Class – XII
MATHEMATICS (041)
SQP Marking Scheme (2019-20)

TIME: 3 Hrs. **Maximum Marks: 80**

	SECTION A	
1	(c) 9	1
2	(a) $3 \times p$	1
3	(b)p=3,q=—	1
4	(b)0.25	1
5	(c) $(2,3)$	1
6	(b)–	1
7	(c) —	1
8	(b) $-\sin$ — $+ c$	1
9	(a) 0	1
10	(b) $\vec{} = -\hat{} + \hat{} + (\hat{}) \hat{}$	1
11	$- - = (-2) = 2$	1
12	2	1
13	$= 2$	1
14	$\frac{-3}{2}$ OR decreasing at rate of 72 units/sec.	1
15	2 units OR $\frac{5}{7}$ $-2\hat{i} - 3\hat{j} + 6k$	1
16	Apply $\rightarrow$ + $\begin{matrix} l+m+n & m+n+l & n+l+m \\ n & l & m \\ 2 & 2 & 2 \end{matrix}$ $= 2(l+m+n) \begin{matrix} 1 & 1 & 1 \\ n & l & m \\ 1 & 1 & 1 \end{matrix}$; yes (l + m + n) is a factor	1
17	$\int (+ 1) = \int ( ) + \int 1 = +$ $= 0 + [ ]$ (As is odd function) =2+2 = 4	1

18	Let $x + \sin x =$ So $(1 + \cos x)$ = $= 3\int — = 3\log\|\ \| +\ = 3\log\|(x + \sin x)\| +$ or directly by writing formula $\frac{(\ )}{(\ )} = \log\|\ (\ )\| +$ **OR** $\int \cos 4\ =— +$	1
19	let $(1 + x) =$ so $2\ =$ $\Rightarrow = -\int\ = -\ +c = -\ (\ )\ +c$	1
20	$\frac{dy}{dx} = e\ e$ $\Rightarrow \frac{dy}{e} = e\,dx$ integrating both sides $\Rightarrow -e\ + = e$ $\Rightarrow\ +\ =$	1
	SECTION B	
21	$= \sin\ \frac{}{\sqrt{}} + \frac{}{\sqrt{}}$ if $- < \ < -$ $=\sin\ \sin\ \cos - + \cos\ \sin -$ if $- + < -\ + - < - + -$ $=\sin\ \sin\ + -$ if $0 <\ + - < -$ i.e. principal values $=\ + -$	1 1
	OR Let 2 divides $-$ and 2 divides $(\ -\ )$: where $,\ ,\ \in$ So 2 divides $(\ -\ ) + (\ -\ )$ 2 divides $(\ -\ )$: Yes relation R is transitive [0] = {0, ± 2, ± 4, ± 6,...}	1 1
22	$y = ae\ + be$ ………………(1) $— = 2ae\ - be$ …………(2) $— = 4ae\ + be$ ………..(3) putting values on LHS $=— - — - 2y$ $=(4ae\ + be\ ) - (2ae\ - be\ ) - 2(ae\ + be\ )$ $=4ae\ + be\ - 2ae\ + be\ - 2ae\ - 2be$ $=0$	1 1

23	x = 2y ………(1) $\Rightarrow 2x$ — = 2 — (given — = —) $\Rightarrow 2x\,\frac{dx}{dt} = 2\,\frac{dx}{dt}$ $\Rightarrow x = 1$ from (1) y = − so point is 1, −	1 1
24	$= \vec{a} - b\ .\ \ b - \vec{c}\ \times (\vec{c} - \vec{a})$ $= \vec{a} - b\ .\ b \times \vec{c} - b \times \vec{a} - \vec{c} \times \vec{c} + \vec{c} \times \vec{a}$ $= \vec{a} - b\ .\ b \times \vec{c} - b \times \vec{a} + \vec{c} \times \vec{a}$ ……($\vec{c} \times \vec{c} = 0$) $= \vec{a} - b\ .\ b \times \vec{c} + \vec{a} \times b + \vec{c} \times \vec{a}$ $= \vec{a}.\ b \times \vec{c}\ + \vec{a}.\ \vec{a} \times b + \vec{a}.(\vec{c} \times \vec{a}) - b.\ b \times \vec{c}\ - b.\ \vec{a} \times b - b.(\vec{c} \times \vec{a})$ $= \vec{a}.\ b \times \vec{c}\ + 0 + 0 - 0 - 0 - b.(\vec{c} \times \vec{a})$ $= \vec{a}.\ b \times \vec{c}\ - b.(\vec{c} \times \vec{a})$ =0 (STP remains same if vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are changed in cyclic order) **OR** $+\ +\ .\ \ +\ +\ = 0$ $\Rightarrow .\ +.\ +.\ +.\ +\ +\ +\ .\ +.\ +.\ +.\ = 0.$ $\Rightarrow$ | | + + | | + 2. + + . = 0 $\Rightarrow 3 + 5 + 7 + 2.\ \ +\ +.\ = 0$ $\Rightarrow 2.\ +\ +.\ = -(9 + 25 + 49)$ $\Rightarrow .\ +\ +.\ = -\ \frac{83}{2}$	 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$
25	Vector in the direction of first line $= 3\hat{\imath} + 4\hat{\jmath} + 5k$ Vector in the direction of second line $= 4\hat{\imath} - 3\hat{\jmath} + 5k$ Angle between two lines is given by cos = $\cos = \frac{3\hat{\imath} + 4\hat{\jmath} + 5k\ .\ 4\hat{\imath} - 3\hat{\jmath} + 5k}{3\hat{\imath} + 4\hat{\jmath} + 5k\ \ 4\hat{\imath} - 3\hat{\jmath} + 5k}$ $\Rightarrow \cos = \frac{12 - 12 + 25}{\sqrt{9 + 16 + 25}\sqrt{9 + 16 + 25}}$ $\Rightarrow \cos = \frac{25}{\sqrt{50}\sqrt{50}}$ $\Rightarrow \cos = \frac{1}{2}$ $\Rightarrow = \frac{}{3}$	 1 $\frac{1}{2}$ $\frac{1}{2}$
26	P(A)=— = − , P(B)=— = — P(Agree)=P(Both speaking truth or both telling lie) = ( )	 1

	= ( ) ( ) ( ) ( ) = - — + - — =—— = — = —— = 74%	1
	SECTION C	
27	Let = f(x) = —— ……………………….(1) Let , ∈ A = R − {3} Let ( ) = ( ) ⇒ $\frac{2 \quad + 3}{\quad - 3} = \frac{2 \quad + 3}{\quad - 3}$ ⇒ (2 + 3)(− 3) = (2 + 3)(− 3) ⇒ (2 − 6 + 3 − 9) = (2 − 6 + 3 − 9) ⇒ −6 + 3 = −6 + 3 ⇒ 9 = 9 ⇒ = Now ( ) = ( ) ⇒ = so ( ) is one-one For onto = $\frac{2x + 3}{x - 3}$ ⇒ − 3 = 2 + 3 ⇒ − 2 = 3 + 3 ⇒ (− 2) = 3(+ 1) ⇒ = $\frac{(\quad)}{(\quad)}$ ……………………….(2) equation (2) is defined for all real values of y except 2 i.e ∈ R − {2} which is same as given set = R − {2} (co-domain=range) Also = f(x) f(x) = f $\frac{3(\quad + 1)}{(\quad - 2)}$ = $\frac{2\frac{(\quad)}{(\quad)} + 3}{\frac{(\quad)}{(\quad)} - 3}$ since f(x) = $\frac{2x + 3}{x - 3}$ $\frac{2(3y + 3) + 3(y - 2)}{3y + 3 - 3y + 6} = \frac{9}{9} =$ Thus for every y ∈ B, there exists x ∈ A such that f(x) = Thus function is onto. Since f(x) is one-one and onto so f(x) is invertible. Inverse is given by = ( ) = $\frac{(\quad)}{(\quad)}$	$\frac{1}{2}$ 1 $1\frac{1}{2}$ 1
28	$\sqrt{1 - x} + \sqrt{1 - y}$ = a (x − y) Let = sin , = sin $\sqrt{1 - \sin}$ + $\sqrt{1 - \sin}$ = a (sin − sin) cos + cos = a (sin − sin) ⇒ 2 cos $\frac{+}{2}$ cos $\frac{-}{2}$ = 2a cos $\frac{+}{2}$ sin $\frac{-}{2}$ ⇒ cos $\frac{-}{2}$ = a sin $\frac{-}{2}$	$\frac{1}{2}$ 1

	$\Rightarrow \cot \frac{-}{2} = a$ $\Rightarrow \frac{-}{2} = \cot$ $\Rightarrow - \quad = 2\cot$ $\Rightarrow \sin \quad - \sin \quad = 2\cot$ differentiating w.r.t. x $\Rightarrow \frac{1}{\sqrt{1-x}} - \frac{1}{1-y} \text{—} = 0$ $\Rightarrow \text{—} = \frac{\overline{1-y}}{\sqrt{1-x}}$ **OR**	1 1 $\frac{1}{2}$
	$x = a(\cos 2\theta + 2\theta \sin 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(-2\sin 2\theta + 2\sin 2\theta + 4\theta \cos 2\theta)$ $\Rightarrow \text{—} = a(4\theta \cos 2\theta)$……………..(1) $y = a(\sin 2\theta - 2\theta \cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(2\cos 2\theta + 4\theta \sin 2\theta - 2\cos 2\theta)$ $\Rightarrow \text{—} = a(4\theta \sin 2\theta)$………………(2) using (1)and (2) $\Rightarrow \text{—} = \frac{a(4\theta \sin 2\theta)}{a(4\theta \cos 2\theta)}$ $\Rightarrow \text{—} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$ Differentiating again with respect to x, we get $\Rightarrow \frac{d\,y}{dx} = 2 \quad 2\theta.\frac{d\theta}{dx}$ $\Rightarrow \frac{d\,y}{dx} = 2 \quad 2\theta.\frac{1}{a(4\theta \cos 2\theta)}$ $\frac{d\,y}{dx}_{-} = 2 \quad \frac{\pi}{4}.\frac{1}{a \quad 4 - \cos -}$ $= \frac{8\sqrt{2}}{}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1
29	$x\frac{dy}{dx} - y = \overline{x \quad + y}$ $\Rightarrow x\frac{dy}{dx} = y + x \overline{\quad + y}$ $\Rightarrow \text{—} = \frac{\overline{\quad}}{}$…………………………………(1) let y = vx differentiating with w.r.t. x $\Rightarrow \text{—} = \quad + \quad \text{—}$ put in (1)	1

	$\Rightarrow + \quad — = \frac{+\sqrt{\quad +}}{x}$ $\Rightarrow + \quad — = \frac{(\ +\sqrt{1+}\ )}{}$ $\Rightarrow \ — = \ +1+\overline{\quad} \ -$ $\Rightarrow \ — = \ \overline{1+}$ $\Rightarrow \frac{}{\sqrt{1+}} = \ —$ integrating both sides $\Rightarrow \ \frac{}{\sqrt{1+}} = \ —$ $\Rightarrow \log \ +1+\overline{\quad} = \log \ +\log$ $\Rightarrow \log \ +1+\overline{\quad} = \log$ $\Rightarrow \ +1+\overline{\quad} \ =$ $\Rightarrow \ -+ \ \ 1+ \ - \ \ =$ $\Rightarrow + \ \overline{\ \ +\ \ } \ =$	1 1– $\frac{1}{2}$
30	Consider I=$\int \mid \ -2 \ \mid$ $\mid \ -2 \ \mid = \begin{cases} -(\ \ -2) & h \ \ 1\leq \ <2 \\ (\ \ -2) & h \ \ 2\leq \ \leq 3 \end{cases}$ I=$\int \mid \ -2 \ \mid \ \ +\int \mid \ -2 \ \mid$ I=$\int -(\ \ -2) \ \ +\int (\ \ -2)$ I=– —– + —– I=– – +– – + – I=– $= 2$	1 1 1 1
31	Let X denotes the smaller of the two numbers obtained So X can take values 1,2,3,4,5,6 P(X=1 is smaller number) P(X=1)=— = — = – (Total cases when two numbers can be selected from first 7 numbers are 7) P(X=2)=— = — P(X=3)=— = — P(X=4)=— = — = – P(X=5)=— = — P(X=6)=— = — <table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>	$\frac{1}{2}$ 2

	$\frac{6}{21}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{1}{21}$
	$\frac{6}{21}$	$\frac{10}{21}$	$\frac{12}{21}$	$\frac{12}{21}$	$\frac{10}{21}$	$\frac{6}{21}$

$\frac{1}{2}$

Mean $=\Sigma$ = — + — + — + — + — + — = —=– 1

OR

Let = event of selecting a two headed coin

= event of selecting a biased coin, which shows 75% times Head

= event of selecting a unbiased coin.

A = event that tossed coin shows head. 1

$\therefore$ ( ) = ( ) = ( ) = $\frac{1}{3}$ $\frac{1}{2}$

. = (coin showing head given that it is two headed coin)

= 1

= (coin showing head given that it is a biased coin)

$= \frac{75}{100} = \frac{3}{4}$ 1

= (coin showing head given that it is unbiased coin)

$= \frac{1}{2}$

By Bayes theorem

(gettingtwo headedcoin when it is known that it shows Head)

= ( ) / [( ) + ( ) + ( )]

= –× / (–× –×– –×–) =–× / (– – –) =– / –×– =–

$1\frac{1}{2}$

Required probability=–

32 Let tailor A works for days and tailor B works for *y* days $\frac{1}{2}$

Objective function :

To minimize labour cost = 150 + 200 (in ₹)

Subject to constraints

6 + 10 ≥ 60 i.e. 3 + 5 ≥ 30

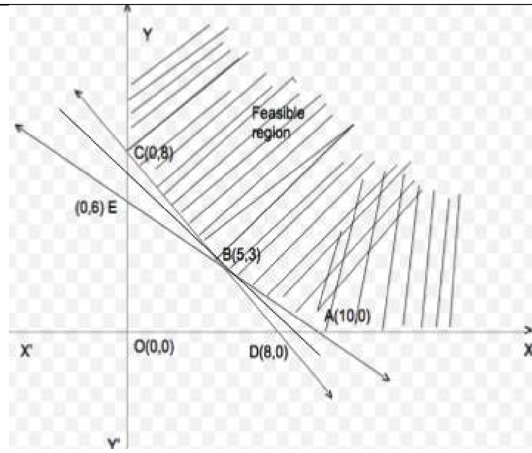
4 + 4 ≥ 32 i.e. + ≥ 8 $1\frac{1}{2}$

≥ 0 , ≥ 0

consider equations to draw the graph and then we will shade feasible region

3 + 5 = 30

+ = 8



1

corner points of feasible region are A(10,0),B(5,3) and C(0,8)
Value of Z at these corner points

Point	$= 150 \quad + 200$ (in ₹)
A(10,0)	=1500+0=1500
B(5,3)	=750+600=1350 (minimum)
C(0,8)	=0+1600=1600

So minimum value of Z is ₹1350 when tailor A works for 5 days and tailor B works for 3 days.

To check draw $150 \quad + 200 \quad < 1350$ i.e $3 \quad + 4 \quad < 27$

As there is no region common with feasible region so minimum value is ₹1350 1

SECTION D

33

(+)
LHS= (+)
(+)
→ − , → −
(+) − (+) − (+) 1
= (+) − 0
0 (+) −
(+) (+ +)(− −) (+ +)(− −) 1
= (+ +)(+ −) 0
0 (+ +)(+ −)
Taking (+ +)common from as well as
(+) (− −) (− −)
=(+ +) (+ −) 0 1
0 (+ −)
Apply → − −

2yz −2 −2
=(+ +) (+ −) 0
0 (+ −)

Apply → and → 1

$= \frac{(\quad)}{\quad}\begin{vmatrix} 2yz & -2 & -2 \\ (\ +\ -\ ) & & 0 \\ 0 & (\ +\ -\ ) & \end{vmatrix}$

Apply → + and → + 1

$= \frac{(\quad)}{\quad}\begin{vmatrix} 2yz & 0 & 0 \\ (\ +\ ) & & \\ & (\ +\ ) & \end{vmatrix}$

expanding along

$= \frac{(\quad)}{\quad}\ 2\ [(\ +\ )(\ +\ ) -\ ]$

$=2(\ +\ +\ )\ [\ +\ +\ +\ -\ ]$

$= 2\ (\ +\ +\ )(\ +\ +\ )$ 1

$= 2\ (\ +\ +\ )$

OR

** $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

1

$|A| = 2(-2) - 3(2-0) + 4(1-0) = -6 \neq 0$

$\therefore A \quad$ exists

Cofactors

$= -2 \qquad = -2 \qquad = 1$

$-2 \qquad = 4 \qquad = -2$

$= 4 \qquad = 4 \qquad = -5$

2

$= \begin{bmatrix} -2 & -2 & 1 \\ -2 & 4 & -2 \\ 4 & 4 & -5 \end{bmatrix}$

$= \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$

$= \frac{\quad}{|\ |} = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ 1

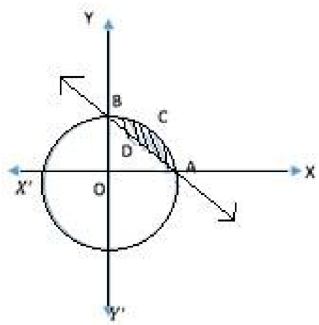
System of equations can be written as $=$ $\frac{1}{2}$

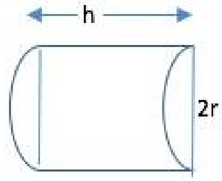
Where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, $=$, $= \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$

Now $=$

$\Rightarrow \ =$

$\Rightarrow \ = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$

	$\Rightarrow \quad = \frac{1}{-6} \begin{matrix} -34-6+28 \\ -34+12+28 \\ 17-\ 6-35 \end{matrix}$ $\Rightarrow \quad = \frac{1}{-6} \begin{matrix} -12 \\ 6 \\ -24 \end{matrix}$ $\Rightarrow \quad = \quad = \begin{matrix} 2 \\ -1 \\ 4 \end{matrix}$ $\Rightarrow \quad = 2, \quad = -1, \quad = 4$	$1\frac{1}{2}$
34	x + y = 1 ……………….(1) x + y = 1 …………………..(2) solving (1) and(2) x + (1 −) = 1 x + x − 2 + 1 = 1 2x − 2 = 0 2 (− 1) = 0 = 0 = 1  Required area = shaded area ACBDA =area(OACBO)– area(OADBO) =∫ (−) 1 − − (1 −) = √ + sin − − — 0 + $\frac{1}{2}$. $\frac{}{2}$ − 0 − 1 − $\frac{1}{2}$ − − − square units	1 1 1 $1\frac{1}{2}$ $1\frac{1}{2}$
35	Let be the radius and h be the height of half cylinder Volume =– h = (constant)……………..(1)	$\frac{1}{2}$( )



Total surface area of half cylinder is $1\frac{1}{2}$

S=2 – + $h + 2\,h$ …………………(2)

From (1) put the value of h in (2) 1

S=( ) + — + 2 —

S=( ) + – — + 2

— = (2) + — — + 2 ……………..(3)

maxima/minima — = 0

$\Rightarrow (2\quad) + \frac{-1}{\quad}\ \frac{4}{\quad} + 2\quad = 0$ 1

$\Rightarrow (2\quad) = \frac{1}{\quad}\ \frac{4\quad + 2}{\quad}$

$\Rightarrow \quad = \frac{2 +}{\quad}$ 1

$\Rightarrow$ V=——………………..(4)

rom (1) and (4)

$\Rightarrow \frac{1}{2}\quad h = \frac{\quad}{+ 2}$ 1

$\Rightarrow \frac{h}{2} = \frac{\quad}{+ 2}$

$\Rightarrow h\quad h : \quad = : \quad + 2$

Differentiating (3) with respect to

—— = (2) + — — + 2 =positive (as all quantities are +ve)

so S is minimum when

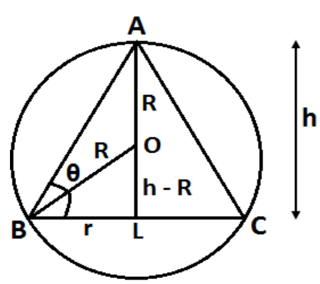
$h\quad h : \quad = : \quad + 2$

OR

Let 2r be the base and h be the height of triangle ,which is inscribed in a circle of radius R

Area of triangle=–( )(h h)

A =–(2)(h) = h ………..(1) 1



$\frac{1}{2}($ $)$

Area being positive quantity, A will be maximum or minimum if is

	maximum or minimum. $= \quad = \quad h$ …………………….(2) ow In triangle OLB $= \quad -$ In ΔOBD $Z = A^2 = r^2 h^2 \quad = \quad - (h - \quad) \Rightarrow \quad = 2h \quad - h$ (2) $= h \ (2h \quad - h \ )$ $\Rightarrow \quad = (2h \quad - h \ )$ $\Rightarrow — = 6h \quad - 4h$ ……………(3) maxima/minima $— = 0$ $\Rightarrow 6h \quad - 4h \quad = 0$ $\Rightarrow 6 \quad = 4h \ (h \neq 0 \ )$ $\Rightarrow h = \frac{3}{2}$ differentiating (3) w.r.t. h $\Rightarrow \frac{}{h} = 12h \quad - 12h$ $\Rightarrow \frac{}{h} \quad \_ = 12 \ \frac{3}{2} \quad - 12 \ \frac{3}{2}$ $= 18 \quad - 27 \quad = -$ so Z= is maximum when $h =$ — $\Rightarrow$ is maximum when $h =$ — when $h =$ — , $= 2h \quad - h \quad = 2 . \ — - \ —$ $= \frac{3}{4}$ $= \frac{\sqrt{3}}{2}$ $\tan \quad = \frac{h}{} = \frac{—}{\sqrt{}} = \sqrt{3} \quad = \frac{}{3}$ Triangle ABC is equilateral triangle	 $\frac{1}{2}$ 1 1 1 1
36	Let (, ,) be any point on the plane in which $(2,1,2)$ and $(4, -2, 1)$ lie. $\therefore$ lie on required plane. Also required plane is perpendicular to given plane $\vec{r}. \ \hat{i} - 2k \quad = 5$ $\therefore$normal to given plane $= \hat{i} \ - 2k$ lie on required plane. $\Rightarrow$, and are coplanar. Where $= (\quad - 2)\hat{i} \ + (y - 1)\hat{j} \ + (z - 2)k$ $= \ =2\hat{i} - 3\hat{j} - k$ $\Rightarrow$Scaler triple product $= 0$	 1 1 1

<table>
<tr><td></td><td>$\Rightarrow \begin{vmatrix} -2 & -1 & -2 \\ 2 & -3 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 0$
$\Rightarrow (\ \ -2)(6-0) - (\ \ -1)(-4+1\ ) + (\ \ -2)(0+3\ ) = 0$
$\Rightarrow 6\ \ -12+3\ \ -3+3-6=0$
$\Rightarrow 2\ \ +\ \ +\ \ =7$ ………………….(1)
Line passing through points $(3,4,1)$ and $(5,1,6)$ is
⇒ — = — = — = …………(2)
⇒General point on the line is $(2\ \ +3, -3\ \ +4, 5\ \ +1)$
As line (2) crosses plane (1) so point Q should satisfy equation(1)
$\therefore 2(2\ \ +3) + (-3\ \ +4) + (5\ \ +1) = 7$
$4\ \ +6-3\ \ +4+5\ \ +1=7$
$6\ \ =-4$
$=-\frac{2}{3}$
$(-+3, 2+4, -\ \ —+1) = \ \ -, 6, --$</td><td>

1

1

1</td></tr>
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