2018
MATHEMATICS

Full marks: 100 Time: 3 hours

General instructions:

i) Approximately 15 minutes is allotted to read the question paper and revise the answers.
ii) The question paper consists of 26 questions. All questions are compulsory.
iii) Marks are indicated against each question.
iv) Internal choice has been provided in some questions.
v) Use of simple calculators (non-scientific and non-programmable) only is permitted.

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) The domain of the function $f(x) = \frac{x}{|x|}$ is **1**

(i) $\mathbf{R} - \{0\}$ (ii) $\mathbf{R}$ (iii) $\mathbf{Z}$ (iv) $\mathbf{W}$

(b) The principal value of $\operatorname{cosec}^{-1}\left(\sqrt{2}\right)$ is **1**

(i) $\frac{-3\pi}{4}$ (ii) $\frac{-\pi}{4}$ (iii) $\frac{\pi}{4}$ (iv) $\frac{3\pi}{4}$

(c) If matrix A is $(a_{ij})_{m \times n}$ and B is $(b_{jk})_{n \times p}$ then the order of the matrix AB is **1**

(i) $m \times n$ (ii) $m \times p$ (iii) $n \times p$ (iv) $p \times n$

(d) If $x = t^2$, $y = t^3$ then $\frac{d^2y}{dx^2}$ is equal to **1**

(i) $\frac{3}{2}$ (ii) $\frac{3}{4t}$ (iii) $\frac{3}{2t}$ (iv) $\frac{3t}{2}$

(e) The angle x which increases twice as fast as its sine is **1**

(i) $\frac{\pi}{3}$ (ii) $\frac{\pi}{2}$ (iii) π (iv) $\frac{3\pi}{2}$

(f) The value of $\int_0^1 e^{x^2} x\, dx$ is equal to **1**

(i) $\frac{1}{3}(e-1)$ (ii) $\frac{1}{2}(e-1)$ (iii) $\frac{1}{3}(e+1)$ (iv) $\frac{1}{2}(e+1)$

(g) The order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is **1**

(i) 0 (ii) 1 (iii) 2 (iv) 3

(h) The angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} + \hat{k}$ is **1**

(i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{4}$ (iii) $\frac{\pi}{3}$ (iv) $\frac{2\pi}{3}$

(i) If P(A) = $\frac{3}{5}$ and P(B) = $\frac{1}{5}$, then the value of P(A $\cap$ B), when A and B are independent events is **1**

(i) $\frac{3}{25}$ (ii) $\frac{3}{28}$ (iii) $\frac{2}{7}$ (iv) $\frac{2}{11}$

(j) Three coins are tossed. The probability of at least one head is **1**

(i) $\frac{1}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{6}{8}$ (iv) $\frac{7}{8}$

Section – B

2. Consider the function $f(x) = \frac{x-3}{x+1}$ defined from $\mathbf{R} - \{-1\}$ to $\mathbf{R} - \{1\}$. Prove that f is both 1-1 and onto. **2**

3. Consider the binary operation on $\mathbf{Q} - \{1\}$ defined by $a * b = a + b - ab$. Find the identity element in $\mathbf{Q} - \{1\}$. **2**

4. Construct a 2 × 2 matrix whose elements are given by, $a_{ij} = \frac{|2i - 3j|}{3}$ **2**

5. Differentiate $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$ with respect to x. **2**

6. Find $\frac{dy}{dx}$ when $\sin^2 x + \cos^2 y = 1$ **2**

7. Using differentials, find the approximate value of $\sqrt{401}$ **2**

8. Evaluate $\int \frac{\sin\left(2\tan^{-1}x\right)}{1+x^2}\,dx$ **2**

9. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^2 x\,dx$ **2**

10. Verify that $y = a\cos x + b\sin x$ is a solution of the differential equation: **2**

$$\frac{d^2y}{dx^2} + y = 0$$

11. Find λ when the scalar projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units. **2**

Section – C

12. Prove that $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{77}{36}\right)$ **4**

13. **a.** Using properties of determinants, prove that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

Or **4**

b. Using elementary row transformation, find the inverse of the matrix $\begin{bmatrix} 2 & 6 \\ 4 & 15 \end{bmatrix}$

14. **a.** Find $\frac{dy}{dx}$ when $y = (\log x)^x + x^{\log x}$

Or **4**

b. Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ in [1, 3]

15. **a.** Evaluate $\int \frac{2x+3}{\sqrt{x^2 + 4x + 3}}\,dx$

Or **4**

b. Evaluate $\int \frac{2x}{(2+x^2)(3+x^2)}\,dx$

16. Evaluate $\int_0^2 \left| x^2 + 2x - 3 \right| dx$ **4**

17. Solve the differential equation $y\,dx + \left(x - y^2\right)dy = 0$ **4**

18. **a.** Find a vector of magnitude 5 which is perpendicular to each of $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

Or **4**

b. Using vectors, find the area of the triangle formed by A(1, –1, 2), B(2, 1, 1) and C(3, –1, 2).

19. **a.** Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+1}{2}$

Or **4**

b. Find the vector and cartesian equations of the plane through the point (0, 7, –7) and containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

20. A company has two plants to manufacture bicycles. The first plant manufactures 60% of the bicycles and the second plant, 40%. Also 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be of standard quality. Find the probability that it comes from the second plant. **4**

21. A bag contains 5 white, 7 red and 8 black balls. If 4 balls are drawn one by one with replacement, what is the probability that: (i) all are white, (ii) at least one is white? **4**

Section – D

22. **a.** If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$, verify that A. (adj A) = (adj A). A = |A| I

Or **6**

b. Using matrix method, solve the following system of linear equations:

$$3x - y + z = 5$$
$$2x - 2y + 3z = 7$$
$$x + y - z = -1$$

23. **a.** Show that the semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$

Or 6

b. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is the maximum possible?

24. **a.** Find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

Or 6

b. Find the area of the region enclosed between the circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$

25. **a.** Find the foot of the perpendicular drawn from the point (1, 2, 3) to the line joining the points (6, 7, 7) and (9, 9, 5).

Or 6

b. Find the direction cosines of the normal to the plane passing through (3, 2, 2) and (1, 0, −1) and parallel to the line $\frac{x-1}{2} = \frac{1-y}{1} = \frac{z-2}{3}$

26. **a.** Two tailors A and B earn ` 300 and ` 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. How many days should each of them work to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost?

Or 6

b. An aeroplane can carry a maximum of 250 passengers. A profit of ` 800 is made on each executive class ticket and a profit of ` 500 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

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