Series HMJ/1

SET-3

कोड नं.
Code No. 65/1/3

रोल नं.
Roll No. 1 7 6 3 3 9 8 5

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

नोट	NOTE
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **15** हैं ।	(I) Please check that this question paper contains **15** printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।	(II) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **36** प्रश्न हैं ।	(III) Please check that this question paper contains **36** questions.
(IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।	(IV) Please write down the Serial Number of the question in the answer-book before attempting it.
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper comprises* **four** *Sections A, B, C and D. This question paper carries* **36** *questions.* **All** *questions are compulsory.*

(ii) **Section A** – *Questions no.* **1** *to* **20** *comprises of* **20** *questions of* **1** *mark each.*

(iii) **Section B** – *Questions no.* **21** *to* **26** *comprises of* **6** *questions of* **2** *marks each.*

(iv) **Section C** – *Questions no.* **27** *to* **32** *comprises of* **6** *questions of* **4** *marks each.*

(v) **Section D** – *Questions no.* **33** *to* **36** *comprises of* **4** *questions of* **6** *marks each.*

(vi) *There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.*

(vii) *In addition to this, separate instructions are given with each section and question, wherever necessary.*

(viii) *Use of calculators is* **not** *permitted.*

SECTION A

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1. The matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible for

$A = A^T$

(A) $\lambda = -1$

(B) $\lambda = 0$

2 -1 3

(C) $\lambda = 1$

(D) $\lambda \in R - \{1\}$

2. The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

(A) 0

(B) 1

(C) 2

(D) 3

3. The value of $\tan^{-1}(\tan \frac{7\pi}{6})$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{7\pi}{6}$

4. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0), then

~~(A)~~ $a = 2b$

(B) $2a = b$

(C) $a = b$

(D) $3a = b$

$2a + 4b = 4a + 0.$

$4b = 2a$

$\boxed{2b = a}$

5. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B' \mid A)$ is equal to

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

~~(C)~~ $\frac{3}{4}$

(D) 1

$\frac{P(B' \cap A)}{(1 - P(B))P(A)} \over P(A)$

$\frac{\frac{3}{4} \times \frac{1}{3}}{\frac{1}{3}}$

$\frac{(1 - \frac{1}{4}) \frac{1}{3}}{\frac{1}{3}}$

6. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to

(A) I

(B) 0

~~(C)~~ $I - A$

(D) $I + A$

$(I - A)(I^2 + A^2 + AI)$

$I - A(I + A + AI)$

7. $\int_1^e \frac{\log x}{x} dx$ is equal to

(A) $\frac{e^2}{2}$

(B) 1

(C) $\frac{1}{2}$

(D) $-\infty$

$\frac{x^2}{2}$ $[\log x]$ $\frac{\log e^2}{2}$

8. A point P lies on the line segment joining the points (– 1, 3, 2) and (5, 0, 6). If x-coordinate of P is 2, then its z-coordinate is

(A) –1

(B) 4

(C) $\frac{3}{2}$

(D) 8

$\frac{\lambda x_1 + 6}{\lambda + 1}$

$2 = \frac{-k+5}{2}$

$4 + 1 = +k$

9. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(A) 0

(B) 1

(C) $\frac{-2}{3}$

(D) $\frac{-3}{2}$

$4 + 3\lambda = 0$

$2 + 3\lambda$

10. The vector equation of the line passing through the point (–1, 5, 4) and perpendicular to the plane z = 0 is

(A) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$

(B) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

(C) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$

(D) $\vec{r} = \lambda\hat{k}$

$-\hat{i} + 5\hat{j}$

Fill in the blanks in question numbers 11 to 15.

11. The position vectors of two points A and B are $\overrightarrow{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is ________ .

12. The equation of the normal to the curve $y^2 = 8x$ at the origin is ________ .

OR

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of ________ cm^2/s.

13. If A is a square matrix of order 3 and A_{ij} is the cofactor of the element a_{ij}, then value of $a_{21} A_{11} + a_{22} A_{12} + a_{23} A_{13}$ is ________ .

OR

If the matrix A is both symmetric and skew symmetric, then A is a __zero__ .

14. A relation R in a set A is called __Sy__, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

15. The greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at x = ________ .

[x]. [a+h] [0+h]

Question numbers 16 to 20 are very short answer type questions.

16. If A is a square matrix of order 3 and $|A| = 2$, then find the value of $|-AA'|$.

3

$|AA'|$ 2? [a+h] 2.

17. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

18. Evaluate :

$$\int_1^3 |2x - 1| \, dx$$

(2^n-1)

19. Find :

$$\int \frac{dx}{\sqrt{9 - 4x^2}}$$

$\frac{1}{2} \quad \frac{dx}{\frac{1}{2}\sqrt{\frac{9}{4} - 4x}}$

20. Find :

$$\int x^4 \log x \, dx$$

OR

Find :

$$\int \frac{2x}{\sqrt[3]{x^2+1}} \, dx$$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

OR

Find the volume of the parallelopiped whose adjacent edges are represented by $2\vec{a}$, $-\vec{b}$ and $3\vec{c}$, where

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k},$$

$$\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}, \text{ and}$$

$$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}.$$

22. If $f(x) = \sqrt{\tan \sqrt{x}}$, then find $f'\left(\frac{\pi^2}{16}\right)$.

23. Using differentials, find the approximate value of $\sqrt{25 \cdot 3}$ up to two places of decimals.

24. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three ?

25. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1} x$, $\frac{1}{\sqrt{2}} \le x \le 1$.

OR

Consider a bijective function $f : R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of f.

26. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

P.T.O.

SECTION C

Question numbers 27 to 32 carry 4 marks each.

27. A furniture trader deals in only two items — chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1,000 and a table costs him ₹ 2,000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

28. If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, $a > 0$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

29. Evaluate $\int_1^3 e^x \, dx$ as limit of the sums.

4 red
3 bla

30. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

col 6C

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

31. Find the general solution of the differential equation

$$y e^y \, dx = (y^3 + 2x e^y) \, dy.$$

OR

Find the particular solution of the differential equation

$$x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right), \text{ given that } y = \frac{\pi}{4} \text{ at } x = 1.$$

32. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) iff ad = bc for all a, b, c, d ∈ N. Show that R is an equivalence relation.

SECTION D

Question numbers 33 to 36 carry 6 marks each.

$H = \frac{h}{3}.$

33. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

34. Using integration, find the area of the region enclosed by the parabola $y = 3x^2$ and the line $3x - y + 6 = 0$.

35. Find the equation of the plane that contains the point A(2, 1, – 1) and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$. Also find the angle between the plane thus obtained and the y-axis.

OR

Find the distance of the point P(– 2, – 4, 7) from the point of intersection Q of the line $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$. Also write the vector equation of the line PQ.

36. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations :

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$

$-4, -4, 7$

$1, -1, 4$