)$ $1\frac{1}{2}$

Equation of AC: $x = y + 3$

Required area $= \int_{-2}^{2}(y+3)dy + \frac{1}{3}\int_{2}^{5}(19-2y)dy - \frac{1}{7}\int_{-2}^{5}(2y+11)dy$ $1\frac{1}{2}$

$\Rightarrow \quad A = \left.\frac{(y+3)^2}{2}\right]_{-2}^{2} + \frac{1}{3}\left.\frac{(19-2y)^2}{-4}\right]_{2}^{5} - \frac{1}{7}\left.\frac{(2y+11)^2}{4}\right]_{-2}^{5}$ 1

$= \frac{1}{2}(25-1) - \frac{1}{12}(81-225) - \frac{1}{28}(441-49) = 10$ sq.units 1

OR

Here $h = \frac{4}{n}$ or $nh = 4$, $f(x) = 3x^2 + 2x + 1$ $\frac{1}{2}$

$\int_0^4 (3x^2 + 2x + 1)dx = \lim_{h \to 0} h[f(0) + f(0+h) + f(0+2h) + ... + f(0 + \overline{n-1}\,h)]$ 1

$= \lim_{h \to 0} h[(1) + (3h^2 + 2h + 1) + (3.2^2h^2 + 2.2h + 1) + ... + (3(n-1)^2h^2 + 2(n-1)h + 1)]$ $1\frac{1}{2}$

$= \lim_{h \to 0} h\left[n + 3h^2\frac{n(n-1)(2n-1)}{6} + 2h\frac{n(n-1)}{2}\right]$ 1

$= \lim_{h \to 0}\left[nh + \frac{(nh)(nh-h)(2nh-h)}{2} + (nh)(nh-h)\right]$ 1

$= 4 + 64 + 16 = 84$ 1

25. $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0 \qquad 1$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow \quad (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \qquad 1+1$$

$\Rightarrow \quad -(a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca) = 0$ 1

$\Rightarrow \quad \dfrac{-1}{2}(a+b+c)\,[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ 1

$\Rightarrow \quad a - b = 0 = b - c = c - a$ as $a + b + c \neq 0$ 1

$\Rightarrow \quad a = b = c$

26. (i) for any $A, B \in P(X)$, $A*B = A \cap B$ and $B*A = B \cap A$

as $A \cap B = B \cap A$ $\therefore$ $A*B = B*A$ 2

$\Rightarrow$ * is commutative

(ii) for any $A, B, C \in P(X)$

$(A*B)*C = (A \cap B)*C = (A \cap B) \cap C$

and $A*(B*C) = A*(B \cap C) = A \cap (B \cap C)$

Since $(A \cap B) \cap C = A \cap (B \cap C) \Rightarrow$ * is associative 2

(iii) for every $A \in P(X)$, $A*X = A \cap X = A$

$X*A = X \cap A = A$ 1

$\Rightarrow$ X is the identity element

(iv) $X*X = X \cap X = X \Rightarrow$ X is the only invertible element. $\because$ it is true only for X. 1

OR

$$f(x) = \frac{4x}{3x+4}$$

for $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$, $f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

$\therefore \quad 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$ 2

$\therefore \quad$ f is a 1 – 1 function.

for $y = \frac{4}{3}$, there is no x such that $f(x) = \frac{4}{3}$

$\therefore \quad$ f is not invertible 1

But $f : R - \left\{-\frac{4}{3}\right\} \to$ Range of f is ONTO so invertible. 1

and $f^{-1}(y) = \frac{4y}{4-3y}$ 2

27. Let given volume of cone be, $V = \frac{1}{3}\pi r^2 h$ ...(i) $\frac{1}{2}$

$\therefore \quad$ Surface area (curved) $S = \pi r l = \pi r\sqrt{r^2 + h^2}$ $\frac{1}{2}$

or $\quad A = S^2 = \pi r^2(r^2 + h^2)$

$$A = S^2 = \pi^2 r^2\left[r^2 + \left(\frac{3V}{\pi r^2}\right)^2\right] \quad \text{[using (i)]}$$

$$= \pi^2\left[r^4 + \frac{9V^2}{\pi^2 r^2}\right]$$ $1\frac{1}{2}$

$$\frac{dA}{dr} = \pi^2\left[4r^3 - \frac{18V^2}{\pi^2 r^3}\right]$$ 1

$$\frac{dA}{dr} = 0 \quad \Rightarrow \quad 4\pi^2 r^6 = 18 \cdot \frac{1}{9} \pi^2 r^4 h^2$$

$\Rightarrow \quad 2r^2 = h^2 \text{ or } h = \sqrt{2}r$ $\qquad 1\frac{1}{2}$

$$\frac{d^2A}{dr^2} = \pi^2 \left[12r^2 + \frac{54V^2}{\pi^2 r^4} \right] > 0 \qquad 1$$

$\Rightarrow$ for least curved surface area, height = $\sqrt{2}$ (radius)

OR

$$x = a\cos\theta + a\theta\sin\theta \quad \Rightarrow \quad \frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta$$

$= a\theta\cos\theta$ $\qquad 1$

$$y = a\sin\theta - a\theta\cos\theta \quad \Rightarrow \quad \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta$$

$= a\theta\sin\theta$ $\qquad 1$

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta \qquad \frac{1}{2}$$

Equation of tangent is

$y - (a\sin\theta - a\theta\cos\theta) = \tan\theta(x - a\cos\theta - a\theta\sin\theta)$ $\qquad 1$

Equation of normal is

$$y - (a\sin\theta - a\theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta) \qquad 1$$

$\Rightarrow \quad y\sin\theta + x\cos\theta = a$ $\qquad \frac{1}{2}$

distance of normal from origin $= \dfrac{|-a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a| = \text{constant}$ $\qquad 1$

28. Let X denote the number of defective bulbs drawn

$\Rightarrow \quad p = \frac{1}{5},\ q = \frac{4}{5}$ $\qquad 1$

X = 0, 1, 2, 3. $\qquad 1$

$$P(X = 0) = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{64}{125}$$ $\frac{1}{2}$

$$P(X = 1) = 3 \times \frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} = \frac{48}{125}$$ $\frac{1}{2}$

$$P(X = 2) = 3 \times \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{12}{125}$$ $\frac{1}{2}$

$$P(X = 3) = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}$$ $\frac{1}{2}$

$$\text{Mean} = \frac{48}{125} + \frac{24}{125} + \frac{3}{125} = \frac{75}{125} = \frac{3}{5}$$ 1

$$\text{Variance} = \left(\frac{48}{125} + \frac{48}{125} + \frac{9}{125}\right) - \left(\frac{3}{5}\right)^2 = \frac{60}{125} = \frac{12}{25}$$ 1

29. Equation of the plane passing through three points is.

$$\begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$ 2

or $2x + 3y - 3z - 5 = 0$ 1

Since $2(3) + 3(1) - 3(3) = 0 \Rightarrow$ lines is parallel to the plane 1

$$\therefore \quad \text{Distance} = \left|\frac{2(3) + 3(5) + (-3)(-2) - 5}{\sqrt{(2)^2 + (3)^2 + (-3)^2}}\right| = \sqrt{22}$$ $1\frac{1}{2} + \frac{1}{2}$

QUESTION PAPER CODE 65/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\int \frac{3x}{3x-1}dx = \int \frac{3x-1+1}{3x-1}dx$ $\frac{1}{2}$

$= x + \frac{1}{3}\log|3x-1| + C$ $\frac{1}{2}$

2. $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 225 \Rightarrow |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 225$ $\frac{1}{2}$

$\Rightarrow \quad (5)^2|\vec{b}|^2 = 225 \Rightarrow |\vec{b}| = 3$ $\frac{1}{2}$

3. $|A^{-1}| = \frac{1}{|A|}$ $\frac{1}{2}$

$= \frac{1}{4}$ $\frac{1}{2}$

4. $\lim\limits_{x\to 0}\frac{\sin\frac{3x}{2}}{2} = \lim\limits_{x\to 0}\frac{3}{2}\cdot\frac{\sin\frac{3x}{2}}{\frac{3x}{2}} = \frac{3}{2}$ $\frac{1}{2}$

$\Rightarrow \quad k = \frac{3}{2}$ $\frac{1}{2}$

SECTION B

5. Let, Number of executive class tickets be x and economy class tickets be y.

$\therefore$ LPP is Maximise Profit $P = 1500x + 1000y$ 1

Subject to: $x + y \le 250$, $x \ge 25$, $y \ge 3x$ 1

6. Given differential equation can be written as

$\frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$ 1

Getting integrating factor $= e^{x - \log x}$ or $\frac{e^x}{x}$ 1

7. $\int \frac{x}{\sqrt{32-x^2}}dx = -\int 1.dt$ where $32 - x^2 = t^2$ 1

$= -t + C = -\sqrt{32-x^2} + C$ 1

8. $f(x) = \sin 2x - \cos 2x$

$\Rightarrow \quad f'(x) = 2\cos 2x + 2\sin 2x$ 1

$f'\left(\frac{\pi}{6}\right) = 2\left[\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right] = (1+\sqrt{3})$ 1

9. Getting $\begin{pmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$ 1

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow \quad x = 2, y = 9$ 1

10. For three vectors to be coplanar

$\begin{vmatrix} 1 & -1 & 1 \\ 3 & 1 & 2 \\ 1 & \lambda & -3 \end{vmatrix} = 0$ 1

$\Rightarrow \quad \lambda = 15$ 1

11. For $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$ $\frac{1}{2}$

Integrating, we get

$\tan^{-1}y = \tan^{-1}x + C.$ $\frac{1}{2}$

As $x = 0, y = \sqrt{3}$ so $\tan^{-1}\sqrt{3} = C \quad \Rightarrow \quad C = \frac{\pi}{3}$ $\frac{1}{2}$

Solution is $\tan^{-1}y = \tan^{-1}x + \frac{\pi}{3}$ $\frac{1}{2}$

12. Radius $= \frac{1}{3}(3x+1)$ $\frac{1}{2}$

$$V = \frac{4}{3}\pi\frac{(3x+1)^3}{27}$$ $\frac{1}{2}$

$$\frac{dV}{dx} = \frac{12\pi\times 3}{81}(3x+1)^2 = \frac{4\pi}{9}(3x+1)^2$$ 1

SECTION C

13. Let the award for regularily be ₹ x and for hard work be ₹ y.

$\therefore$ $x + y = 6000$ and 1

$x + 3y = 11000$

$\Rightarrow$ $\begin{pmatrix}1 & 1\\1 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6000\\11000\end{pmatrix}$ or $A.X = B$ 1

$\therefore$ $X = A^{-1}B$ as $|A| = 2 \neq 0$.

$\Rightarrow$ $\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{2}\begin{pmatrix}3 & -1\\-1 & 1\end{pmatrix}\begin{pmatrix}6000\\11000\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3500\\2500\end{pmatrix}$ $\therefore$ x = ₹ 3500, y = ₹ 2500 1

Any two values like obedience, respect for elders,... 1

14. Let the events be

E_1: tansferring a red ball from A to B

E_2: transferring a black ball from A to B

A: Getting a red ball from bag B $\frac{1}{2}$

$P(E_1) = \frac{3}{5}$, $P(E_2) = \frac{2}{5}$ $\frac{1}{2}$

$P(A/E_1) = \frac{1}{2}$, $P(A/E_2) = \frac{1}{3}$ 1

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$ $\frac{1}{2}$

$$= \frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2}+\frac{2}{5}\cdot\frac{1}{3}} = \frac{9}{13}$$ $1+\frac{1}{2}$

OR

Required probability $= P(A \cup B)$ 1

$= P(A) + P(B) - P(A) \cdot P(B)$ $\frac{1}{2}$

$= P(A)\,[1 - P(B)] + 1 - P(B')$ $\frac{1}{2}$

$= P(A)\,P(B') - P(B') + 1$ 1

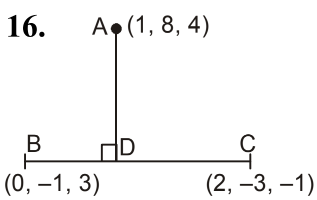
$= (1 - P(B'))\,(1 - P(A)) = 1 - P(A')\,P(B')$ 1

15. Given equation can be written as $\tan^{-1}(1) - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$ $1\frac{1}{2}$

$\Rightarrow \quad \frac{3}{2}\tan^{-1}x = \frac{\pi}{4} \quad \text{or} \quad \tan^{-1}x = \frac{\pi}{6}$ $1\frac{1}{2}$

$\Rightarrow \quad x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ 1

16.



Equation of line passing through B and C is

$$\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} \quad \text{or} \quad \frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2}$$ 1

Any point D on BC can be

$[\lambda, -\lambda - 1, -2\lambda + 3]$ for some value of λ. 1

$\therefore$ Direction ratios of AD are $\langle \lambda - 1, -\lambda - 9, -2\lambda - 1 \rangle$ $\frac{1}{2}$

$AD \perp BC \Rightarrow 1(\lambda - 1) - 1(-\lambda - 9) - 2(-2\lambda - 1) = 0$ $\frac{1}{2}$

$\Rightarrow \quad \lambda = -\frac{5}{3}$ $\frac{1}{2}$

$\therefore$ D is $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ $\frac{1}{2}$

17. Let the vector $\vec{p} = (2\vec{a} + \vec{b} + 2\vec{c})$ makes angles α, β, γ respectively with the vector $\vec{a}, \vec{b}, \vec{c}$

Given that $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert$ and $\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{a} = 0$	1
$\cos\alpha = \dfrac{(2\vec{a} + \vec{b} + 2\vec{c})\cdot\vec{a}}{\lvert 2\vec{a} + \vec{b} + 2\vec{c}\rvert\lvert\vec{a}\rvert}$	$\frac{1}{2}$
$= \dfrac{2\lvert a\rvert^2}{3\lvert\vec{a}\rvert\lvert\vec{a}\rvert} = \dfrac{2}{3} \quad\Rightarrow\quad \alpha = \cos^{-1}\dfrac{2}{3}$	1
$\cos\beta = \dfrac{(2\vec{a} + \vec{b} + 2\vec{c})\cdot\vec{b}}{\lvert 2\vec{a} + \vec{b} + 2\vec{c}\rvert\lvert\vec{b}\rvert} = \dfrac{\lvert\vec{b}\rvert^2}{3\lvert\vec{b}\rvert\lvert\vec{b}\rvert} = \dfrac{1}{3} \quad\Rightarrow\quad \beta = \cos^{-1}\dfrac{1}{3}$	1
$\cos\gamma = \dfrac{(2\vec{a} + \vec{b} + 2\vec{c})\cdot\vec{c}}{\lvert 2\vec{a} + \vec{b} + 2\vec{c}\rvert\lvert\vec{c}\rvert} = \dfrac{2\lvert\vec{c}\rvert^2}{3\lvert\vec{c}\rvert\lvert\vec{c}\rvert} = \dfrac{2}{3} \quad\Rightarrow\quad \gamma = \cos^{-1}\dfrac{2}{3}$	$\frac{1}{2}$

18.

$f'(x) = 6x^2 - 6x - 36$	$\frac{1}{2}$
$= 6(x^2 - x - 6) = 6(x-3)(x+2)$	
$f'(x) = 0 \Rightarrow x = -2, x = 3$	1
$\therefore$ the intervals are $(-\infty, -2), (-2, 3), (3, \infty)$	$\frac{1}{2}$
getting $f'(x)$ +ve in $(-\infty, -2) \cup (3, \infty)$ and –ve in $(-2, 3)$	$1\frac{1}{2}$
$\therefore$ f(x) is strictly increasing in $(-\infty, -2) \cup (3, \infty)$, and strictly decreasing in $(-2, 3)$	$\frac{1}{2}$

19.

For $\displaystyle\int \frac{x^2 + x + 1}{(x+1)^2(x+2)}dx = \int\left[\frac{3}{x+2} - \frac{2}{x+1} + \frac{1}{(x+1)^2}\right]dx$	$2\frac{1}{2}$
$= 3\log\lvert x+2\rvert - 2\log\lvert x+1\rvert - \dfrac{1}{x+1} + C$	$1\frac{1}{2}$

OR

$$I = \int (x-3)\sqrt{3-2x-x^2}\,dx = \int\left[-\frac{1}{2}(-2-2x)-4\right]\sqrt{3-2x-x^2}\,dx \qquad 1$$

$$= -\frac{1}{2}\int(-2-2x)\sqrt{3-2x-x^2}\,dx - 4\int\sqrt{4-(x+1)^2}\,dx \qquad \frac{1}{2}+1$$

$$= -\frac{1}{3}(3-2x-x^2)^{3/2} - 4\left[\frac{(x+1)}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right)\right] + C \qquad \frac{1}{2}+1$$

20. Let $u = (\cos x)^x \Rightarrow \log u = x.\log\cos x$ $\qquad \frac{1}{2}$

$$\Rightarrow \quad \frac{du}{dx} = (\cos x)^x.[-x\tan x + \log\cos x] \qquad 1\frac{1}{2}$$

$$\therefore \quad y = (\cos x)^x + \sin^{-1}\sqrt{3x} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-3x}}.\frac{\sqrt{3}}{2\sqrt{x}} \qquad 1\frac{1}{2}$$

$$\therefore \quad \frac{dy}{dx} = (\cos x)^x[-x\tan x + \log\cos x] + \frac{\sqrt{3}}{2\sqrt{x}}.\frac{1}{\sqrt{1-3x}} \qquad \frac{1}{2}$$

OR

$$y = (\sec^{-1}x)^2 \quad \Rightarrow \quad \frac{dy}{dx} = 2\sec^{-1}x.\frac{1}{x\sqrt{x^2-1}} \qquad 1$$

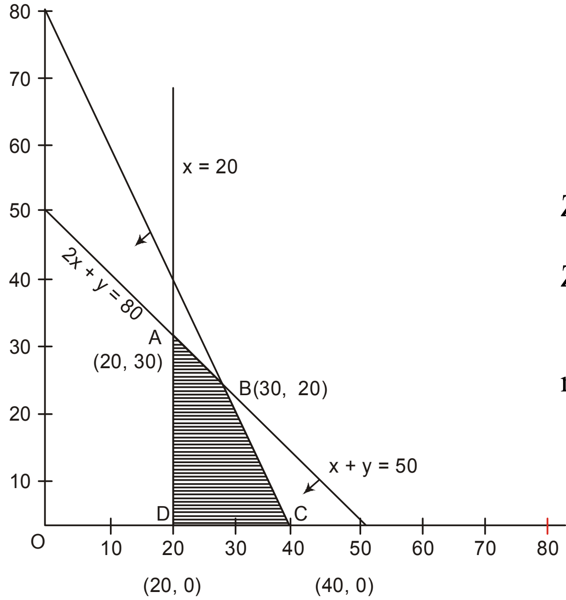
$$\therefore \quad x\sqrt{x^2-1}\cdot\frac{dy}{dx} = 2\sec^{-1}x \qquad \frac{1}{2}$$

$$\Rightarrow \quad x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}\right) = \frac{2}{x\sqrt{x^2-1}} \qquad 1\frac{1}{2}$$

$$\Rightarrow \quad x^2(x^2-1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot x(2x^2-1) = 2 \qquad 1$$

$$\text{i.e.,} \quad x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3-x)\frac{dy}{dx} = 2$$

21.



Correct lines $1\frac{1}{2}$

Correct shading 1

Z(20, 0) = 2100, Z(40, 0) = 4200, Z(20, 30) = 4800 1

Z(30, 20) = 4950

max. value of Z is 4950 when x = 30, y = 20 $\frac{1}{2}$

22. Given differential equation can be written as,

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2} \qquad \frac{1}{2}$$

Put y = vx, $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

so, $v + x\frac{dv}{dx} = \frac{2v + v^2}{2}$ $\frac{1}{2}$

$$\int \frac{2}{v^2} dv = \int \frac{dx}{x} \qquad \frac{1}{2}$$

$$\frac{-2}{v} = \log|x| + C \Rightarrow \frac{-2x}{y} = \log|x| + C \qquad \frac{1}{2}$$

y = 2, x = 1 gives C = –1 $\frac{1}{2}$

Solution is $\frac{2x}{y} = 1 - \log|x|$ or $y = \frac{2x}{1 - \log|x|}$ where, $x \neq 0, e$ $\frac{1}{2}$

23. $I = \int_0^{\pi} \frac{x}{1+\sin x} dx = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} dx$ $\frac{1}{2}$

$\Rightarrow \quad I = \pi \int_0^{\pi} \frac{dx}{1+\sin x} - I$ 1

$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{dx}{1+\sin x}$ $\frac{1}{2}$

$= \pi \int_0^{\pi} \frac{(1-\sin x)}{\cos^2 x} dx = \pi \left[\int_0^{\pi} \sec^2 x \, dx - \int_0^{\pi} \sec x \tan x \, dx \right]$ $\frac{1}{2}$

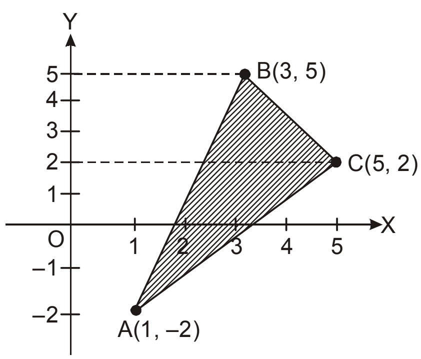
$= \pi \,|\tan x|_0^{\pi} - \pi \,|\sec x|_0^{\pi}$ $\frac{1}{2}$

$= 0 - \pi(-2)$ $\frac{1}{2}$

$\Rightarrow \quad I = \pi$ $\frac{1}{2}$

SECTION D

24. **For Correct Figure** 1



Equation of AB: $x = \frac{1}{7}(2y + 11)$

Equation of BC: $x = \frac{1}{3}(19 - 2y)$ $1\frac{1}{2}$

Equation of AC: $x = y + 3$

Required area $= \int_{-2}^{2} (y+3)dy + \frac{1}{3}\int_2^5 (19 - 2y)dy - \frac{1}{7}\int_{-2}^5 (2y + 11)dy$ $1\frac{1}{2}$

$\Rightarrow A = \left.\frac{(y+3)^2}{2}\right]_{-2}^{2} + \frac{1}{3}\left.\frac{(19-2y)^2}{-4}\right]_2^5 - \frac{1}{7}\left.\frac{(2y+11)^2}{4}\right]_{-2}^5$ 1

$= \frac{1}{2}(25 - 1) - \frac{1}{12}(81 - 225) - \frac{1}{28}(441 - 49) = 10$ sq.units 1

OR

Here $h = \frac{4}{n}$ or $nh = 4$, $f(x) = 3x^2 + 2x + 1$ $\frac{1}{2}$

$$\int_0^4 (3x^2 + 2x + 1)dx = \lim_{h \to 0} h[f(0) + f(0 + h) + f(0 + 2h) + ... + f(0 + \overline{n-1}\,h)] \qquad 1$$

$$= \lim_{h \to 0} h[(1) + (3h^2 + 2h + 1) + (3.2^2 h^2 + 2.2h + 1) + ... + (3(n-1)^2 h^2 + 2(n-1)h + 1)] \qquad 1\frac{1}{2}$$

$$= \lim_{h \to 0} h\left[n + 3h^2 \frac{n(n-1)(2n-1)}{6} + 2h\frac{n(n-1)}{2}\right] \qquad 1$$

$$= \lim_{h \to 0}\left[nh + \frac{(nh)(nh-h)(2nh-h)}{2} + (nh)(nh-h)\right] \qquad 1$$

$= 4 + 64 + 16 = 84$ 1

25. Let given volume of cone be, $V = \frac{1}{3}\pi r^2 h$ ...(i) $\frac{1}{2}$

$\therefore$ Surface area (curved) $S = \pi rl = \pi r\sqrt{r^2 + h^2}$ $\frac{1}{2}$

or $A = S^2 = \pi r^2(r^2 + h^2)$

$$A = S^2 = \pi^2 r^2\left[r^2 + \left(\frac{3V}{\pi r^2}\right)^2\right] \qquad \text{[using (i)]}$$

$$= \pi^2\left[r^4 + \frac{9V^2}{\pi^2 r^2}\right] \qquad 1\frac{1}{2}$$

$$\frac{dA}{dr} = \pi^2\left[4r^3 - \frac{18V^2}{\pi^2 r^3}\right] \qquad 1$$

$$\frac{dA}{dr} = 0 \Rightarrow 4\pi^2 r^6 = 18 \cdot \frac{1}{9}\pi^2 r^4 h^2$$

$\Rightarrow$ $2r^2 = h^2$ or $h = \sqrt{2}r$ $1\frac{1}{2}$

$$\frac{d^2A}{dr^2} = \pi^2\left[12r^2 + \frac{54V^2}{\pi^2 r^4}\right] > 0 \qquad 1$$

$\Rightarrow$ for least curved surface area, height = $\sqrt{2}$ (radius)

OR

$$x = a\cos\theta + a\theta\sin\theta \Rightarrow \frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta$$

$= a\theta\cos\theta$ 1

$$y = a\sin\theta - a\theta\cos\theta \Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta$$

$= a\theta\sin\theta$ 1

$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$ $\frac{1}{2}$

Equation of tangent is

$y - (a\sin\theta - a\theta\cos\theta) = \tan\theta(x - a\cos\theta - a\theta\sin\theta)$ 1

Equation of normal is

$y - (a\sin\theta - a\theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta)$ 1

$\Rightarrow$ $y\sin\theta + x\cos\theta = a$ $\frac{1}{2}$

distance of normal from origin $= \frac{|-a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a| = \text{constant}$ 1

26. (i) for any A, B $\in$P(X), A*B = A$\cap$B and B*A = B$\cap$A

as A$\cap$B = B$\cap$A $\therefore$ A*B = B*A 2

$\Rightarrow$ * is commutative

(ii) for any A, B, C $\in$P(X)

(A*B)*C = (A $\cap$ B)*C = (A $\cap$ B) $\cap$ C

and A*(B*C) = A*(B $\cap$ C) = A $\cap$ (B $\cap$ C)

Since (A $\cap$ B) $\cap$ C = A $\cap$ (B $\cap$ C) $\Rightarrow$ * is associative 2

(iii) for every A $\in$ P(X), A*X = A $\cap$ X = A

X*A = X $\cap$ A = A 1

$\Rightarrow$ X is the identity element

(iv) X*X = X $\cap$ X = X $\Rightarrow$ X is the only invertible element. $\because$ it is true only for X. 1

OR

$$f(x) = \frac{4x}{3x+4}$$

for $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$, $f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

$\therefore \quad 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$ 2

$\therefore \quad$ f is a 1 – 1 function.

for $y = \frac{4}{3}$, there is no x such that $f(x) = \frac{4}{3}$

$\therefore \quad$ f is not invertible 1

But $f : R - \left\{-\frac{4}{3}\right\} \rightarrow$ Range of f is ONTO so invertible. 1

and $f^{-1}(y) = \frac{4y}{4-3y}$ 2

27. $C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$ 1

$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \quad (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$$ 1+1

$\Rightarrow \quad -(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$ 1

$\Rightarrow \quad \frac{-1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ 1

$\Rightarrow \quad a - b = 0 = b - c = c - a$ as $a + b + c \neq 0$ 1

$\Rightarrow \quad a = b = c$

28. Equation of plane passing through A(1, –2, 2), B(4, 2, 3) and C(3, 0, 2) is

$$[\vec{r} - (\hat{i} - 2\hat{j} + 2\hat{k})]\cdot[(3\hat{i} + 4\hat{j} + \hat{k}) \times (2\hat{i} + 2\hat{j})] = 0$$ $1\frac{1}{2}$

$\Rightarrow$ $\vec{r}\cdot(\hat{i} - \hat{j} + \hat{k}) = 5$ ...(i) $1\frac{1}{2}$

Any point on the given line is $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$ 1

$\Rightarrow$ When the line intersects the plane

$$((2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k})\cdot(\hat{i} - \hat{j} + \hat{k}) = 5$$ 1

$\Rightarrow$ $\lambda = 0$ $\frac{1}{2}$

$\Rightarrow$ The required point is (2, –1, 2) $\frac{1}{2}$

29. P(probability of getting 4) $= \frac{1}{10}$

P(probability of not getting 4) $= \frac{9}{10}$ 1

X	0	1	2	$\frac{1}{2}$
P(X)	$\left(\frac{9}{10}\right)^2 = \frac{81}{100}$	$2 \times \frac{9}{10} \times \frac{1}{10} = \frac{18}{100}$	$\left(\frac{1}{10}\right)^2 = \frac{1}{100}$	$1\frac{1}{2}$
XP(X)	0	$\frac{18}{100}$	$\frac{2}{100}$	1
X^2P(X)	0	$\frac{18}{100}$	$\frac{4}{100}$	1

Variance $= \Sigma X^2P(X) - [\Sigma XP(X)]^2$

$$= \frac{22}{100} - \left(\frac{20}{100}\right)^2 = \frac{18}{100} = 0.18$$ 1