General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper comprises* **four** *Sections A, B, C and D. This question paper carries* **36** *questions.* **All** *questions are compulsory.*

(ii) **Section A** – *Questions no.* **1** *to* **20** *comprises of* **20** *questions of* **1** *mark each.*

(iii) **Section B** – *Questions no.* **21** *to* **26** *comprises of* **6** *questions of* **2** *marks each.*

(iv) **Section C** – *Questions no.* **27** *to* **32** *comprises of* **6** *questions of* **4** *marks each.*

(v) **Section D** – *Questions no.* **33** *to* **36** *comprises of* **4** *questions of* **6** *marks each.*

(vi) *There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.*

(vii) *In addition to this, separate instructions are given with each section and question, wherever necessary.*

(viii) *Use of calculators is* **not** *permitted.*

SECTION A

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1. If A is a 3×3 matrix and $|A| = -2$, then value of $|A\,(\text{adj}\,A)|$ is

$|A|^2$

(A) -2

(B) 2

(C) -8

(D) 8

2. The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

(A) 0

(B) 1

(C) 2

(D) 3

3. The principal value of $\cos^{-1}(\cos \frac{13\pi}{6})$ is

$2\pi + \pi/6$

(A) $\frac{13\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

4. The corner points of the feasible region determined by the system of linear inequalities are $(0, 0)$, $(4, 0)$, $(2, 4)$ and $(0, 5)$. If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both $(2, 4)$ and $(4, 0)$, then

(A) $a = 2b$

(B) $2a = b$

(C) $a = b$

(D) $3a = b$

$2a + 4b = 4a$

$2a = 4b$

$a = 2b$.

5. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B' \mid A)$ is equal to

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{3}{4}$

(D) 1

$\frac{P(B' \cap A)}{P(A)} = P(B')$

$\frac{3}{4}$

6. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to

(A) I

(B) 0

(C) $I - A$

(D) $I + A$

$A^2 = A$

$A = I$

7. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$, where $x \neq 0$, is equal to

$-\frac{1}{x^2} \sin \frac{1}{x}$

(A) -2

(B) 0

(C) 1

(D) π

8. The image of the point $(2, -1, 5)$ in the plane $\vec{r} \cdot \hat{i} = 0$ is

(A) $(-2, -1, 5)$

(B) $(2, 1, -5)$

(C) $(-2, 1, -5)$

(D) $(2, 0, 0)$

9. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(A) 0

(B) 1

(C) $\frac{-2}{3}$

(D) $\frac{-3}{2}$

10. The vector equation of the line passing through the point $(-1, 5, 4)$ and perpendicular to the plane $z = 0$ is

(A) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$

(B) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

(C) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$

(D) $\vec{r} = \lambda\hat{k}$

Fill in the blanks in question numbers 11 to 15.

11. The position vectors of two points A and B are $\vec{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is $2i - j + k$ __________.

12. The equation of the normal to the curve $y^2 = 8x$ at the origin is __________.

OR

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of __________ cm^2/s.

$\frac{dr}{dt} = 3$ cm/s

13. On applying elementary column operation $C_2 \to C_2 - 3C_1$ in the matrix equation $\begin{bmatrix} 4 & -2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, the RHS (Right Hand Side) of the equation becomes ____________ .

OR

A square matrix A is said to be symmetric if $A = A^T$.

14. A relation R in a set A is called symm, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

15. The greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at x = 1 .

Question numbers 16 to 20 are very short answer type questions.

16. If A is a non-singular square matrix of order 3 and $A^2 = 2A$, then find the value of $|A|$. 8

17. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black. 26/51

18. Evaluate :

$$\int_1^3 |2x - 1|\, dx$$

6

19. Find :

$$\int \frac{dx}{\sqrt{9 - 4x^2}}$$

$\frac{1}{2} \sin^{-1} \frac{2x}{3} + C$

20. Find :

$$\int x^4 \log x\, dx$$

OR

Find :

$$\int \frac{2x}{\sqrt[3]{x^2 + 1}}\, dx$$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

OR

Find the volume of the parallelopiped whose adjacent edges are represented by $2\vec{a}$, $-\vec{b}$ and $3\vec{c}$, where

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k},$$

$$\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}, \text{ and}$$

$$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}.$$

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22. Examine the applicability of Rolle's theorem for the function $f(x) = \sin 2x$ in $[0, \pi]$. Hence find the points where the tangent is parallel to x-axis. 2/4

23. Find the values of x for which the function $f(x) = 2 + 3x - x^3$ is decreasing.

24. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three ? 0

25. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$, $\frac{1}{\sqrt{2}} \le x \le 1$.

OR

Consider a bijective function $f : R_+ \rightarrow (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of f.

26. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

$\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$

SECTION C

Question numbers 27 to 32 carry 4 marks each.

27. A furniture trader deals in only two items — chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1,000 and a table costs him ₹ 2,000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

28. If $x = a \sec^3 \theta,\ y = a \tan^3 \theta$, then find $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{4}$.

29. Find :

$$\int \frac{2x+1}{\sqrt{3+2x-x^2}}\,dx$$

30. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

31. Find the general solution of the differential equation

$$y\,e^y\,dx = (y^3 + 2x\,e^y)\,dy.$$

OR

Find the particular solution of the differential equation

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right),\ \text{given that } y = \frac{\pi}{4} \text{ at } x = 1.$$

32. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) iff ad = bc for all a, b, c, d ∈ N. Show that R is an equivalence relation.

SECTION D

Question numbers 33 to 36 carry 6 marks each.

33. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

34. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 9,\ x + y \geq 3\}$.

35. Find the equation of the plane that contains the point $A(2, 1, -1)$ and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$. Also find the angle between the plane thus obtained and the y-axis.

OR

Find the distance of the point $P(-2, -4, 7)$ from the point of intersection Q of the line $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$. Also write the vector equation of the line PQ.

36. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations :

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$