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Senior Secondary School Certificate Examination

July 2017 (Compartment)

Marking Scheme — Mathematics 65/1/1, 65/1/2, 65/1/3 [Delhi Region]

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

5. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

6. Separate Marking Scheme for all the three sets has been given.

7. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Order of AB is 3×4 1

2. $\frac{dy}{dx} = \cos x$ $\frac{1}{2}$

Slope of tangent at (0, 0) is 1

Equation of tangent is $y = x$ $\frac{1}{2}$

3. Putting $(1 + \log x)$ or $\log x = t$ $\frac{1}{2}$

$\log |1 + \log x| + C$ $\frac{1}{2}$

4. π 1

SECTION B

5. $R_2 \to R_2 + R_1$ implies

$$\begin{pmatrix} 2 & 3 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -3 \\ 17 & -7 \end{pmatrix}$$ 1+1

1 mark for pre matrix on LHS and 1 mar for matrix on RHS

6. $\lim_{x \to 2} f(x) = f(2)$ $\frac{1}{2}$

$$\lim_{x \to 2} \frac{(x+5)\cancel{(x-2)}}{\cancel{x-2}} = k$$ 1

$\therefore \quad k = 7$ $\frac{1}{2}$

7. $\frac{dr}{dt} = -3$ cm/min, $\frac{dh}{dt} = 2$ cm/min $\frac{1}{2}$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left[r^2\frac{dh}{dt} + 2hr\frac{dr}{dt}\right] \qquad 1$$

$$\left(\frac{dV}{dt}\right)_{\text{at } r=9,\, h=6} = -54\pi \text{ cm}^3/\text{min} \qquad \frac{1}{2}$$

$\Rightarrow$ Volume is decreasing at the rate 54π cm^3/min.

8. $I = \int\sqrt{(x-1)^2 - 1^2}\,dx$ 1

$$= \frac{(x-1)}{2}\sqrt{x^2-2x} - \frac{1}{2}\log\left|x-1+\sqrt{x^2-2x}\right| + C \qquad 1$$

9. Differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a \qquad 1$$

Eliminating 4a, we get

$$y^2 = 2y\frac{dy}{dx}\cdot x$$

or $2xy\frac{dy}{dx} - y^2 = 0$ 1

10. Integrating factor is $e^{\int 2dx} = e^{2x}$ $\frac{1}{2}$

$\therefore$ Required solution is

$$y.e^{2x} = \int e^{3x}\cdot e^{2x}dx \qquad \frac{1}{2}$$

$$y.e^{2x} = \frac{e^{5x}}{5} + C \qquad 1$$

or $y = \frac{e^{3x}}{5} + Ce^{-2x}$

11. Let A be $10\hat{i} + 3\hat{j}$, B be $12\hat{i} - 5\hat{j}$, C be $\lambda\hat{i} + 11\hat{j}$

$\overrightarrow{AB} = 2\hat{i} - 8\hat{j}$ $\frac{1}{2}$

$\overrightarrow{AC} = (\lambda - 10)\hat{i} + 8\hat{j}$ $\frac{1}{2}$

As $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear

$$\frac{2}{\lambda - 10} = \frac{-8}{8}$$ $\frac{1}{2}$

So $\lambda = 8$ $\frac{1}{2}$

12. Let number of large vans = x

and number of small vans = y

Minimize cost z = 400x + 200y $\frac{1}{2}$

Subject to constraints

$$\left.\begin{aligned} &200x + 80y \geq 1200 \text{ or } 5x + 2y \geq 30 \\ &x \leq y \\ &400x + 200y \leq 3000 \text{ or } 2x + y \leq 15 \end{aligned}\right\}$$ $1\frac{1}{2}$

$x \geq 0, y \geq 0$

SECTION C

13. Putting $x = \cos\theta$ 1

LHS becomes

$$\tan^{-1}\left(\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta/2 - \sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2 + \sqrt{2}\sin\theta/2}\right)$$ 1

$$= \tan^{-1}\left(\frac{1-\tan\theta/2}{1+\tan\theta/2}\right)$$ 1

$$= \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$ $\frac{1}{2}$

$$= \frac{\pi}{4}-\frac{\theta}{2}$$

$$= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x = \text{RHS}$$ $\frac{1}{2}$

14. Taking x, y, z common from C_1, C_2, C_3 respectively, we get

$$xyz\begin{vmatrix} a/x & b/y-1 & c/z-1 \\ a/x-1 & b/y & c/z-1 \\ a/x-1 & b/y-1 & c/z \end{vmatrix} = 0$$ 1

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a/x+b/y+c/z-2 & b/y-1 & c/z-1 \\ a/x+b/y+c/z-2 & b/y & c/z-1 \\ a/x+b/y+c/z-2 & b/y-1 & c/z \end{vmatrix} = 0$$ 1

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 1 & b/y & c/z-1 \\ 1 & b/y-1 & c/z \end{vmatrix} = 0$$ $\frac{1}{2}$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$ 1

$\therefore$ $$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\cdot 1 = 0 \Rightarrow \frac{a}{x}+\frac{b}{y}+\frac{c}{z} = 2$$ $\frac{1}{2}$

OR

We know that

$$IA = A \qquad 1$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix} \qquad 1$$

$R_2 \to \dfrac{R_2}{-5}$

$$\begin{pmatrix} 1 & 0 \\ 2/5 & -1/5 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \qquad \frac{1}{2}$$

$R_1 \to R_1 - 2R_2$

$$\begin{pmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad 1$$

$$\therefore \quad A^{-1} = \begin{pmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{pmatrix} \qquad \frac{1}{2}$$

Full marks for finding correct A^{-1} using column transformations with $AI = A$

15. $\dfrac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta)$

$$= a\,\theta\cos\theta \qquad 1\frac{1}{2}$$

$$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$$

$$= a\,\theta\sin\theta \qquad 1$$

$$\therefore \quad \frac{dy}{dx} = \tan\theta \qquad \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \sec^2\theta \times \frac{d\theta}{dx} = \frac{\sec^3\theta}{a\theta} \qquad 1$$

16. Differentiating y = cos (x + y) wrt x we get

$$\frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$$ 1

Slope of given line is $\frac{-1}{2}$ $\frac{1}{2}$

As tangent is parallel to line x + 2y = 0

$$\therefore \quad \frac{-\sin(x+y)}{1+\sin(x+y)} = \frac{-1}{2}$$

$\Rightarrow \quad \sin(x+y) = 1$

$\Rightarrow \quad x + y = n\pi + (-1)^n \frac{\pi}{2}, n \in Z$ ...(1) 1

Putting (1) in y = cos (x + y)

we get y = 0

$\Rightarrow \quad x = n\pi + (-1)^n \pi/2, n \in Z$

$x = \frac{-3\pi}{2} \in [-2\pi, 0]$ $\frac{1}{2}$

$\therefore$ Required equation of tangent is

$$y = \frac{-1}{2}\left(x + \frac{3\pi}{2}\right)$$

or $2y + x + \frac{3\pi}{2} = 0$ 1

17. Given integral $= \int \frac{x+5}{(x+5)(3x-2)} dx$ 2

$$= \int \frac{1}{3x-2} dx$$

$= \frac{1}{3} \log|3x-2| + C$ 2

OR

$$\text{Let } I = \int_0^{\pi/4} \frac{1}{\cos^2 x + 4\sin^2 x} dx$$

$$= \int_0^{\pi/4} \frac{\sec^2 x}{1 + 4\tan^2 x} dx$$ 1

Let $\tan x = t$, $\sec^2 x\, dx = dt$ $\frac{1}{2}$

$$I = \int_0^1 \frac{1}{1+4t^2} dt$$ 1

$$= \frac{1}{2} \tan^{-1} 2t \Big|_0^1$$ 1

$$= \frac{1}{2} \tan^{-1} 2$$ $\frac{1}{2}$

18. Let $\dfrac{x^2}{(x-1)(x^2+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1}$ 1

$$A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$$ $1\frac{1}{2}$

Thus integral becomes

$$\frac{1}{2}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{x dx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{2}\log|x-1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1} x + C$$ $1\frac{1}{2}$

19. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y\cos\frac{y}{x} + x}{x\cos\frac{y}{x}} \qquad ....(i)$$ $\frac{1}{2}$

Clearly it is homogenous

Let $\dfrac{y}{x} = v$, $\dfrac{dy}{dx} = v + \dfrac{dv}{dx}$ 1

(1) becomes

$$v + x\frac{dv}{dx} = v + \sec v$$

$\Rightarrow \quad \cos v\, dv = \frac{dx}{x}$ 1

integrating both sides we get

$\sin v = \log|x| + C$ 1

$\sin\frac{y}{x} = \log|x| + C$ $\frac{1}{2}$

20. $\overrightarrow{AB} = \hat{i} + (x-3)\hat{j} + 4\hat{k}$

$\overrightarrow{AC} = \hat{i} - 3\hat{k}$

$\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ $1\frac{1}{2}$

As A, B, C & D are coplanar

$$\left.\begin{array}{l} \therefore \quad \overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD}) = 0 \\ \text{i.e.} \quad \begin{vmatrix} 1 & x-3 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0 \end{array}\right\}$$ $1\frac{1}{2}$

which gives

$x = 6$ 1

21. Given equation of lines can be written as

$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{1}$ ...(1) 1

$\frac{x-1}{-3p/7} = \frac{y-5}{-1} = \frac{z-11}{-7}$ ...(2) 1

(1) & (2) are perpendicular

So $-3\left(\frac{-3p}{7}\right) + \frac{2p}{7}(-1) + 1(-7) = 0$ 1

which gives $p = 7$ 1

OR

Required equation of plane is $x + y + z - 1 + \lambda(2x + 3y + 4z - 5) = 0$ for some λ. 1

i.e. $(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z = 1 + 5\lambda$

according to question

$$2\left(\frac{1+5\lambda}{1+3\lambda}\right) = 3\left(\frac{1+5\lambda}{1+4\lambda}\right)$$ 1

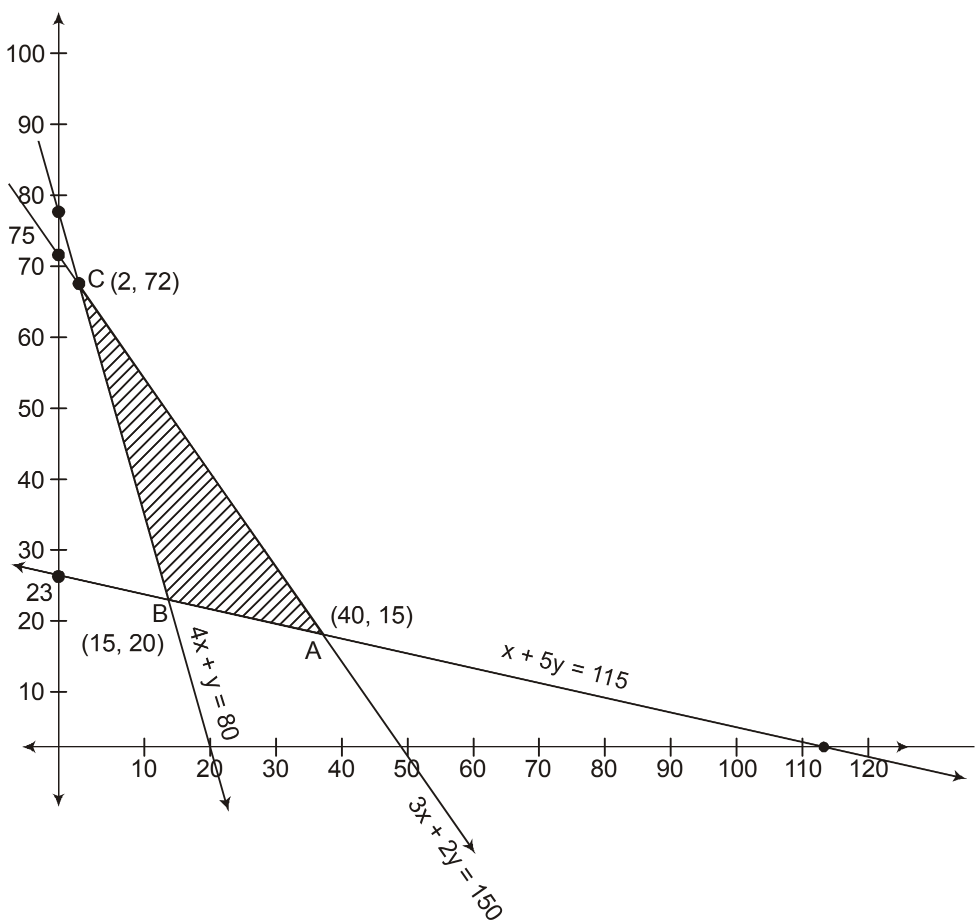
Solving we get $\lambda = -1$ 1

Thus the equation of required plane is

$-x - 2y - 3z = -4$

or $x + 2y + 3z = 4$ 1

22.



Correct lines $1\frac{1}{2}$

Correct shading 1

Corner points	Value of z	
A(40, 15)	285	
B(15, 20)	$150 \rightarrow$ minimum	1
C(2, 72)	228	

minimum z = 150

when x = 15, y = 20 $\frac{1}{2}$

23. E_1: Student selected from category A

E_2: Student selected from category B

E_3: Student selected from category C

S: Student could not get good marks

$$P(E_1) = \frac{1}{6} \quad P(E_2) = \frac{3}{6} \quad P(E_3) = \frac{2}{6}$$ 1

$P(S/E_1) = 0.002$ $P(S/E_2) = 0.02$, $P(S/E_3) = 0.2$

$$P(E_3/S) = \frac{P(E_3)\, P(S/E_3)}{P(E_1)\, P(S/E_1) + P(E_2)\, P(S/E_2) + P(E_3)\, P(S/E_3)}$$

$$= \frac{\frac{2}{6} \times 0.2}{\frac{1}{6} \times .002 + \frac{3}{6} \times .02 + \frac{2}{6} \times 0.2}$$ 1

$$= \frac{200}{231}$$ 1

Value: Hardwork and Regularity 1

SECTION D

24. For one-one

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ such that

$f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{4x_1}{3x_1 + 4} = \frac{4x_2}{3x_2 + 4}$

$\Rightarrow \quad 12 \cancel{x_1} x_2 + 16x_1 = 12 \cancel{x_1} x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one 3

Clearly f: $R - \left\{-\frac{4}{3}\right\} \rightarrow$ Range f is onto 1

Let $f(x) = y$

i.e. $\frac{4x}{3x + 4} = y$

$\Rightarrow \quad x = \frac{4y}{4 - 3y}$ 1

So f^{-1}: Range $f \rightarrow R - \left\{-\frac{4}{3}\right\}$ is

$f^{-1}(y) = \frac{4y}{4 - 3y}$ 1

OR

(a, b) * (c, d) = (a + c, b + d)

(c, d) * (a, b) = (c + a, d + b)

(a, b) * (c, d) = (c, d) * (a, b)

$\therefore$ * is commutative 2

((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)

(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)

As $((a, b) * (c, d)) * (e, f) = (a, b) * ((c, d) * (e, f))$

$\therefore$ * is associative 2

Let (e_1, e_2) be identity

$(a, b) * (e_1, e_2) = (a, b)$

$(a + e_1, b + e_2) = (a, b)$

$e_1 = 0, e_2 = 0$

$(0, 0) \in R \times R$ is the identity element. 2

25. Clearly order of A is 2×3 1

Let $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ 1

So $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

gives

$$\left.\begin{array}{l} 2a - d = -1,\ 2b - e = -8,\ 2c - f = -10 \\ a = 1,\ b = -2,\ c = -5 \end{array}\right\}$$ 2

$\Rightarrow$ $d = 3, e = 4, f = 0$ 1

Thus $A = \begin{pmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{pmatrix}$ 1

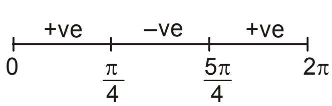
26. $f(x) = \sin x + \cos x \qquad 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$ 1

$f'(x) = 0 \Rightarrow \cos x = \sin x$ 1

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ 1

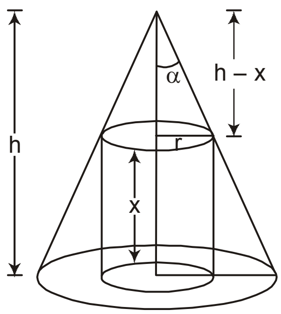
Sign of $f^1(x)$



2

So f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ and strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ 1

OR



For Figure **1**

$$\frac{r}{h-x} = \tan\alpha$$ 1

$$r = (h-x)\tan\alpha$$

Volume of cylinder

$$V = \pi r^2 x$$

$$V = \pi(h-x)^2 x \tan^2\alpha$$ $\frac{1}{2}$

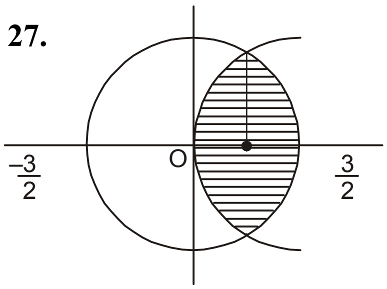
$$\frac{dV}{dx} = \pi\tan^2 (h-x)(h-3x)$$

$$\frac{dV}{dx} = 0 \Rightarrow h = x \text{ or } h = 3x$$

i.e. $x = \frac{h}{3}$ $1\frac{1}{2}$

$$\left.\begin{aligned} &\frac{d^2V}{dx^2} = \pi\tan^2\alpha\,(6x-4h) \\ \therefore\quad &\frac{d^2V}{dx^2} < 0 \quad \text{at} \quad x = \frac{h}{3} \\ \therefore\quad &V \text{ is maximum at } x = \frac{h}{3} \end{aligned}\right\}$$ 1

and maximum volume is $V = \frac{4}{27}\pi h^3 \tan^2 \propto$ 1

27. 

x coordinate of point of intersection is, $x = \frac{1}{2}$ 1

For Figure 1

Required area

$$= 2\left(\int_0^{\frac{1}{2}} 2\sqrt{x}\, dx + \int_{\frac{1}{2}}^{3/2} \sqrt{\frac{9}{4} - x^2}\, dx\right)$$ 2

$$= 2\left[\frac{4}{3}x^{3/2}\Big|_0^{\frac{1}{2}} + \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Big|_{\frac{1}{2}}^{3/2}\right]$$ $\frac{1}{2}+1$

$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right) \quad \text{or} \quad \frac{\sqrt{2}}{6} + \frac{9}{4}\cos^{-1}\frac{1}{3}$$ $\frac{1}{2}$

28. Clearly required plane passes through point (8, –19, 10) and normal to plane is perpendicular to given lines so equation of plane is given by

$$\begin{vmatrix} x-8 & y+19 & z-10 \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 0$$ 3

$24(x - 8) + 36(y + 19) + 72(z - 10) = 0$ 2

or $2(x - 8) + 3(y + 19) + 6(z - 10) = 0$

whichi gives

$2x + 3y + 6z = 19$ 1

29. $n = 8, P = \frac{1}{2}, \quad q = \frac{1}{2}$ 1

(i) $P(X = 5) = 8C_5\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 = \frac{7}{32}$ $1\frac{1}{2}$

(ii) $P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8)$

$$= {}^8C_6\left(\frac{1}{2}\right)^8 + {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8$$ 2

$$= \frac{37}{256}$$ $\frac{1}{2}$

(iii) $P(X \le 6) = 1 - [P(X = 7) + P(X = 8)]$

$$= 1 - \frac{9}{256} = \frac{247}{256}$$ 1

OR

Let X denote number of red cards drawn

X(xi)	P(X)	p_i	$p_i x_i$	$p_i x_i^2$	
0	${}^3C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	0	0	
1	${}^3C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	Correct table 4
2	${}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1$	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$	
3	${}^3C_3 \left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$	

$$\text{mean} = \Sigma p_i x_i = \frac{12}{8} = \frac{3}{2}$$ 1

$$\text{Variance} = \Sigma p_i x_i^2 - (\text{mean})^2$$

$$= 3 - \frac{9}{4} = \frac{3}{4}$$ 1

QUESTION PAPER CODE 65/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx} = \cos x$ $\frac{1}{2}$

Slope of tangent at (0, 0) is 1

Equation of tangent is $y = x$ $\frac{1}{2}$

2. π 1

3. Order of AB is 3×4 1

4. Putting $(1 + \log x)$ or $\log x = t$ $\frac{1}{2}$

$\log |1 + \log x| + C$ $\frac{1}{2}$

SECTION B

5. Differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a$$ 1

Eliminating 4a, we get

$$y^2 = 2y\frac{dy}{dx} \cdot x$$

or $$2xy\frac{dy}{dx} - y^2 = 0$$ 1

6. Let number of large vans = x

and number of small vans = y

Minimize cost $z = 400x + 200y$ $\frac{1}{2}$

Subject to constraints

$$\left.\begin{array}{l} 200x + 80y \geq 1200 \text{ or } 5x + 2y \geq 30 \\ x \leq y \\ 400x + 200y \leq 3000 \text{ or } 2x + y \leq 15 \end{array}\right\}$$ $1\frac{1}{2}$

$$x \geq 0, y \geq 0$$

7. $R_2 \to R_2 + R_1$ implies

$$\begin{pmatrix} 2 & 3 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -3 \\ 17 & -7 \end{pmatrix}$$ 1+1

1 mark for pre matrix on LHS and 1 mar for matrix on RHS

8. Integrating factor is $e^{\int 2dx} = e^{2x}$ $\frac{1}{2}$

$\therefore$ Required solution is

$$y.e^{2x} = \int e^{3x} \cdot e^{2x} dx$$ $\frac{1}{2}$

$$y.e^{2x} = \frac{e^{5x}}{5} + C$$ 1

or $y = \dfrac{e^{3x}}{5} + Ce^{-2x}$

9. $\dfrac{dr}{dt} = -3$ cm/min, $\dfrac{dh}{dt} = 2$ cm/min $\frac{1}{2}$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left[r^2\frac{dh}{dt} + 2hr\frac{dr}{dt}\right]$$ 1

$$\left(\frac{dV}{dt}\right)_{\text{at } r=9,\, h=6} = -54\pi \text{ cm}^3/\text{min}$$ $\frac{1}{2}$

$\Rightarrow$ Volume is decreasing at the rate 54π cm^3/min.

10. $11\hat{i} - 3\hat{j} = \dfrac{3(12\hat{i} + \mu\hat{j}) + 1(\lambda\hat{i} + 3\hat{j})}{4}$ 1

$44 = 36 + \lambda, -12 = 3\mu + 3$

$\lambda = 8, \mu = -5$ $\frac{1}{2}+\frac{1}{2}$

11. Given integral becomes

$$\int \sqrt{1-(x-1)^2}\,dx \qquad 1$$

$$= \frac{(x-1)}{2}\sqrt{2x-x^2} + \frac{1}{2}\sin^{-1}(x-1) + C \qquad 1$$

12. $\lim_{x \to 0} f(x) = f(0)$ $\qquad \frac{1}{2}$

$$\lim_{x \to 0} \frac{4 \times 2\sin^2 2x}{4x^2} = p \qquad 1\frac{1}{2}$$

$$p = 8 \qquad \frac{1}{2}$$

SECTION C

13. Let $\frac{x^2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ $\qquad 1$

$$A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2} \qquad 1\frac{1}{2}$$

Thus integral becomes

$$\frac{1}{2}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{x\,dx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{2}\log|x-1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C \qquad 1\frac{1}{2}$$

14. $\overrightarrow{AB} = \hat{i} + (x-3)\hat{j} + 4\hat{k}$

$\overrightarrow{AC} = \hat{i} - 3\hat{k}$

$\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ $\qquad 1\frac{1}{2}$

As A, B, C & D are coplanar

$$\left.\begin{array}{l} \therefore \quad \overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD}) = 0 \\ \text{i.e.} \quad \begin{vmatrix} 1 & x-3 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0 \end{array}\right\} \qquad 1\frac{1}{2}$$

which gives

$$x = 6 \qquad 1$$

15. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y\cos\frac{y}{x} + x}{x\cos\frac{y}{x}} \qquad ....(i)$$ $\frac{1}{2}$

Clearly it is homogenous

Let $\frac{y}{x} = v, \frac{dy}{dx} = v + \frac{dv}{dx}$ 1

(1) becomes

$$v + x\frac{dv}{dx} = v + \sec v$$

$\Rightarrow \quad \cos v\, dv = \frac{dx}{x}$ 1

integrating both sides we get

$$\sin v = \log|x| + C$$ 1

$$\sin\frac{y}{x} = \log|x| + C$$ $\frac{1}{2}$

16. Putting $x = \cos\theta$ 1

LHS becomes

$$\tan^{-1}\left(\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta/2 - \sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2 + \sqrt{2}\sin\theta/2}\right)$$ 1

$$= \tan^{-1}\left(\frac{1-\tan\theta/2}{1+\tan\theta/2}\right)$$ 1

$$= \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$ $\frac{1}{2}$

$$= \frac{\pi}{4} - \frac{\theta}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x = \text{RHS}$$ $\frac{1}{2}$

17. Taking x, y, z common from C_1, C_2, C_3 respectively, we get

$$xyz\begin{vmatrix} a/x & b/y-1 & c/z-1 \\ a/x-1 & b/y & c/z-1 \\ a/x-1 & b/y-1 & c/z \end{vmatrix} = 0$$ 1

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a/x+b/y+c/z-2 & b/y-1 & c/z-1 \\ a/x+b/y+c/z-2 & b/y & c/z-1 \\ a/x+b/y+c/z-2 & b/y-1 & c/z \end{vmatrix} = 0$$ 1

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 1 & b/y & c/z-1 \\ 1 & b/y-1 & c/z \end{vmatrix} = 0$$ $\frac{1}{2}$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$ 1

$$\therefore \quad \left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\cdot 1 = 0 \Rightarrow \frac{a}{x}+\frac{b}{y}+\frac{c}{z} = 2$$ $\frac{1}{2}$

OR

We know that

$IA = A$ 1

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix}$$ 1

$R_2 \to \dfrac{R_2}{-5}$

$$\begin{pmatrix} 1 & 0 \\ 2/5 & -1/5 \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$ $\frac{1}{2}$

$R_1 \to R_1 - 2R_2$

$$\begin{pmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ 1

$\therefore \quad A^{-1} = \begin{pmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{pmatrix}$ $\frac{1}{2}$

Full marks for finding correct A^{-1} using column transformations with $AI = A$

18. Differentiating $y = \cos(x + y)$ wrt x we get

$$\frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$$ 1

Slope of given line is $\frac{-1}{2}$ $\frac{1}{2}$

As tangent is parallel to line $x + 2y = 0$

$$\therefore \quad \frac{-\sin(x+y)}{1+\sin(x+y)} = \frac{-1}{2}$$

$\Rightarrow \quad \sin(x + y) = 1$

$\Rightarrow \quad x + y = n\pi + (-1)^n \frac{\pi}{2}, n \in Z \qquad ...(1)$ 1

Putting (1) in $y = \cos(x + y)$

we get $y = 0$

$\Rightarrow \quad x = n\pi + (-1)^n \pi/2, n \in Z$

$x = \frac{-3\pi}{2} \in [-2\pi, 0]$ $\frac{1}{2}$

$\therefore$ Required equation of tangent is

$$y = \frac{-1}{2}\left(x + \frac{3\pi}{2}\right)$$

or $\quad 2y + x + \frac{3\pi}{2} = 0$ 1

19. Given equation of lines can be written as

$$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{1} \quad ...(1) \qquad 1$$

$$\frac{x-1}{-3p/7} = \frac{y-5}{-1} = \frac{z-11}{-7} \quad ...(2) \qquad 1$$

(1) & (2) are perpendicular

So $\quad -3\left(\frac{-3p}{7}\right) + \frac{2p}{7}(-1) + 1(-7) = 0$ 1

which gives p = 7 1

OR

Required equation of plane is $x + y + z - 1 + \lambda(2x + 3y + 4z - 5) = 0$ for some λ. 1

i.e. $\quad (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z = 1 + 5\lambda$

according to question

$$2\left(\frac{1+5\lambda}{1+3\lambda}\right) = 3\left(\frac{1+5\lambda}{1+4\lambda}\right) \qquad 1$$

Solving we get $\lambda = -1$ 1

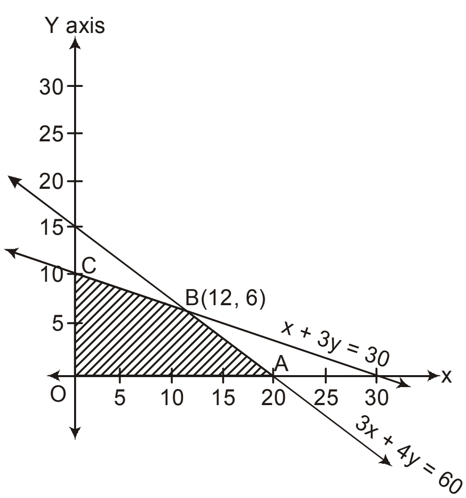
Thus the equation of required plane is

$$-x - 2y - 3z = -4$$

or $\quad x + 2y + 3z = 4$ 1

20. **Correct lines** 2

Correct shading 1



Corner points	Value of Z	
O(0, 0)	0	
A(20, 0)	160000	
B(12, 6)	168000 → max	$\frac{1}{2}$
C(0, 10)	120000	

Maximum Z = 168000

at x = 12, y = 6 $\frac{1}{2}$

21. Given integral $= \int \frac{x+7}{(3x+4)(x+7)} dx$ 2

$= \int \frac{1}{3x+4} dx$

$= \frac{1}{3} \log|3x+4| + C$ 2

22. $\frac{dx}{dt} = -3 \sin t + 6\cos^2 t \sin t$

$= +3 \sin t \cos 2t$ $1\frac{1}{2}$

$\frac{dy}{dt} = 3\cos t - 6\sin^2 t - \cos t$

$= 3\cos t \cos 2t$ 1

$\frac{dy}{dx} = \cot t$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2 t \frac{dt}{dx} = \frac{-1}{3} \frac{\operatorname{cosec}^3 t}{\cos 2t}$ 1

23. E_1: Student selected from category A

E_2: Student selected from category B

E_3: Student selected from category C

S: Student could not get good marks

$P(E_1) = \frac{1}{6}$ $P(E_2) = \frac{3}{6}$ $P(E_3) = \frac{2}{6}$ 1

$P(S/E_1) = 0.002$ $P(S/E_2) = 0.02$, $P(S/E_3) = 0.2$

$$P(E_3/S) = \frac{P(E_3)\, P(S/E_3)}{P(E_1)\, P(S/E_1) + P(E_2)\, P(S/E_2) + P(E_3)\, P(S/E_3)}$$

$$= \frac{\frac{2}{6} \times 0.2}{\frac{1}{6} \times .002 + \frac{3}{6} \times .02 + \frac{2}{6} \times 0.2}$$ 1

$= \frac{200}{231}$ 1

Value: Hardwork and Regularity 1

SECTION D

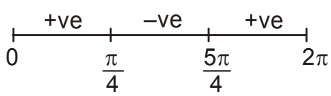
24. $f(x) = \sin x + \cos x \qquad 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$ 1

$f'(x) = 0 \Rightarrow \cos x = \sin x$ 1

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ 1

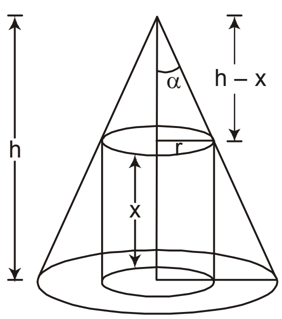
Sign of $f^1(x)$



2

So f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ and strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ 1

OR



For Figure 1

$\frac{r}{h - x} = \tan \alpha$ 1

$r = (h - x) \tan \alpha$

Volume of cylinder

$V = \pi r^2 x$

$V = \pi(h - x)^2 x \tan^2\alpha$ $\frac{1}{2}$

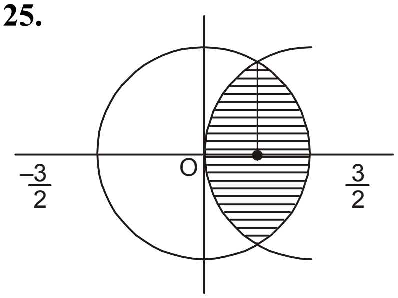
$\frac{dV}{dx} = \pi \tan^2 (h - x)(h - 3x)$

$\frac{dV}{dx} = 0 \Rightarrow h = x \text{ or } h = 3x$

i.e. $x = \frac{h}{3}$ $1\frac{1}{2}$

$$\left.\begin{array}{l} \dfrac{d^2V}{dx^2} = \pi\tan^2\alpha\,(6x - 4h) \\ \\ \therefore \quad \dfrac{d^2V}{dx^2} < 0 \quad \text{at} \quad x = \dfrac{h}{3} \\ \\ \therefore \quad \text{V is maximum at } x = \dfrac{h}{3} \end{array}\right\} \qquad 1$$

and maximum volume is $V = \frac{4}{27}\pi h^3 \tan^2 \alpha$ 1

25. 

x coordinate of point of intersection is, $x = \frac{1}{2}$ 1

For Figure 1

Required area

$$= 2\left(\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{3/2} \sqrt{\frac{9}{4} - x^2}\,dx\right) \qquad 2$$

$$= 2\left[\frac{4}{3}x^{3/2}\Big|_0^{\frac{1}{2}} + \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Big|_{\frac{1}{2}}^{3/2}\right] \qquad \frac{1}{2} + 1$$

$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right) \quad \text{or} \quad \frac{\sqrt{2}}{6} + \frac{9}{4}\cos^{-1}\frac{1}{3} \qquad \frac{1}{2}$$

26. $n = 8,\ P = \frac{1}{2},\ \ q = \frac{1}{2}$ 1

(i) $P(X = 5) = 8C_5\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 = \frac{7}{32}$ $1\frac{1}{2}$

(ii) $P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8)$

$$= {}^8C_6\left(\frac{1}{2}\right)^8 + {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8 \qquad 2$$

$$= \frac{37}{256} \qquad \frac{1}{2}$$

(iii) $P(X \le 6) = 1 - [P(X = 7) + P(X = 8)]$

$$= 1 - \frac{9}{256} = \frac{247}{256}$$ 1

OR

Let X denote number of red cards drawn

X(xi)	P(X)	p_i	$p_i x_i$	$p_i x_i^2$
0	${}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	0	0
1	${}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	${}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	${}^3C_3\left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$

Correct table 4

$$\text{mean} = \Sigma p_i x_i = \frac{12}{8} = \frac{3}{2}$$ 1

$$\text{Variance} = \Sigma p_i x_i^2 - (\text{mean})^2$$

$$= 3 - \frac{9}{4} = \frac{3}{4}$$ 1

27. For one-one

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ such that

$f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{4x_1}{3x_1 + 4} = \frac{4x_2}{3x_2 + 4}$

$\Rightarrow \quad 12\cancel{x_1}x_2 + 16x_1 = 12\cancel{x_1}x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore \quad$ f is one-one 3

Clearly f: $R - \left\{-\frac{4}{3}\right\} \to$ Range f is onto 1

Let $f(x) = y$

i.e. $\frac{4x}{3x+4} = y$

$\Rightarrow \quad x = \frac{4y}{4-3y}$ 1

So f^{-1}: Range f $\to R - \left\{-\frac{4}{3}\right\}$ is

$f^{-1}(y) = \frac{4y}{4-3y}$ 1

OR

$(a, b) * (c, d) = (a + c, b + d)$

$(c, d) * (a, b) = (c + a, d + b)$

$(a, b) * (c, d) = (c, d) * (a, b)$

$\therefore$ * is commutative 2

$((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)$

$(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)$

As $((a, b) * (c, d)) * (e, f) = (a, b) * ((c, d) * (e, f))$

$\therefore$ * is associative 2

Let (e_1, e_2) be identity

$(a, b) * (e_1, e_2) = (a, b)$

$(a + e_1, b + e_2) = (a, b)$

$e_1 = 0, e_2 = 0$

$(0, 0) \in R \times R$ is the identity element. 2

28. Clearly order of X is 3×2 1

Let $X = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ 1

So $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \\ 11 & 10 & 9 \end{pmatrix}$

$$\left.\begin{array}{lll} a+4b=-7 & c+4d=2 & e+4f=11 \\ 2a+5b=-8 & 2c+5d=4 & 2e+5f=10 \end{array}\right\}$$ 2

Solving we get

$a = 1, \quad b = -2, \quad c = 2, \quad d = 0, \quad e = -5 \quad f = 4$ 1

Thus $X = \begin{pmatrix} 1 & -2 \\ 2 & 0 \\ -5 & 4 \end{pmatrix}$

29. Consider $\begin{vmatrix} -1+3 & 2-1 & 5-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$ 2

$$= \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$$ 1

$\therefore$ Lines are coplanar.

Equation of plane is given by

$$\begin{vmatrix} x+3 & y-1 & z-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$$ 2

Which gives

$-x + 2y - 3 = 0$ or $x - 2y + z = 0$ 1

QUESTION PAPER CODE 65/1/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. π 1

2. Order of AB is 3×4 1

3. $\frac{dy}{dx} = \cos x$ $\frac{1}{2}$

Slope of tangent at (0, 0) is 1

Equation of tangent is $y = x$ $\frac{1}{2}$

4. Putting $(1 + \log x)$ or $\log x = t$ $\frac{1}{2}$

$\log |1 + \log x| + C$ $\frac{1}{2}$

SECTION B

5. Let A be $10\hat{i} + 3\hat{j}$, B be $12\hat{i} - 5\hat{j}$, C be $\lambda\hat{i} + 11\hat{j}$

$\overrightarrow{AB} = 2\hat{i} - 8\hat{j}$ $\frac{1}{2}$

$\overrightarrow{AC} = (\lambda - 10)\hat{i} + 8\hat{j}$ $\frac{1}{2}$

As $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear

$\frac{2}{\lambda - 10} = \frac{-8}{8}$ $\frac{1}{2}$

So $\lambda = 8$ $\frac{1}{2}$

6. Let number of large vans = x

and number of small vans = y

Minimize cost $z = 400x + 200y$ $\frac{1}{2}$

Subject to constraints

$$\left.\begin{array}{l} 200x + 80y \geq 1200 \text{ or } 5x + 2y \geq 30 \\ x \leq y \\ 400x + 200y \leq 3000 \text{ or } 2x + y \leq 15 \end{array}\right\} \quad 1\frac{1}{2}$$

$$x \geq 0,\ y \geq 0$$

7. $R_2 \to R_2 + R_1$ implies

$$\begin{pmatrix} 2 & 3 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -3 \\ 17 & -7 \end{pmatrix} \quad 1+1$$

1 mark for pre matrix on LHS and 1 mar for matrix on RHS

8. $\frac{dr}{dt} = -3$ cm/min, $\frac{dh}{dt} = 2$ cm/min $\quad \frac{1}{2}$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left[r^2 \frac{dh}{dt} + 2hr\frac{dr}{dt}\right] \quad 1$$

$$\left(\frac{dV}{dt}\right)_{\text{at } r=9,\, h=6} = -\,54\pi \text{ cm}^3\text{/min} \quad \frac{1}{2}$$

$\Rightarrow$ Volume is decreasing at the rate 54π cm^3/min.

9. Differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a \quad 1$$

Eliminating 4a, we get

$$y^2 = 2y\frac{dy}{dx} \cdot x$$

or $\quad 2xy\frac{dy}{dx} - y^2 = 0 \quad 1$

10. $\lim_{x \to \pi/4} f(x) = f(\pi/4)$

$$\lim_{x \to \pi/4} \frac{\sqrt{2}\sin(x - \pi/4)}{4(x - \pi/4)} = k \quad 1\frac{1}{2}$$

$\therefore \quad k = \frac{\sqrt{2}}{4} \quad \frac{1}{2}$

11. Given integral $= \int \frac{1}{\sqrt{(x-2)^2 - 2^2}} dx$ 1

$= \frac{(x-2)}{2}\sqrt{x^2 - 4x} - 2\log|x - 2 + \sqrt{x^2 + 4x}| + C$ 1

12. Integrating factor is $e^{\int \frac{2}{x}dx} = x^2$ $\frac{1}{2}$

Solution is $y.x^2 = \int x \cdot x^2 dx + C$ $\frac{1}{2}$

$$\left.\begin{array}{ll} & y \cdot x^2 = \frac{x^4}{4} + C \\ \text{or} & y = \frac{x^2}{4} + \frac{C}{x^2} \end{array}\right\}$$ 1

SECTION C

13. $\frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta)$

$= a\,\theta\cos\theta$ $1\frac{1}{2}$

$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$

$= a\,\theta\sin\theta$ 1

$\therefore \quad \frac{dy}{dx} = \tan\theta$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \sec^2\theta \times \frac{d\theta}{dx} = \frac{\sec^3\theta}{a\theta}$ 1

14. Differentiating $y = \cos(x+y)$ wrt x we get

$\frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$ 1

Slope of given line is $\frac{-1}{2}$ $\frac{1}{2}$

As tangent is parallel to line $x + 2y = 0$

$$\therefore \quad \frac{-\sin(x+y)}{1+\sin(x+y)} = \frac{-1}{2}$$

$$\Rightarrow \quad \sin(x+y) = 1$$

$$\Rightarrow \quad x + y = n\pi + (-1)^n \frac{\pi}{2}, n \in Z \qquad ...(1) \qquad 1$$

Putting (1) in $y = \cos(x+y)$

we get $y = 0$

$$\Rightarrow \quad x = n\pi + (-1)^n \pi/2, n \in Z$$

$$x = \frac{-3\pi}{2} \in [-2\pi, 0] \qquad \frac{1}{2}$$

$\therefore$ Required equation of tangent is

$$y = \frac{-1}{2}\left(x + \frac{3\pi}{2}\right)$$

or $\quad 2y + x + \dfrac{3\pi}{2} = 0$ 1

15. Let $\dfrac{x^2}{(x-1)(x^2+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1}$ 1

$$A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2} \qquad 1\frac{1}{2}$$

Thus integral becomes

$$\frac{1}{2}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{2}\log|x-1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C \qquad 1\frac{1}{2}$$

16. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y\cos\frac{y}{x} + x}{x\cos\frac{y}{x}} \qquad ....(i) \qquad \frac{1}{2}$$

Clearly it is homogenous

Let $\frac{y}{x} = v,\ \frac{dy}{dx} = v + \frac{dv}{dx}$ 1

(1) becomes

$$v + x\frac{dv}{dx} = v + \sec v$$

$\Rightarrow \quad \cos v\, dv = \frac{dx}{x}$ 1

integrating both sides we get

$\sin v = \log|x| + C$ 1

$\sin\frac{y}{x} = \log|x| + C$ $\frac{1}{2}$

17. $\overrightarrow{AB} = \hat{i} + (x-3)\hat{j} + 4\hat{k}$

$\overrightarrow{AC} = \hat{i} - 3\hat{k}$

$\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ $1\frac{1}{2}$

As A, B, C & D are coplanar

$$\left.\begin{array}{l} \therefore \quad \overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD}) = 0 \\ \text{i.e.} \quad \begin{vmatrix} 1 & x-3 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0 \end{array}\right\}$$ $1\frac{1}{2}$

which gives

$x = 6$ 1

18. Given equation of lines can be written as

$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{1}$ ...(1) 1

$\frac{x-1}{-3p/7} = \frac{y-5}{-1} = \frac{z-11}{-7}$ ...(2) 1

(1) & (2) are perpendicular

So $\quad -3\left(\frac{-3p}{7}\right)+\frac{2p}{7}(-1)+1(-7) = 0$ 1

which gives p = 7 1

OR

Required equation of plane is $x + y + z - 1 + \lambda(2x + 3y + 4z - 5) = 0$ for some λ. 1

i.e. $\quad (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z = 1 + 5\lambda$

according to question

$$2\left(\frac{1+5\lambda}{1+3\lambda}\right) = 3\left(\frac{1+5\lambda}{1+4\lambda}\right)$$ 1

Solving we get $\lambda = -1$ 1

Thus the equation of required plane is

$$-x - 2y - 3z = -4$$

or $\quad x + 2y + 3z = 4$ 1

19. Taking x, y, z common from C_1, C_2, C_3 respectively, we get

$$xyz\begin{vmatrix} a/x & b/y-1 & c/z-1 \\ a/x-1 & b/y & c/z-1 \\ a/x-1 & b/y-1 & c/z \end{vmatrix} = 0$$ 1

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a/x+b/y+c/z-2 & b/y-1 & c/z-1 \\ a/x+b/y+c/z-2 & b/y & c/z-1 \\ a/x+b/y+c/z-2 & b/y-1 & c/z \end{vmatrix} = 0$$ 1

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 1 & b/y & c/z-1 \\ 1 & b/y-1 & c/z \end{vmatrix} = 0$$ $\frac{1}{2}$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix}1 & b/y-1 & c/z-1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix} = 0$$ 1

$\therefore$ $$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\cdot 1 = 0 \Rightarrow \frac{a}{x}+\frac{b}{y}+\frac{c}{z} = 2$$ $\frac{1}{2}$

OR

We know that

$IA = A$ 1

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}A = \begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix}A = \begin{pmatrix}1 & 2\\ 0 & -5\end{pmatrix}$$ 1

$R_2 \to \frac{R_2}{-5}$

$$\begin{pmatrix}1 & 0\\ 2/5 & -1/5\end{pmatrix}A = \begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$$ $\frac{1}{2}$

$R_1 \to R_1 - 2R_2$

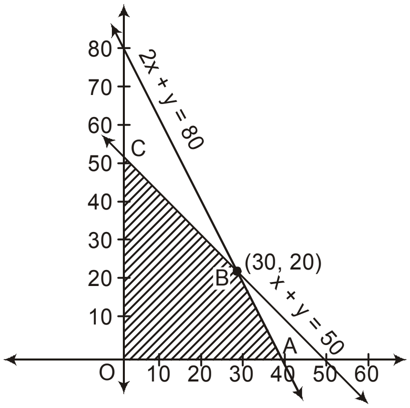
$$\begin{pmatrix}1/5 & 2/5\\ 2/5 & -1/5\end{pmatrix}A = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$ 1

$\therefore$ $$A^{-1} = \begin{pmatrix}1/5 & 2/5\\ 2/5 & -1/5\end{pmatrix}$$ $\frac{1}{2}$

Full marks for finding correct A^{-1} using column transformations with $AI = A$

20. **Correct lines** 1

Correct shading 1



Corner points	Value of Z	
O(0 , 0)	0	
A(40, 0)	4200	
B(30, 20)	4950 maximum	$\frac{1}{2}$
C(0, 50)	4500	

Maximum Z = 4950

at x = 30, y = 20 $\frac{1}{2}$

21. Let $x + 3 = A(2x - 4) + B$

which gives $A = \frac{1}{2}, \ B = 5$ 1

Given integral becomes

$$I = \frac{1}{2}\int \frac{2x-4}{\sqrt{5-4x+x^2}}dx + 5\int \frac{dx}{\sqrt{5-4x+x^2}}$$ 1

$$= \sqrt{5-4x+x^2} + 5\int \frac{dx}{\sqrt{(x-2)^2+1^2}}$$ $\frac{1}{2}+\frac{1}{2}$

$$= \sqrt{5-4x+x^2} + 5\log\left|x-2+\sqrt{5-4x+x^2}\right| + C$$ 1

22. LHS becomes

$$\cot^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$$ 2

$$= \cot^{-1}\left[\frac{2\cos x/2}{2\sin x/2}\right]$$ 1

$$= \cot^{-1}\cot\frac{x}{2}$$

$$= \frac{x}{2}$$ 1

23. E_1: Student selected from category A

E_2: Student selected from category B

E_3: Student selected from category C

S: Student could not get good marks

$P(E_1) = \frac{1}{6} \quad P(E_2) = \frac{3}{6} \quad P(E_3) = \frac{2}{6}$ 1

$P(S/E_1) = 0.002\ P(S/E_2) = 0.02,\ P(S/E_3) = 0.2$

$$P(E_3/S) = \frac{P(E_3)\,P(S/E_3)}{P(E_1)\,P(S/E_1) + P(E_2)\,P(S/E_2) + P(E_3)\,P(S/E_3)}$$

$$= \frac{\frac{2}{6} \times 0.2}{\frac{1}{6} \times .002 + \frac{3}{6} \times .02 + \frac{2}{6} \times 0.2}$$ 1

$$= \frac{200}{231}$$ 1

Value: Hardwork and Regularity 1

SECTION D

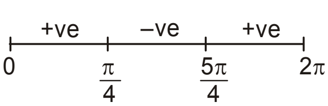
24. $f(x) = \sin x + \cos x \qquad 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$ 1

$f'(x) = 0 \Rightarrow \cos x = \sin x$ 1

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ 1

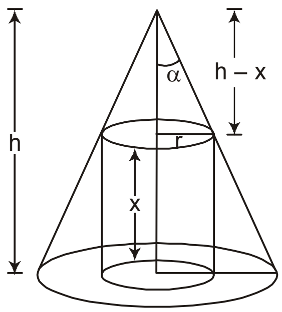
Sign of $f^1(x)$



2

So f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ and strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ 1

OR



For Figure 1

$$\frac{r}{h-x} = \tan\alpha$$ 1

$$r = (h-x)\tan\alpha$$

Volume of cylinder

$$V = \pi r^2 x$$

$$V = \pi(h-x)^2 x \tan^2\alpha$$ $\frac{1}{2}$

$$\frac{dV}{dx} = \pi\tan^2 (h-x)(h-3x)$$

$$\frac{dV}{dx} = 0 \Rightarrow h = x \text{ or } h = 3x$$

i.e. $x = \frac{h}{3}$ $1\frac{1}{2}$

$$\frac{d^2V}{dx^2} = \pi\tan^2\alpha\,(6x-4h)$$

$\therefore$ $\frac{d^2V}{dx^2} < 0$ at $x = \frac{h}{3}$

$\therefore$ V is maximum at $x = \frac{h}{3}$ 1

and maximum volume is $V = \frac{4}{27}\pi h^3 \tan^2 \propto$ 1

25. $n = 8, P = \frac{1}{2}, \quad q = \frac{1}{2}$ 1

(i) $P(X = 5) = 8C_5\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 = \frac{7}{32}$ $1\frac{1}{2}$

(ii) $P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8)$

$$= {}^8C_6\left(\frac{1}{2}\right)^8 + {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8$$ 2

$$= \frac{37}{256}$$ $\frac{1}{2}$

(iii) $P(X \leq 6) = 1 - [P(X = 7) + P(X = 8)]$

$$= 1 - \frac{9}{256} = \frac{247}{256}$$ 1

OR

Let X denote number of red cards drawn

X(xi)	P(X)	p_i	$p_i x_i$	$p_i x_i^2$
0	${}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	0	0
1	${}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	${}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	${}^3C_3\left(\frac{1}{2}\right)^3$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$

Correct table 4

$$\text{mean} = \Sigma p_i x_i = \frac{12}{8} = \frac{3}{2}$$ 1

$$\text{Variance} = \Sigma p_i x_i^2 - (\text{mean})^2$$

$$= 3 - \frac{9}{4} = \frac{3}{4}$$ 1

26. For one-one

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ such that

$$f(x_1) = f(x_2)$$

$\Rightarrow \quad \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

$\Rightarrow \quad 12\cancel{x_1}x_2 + 16x_1 = 12\cancel{x_1}x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one 3

Clearly f: $R - \left\{-\frac{4}{3}\right\} \rightarrow$ Range f is onto 1

Let f(x) = y

i.e. $\frac{4x}{3x+4} = y$

$\Rightarrow \quad x = \frac{4y}{4-3y}$ 1

So f^{-1}: Range f $\rightarrow R - \left\{-\frac{4}{3}\right\}$ is

$f^{-1}(y) = \frac{4y}{4-3y}$ 1

OR

(a, b) * (c, d) = (a + c, b + d)

(c, d) * (a, b) = (c + a, d + b)

(a, b) * (c, d) = (c, d) * (a, b)

$\therefore$ * is commutative 2

((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)

(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)

As ((a, b) * (c, d)) * (e, f) = (a, b) * ((c, d) * (e, f))

$\therefore$ * is associative 2

Let (e_1, e_2) be identity

$(a, b) * (e_1, e_2) = (a, b)$

$(a + e_1, b + e_2) = (a, b)$

$e_1 = 0, e_2 = 0$

$(0, 0) \in R \times R$ is the identity element. 2

27. Clearly order of A is 2×3 1

Let $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ 1

So $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

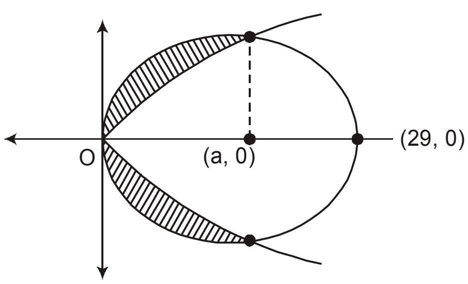
gives

$$\left.\begin{array}{l} 2a - d = -1,\ 2b - e = -8,\ 2c - f = -10 \\ a = 1,\ b = -2,\ c = -5 \end{array}\right\}$$ 2

$\Rightarrow$ $d = 3,\ e = 4,\ f = 0$ 1

Thus $A = \begin{pmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{pmatrix}$ 1

28. **Correct figure** 1



x-coordinate of point of intersection is, $x = a$ 1

Required area $= 2\left[\int_0^a \left(\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}\right) dx\right]$ 2

$$= 2\left[\frac{x-a}{2}\sqrt{a^2 - (x-a)^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a} - 2\sqrt{a}\,\frac{x^{\frac{3}{2}}}{3}\right]_0^a$$ 1

$$= \left(\frac{\pi}{2} - \frac{4}{3}\right)a^2$$ 1

29. Required equation of plane is given by

$$\begin{vmatrix} x+1 & y-3 & z-2 \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$$ 3

$\Rightarrow$ $(x+1)(-7) - (y-3)(-8) + (z-2)(-3) = 0$ 2

$\Rightarrow$ $-7x + 8y - 3z = 25$ 1

or $7x - 8y + 3z + 25 = 0$