SET – 3

Series : HMJ/4

कोड नं.
Code No. **65/4/3**

रोल नं.
Roll No. 1 3 6 4 6 0 7 6

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Code on the title page of the answer-book.

	नोट		NOTE
(I)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **15** हैं ।	(I)	Please check that this question paper contains **15** printed pages.
(II)	प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।	(II)	Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में **36** प्रश्न हैं ।	(III)	Please check that this question paper contains **36** questions.
(IV)	**कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**	(IV)	**Please write down the Serial Number of the question in the answer-book before attempting it.**
(V)	इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका में कोई उत्तर नहीं लिखेंगे ।	(V)	15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper comprises* ***four*** *sections – A, B, C and D.*
This question paper carries ***36*** *questions.* ***All*** *questions are compulsory.*

(ii) ***Section A*** *– Question no.* ***1*** *to* ***20*** *comprises of* ***20*** *questions of* ***one*** *mark each.*

(iii) ***Section B*** *– Question no.* ***21*** *to* ***26*** *comprises of* ***6*** *questions of* ***two*** *marks each.*

(iv) ***Section C*** *– Question no.* ***27*** *to* ***32*** *comprises of* ***6*** *questions of* ***four*** *marks each.*

(v) ***Section D*** *– Question no.* ***33*** *to* ***36*** *comprises of* ***4*** *questions of* ***six*** *marks each.*

(vi) *There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.*

(vii) *In addition to this, separate instructions are given with each section and question, wherever necessary.*

(viii) *Use of calculators is not permitted.*

Section – A

Question numbers **1** to **10** are multiple choice questions of **1** mark each. You have to select the correct choice :

1. The two lines $x = ay + b$, $z = cy + d$; and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other, if

(a) $\frac{a}{a'} + \frac{c}{c'} = 1$ (b) $\frac{a}{a'} + \frac{c}{c'} = -1$ (c) $aa' + cc' = 1$ (d) $aa' + cc' = -1$

2. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is

(a) 3 (b) 0 (c) -1 (d) 1

3. In an LPP, if the objective function $z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is

(a) 0 (b) 2 (c) finite (d) infinite

4. From the set $\{1,2,3,4,5\}$, two numbers a and b ($a \neq b$) are chosen at random. The probability that $\frac{a}{b}$ is an integer is :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

5. $\int_0^{\pi/8} \tan^2(2x)$ is equal to

(a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$

6. If $\vec{a} \cdot \vec{b} = \frac{1}{2} |\vec{a}| \, |\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) 0° (b) 30° (c) 60° (d) 90°

7. A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{12}$ (d) $\frac{1}{24}$

8. The value of $\tan^{-1}\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is

(a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{3-\sqrt{5}}{2}$ (c) $\frac{-3+\sqrt{5}}{2}$ (d) $\frac{-3-\sqrt{5}}{2}$

9. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det (adj A) equals

(a) a^{27} (b) a^9 (c) a^6 (d) a^2

10. The line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is parallel to the plane

(a) $2x + 3y + 4z = 0$
(b) $3x + 4y - 5z = 7$
(c) $2x + y - 2z = 0$
(d) $x - y + z = 2$

In Q. Nos. **11** to **15,** fill in the blanks with correct word / sentence :

11. The slope of the tangent to the curve $y = x^3 - x$ at the point (2, 6) is _____.

OR

The rate of change of the area of a circle with respect to its radius r, when r = 3 cm, is _____.

12. If $f : R \to R$ be given by $f(x) = (3 - x^3)^{1/3}$, then $fof(x)$ = _____

13. If $\vec{a}$ is a non-zero vector, then $(\vec{a}.\hat{i})\,\hat{i} + (\vec{a}.\hat{j})\,\hat{j} + (\vec{a}.\hat{k})\,\hat{k}$ equals _____.

OR

The projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ is _____.

14. If $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$, then $x \cdot y$ = _____

15. If $f(x) = x \mid x \mid$, then $f'(x)$ = _____.

Q. Nos. **16** to **20** are very short answer type questions.

16. Show that the function $y = ax + 2a^2$ is a solution of the differential equation $2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$.

17. Find adj A, if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

18. If $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$, then find the value of a.

OR

Find $\int \frac{dx}{\sqrt{x}+x}$

19. Find $\int \frac{1}{x(1+x^2)}\,dx$

20. If $[x]$ denotes the greatest integer function, then find $\int_0^{3/2} [x^2]\,dx$

Section – B

Q. Nos. **21** to **26** carry **2** marks each.

21. Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 2|\vec{b}|$ and $(\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b}) = 12$.

OR

Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

22. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$, if $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$.

23. Find the equation of the plane with intercept 3 on the y-axis and parallel to xz – plane.

24. Check if the relation R on the set A = { 1, 2, 3, 4, 5, 6 } defined as R = { (x, y) : y is divisible by x } is (i) symmetric (ii) transitive

OR

Prove that :

$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

25. Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the intervals $(-3, 0) \cup (0, 3)$.

26. Three distinct numbers are chosen randomly from the first 50 natural numbers. Find the probability that all the three numbers are divisible by both 2 and 3.

Section – C

Q. Nos. **27** to **32** carry **4** marks each.

27. A manufacturer has three machines I, II and III installed in his factory. Machine I and II are capable of being operated for atmost 12 hours whereas machine III must be operated for atleast 5 hours a day. He produces only two items M and N each requiring the use of all the three machines.

The number of hours required for producing 1 unit of M and N on three machines are given in the following table :

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on one unit of items M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produced. What will be the maximum profit ?

28. Prove that the relation R on Z, defined by R $\{(x, y) : (x - y)$ is divisible by $5\}$ is an equivalence relation.

29. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails.

OR

Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

30. If $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

OR

Verify the Rolle's Theorem for the function $f(x) = e^x \cos x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

31. Evaluate $\int_0^1 \sqrt{3-2x-x^2}\, dx$

32. Find the general solution of the differential equation $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x}$.

Section – D

Q. Nos. **33** to **36** carry **6** marks each.

33. Find the area of the region lying in the first quadrant and enclosed by the x – axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

34. Using properties of determinates prove that :

$$\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.$$

OR

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence find A^{-1}.

35. Find the intervals on which the function $f(x) = (x-1)^3 (x-2)^2$ is (a) strictly increasing (b) strictly decreasing.

OR

Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume.

36. Find the image of the point (–1, 3, 4) in the plane $x - 2y = 0$.